

I. Find the following

1. If $y = \cos t$, then find $\frac{d^2y}{dx^2} + y$
2. Find $\frac{dy}{dx}$ if $y = \log_x a$
3. If $y = x^{xx^x \dots}$ find $\frac{dy}{dx}$
4. Differentiate e^{x^3} w.r.t. $\log x$
5. Show that $f(x) = \cos(x^2)$ is a continuous function.

II. Solve

$$1. f(x) = \frac{1 - \cos 4x}{x^2}, \quad x < 0$$

$$= a, \quad x = 0$$

$$= \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, \quad x > 0$$

Find the value of 'a' if $f(x)$ is continuous at $x = 0$

2. Show that $f(x) = [x] + [-x]$ has discontinuity for integral values of x
3. Show that the function $f(x) = |x - 1|$ for $x \in R$ is not differentiable at $x = 1$
4. Find the derivative w.r.t. x
 $\cot^{-1} \left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right] \quad 0 < x < \frac{\pi}{2}$
5. Verify Meanvalue theorem if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$
6. If $\sin y = x \sin(a + y)$ show that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin(a)}$
7. If $y = (\tan^{-1} x)^2$ show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$
8. If $x^y = e^{(x-y)}$ show that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

9. $x\sqrt{1+y} + y\sqrt{1+x} = 0$ $y \neq x$, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$
10. Differentiate $\sin^{-1} \frac{2x}{1+x^2}$ w.r.t. $\cos^{-1} \frac{1-x^2}{1+x^2}$