

**WAVELET TRANSFORM APPROACH TO FITTING FINANCIAL TIME SERIES DATA**

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المخلص :

هذه الدراسة تبين تقنية مطورة حديثاً لدراسة العلاقة الديناميكية بين السلاسل الزمنية المالية؛ وذلك بدمج تحويل الموجات المنفصل Discrete Wavelet Transform (DWT) ونموذج تصحيح الخطأ المتغير Markov-Switching Model (MS-VECM). حيث تم استخدام تحويل الموجات المنفصل لتقليل الضوضاء في مجموعة البيانات اليومية لسوق الأسهم في المملكة المتحدة (UK) وأربعة أسواق أسهم في منطقة الشرق الأوسط وشمال أفريقيا (MENA) وهي مصر والمغرب والمملكة العربية السعودية واسطنبول، وتمثلت البيانات عن الفترة الممتدة من 1 مايو 2001 إلى 30 ديسمبر 2011. حيث تم تحليل جزء التردد المنخفض من السلسلة الأصلية التي تم إنشاؤها بواسطة تحويل الموجات المنفصل، وبعدها تم تطبيق كلا من اختبار التكامل المشترك ونموذج تصحيح الخطأ المتغير (MS-VECM) لتحديد العلاقات طويلة وقصيرة الأجل بين أسواق الأسهم المذكورة أعلاه. توصلت الدراسة عن طريق اختبارات المقارنة إلى أن النموذج المقترح (DWT-MS-VECM) أفضل من النموذج التقليدي (MS-VECM) من حيث الأداء ويتناسب مع سلسلة سوق الأوراق المالية. وبالإضافة إلى ذلك، نموذج DWT-MS-VECM تمكن من توضيح الوقت لفترة الأزمات التي أثرت على هذه أسواق الأسهم، ويقدم معلومات حقيقية عن العلاقات بين أسواق الأسهم المدروسة. كما أظهرت نتائج اختبار التكامل المشترك وجود علاقة طويلة الأجل بين أسواق الأسهم المدروسة. وبالإضافة إلى ذلك، يبين النموذج المقترح وجود علاقة قصيرة الأجل بين سوق الأوراق المالية في المملكة المتحدة وجميع أسواق الأوراق المالية الأخرى باستثناء سوق المملكة العربية السعودية.

## Abstract

This study explores a newly developed technique; combining wavelet transform and Markov-Switching Vector Error Correction model (MS-VECM), to investigate the dynamic relationship among financial time series. Wavelet filter has been used to annihilate noise data in daily data set of the UK stock market and four stock markets of Middle East and North Africa (MENA) region namely, Egypt, Morocco, Saudi Arabia, and Istanbul using the stock prices data from May1, 2001 to December 30, 2011. The series generated by the discrete wavelet transform is then analyzed to determine the long-term and short-term relationships between the stock markets by using a cointegration test and a MS-VECM model. The comparison between the proposed and traditional models demonstrates that the former dominates the latter in performance and fitting the financial stock market series. The cointegration test affirms the existence of long-term relationship between the studied series. The proposed model also shows the existence of a short-term relationship between stock market in UK and all other stock markets except that of Saudi Arabia. In additional, the DWT-MS-VECM model manages to capture a satisfactory timing of the crises period that had affected those stock markets and provides positive information on the relationships among these stock markets.

*Keywords-component; Wavelet transform; MS-VECM model; MENA stock market;*

## Introduction

During these years, increasing economic integration of the global economy has become a popular research topic in economy which has encouraged the scholars and investors to focus on the issue of studying a dynamic relationship among the stock market indices around the globe. There exist several different models describing this type of relationship, such as linear-models (for example; VAR and GARCH models) and non linear-models (such as Markov -Switching Models) which allow using conditional information in the forecasting process. Since these models produce a time conditional forecasted distribution, an alternative to an unconditional forecasted distribution, so can be utilized to model and explain time series dynamics, to display the presence of regime persistence and to get a better forecast (Dieobol: 1986). Therefore, researchers have previously explored the causal relationship between developed and developing stock markets using different models, for example,

(Majid & Shabri:2018) used daily data from 1999 to 2016 to investigate the dominance of world Islamic stock markets of Japan, the UK, and the US over the Islamic stock market of Indonesia. From the Granger causalities based on Vector Error Correction Model (VECM) analysis it is found that, the Islamic stock markets of Indonesia, the UK, Japan, and the US are moving towards a greater level of integration. (Cevik & Bagan:2018) used the MS-VAR model to examine the relationship of stock market returns between the Islamic and conventional stock markets from July 1999 until February 2016. Monthly stock market returns were used for this study. The findings provide strong evidence in favor of nonlinear relation between the conventional and Islamic stock markets. The regime-dependent Granger causality test and impulse-responses analysis results suggest that Islamic stock market is affected from conventional stock markets in both the bear and bull markets regimes. These results indicated that Islamic financial markets provide diversification benefits and they are safe havens during financial distressed periods cannot be supported empirically.

(Ismail & Isa:2008) used monthly data (1990 to 2004) to investigate the relationship among the return of three markets of Southeast Asian region, Malaysia, Singapore, and Thailand, based on co-integration test, VAR model, and MS-VAR model. The study showed that there was no long-run relationship between the stock markets of ASEAN region, while in short-term, the MS-VAR had showed co-movements among the three returns stock markets. Additionally, they explored that the performance and fitting data of MS-VAR model are much better than that of VAR model.

(Fan & Wang:2009) used the weekly closing price for stock market indices of the US, UK, Japan, Hong Kong and Mainland China regions to study the long and short-run relationships between the China and International main stock markets from June 5,1992 to December 26,2008. Their study was based on co-integration test, Vector Error Correction (VEC) and Markov-Switching Vector Error Correction (MS-VEC) models. They found a long-run relationship between the China and International main stock markets since 1999. In case of short-term relationship, they observed that the International main stock markets have been effecting on China stock market directly or indirectly. In comparison between VEC and MS-VEC models, it was studied that the MS-VEC model has less AIC, HQ, and SC. Therefore MS-VEC model is better to provide

effective information about these relationships among these stock markets. (Tan:2012) used the MS-VAR model to examine the relationship of stock market returns between the International stock markets (specifically, US, UK, Singapore, and China) and the stock market of the Philippines from 2000 until 2010. Weekly stock market returns were used for this study. The findings revealed that, the Philippines stock market is most correlated with stock markets of Hong Kong and Singapore compared to stock markets of US and UK in both regimes. These results indicated that there is stronger relationship between economies in the same region. (Marashdeh:2005) employed the monthly stock market indices of four major emerging markets of the MENA region Egypt, Turkey, Jordan, and Morocco and the three developed stock markets of the United States, United Kingdom, and Germany to examine the financial integration between these market indices. The research was based on a newly proposed autoregressive distributed lag (ARDL) approach. The results demonstrated the existence of integration among the stock markets in the MENA countries but not between the developed and MENA markets. In general, there is long and short term causal relationship among developed stock markets and MENA stock markets. Therefore, the stock markets of Morocco and Egypt are one that offers unrestricted access to foreign investors, while the stock markets of Saudi Arabia and Istanbul have the largest market capitalization within the MENA region (Yu& Hassan:2008).

In previous work, MS-Markov switching models, VEC, VAR, GARCH, and ARDL have been to explore the relationship among the stock markets of the world. However, the problem with these models is that they are sensitive to the noise in the series. This well obtains inaccurate information about causal relationships among stock markets. Due to the noise in financial time series can be lead to the biased estimation of parameters, erroneous or invalid inferences, and poor volatility forecasts (Jammazi& Aloui:2010). Therefore, in the present study the aim is to develop a model using new technique, based on the combination of the DWT and MS-VECM model. It is a novel technique as to the limit of author knowledge there does not exist any work in literature using the proposed model (DWT with MS-VECM model) to study the relationship between stock market of UK and selected stock markets of the MENA region. It also compares the proposed and traditional model in terms of their performance and fit with the financial stock market series.

## METHODOLOGY

### A. Discrete Wavelet Transform (DWT)

DWT aims to decompose the discrete time signal to basic functions called the wavelets to provide a clear analytic view of the analyzed signal. Proposed by (Gencay & Whitcher:2001), the DWT was implemented through pyramid algorithm to find the wavelet coefficients of a discrete series.

The DWT can be defined in terms of two filters, namely, high-pass filter denoted as  $H=\{h_l\}$  and low-pass filter denoted as  $G=\{g_l\}$ . The  $\{h_l\}$  and  $\{g_l\}$  are the coefficients of the filters and are defined as follows:

$$g_l = \frac{1}{\sqrt{2}} \int \varphi(t) \varphi(2t-l) dt$$

$$h_l = \frac{1}{\sqrt{2}} \int \psi(t) \varphi(2t-l) dt$$

where the father wavelet  $\varphi(t)$  and mother  $\psi(t)$  functions are defined as:

$$\varphi(t) = \sqrt{2} \sum g_l \varphi(2t-l)$$

$$\psi(t) = \sqrt{2} \sum h_l \varphi(2t-l)$$

FIGURE 1 shows a flow diagram for the first step of the pyramid algorithm where the symbol  $\downarrow 2$  denotes downsampling (elimination of every other value of input vector) by 2.

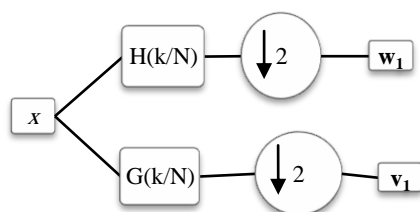


Figure 1. Schematic illustrating of DWT decomposition using the pyramid algorithm

where  $x$  is a length vector  $(N=2^J)$  of observations. At each iteration of the pyramid algorithm, the filtering requires three objects: the data  $x$ , the high-

pass filter and low-pass filter. The first step is filtering the data with each filter to produce detail coefficients and scaling coefficients, subsample both filter coefficients to half their original lengths, keep the subsampled output from  $h_1$  filter and repeat the filtering procedure for the subsampled output from  $g_1$  filter as follows in (Gencay & Whitcher:2001):

$$w_{1,t} = \sum_{l=0}^{L-1} h_l x_{2t+1-l \bmod N} \quad \text{and} \quad v_{1,t} = \sum_{l=0}^{L-1} g_l x_{2t+1-l \bmod N}$$

where the  $t=0,1,\dots,\frac{N}{2}-1$  and the downsampling operation has been included in the filtering step through the subscript  $x_t$ . The  $x$  series has been filtered high-as well as low- pass to produce  $\frac{N}{2}$  coefficients associated with this information. Repeating this procedure up to  $J$  times, the number of multi-resolution levels such that  $J = \log_2(N)$ , the DWT coefficients for scales can be computed as:

$$w_{j,t} = \sum_{l=0}^{L-1} h_l v_{j-1, 2t+1-l \bmod N}$$

$$v_{j,t} = \sum_{l=0}^{L-1} g_l v_{j-1, 2t+1-l \bmod N}$$

## B. Unit Root Test

Financial data are often displayed as non-stationary data or the data possessing time varying mean, variance, and covariance. Therefore, the unit root test is used to test for stationarity in order to avoid spurious regression. The two types of unit root tests, the Augmented Dickey Fuller (ADF) (Dikey & Fuller:1981) and Phillips Perron (PP) (Philips & Perron:1988) are used for this purpose, and are defined as follows, respectively

- (ADF)  $\Delta y_t = (P_\alpha - 1)y_{t-1} + \varepsilon_t$
- (PP)  $\Delta y_t = \alpha + \beta y_t + \varepsilon_t$

where  $\Delta y_t$  is the first difference operator and  $\varepsilon_t$  is a white noise error term. In these tests, the null hypothesis, the series have unit root if  $P_\alpha = 1$  and  $\alpha = 0$ , is tested against the alternative hypothesis that they do not. The PP test is

different from the ADF test in that the PP test tends to move robust to a wide range of serial correlations and time-dependent heteroskedasticity.

### c. Co-integration Test

The two or more time series data that are non stationary in levels but stationary in log first differences are said to be co-integrated of order one  $I(1)$ . Thus, non-stationary time series are cointegrated if a linear combination of these variables is stationary. The stationary linear combinations called equations are well explored as long-run equilibrium relationships among the variables. In sum, when there is co-integration among the variables then there are common forces that co-move the variables over time. (Johansen & Juselius:1990) implemented the co-integration based on vector autoregression model, which provides more robust results when there were more than two variables (Gonzalo:1994) and when the number of observations was greater than 100 (Hargreaves:1994). The Johansen approach considers a vector of  $n$  variables, formally:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Gamma_{k-1} \Delta X_{t-k-1} + \Pi X_{t-k} + \mu + \Psi D_t + \varepsilon_t$$

where  $\Gamma_i = -I + \Pi_1 + \Pi_2 + \dots + \Pi_i$  for  $i = 1, 2, \dots, k-1$

and  $\Pi = -I + \Pi_1 + \Pi_2 + \dots + \Pi_k$

where  $I$  is an indent matrix,  $\Gamma_i$  incorporates the short run adjustment parameters and the matrix  $\Pi = \alpha\beta'$  contains the long run relationships in the data with  $\alpha$  and  $\beta'$  as  $n \times r$  matrices such that  $\alpha$  represents the speed of adjustment to disequilibrium and  $\beta'$  contains  $r$  cointegration vectors. The vector of constant  $\mu$  signals the possibility deterministic drift in the series, dummy variables in  $D_t$  account for the short-run (shocks) that affect the entire system and  $k$  is the maximum number of lag length. Two different likelihood ratio tests were developed by Johansen, the Trace test and the Maximum Eigenvalue test to determine the number of co-integration vectors. The Johansen tests are known to be sensitive to the lag length selection. The lag length of  $k$  was chosen based on the modified Likelihood-Ratio (LR) test, Final Prediction Error (FPE), Akaike Information Criterion (AIC), Schwarz Information

Criterion (SIC) and Hannan-Quinn Information Criterion(HQ). In this work, the optimal length is selected based on the most common lags resulting from those criteria.

#### D. Wavelet With Markov-Switching Vector Error Correction Models Ms-Vecm

In this section, the DWT and MS-VECM model are combined to study the relationship among the stock markets. As a primary step, the DWT is used to filter the data and then MS-VECM model is applied to obtain the objective.

The MS-VECM serves as an error correction mechanism in each imbalance regime (Krolzig:1997), where the regimes are created by stationary irreducible Markov chain. Error arising out from regime shifts will be corrected towards the stationary distribution of the regimes by MS-VECM.

MS-VECM with shifts in mean and the long-run equilibrium  $\mu(s_t)$  is expressed with a co-integration as follows:

Let  $x_t$ ,  $t \in (1, \dots, T)$  is N-dimension vector scaling coefficients series (approximation series) which are generated by DWT and  $s_t, s_t \in (1, \dots, M)$  is unobservable regime of  $M$  states.  $s_t$  is governed by discrete stat Markov chain stochastic process with the transition probabilities:

$$G = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ g_{M1} & g_{M2} & \cdots & g_{MM} \end{pmatrix}$$

The transition probabilities  $g_{ij}$  in  $G$  are constant and defined by:

$$g_{ij} = \Pr(s_{t+1} = j | s_t = i) \quad \sum_{j=1}^M g_{ij} = 1, \forall i, j \in \{1, \dots, M\}$$



The mean  $\mu(s_t)$  is function describing the dependence of the parameter  $\mu$  on the realized regime  $s_t$

$$\mu(s_t) = \begin{cases} \mu_1 & \text{if } s_t = 1 \\ \mu_2 & \text{if } s_t = 2 \\ \vdots & \\ \mu_M & \text{if } s_t = M \end{cases}$$

And if  $r$  is the rank of co-integration and  $0 \leq r \leq n$ , and the  $z_t$  is the co-integration term with  $z_t = \beta' x_t - \mu(s_t) - \gamma_t$

Then the MS-VECM model can be written as:

$$\Delta x_t = \sum_{k=1}^p A_k(s_t) \Delta x_{t-k} + \alpha z_{t-1} + \mu(s_t) + u_t$$

Where the innovation  $u_t | s_t \sim NID(0, \Sigma(s_t))$ , and  $\mu(s_t)$  is the shift in the mean, and  $\Delta$  is the differencing operation,  $\Delta x_t = x_t - x_{t-1}$ , and  $A_k(s_t)$  are  $N \times N$  matrices, and  $\alpha$  is an  $N \times r$  matrix. The Expectation -Maximization EM algorithm or numerical algorithms in (Krolzig:1997) can be used to estimate the model parameters.

## Empirical Results And Discussion

### E. Dtata

For the present study, the data consists of the daily price of stock market indices of UK stock market and four stock markets of Middle East and North Africa (MENA) region, Egypt, Morocco, Saudi Arabia, and Istanbul. Stock prices data were taken from DataStream which covers the period from 1 May 2001 to 30 December 2011.

### F. Wavelet decompositions

The stock market indices were decomposed using Daubechies (db2) basic function based on DWT, which is better than the MODWT for producing a less noisy series for a given financial dataset (Ahmed&Ismail:2013). The level of the wavelet decomposition corresponds to a level-2 decomposition of each stock

market index that results in reasonable smoothing and produces a signal that identifies its identity without losing the basic characteristics of the original series. The wavelet decompositions are illustrated in Fig. 1 to 5.

These figures below display that the behavior of most of the stock markets is quite similar. For example, the prices in 2001 were low because of the September 11 attacks in the United States. Prices started to increase at the end of 2002 and hit a peak at the end of 2007; they then declined in 2008 and reached a low point at the beginning of 2009 because of the global financial crisis of 2008 to 2009 or the Lehman shock in the United States. After that, the price started to increase again at the end of 2009 until the end of 2011.

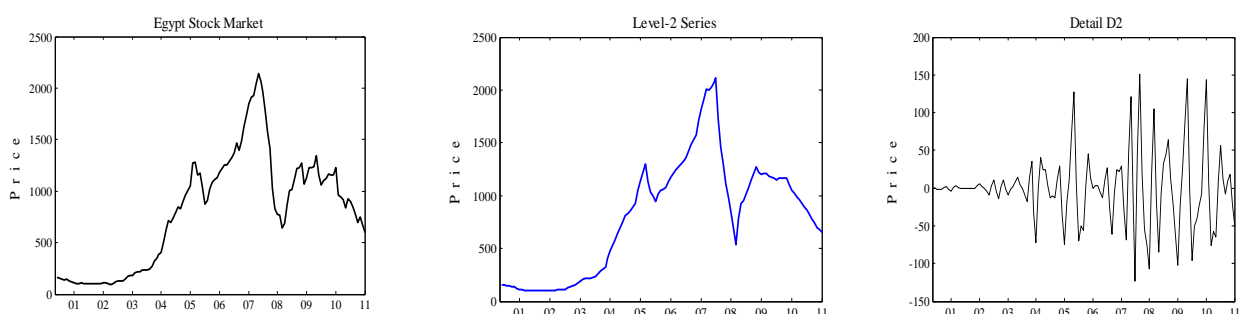


Figure 1. Decomposition of Egypt Stock Market by DWT (db2)

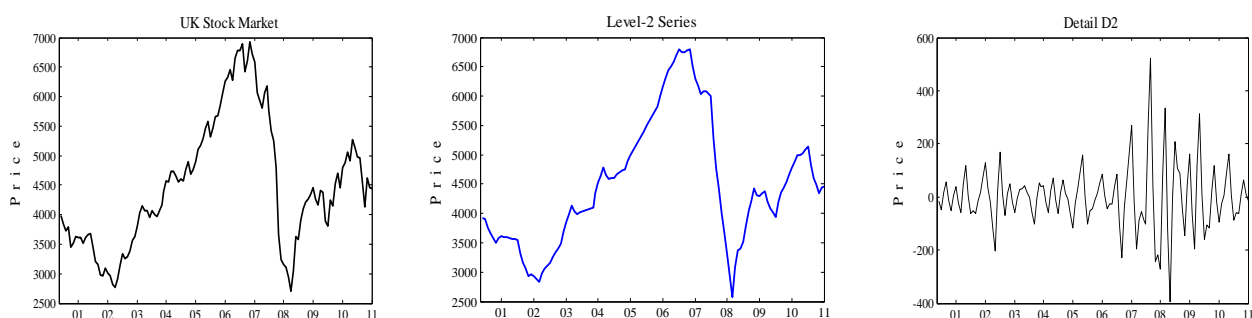


Figure 2. Decomposition of UK Stock Market by DWT (db2)

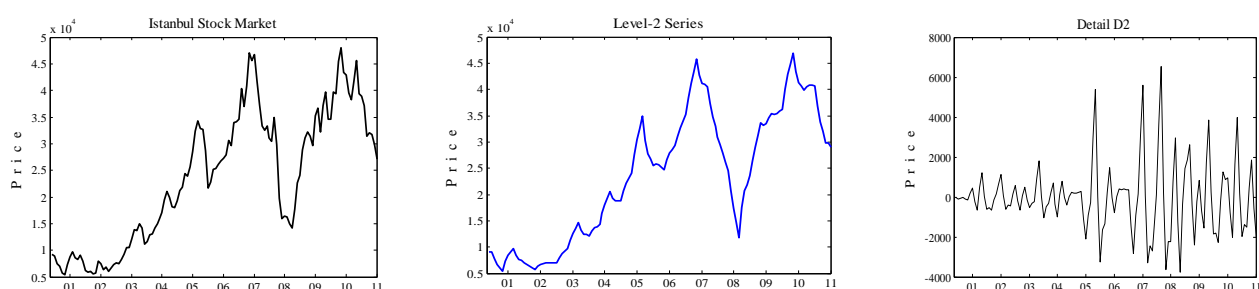


Figure 3. Decomposition of Istanbul Stock Market by DWT (db2)

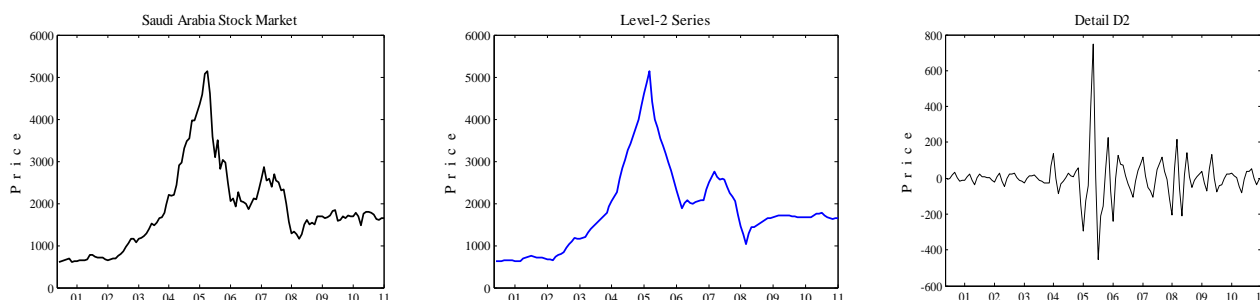


Figure 4. Decomposition of Saudi Arabia Stock Market by DWT (db2)

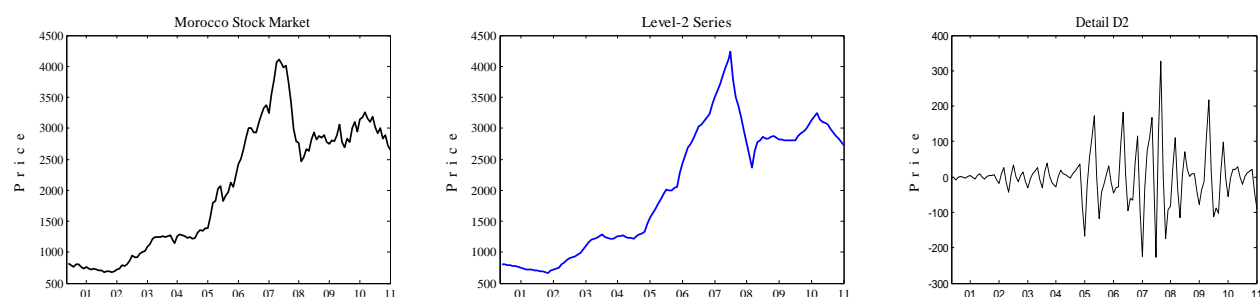


Figure 5. Decomposition of Morocco Stock Market by DWT (db2)

## G. Stationarity tests

Stationarity tests were performed on the original and level-2 approximation series to test whether or not the data have a unit root. Table I shows the results of the stationarity tests for the original series and indicated that the original series are non-stationary time series processes. However, they are stationary at the first difference with the order of integration as  $I(1)$ .

TABLE I. Unit Root Test Results of Original Series

Country	Augmented Dickey Fuller test (ADF)		Phillips-Perron test (PP)	
	<i>p-value at level</i>	<i>p-value at first difference</i>	<i>p-value at level</i>	<i>p-value at first difference</i>
UK	0.4888	0.000*	0.4345	0.000*
Egypt	0.4858	0.000*	0.4830	0.000*
Istanbul	0.3987	0.000*	0.4952	0.000*
Morocco	0.6645	0.000*	0.6674	0.000*
Saudi Arabia	0.4255	0.000*	0.3957	0.000*

\* Denotes rejection of the hypothesis at the 0.05 level

Table II shows the results of the unit root test of the level-2 approximation series of the stock market indices. These results indicate a P-value greater than 0.05, which means that the level-2 approximation series of all the stock market indices are non-stationary time series processes and that they are stationary series at the first difference.

TABLE II. Unit Root Test Results of Level -2 Series

Country	Augmented Dickey Fuller test (ADF)		Phillips-Perron test (PP)	
	<i>p-value at level</i>	<i>p-value at first difference</i>	<i>p-value at level</i>	<i>p-value at first difference</i>
UK	0.3513	0.000*	0.4455	0.000*
Egypt	0.4169	0.000*	0.5761	0.000*
Istanbul	0.3840	0.000*	0.5094	0.000*
Morocco	0.6763	0.000*	0.5518	0.000*
Saudi arabia	0.2993	0.000*	0.3816	0.000*

\* Denotes rejection of the hypothesis at the 0.05 level

## H. Co-integration test

In the case of original series, as reported in table I, the logarithm price sequences are  $I(1)$ . So the long-term relationship among variables can be investigated through a co-integration test Johansen (1990). The results of using these criteria (LR, FPE, AIC, and HQ) reveal that the optimal lag order for the return original series is 7. The results for the co-integration test are given in Table III.

TABLE III. Johansen –Juselius Co-integration Test for original series

No.of cointegration vector	Eigen-value	Trace		
		<i>Statistic</i>	<i>5% Critical Value</i>	<i>Prob</i>
None *	0.316871	100.1452	69.81889	0.0000
At most 1 *	0.181515	54.79763	47.85613	0.0097
At most 2 *	0.135276	30.96195	29.79707	0.0366
At most 3	0.094129	13.66590	15.49471	0.0926

\* Denotes rejection of the hypothesis at the 0.05 level

For level-2 series, shown in Table II, the logarithm price sequences are  $I(1)$ . Therefore, Johansen test can be used to test the co-integration relationship among the variables. The results thus obtained show that the optimal lag order for the return level-2 series is 2. The results for the co-integration test are given in Table IV.

TABLE IV. Johansen –Juselius Co-integration Test for Level-2 series

No.of cointegration	Eigen- value	Trace		
		<i>Statistic</i>	<i>5% Critical Value</i>	<i>Prob</i>
None *	0.343214	86.12121	69.81889	0.0015
At most 1	0.116727	33.57158	47.85613	0.5254
At most 2	0.085959	18.05647	29.79707	0.5619

\* Denotes rejection of the hypothesis at the 0.05 level

## I. Performance comparison

As shown in Table V for the stock markets of the UK and MENA region, the Akaike information criterion (AIC), Schwarz criterion (SBC), and HQ criterion produced a minimum values for the proposed model (DWT-MS-VECM). These results indicate that the performance and fit the financial series of the proposed model is better than the MS-VECM.

TABLE V. Comparison of the Performance of the Models

	DWT-MS-VECM	MS-VECM
AIC	32.4842	33.8644
HQ	33.1920	36.0171
SC	34.2264	39.1657

The results of the comparison indicate that the proposed model (DWT with VECM model) is better in terms of its fit with the stock market series and therefore provides investors real and good information on the co-movement in the stock markets. As such, this study chooses the proposed model in determining the relationships among these stock markets.

## J. Estimation and Analysis of DWT-MS-VECM

In this section the results in Table VI indicate that regime2 (boom) has the highest probability and the highest duration, which are about 0.9017 and 10.17 months respectively, thus showing the most stability of this regime.

TABLE VI. Matrix of transition probability and regime properties

	Depression	Boom	Duration	Prop
Depression	0.8351	0.1649	6.06	0.3735
Boom	0.0983	0.9017	10.17	0.6265

In addition, the time path of smoothed probabilities is illustrated in Figure 6 which indicates that the regime 2 (boom) plays a dominated role with frequency of 0.6265 to identify crises periods around the world that has affected these stock markets. As shown on figure 6, the probabilities of regime 2 (boom) are near to zero in the end of 2001, early of 2002, during the end of 2007 to 2009 and end of 2011. These periods are inline the world financial crises such as September 11 attacks and Downturn in stock prices during 9 October 2002 in stock markets across (US, Canada, Asia, and Europe). Next, the global financial crisis and finally August stock markets crisis was the sharp drop in stock prices in stock markets across the US, MENA, Europe and Asia. This was due to fears of contagion of the European sovereign debt crisis to Spain and Italy. On the contrary, the smooth probabilities of regime 1(depression) are near to one just after the smooth probabilities of regime 2 are near to zero. This mean regime 1 is depression and fall period for the stock market indices.

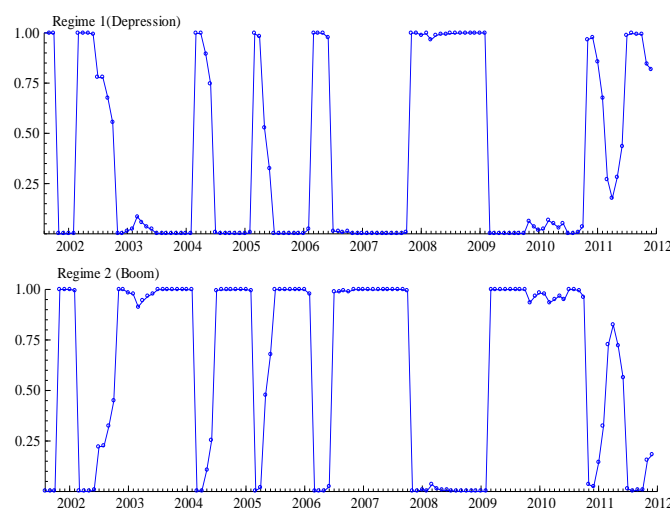


Fig 6: Time path of regime smoothed probabilities

Finally, the outcomes of DWT-MS-VECM are presented in Table VII. The estimated coefficients of the mean suggest that the behavior of MENA stock markets follow the behavior of the stock market in UK. The first regime ( $s_t=1$ ) indicates that all the stock markets are in the boom phase with positive sign. While, the second regime ( $s_t=2$ ) indicates that all the stock markets are in the depression phase with negative sign. The estimated coefficients of the DWT-VECM in Table 6 suggest that the stock market of UK is affected by stock market of Egypt at 10%, and vice versa at 1%. Also the stock market of UK affects stock market of Istanbul at 5% level and stock market of Morocco at 1%



level. However, there is no relationship between stock market of UK and stock market of Saudi Arabia.

TABLE VII. Estimates of DWT-MS-VECM Model

	UK	EGYPT	ISTANBUL	MOROCCO	SAUDI
$\mu(s_t = 1)$	-1.942027	-2.423585	-6.584330	-0.670383	-0.186957
$\mu(s_t = 2)$	1.412073	1.708947	5.639206	0.889842	1.205291
UK(-1)	0.612783*	1.028027#	0.165120	0.471546	0.066178
UK(-2)	-0.029333	-1.332680	0.072396	-0.434167	-0.099011
Egypt(1)	-0.083934**	-0.356785*	0.002058	-0.294399**	-0.021537
Egypt(2)	0.076964*	0.512458**	0.062845	0.339827**	0.167435**
Istanbul(1)	-0.087892*	-0.542987*	0.086952	-0.220747	-0.076414
Istanbul(2)	-0.070630#	-0.695472**	-0.254738**	-0.654334**	-0.070292
Morocco(1)	0.130553**	0.578374*	-0.016610	0.444272**	0.060449
Morocco(2)	-0.135755**	-0.657977*	-0.093202	-0.453883**	-0.247037
Saudi Arabia (1)	0.001867	0.517534#	0.066609	0.005179	0.540312*
Saudi Arabia (2)	0.019217	-0.245295	0.038106	-0.091975	0.023584
SE	2.961324	15.353739	5.573658	10.012940	4.812178

# denoted weak significance at 10%, \* denoted strong significance at 5%, \*\* denoted very strong significance at 1%

## Conclusion

Determining the relationship between the stock markets of developed countries and those of the MENA region is significantly important for international investors in making the financial decisions. MS-Markov Switching models are one of the most popular approaches in multivariate time series analysis and have been frequently used in previous studies. This study employs a new technique to investigate the relationship between the stock markets of MENA region and UK by combining the wavelet transform and MS-VECM model. Daubechies2 (db2) based on the DWT technique has been used to separate low-frequency signals from high-frequency noise in UK and MENA index series and there by eliminate noise in the stock price signal. The low-frequency data at level-2 has been used to examine the long- and short-term relationships between the stock market of the United Kingdom and four of the stock markets of the MENA region explicitly, Egypt, Istanbul, Morocco, and Saudi Arabia, using co-integration test and the proposed model (DWT-MS-VECM model) for the period from 5/1/2001 to 12/30/2011. The results of the comparison between the proposed and traditional model show that DWT-MS-VECM model outperforms the MS-VECM model in terms of performance and fit with the financial stock market series. The co-integration test for the original data and low-frequency data at level-2 reveal a long-term relationship between the stock market of the UK and those of the MENA region. In short-term, the stock market in UK has relationship with these stock markets except that in Saudi Arabia. Additionally, the DWT-MS-VECM model manages to capture a satisfactory timing of the crises period that has affected those stock markets and provides fine information on the relationships among these stock markets.

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