THE METRIC SYSTEM

The metric system or SI (International System) is the most common system of measurements in the world, and the easiest to use. The base units for the metric system are the units of:

- length, measured in meters (m);
- time, measured in seconds (s);
- mass, measured in grams (g); and
- temperature, measured in Celsius (°C).

In the metric system, prefixes are used to describe multiples or fractions of the base units. The most common metric system prefixes are:

- nano (n) .... / 100,000,000
- micro (μ) .... / 1,000,000
- milli (m) .... / 1,000
- centi, (c) .... / 100
- deci (d) .... / 10
- deca (da).....................................x 10
- hecto (h) ....x 100
- kilo (k)...................................x 1,000
- mega (M).........................x 1,000,000
- giga (G) ....x 1,000,000,000

Using the base unit for length:

1 nanometer (nm)	=	1 m / 100,000,000	=	0.000 000 001 m
1 micrometer (μm)	=	1 m / 1,000,000	=	0.000 001 m
1 millimeter (mm)	=	1 m / 1,000	=	0.001 m
1 centimeter (cm)	=	1 m / 100	=	0.01 m
1 decimeter (dm)	=	1 m / 10	=	0.1 m
1 decameter (dam)	=	1 m x 10	=	10 m
1 hectometer (hm)	=	1 m x 100	=	100 m
1 kilometer (km)	=	1 m x 1,000	=	1,000 m
1 megameter (Mm)	=	1 m x 1,000,000	=	1,000,000 m
1 gigameter (Gm)	=	1 m x 1,000,000,000	=	1,000,000,000 m

Similarly for mass, you will often see the following units:

1 milligram (mg)	=	1 g / 1,000	=	0.001 g
1 kilogram (kg)	=	1 g x 1,000	=	1,000 g

THE IMPERIAL SYSTEM

The Imperial system is more complicated than the metric system, as it does not work in multiples of 10 as the metric system does. The Imperial system is used in England and the United States, and you will probably recognize many of the units used.

In the Imperial system the base units are:

- length, commonly measured in inches (in), feet (ft), yards and miles;
- time, commonly measured in seconds (s), hours (hr), days (d), weeks and years (yr);
- weight, commonly measured pounds (lb); and
- temperature, measured in Fahrenheit (°F).

Note that the Imperial system commonly uses weight, rather than mass. Weight refers to the gravitational pull on an object, whereas mass refers to the amount of matter in the object. An object would have the same mass on the moon as it does on the earth, but it would weight less on the moon as the gravitational pull of the moon is less than the gravitational pull of the earth. Astronauts have the same mass on the moon as they do on the earth, but they can jump higher on the moon because they weigh less there! This difference does not affect us much as all the problems we will be solving assume we are on the earth, but you should be aware of it.

Conversions between the common units of length used in the Imperial system are listed below

- 12 in = 1 ft
- 3 ft = 1 yard
- 1760 yards = 1 mile

The units of time are generally accepted for use with the metric system, as well as the Imperial system. Conversions between the common units of time are listed below:

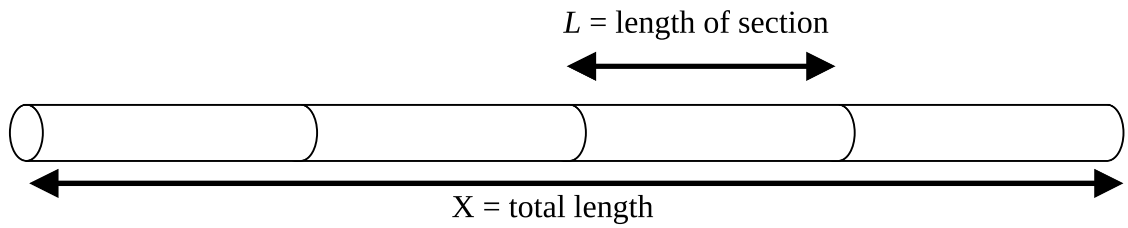
- 60 s = 1 min
- 24 hrs = 1 day
- 7 days = 1 week
- 52 weeks = 1 yr

Conversion between Metric and Imperial

The tables on the next two pages give the conversions between Metric and Imperial. These tables come attached to the exam, and you should know how to use them. Math questions on the exam generally give both metric and imperial units, allowing you to work in whichever system you are most comfortable in. It helps if you can convert between the two systems to check your answers!

LENGTH

N = number of sections



Units of Measurement:

Metric (SI):
- mm = millimeters
- cm = centimeters
- m = meters
- km = kilometers

Imperial:
- in. = inches
- ft = feet
- yards
- miles

Conversion:

Metric (SI):

1 cm = 10 mm
- cm x 10 = mm
- mm x 0.1 = cm

1 m = 100 cm
- m x 100 = cm
- cm x 0.01 = m

1 m = 1 000 mm
- m x 1 000 = mm
- mm x 0.001 = m

1 km = 1 000 m
- km x 1 000 = m
- m x 0.001 = km

Metric to Imperial (and back):

1 in = 2.54 cm
- cm x 0.393 7 = inches
- inches x .54 = cm

1 ft = 30.48 cm
- cm x 0.032 8 = feet
- feet x 30.48 = cm

1 m = 3.281 ft
- m x 3.281 = feet
- feet x 0.304 8 = m

1 mile = 1.609 km
- km x 0.621 4 = miles
- miles x 1.609 = km

Length: Questions

1. If I have 7 sections of pipe, each 3.0 m long, what is the total length of line I can replace?

   Given: L = ____________

   N = ____________ $X = L \times N$

   Find: X = ____________

2. If I have to replace 21 m of pipeline, and each pipe is 3.0 m long, how many sections of pipe will I need?

   Given: X = ____________

   L = ____________ $N = \frac{X}{L}$

   Find: N = ____________

3. A line has failed and 23 m must be replaced. How many 3.0 m long section of pipe will be needed to repair the line?

   Given: X = ____________

   L = ____________ $N = \frac{X}{L}$

   Find: N = ____________

4. A line has failed and 1.6 km must be replaced. How many 6 m long section of pipe will be needed to repair the line?

   Given: X = ____________

   L = ____________ $N = \frac{X}{L}$

   Find: N = ____________

5. A line has failed and 0.7 km must be replaced. How many 3.5 m long section of pipe will be needed to repair the line?

   Given: X = ____________

   L = ____________ $N = \frac{X}{L}$

   Find: N = ____________

Length: Discussion and Answers

The first 5 questions on length are designed to introduce the concepts of basic problems solving, using and rearranging simple equations, identifying given parameters and calculating unknown parameters, knowing whether to round up or down, and converting between different units within the metric system.

Question 1: Using Equations

The basic equation is: $X = L \times N$
where: N = number of sections
L = length of section
X = total length

In plain English the equation reads: *The total length of the pipe equals the length of one section multiplied by the number of sections.*

IMPORTANT: The student should always remember to note units!

1. If I have 7 sections of pipe, each 3.0 m long, what is the total length of line I can replace?

   Given: $L =$ 3.0 m
   N = 7 (no units)
   Find: X
   Solution: X $= L \times N$
   $= 3.0\ \text{m} \times 7$
   $= 21.0\ \text{m}$

Question 2: Rearranging Equations

2. If I have to replace 21 m of pipeline, and each pipe is 3.0 m long, how many sections of pipe will I need?

   Given: X = 21 m
   $L =$ 3.0 m

   Find: N

   As with the first question, the basic equation is:

   $X = L \times N$

   In Question 2, we are given the total length (X) and the length of each section (L), and we have to find out how many sections we need (or solve for N).

When we rearrange an equation, we have to do the same thing to both sides to keep the equation equal. The first step in rearranging is to identify which parameter we have to solve for, and "isolate" that parameter on the left side. In this case we are solving for N, so we divide both sides of the equation by L.

After rearranging, the equations becomes:

$$\frac{X}{L} = \frac{LxN}{L}$$

Since any number divided by itself equals one, L/L cancels out. Switching sides, the equation becomes:

$$N = \frac{X}{L}$$

In plain English the equation reads: *The number of sections required is equal to the total length of the pipe divided by the length of one section.*

Solution: $N = \frac{X}{L}$

$$= \frac{21m}{3.0m}$$

$$= 7$$

Question 3: Rounding

3. A line has failed and 23 m must be replaced. How many 3.0 m long section of pipe will be needed to repair the line?

Given:	X =	23 m
	L =	3.0 m
Find:	N	

The equation used in Questions 3 to 5 is the same as that used in Question 2, but here the answer does not come out to an even number. The operator should know that they need to round the answer up, otherwise they would not have enough pipe to replace the full length of the failed section.

Solution: $N = \frac{X}{L}$

$$= \frac{23m}{3m}$$

$$=7.67\text{m}$$

You will need eight sections of pipe to fix the line.

Questions 4 and 5: Converting to Consistent Units

4. A line has failed and 1.6 km must be replaced. How many 6 m long section of pipe will be needed to repair the line?

   The operator should know they should always be working in the same or compatible units. Here, X is given in kilometers and *L* in meters. The operator must convert either X or L.

Given:	X	= 1.6 km
		= 1,600 m
	L	= 6 m
Find:	N	

Solution: $N = \frac{X}{L}$

$$= \frac{1{,}600m}{6m}$$

$$= 266.7$$

You will need 267 sections of pipe to repair the line.

5. A line has failed and 0.7 km must be replaced. How many 3.5 m long section of pipe will be needed to repair the line?

Given:	X	= 0.7 km
		= 700 m
	L	= 3.5 m
Find:	N	

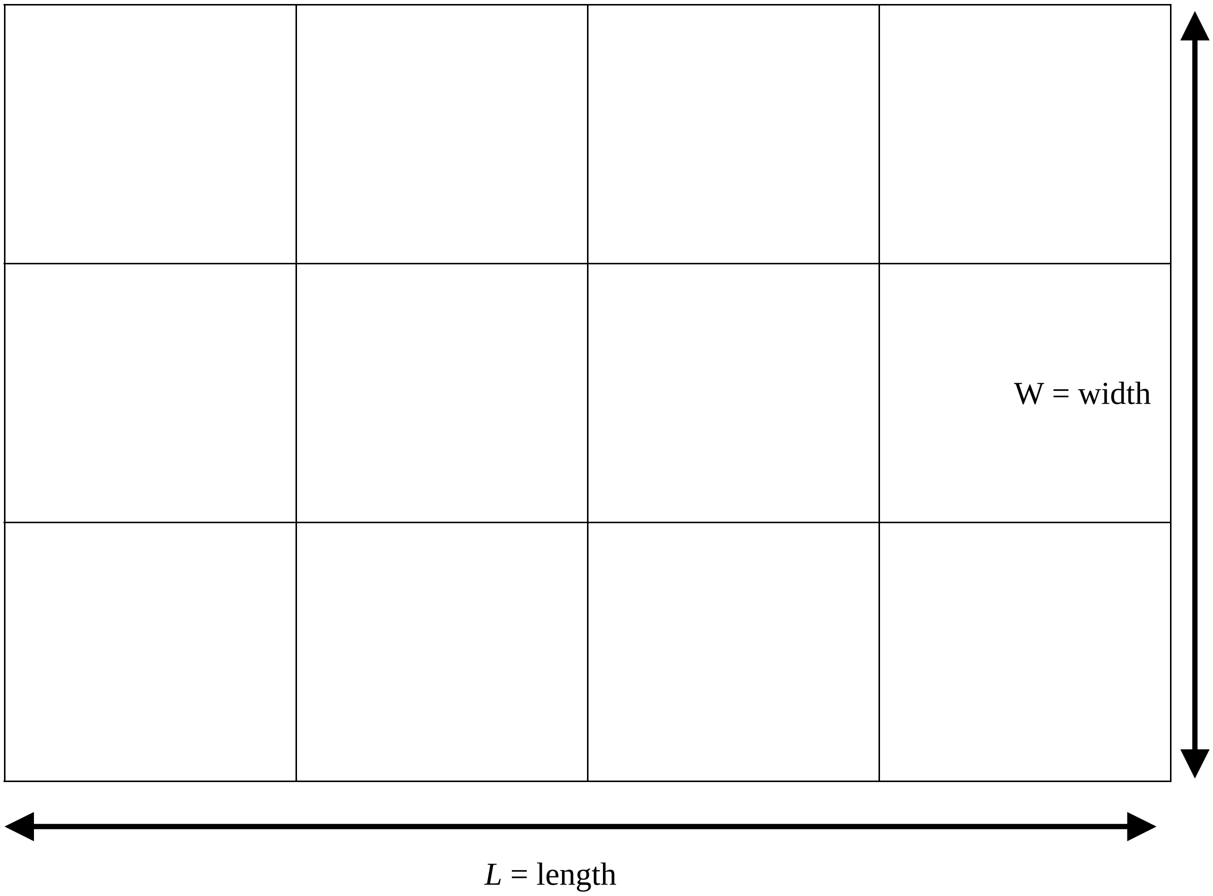
Solution: $N = \frac{X}{L}$

$$= \frac{700m}{3.5m}$$

$$= 200$$

You will need at least 200 sections of pipe to repair the line.

AREA



Units of Measurement:

Metric (SI):

m^2 = square meters
ha = hectares

Imperial:

ft^2 = square feet = ft. sq.
square yards
acres

Conversion:

Metric (SI):

1 m^2 = 10 000 cm^2
m^2 x 10 000 = cm^2
cm^2 x 0.000 1 = m^2

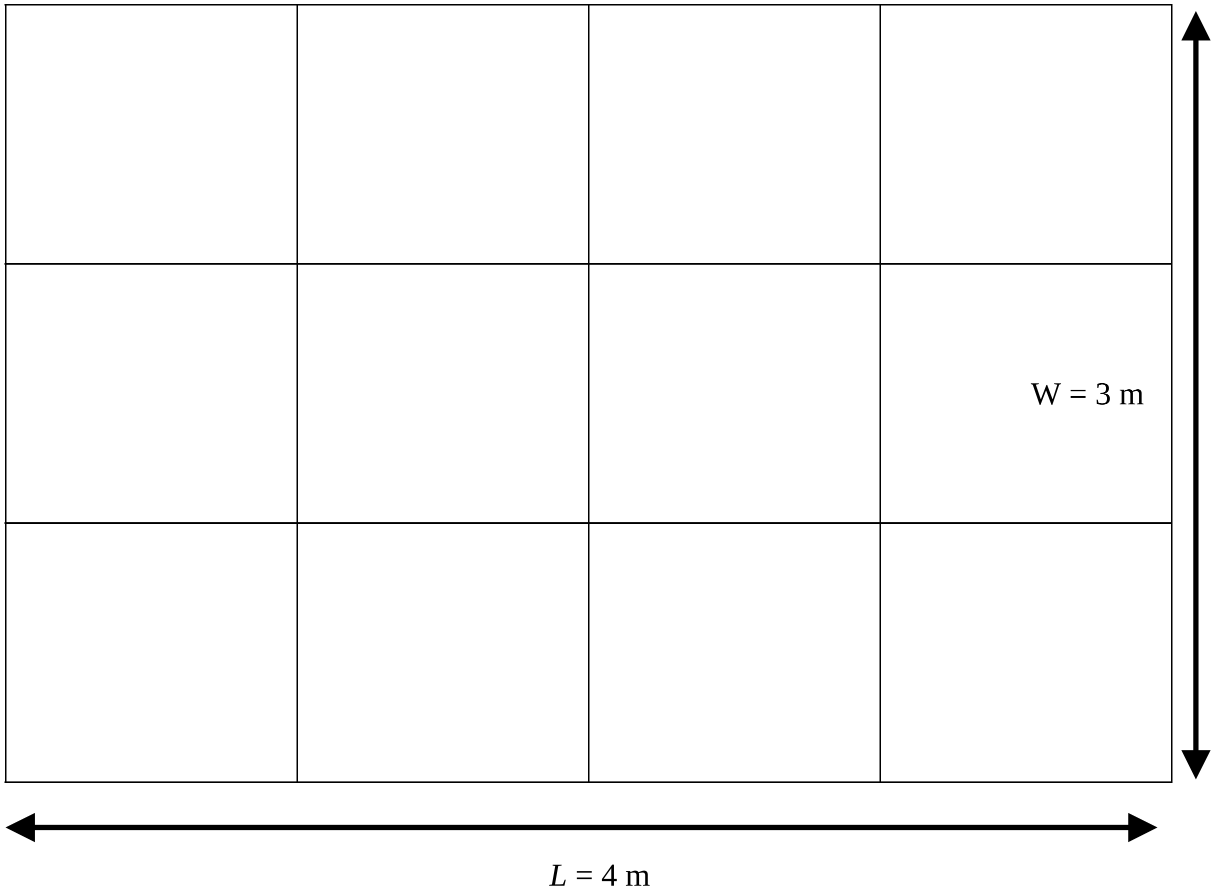
1 ha = 10 000 m^2
ha x 10 000 = m^2
m^2 x 0.000 1 = m^2

Metric to Imperial (and back):

1 m^2 = 10.763 9 square feet
m^2 x 10.763 9 = square feet
feet square x 0.0929 = m^2

1 ha = 2.471 acres
ha x 2.471 = acres
acres x 0.404 69 = ha

Area of a Rectangle: Questions



1. A floor measures 4 m by 3 m. What is the area of the floor?

Given: L = ______________

W = ______________ $\qquad A = L \times W$

Find: A = ______________

2. A floor has a length of 4 m and an area of 12 m^2, what is the width of the floor?

Given: A = ______________

L = ______________ $\qquad W = \dfrac{A}{L}$

Find: W = ______________

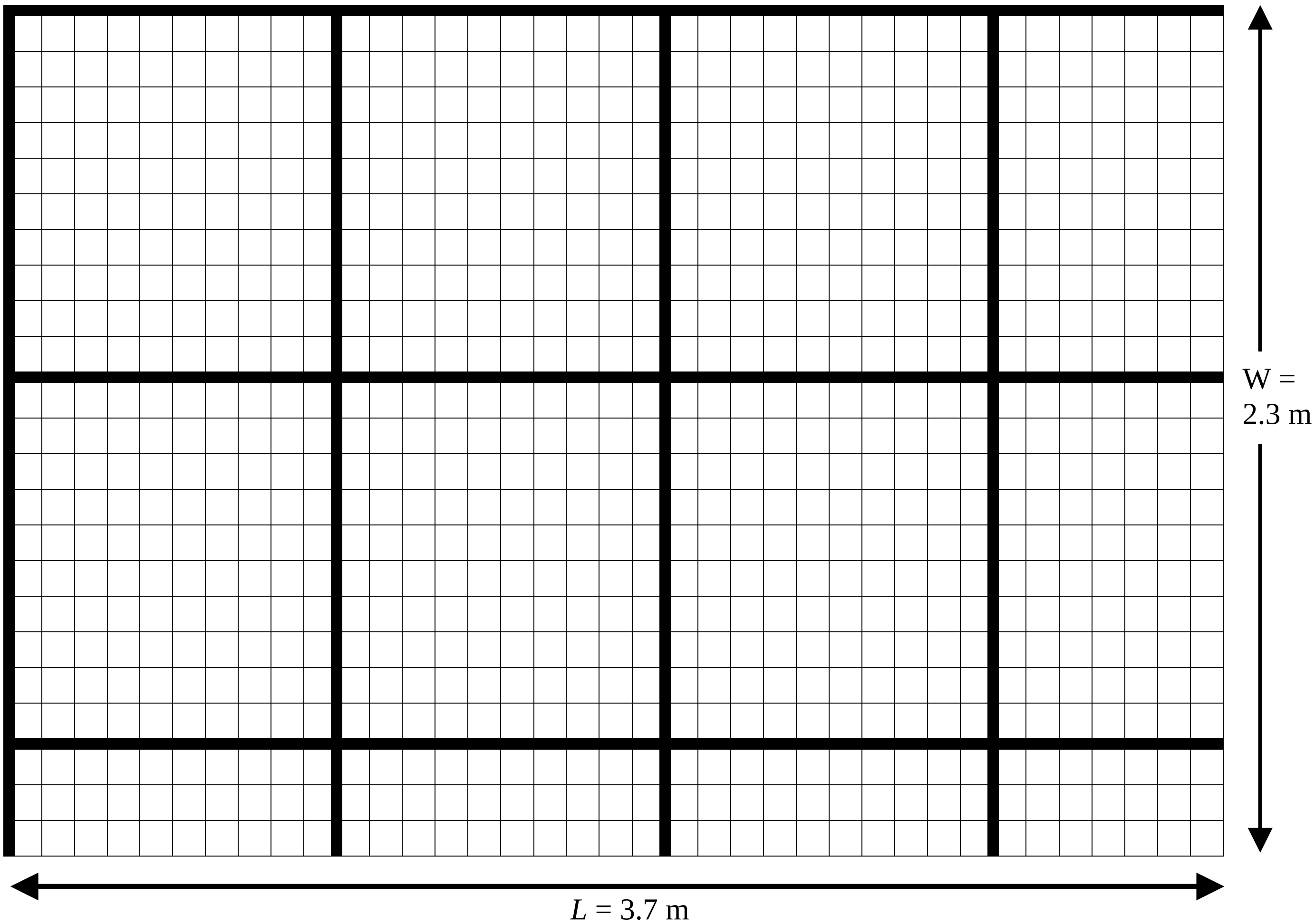
3. A floor has a width of 3 m and an area of 12 m^2, what is the length of the floor?

Given: A = ______________

W = ______________ $\qquad L = \dfrac{A}{W}$

Find: L = ______________

Area of a Rectangle: Questions



4. A floor measures 3.7 m by 2.3 m. What is the area of the floor?
   Given: L = ____________ W = ____________
   Find: A = ____________

5. A floor has a length of 3.7 m and an area of 8.5 m^2, what is the width of the floor?
   Given: A = ____________ L = ____________
   Find: W = ____________

6. A floor has a width of 2.3 m and an area of 8.5 m^2, what is the length of the floor?
   Given: A = ____________ W = ____________
   Find: L = ____________

Area of a Rectangle: Discussion

The basic equation for calculating the area of a square or rectangle is:

$A = L \times W$

In plain English the equation reads: "*The area of a rectangle is equal to the length times the width*", or "*the area of a rectangle is equal to the length multiplied by the width*".

Note that a square is simple the special case of a rectangle where the length is equal to the width. The more general term rectangle is therefore used in the discussion.

The questions relating to area of a rectangle reinforce the concepts of basic problems solving, identifying given parameters and calculating unknown parameters and using and rearranging simple equations.

The student should clearly understand the meaning of the unit m^2 (meters square), in the sense that:

$1\ m^2 = 1\ m \times 1\ m = 1\ m \times m$

Similarly, the units square inch or square feet, where:

1 sq. in. $= 1\ in^2 = 1\ in \times 1\ in = 1\ in \times in$

1 sq. ft. $= 1\ ft^2 = 1\ ft \times 1\ ft = 1\ ft \times ft$

In addition, the student should be familiar with different formats for expressing multiplication. Note that the equation:

$A = L \times W$

can also be written in the following ways:

$A = (L)(W)$

$A = LW$

$A = L \cdot W$

$A = L * W$

Area of a Rectangle: Answers

1. A floor measures 4 m by 3 m. What is the area of the floor?

   Given: $L = 4$ m

   W = 3 m

   Find: A

   Solution: A = L x W

   = 4 m x 3 m

   = 12 m x m

   = 12 m^2

2. A floor has a length of 4 m and an area of 12 m^2, what is the width of the floor?

   Given: A = 12 m^2

   L = 4 m

   Find: W

   Solution: $W = \frac{A}{L}$

   $= \frac{12mxm}{4m}$

   =3 m

3. A floor has a width of 3 m and an area of 12 m^2, what is the length of the floor?

   Given: A = 12 m^2

   W = 3 m

   Find L

   Solution: $L = \frac{A}{W}$

   $= \frac{12mxm}{3m}$

   = 4 m

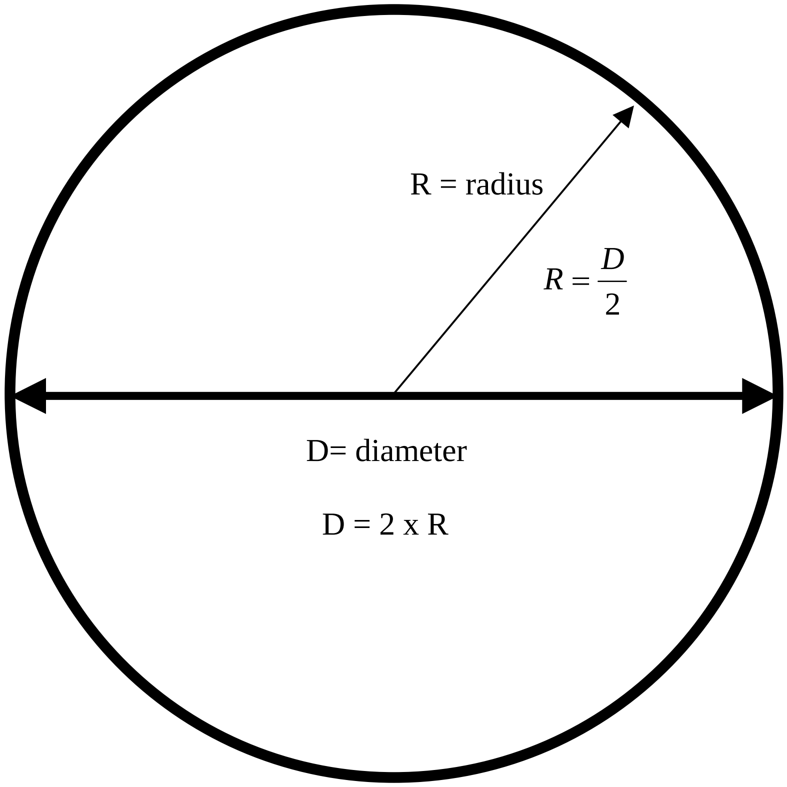
Questions 4 to 6 are similar but the student should be comfortable working with decimal places and should know when to round up and when to round down.

4. A = 8.5 m^2

5. W = 2.3m

6. W = 3.7m

Area of a Circle: Questions



$A = \pi R^2$

1. A circle has a radius of 3 m. What is the area of the circle?
   Given: R = _____________

   Find: A = _____________

2. A circle has a diameter of 6 m. What is the area of the circle?
   Given: D= _____________

   Find: R = _____________

   A = _____________

3. A circle has a radius of 3.8 m. What is the area of the circle?
   Given: R = _____________

   Find: A = _____________

Area of a Circle: Discussion

The water treatment plant operator often works with circular containers, and must be able to calculate the area and volume of these containers. This section introduces the formula used to calculate the area of a circle, and the concepts of Pi (π) and radius squared (R^2).

The radius of a circle is the length from the centre of the circle to the edge of the circle. The units of radius (R) are the units of length. The term R^2 means:

$$R^2 = R \times R$$

The units of R^2 will be a unit of length squared such as m^2, in^2 or ft^2:

Pi (π) is a number that relates the radius of a circle to its area. Pi is always equal to:

$$\pi = 3.141592654.......$$

This is typically rounded to:

$$\pi = 3.14$$

The basic formula for calculating the area of a circle is:

$$A = \pi R^2$$

In plain English the equation reads: *The area of a circle is equal to Pi(3.14) times the radius squared.*

The diameter of a circle (D) is the length from one side of a circle to the other. Diameter (D) is equal to twice the radius, or:

$$D = 2R$$

Similarly, by rearranging the equation, we know the radius is equal to half the diameter, or

$$R = \frac{D}{2}$$

If a question gives the diameter of the circle, the student should calculate the radius first and then the area.

Area of a Circle: Answers

1. A circle has a radius of 3 m. What is the area of the circle?

   Given: $R = 3$ m

   Find: A

   Answer: $A = \pi R^2$

   $= 3.14\ (3\text{ m})(3\text{ m})$

   $= 28.26\ (\text{m})(\text{m})$

   $= 28.3\text{ m}^2$

2. A circle has a diameter of 6 m. What is the area of the circle?

   Given: $D = 6$ m

   Find: A

   Answer: $R = \dfrac{D}{2}$

   $= \dfrac{6m}{2}$

   $= 3$ m

   $A = \pi R^2$

   $= (3.14)(3\text{ m})(3\text{ m})$

   $= 28.3\text{ m}^2$

3. A circle has a radius of 3.8 m. What is the area of the circle?

   Given: $R = 3.8$ m

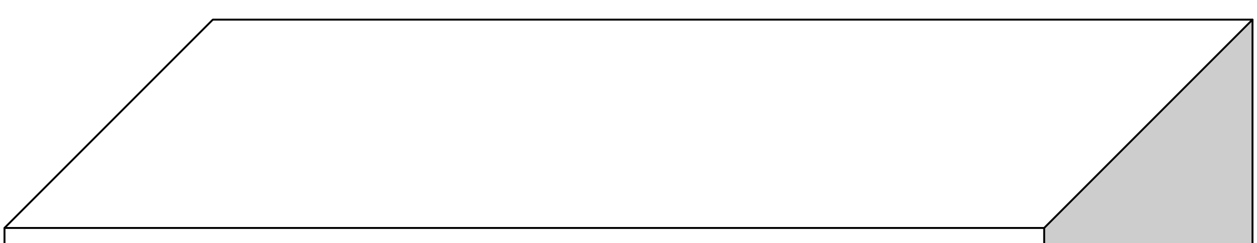
   Find: A

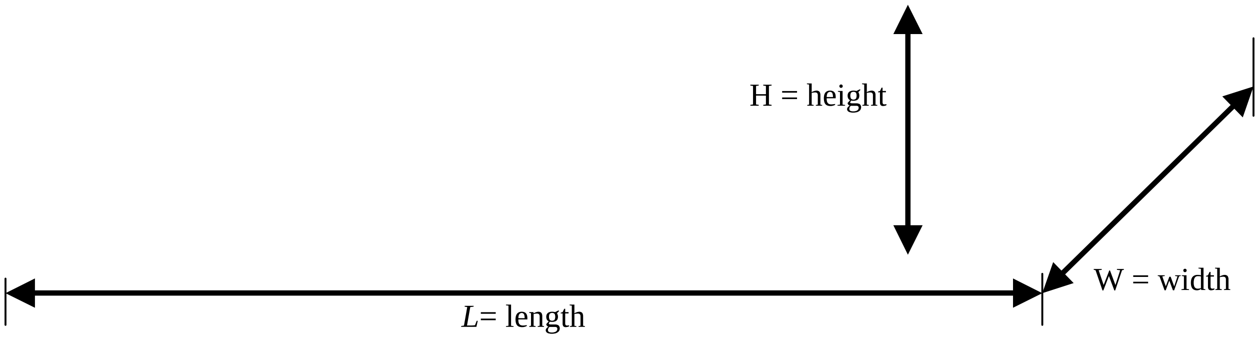
   Answer: $A = \pi R^2$

   $= 3.14\ (3.8\text{ m})(3.8\text{ m})$

   $= 45.3\text{ m}^2$

VOLUME





Units of Measurement:

Metric (SI):

- cm^3 = cubic centimeters
- m^3 = cubic meters = cu m
- L = liters
- mL = milliliters

Imperial:

- ft^3 = cubic feet = ft. cu.
- gallons (Imperial) = Imp. gal. = ig
- US gallons

Conversion:

Metric (SI):

1 m^3 = 1 000 000 cm^3
- m^3 x 1 000 000 = cm^3
- cm^3 x 0.000 001 = m^3

1 m^3 = 1 000 L
- m^3 x 1 000 = L
- l x 0.001 = m^3

1 l = 1 000 mL
- L x 1 000 = mL
- m x 0.001 = L

1 mL = 1 cm^3
- mL x 1 = cm^3
- cm^3 x 1 = mL

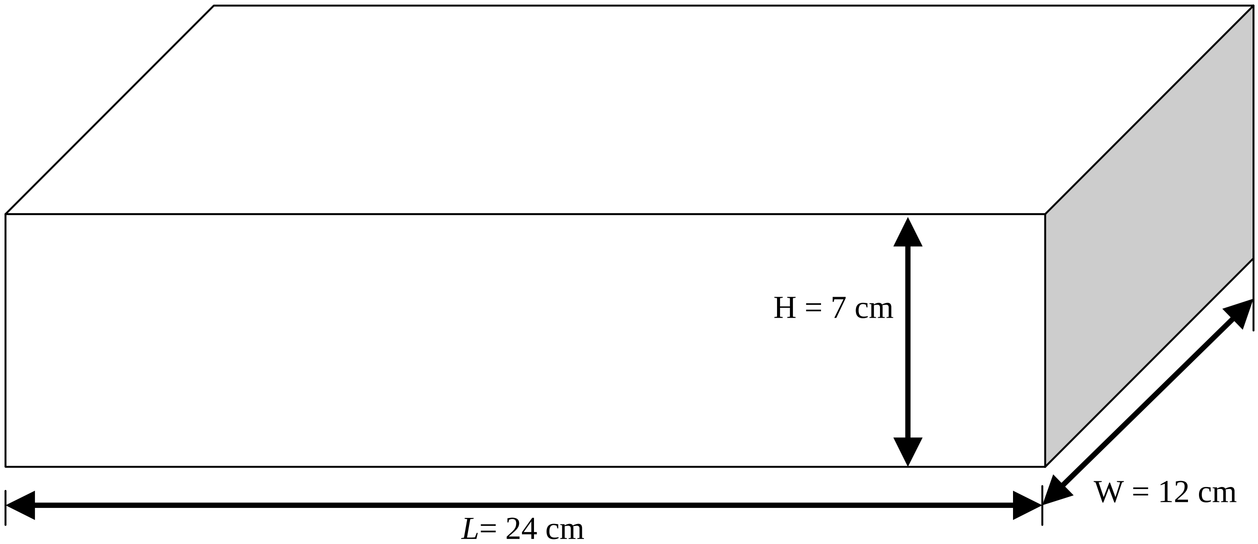
Metric to Imperial (and back):

1 m^3 = 35.315 cubic feet
- m^3 x 35.315 = cubic feet
- cubic feet x 0.028 32 = m^3

1 gallon (Imperial) = 4.546 L
- l x 0.219 9 = gallons (Imperial)
- gallons (Imperial) x 4.546 = L

1 gallon (Imperial) = 1.2001 US gallons
- US gallons x 0.8327 = Imperial gallons
- Imperial gallons x 1.2001 = US gallons

Volume of a Box: Questions



1. A box has a top area of 1.92 m^2 and a height of 1.2 m. What is the volume of the box?

2. A box of has a length of 24 cm, a width of 12 cm and a height of 7 cm. What is the area of the top of the box and the volume of the box?

3. A box of has a length of 1.6 m, a width of 92 cm and a height of 1.2m. What is the volume of the box in cm^3, m^3 and L?

4. A Foreman places 3 m^3 of sand in a container 2.4 m long and 1.8 m wide. What is the height of sand in the container?

5. A Foreman places 60 L of water in a container 2.4 m long and 1.8 m wide. What is the height of water in the container?

6. A water tank 2.4 m long and 1.8 m wide has 1.2 m of water in it in the morning. By the end of the day, the container has 1.7 m of water in it. How much water was added to the container that day?

7. A water tank 2.4 m long and 1.8 m wide has 1.2 m of water in it in the morning. If we add another 850 L of water to the tank, what will be the height of water in the tank?

Volume of a Box: Discussion

By now, the student should be familiar with the basic problem solving technique: identifying what is given, what they need to find, what equation to use, whether they need to rearrange the equation, and substituting the given values into the equation to find the required value.

The basic equation for calculating volume, is area of the top or base times the height. In the case of a box, the area of the top or base is equal to the length times the width:

V= A x H

where H = height

A = *L* x W

Substituting the equation for area into the equation for volume, we get:

V = (*L* x W) x H

or:

V= *L* x W x H

In plain English the equation reads: "*The volume of a box is equal to the length times the width times the height*".

The student should understand the meaning of the unit m^3 (cubic meters), in the sense that:

1 m^3 = 1 m x 1 m x 1 m= 1 m x m x m

Similarly, the units cubic inch or cubic feet, where:

1 cu. in. = 1 in^3 = 1 in x 1 in x 1 in = 1 in x in x in

1 cu. ft. = 1 ft^3 = 1 ft x 1 ft = 1 ft x ft x ft

In the metric system, volume, particularly liquid volume, is often measured in L or mL. Keep in mind that:

1 l = 1 dm3 = 1 dm x 1 dm x 1 dm

1 ml = 1 cm^3 = 1 cm x 1 cm x 1 cm

1 m^3 = 1,000 L

1 L = 1,000 mL

Volume of a Box: Answers

1. A box has a top area of 1.92 m^2 and a height of 1.2 m. What is the volume of the box?

   Given: $A = 1.92\ m^2$

   $H = 1.2\ m$

   Find: V

   Solution: $V = A \times H$

   $= (1.92\ m^2)(1.2\ m)$

   $= 2.304\ (m \times m)(m)$

   $= 2.3\ m^3$

2. A box of has a length of 24 cm, a width of 12 cm and a height of 7 cm. What is the area of the top of the box and the volume of the box?

   Given: $L = 24\ cm$

   $W = 12\ cm$

   $H = 7\ cm$

   Find: A and V

   Solution: $A = L \times W$

   $= (24\ cm)(12\ cm)$

   $=288\ cm^2$

   $V = A \times H$

   $= (288 cm^2)(7\ cm)$

   $= 2016\ cm^3$

3. A box of has a length of 1.6 m, a width of 92 cm and a height of 1.2m. What is the volume of the box in cm^3, m^3 and L?

   Given: $L = 1.6\ m$

   $W = 92\ cm$

   $H = 1.2\ m$

   Find: V in cm^3, m^3 and L

   Solution: $V = L \times W \times H$

   First convert all given parameters to consistent units. In this case you can convert to either cm or m. If we convert to m:

   $cm \times 0.01 = m$

   $W = 92\ cm \times 0.01 = 0.92\ m$

   $V = L \times W \times H$

   $= (1.6\ m)(0.92\ m)(1.2\ m)$

   $= 1.77\ m^3$

m^3 x 1 000 000 = cm^3
V = 1.77 m^3 x 1 000 000 = 1,770,000 cm^3

m^3 x 1 000 = L
V = 1.77 m^3 x 1 000 = 1,770 L

4. A Foreman places 3 m^2 of sand in a container 2.4 m long and 1.8 m wide. What is the height of sand in the container?

Given: V = 3 m^2
L = 2.4 m
W = 1.8 m

Find: H

Solution: W = L x W x H
To isolate H, we divide both sides of the equation by L x W

$$\frac{V}{LxW} = \frac{LxWxH}{LxW}$$

$\frac{LxW}{LxW} = 1$, therefore $\frac{V}{LxW} = 1H$, or

$$H = \frac{V}{LxW}$$

$$= \frac{3\,\mathrm{m}^3}{(2.4\mathrm{m})(1.8\ \mathrm{m})}$$

= 0.69 m

5. A Foreman places 60 l of water in a container 2.4 m long and 1.8 m wide. What is the height of water in the container?

Given: V = 60 L
L = 2.4 m
W = 1.8 m

Find: H

Solution: First convert all parameters to consistent units, in this case m,
L x 0.001 = m^3
V = 60 L x 0.001 = 0.06 m^3

$$H = \frac{V}{LxW}$$

$$= \frac{0.06m^3}{(2.4m)(1.8m)}$$

= 0.014 m

m x 100 = cm
H = 0.014 m x 100 = 1.4 cm

6. A water tank 2.4 m long and 1.8 m wide has 1.2 m of water in it in the morning. By the end of the day, the container has 1.7 m of water in it. How much water was added to the container that day?

Given: L = 2.4 m
W = 1.8 m
H_1 = 1.2 m
H_2 = 1.7 m

Find: V

Solution: Note all measurements are given in m, so we do not have to convert.
In this case we are interested only in the volume of water added to the tank, so the height we need is the difference between the height of water in the tank in the morning and the height of water in the tank in the evening.
$H = H_2 – H_1$
= 1.7 m – 1.2 m
= 0.5 m
V = L x W x H
= (2.4 m)(1.8 m)(0.5 m)
= 4.82 m^3
m^3 x 1 000 = L
V = 4.82 m^3 x 1,000 = 4820 L

7. A water tank 2.4 m long and 1.8 m wide has 1.2 m of water in it in the morning. If we add another 850 l of water to the tank, what will be the height of water in the tank?

Given: L = 2.4 m
W = 1.8 m
H_1 = 1.2 m
V = 850 L

Find: H_2

Solution: First convert all parameters to consistent units:
L x 0.001 = m^3
V = 850 l x 0.001 = 0.85 m^3

$$H = \frac{V}{LxW}$$

$$= \frac{0.85m^3}{(2.4m)(1.8m)}$$

= 0.20 m

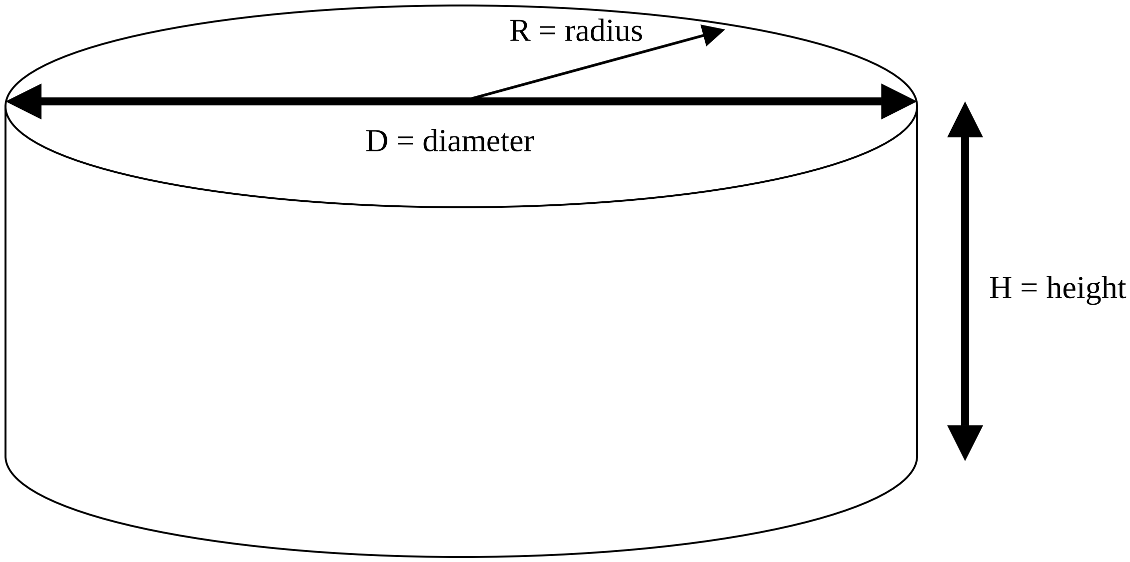
In this case H is the difference between the height of water in the tank in the morning, and the height of water in the tank after adding another 850 l. To find the total height of water in the tank after adding 850 l, we have to add the original height to height difference, or:

$H_2 = H_1 + H$

= 1.2 m + 0.2 m

= 1.4 m

Volume of a Cylinder: Questions



1. An upright cylindrical drum has a radius of 3 m and a height of 2 m. What is the area of the lid and the volume of the cylinder in m^3?

2. a) An upright cylinder has a diameter of 0.5 m and a height of 2.6 m. What is the volume of the cylinder in m^3 and L?

   b) The cylinder from question 2a is filled with water to a depth of 1.8 m. How much water is in the cylinder?

3. An upright cylindrical tank with a diameter of 3 m, was filled to a depth of 2.3 m in the morning and 1.5 m at the end of the day. Assuming there was no water added to the tank that day, how much water was taken out?

4. A water treatment plant operator has a chlorine mixing tank with a diameter of 0.8 m, and a depth of 1.1 m. If there is 0.2 m of chlorine solution left in the tank, and the operator decides not to empty the tank before adding an addition 150 L of water, what will be the depth of liquid in the tank after adding the extra water?

5. How many litres of water can be held in 15 m of 200 mm pipe?

Volume of a Cylinder: Answers

Questions 1 to 4: Cylindrical Tanks

1. An upright cylindrical drum has a radius of 3 m and a height of 2 m. What is the area of the lid and the volume of the cylinder in m^3?

   Given: R =3 m
   H = 2 m

   Find: A and V

   Solution: $A = \pi R^2$
   $= (3.14)(3\ m)^2$
   $= (3.14)(3\ m)(3\ m)$
   $= 28.3\ m^2$
   V = A x H
   $= (28.26\ m^2)(2\ m)$
   $= 56.5\ m^3$

2. a) An upright cylinder has a diameter of 0.5 m and a height of 2.6 m. What is the volume of the cylinder in m^3 and l?

   Given: D = 0.5 m
   H = 2.6 m

   Find: V in m^3 and L

   Solution $R = \frac{D}{2}$
   $= \frac{0.5m}{2}$
   $= 0.25$ m
   V = A x H
   $= \pi R^2 H$
   = (3.14)(0.25 m)(0.25 m)(2.6 m)
   $= 0.51\ m^3$
   m^3 x 1 000 = L
   V = 0.51 m^3 x 1 000 = 510 L

2. b) The cylinder from question 2a is filled with water to a depth of 1.8 m. How much water is in the cylinder?

Given: $D = 0.5$ m
$R = 0.25$ m (calculated above)
$H = 1.8$ m

Find: V in m^3 and L

Solution $V = \pi R^2 H$

$= (3.14)(0.25\text{ m})(0.25\text{ m})(1.8\text{ m})$

$= 0.35\text{ m}^3$

$m^3 \times 1\,000 = L$

$V = 0.35\text{ m}^3 \times 1\,000 = 350\text{ L}$

3. An upright cylindrical tank with a diameter of 3 m, was filled to a depth of 2.3 m in the morning and 1.5 m at the end of the day. Assuming there was no water added to the tank that day, how much water was taken out?

Given: $D = 3$ m
$H_1 = 2.3$ m
$H_2 = 1.5$ m

Find: V

Solution $R = \frac{D}{2}$

$= \frac{3m}{2}$

$= 1.5$ m

$H = H_1 - H_2$

$= 2.3\text{ m} - 1.5\text{ m}$

$= 0.8$ m

$V = \pi R^2 H$

$= (3.14)(1.5\text{ m})(1.5\text{ m})(0.8\text{ m})$

$= 5.65\text{ m}^3$

$m^3 \times 1\,000 = L$

$V = 5.65\text{ m}^3 \times 1{,}000 = 5{,}650\text{ L}$

4. A water treatment plant operator has a chlorine mixing tank with a diameter of 0.8 m, and a depth of 1.1 m. If there is 0.2 m of chlorine solution left in the tank, and the operator decides not to empty the tank before adding an addition 150 L of water, what will be the depth of liquid in the tank after adding the extra water?

Given: $D = 0.8$ m

$H_1 = 0.2$ m

$V = 150$ L

Find: H_2

Solution

$$R = \frac{D}{2}$$

$$= \frac{0.8m}{2}$$

$$=0.4 \text{ m}$$

The basic equation is: $V = \pi R^2 H$

Rearranging for H, where H is the extra height of the water added

$$H = \frac{V}{\pi R^2}$$

Converting V to get consistent units,

L x 0.001 = m^3

V = 150 l x 0.001 = 0.15 m^3

$$H = \frac{0.15m^3}{(3.14)(0.4m)(0.4m)}$$

$$= 0.3 \text{ m}$$

The total depth of water in the tank will be equal to the initial depth of water plus the height of the water added.

$H_2 = H_1 + H$

$= 0.2 \text{ m} + 0.3 \text{ m}$

$= 0.5 \text{ m}$

Questions 5: Pipes

Questions relating to pipes are similar to those relating to cylindrical tanks. The basic equation is the same, except that in this case H, refers to the length of the pipe.

5. How many litres of water can be held in 15 m of 200 mm pipe?

Given: D =200 mm
H =15 m

Find: V in L

Solution First we have to convert to consistent units, in this case, m:

mm x 0.001 = m

D = 200 mm x 0.001 = 0.2 m

$$R = \frac{D}{2}$$

$$= \frac{0.2m}{2}$$

=0.1 m

V = A x H

= π R^2 H

= (3.14)(0.1 m)(0.1 m)(15 m)

= 0.47 m^3

m^3 x 1 000 = L

V = 0.47 m^3 x 1 000 = 470 L

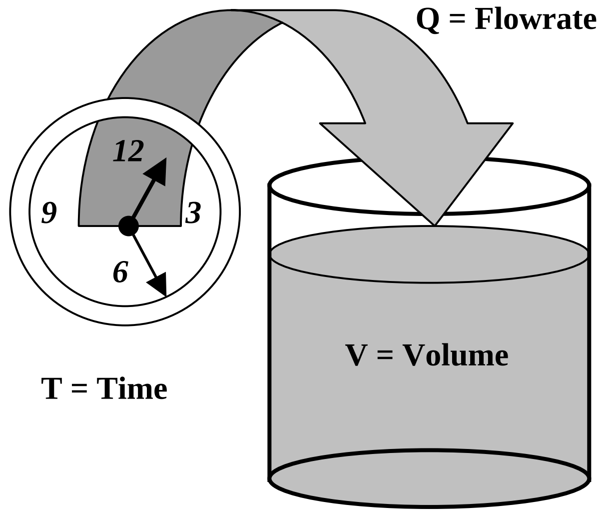
FLOW RATE

Definition:

The volume of liquid passing through a specific point over a given period of time.

For example, the volume of water coming out the end of a tap in one minute or the volume of water passing through a pump in a day or an hour, or the volume of water pumped through a water treatment plant in a year, a month or a week.

$$Q = \frac{V}{T}$$



Units of Measurement:

Metric (SI):

- L/s = litres per second
- L/min = litres per minute
- L/hr = litres per hour
- m^3/s = meters cubed per second
- m^3/day = meters cubed per day
- ML/day = million litres per day
- ML/yr = million litres per year

Imperial:

- ft^3/s = cubic feet per second
- GPM (Imp.) = gallons (Imperial) per minute
- US GPM= gallons (US) per minute
- ig/min = gallons (Imperial) per minute
- ig/d = gallons (Imperial) per day

Conversion:

Metric (SI):

1 L/min = 60 L/s
- L/min x 60 = L/s
- L/s x 0.0166 = L/min

1 L/day = 24 L/hr
- L/day x 24 = L/hr
- L/hr x 0.0416 7 = L/day

1 m^3/day = 1 000 L/day
- m^3/day x 10 000 = L/day
- L/day x 0.000 1 = m^3/day

Metric to Imperial (and back):

1 gallon (Imperial)/min = 4.546 L/min
- L/min x 0.219 9 = gallons (Imp.) per minute
- gallons (Imp.) per minute x 4.546 = L/min

1 gallon per day = 4.545 9 L/d
- L/d x 0.219 975 = gallons (Imp.) per day
- gallons (Imp.) per day x 4.545 9 = L/d

1 m^3/s = 35.315 cubic feet per second
- m^3/s x 35.315 = cubic feet per second
- cubic feet per second x 0.028 32 = m^3/s

Flowrate: Questions

1. It takes 3 min to fill An 12 L jar. What is the flowrate of the water?

2. Water is flowing at a rate of 4 L/min and it takes 3 min to fill a jar. How big is the jar?

3. At a flowrate of 4 L/min, how long does it take to fill a 12 L jar?

4. Water is flowing at a rate of 3.7 L/min and it takes 2.3 min to fill a jar. How big is the jar?

5. It takes 2.3 min to fill An 8.5 L jar. What is the flowrate of the water?

6. At a flowrate of 3.7 L/min, how long does it take to fill an 8.5 L jar?

7. A total of 3,105 m^3 passes through a flow meter in 3 days. What is the flowrate in m^3/day and l/day?

8. A flowmeter reads 134 567 030 l on Monday morning and 137 672 428 l on Thursday morning the same week. What is the average daily flow?

9. At a flow rate of 1,000 l/min, how many days would it take to fill a reservoir that 50 m long, 20 m wide and 2 m deep?

10. At a flow rate of 16 m^3/hr, how long would it take to fill an upright cylindrical tank with a diameter of 5 m and a height of 3.6 m?

Flowrate: Answers

1. It takes 3 min to fill An 12 l jar. What is the flowrate of the water?

Given: V = 12 l
T = 3 min

Find: Q

Solution: $Q = \frac{V}{T}$

$= \frac{12\ell}{3\,\text{min}}$

= 4 L/min

2. Water is flowing at a rate of 4 L/min and it takes 3 min to fill a jar. How big is the jar?

Given: Q = 4 L/min
T = 3 min

Find: V

Solution: The basic equation is: $Q = \frac{V}{T}$

Multiply both sides of the equation by T to isolate V,

V = Q x T

= 4 L/min x 3 min

= 12 L

3. At a flowrate of 4 L/min, how long does it take to fill a 12 L jar?

Given: Q = 4 L/min
V = 12 L

Find: T

Solution: Multiply both sides of the basic equation by $\frac{T}{Q}$ to isolate T,

$T = \frac{V}{Q}$

$= \frac{12L}{4L/\min}$

= 3 min

4. Water is flowing at a rate of 3.7 L/min and it takes 2.3 min to fill a jar. How big is the jar?

Given: Q = 3.7 L/min
T = 2.3 min

Find: V

Solution: V = Q x T
= (3.7 L/min)(2.3 min)
= 8.5 L

5. It takes 2.3 min to fill An 8.5 L jar. What is the flowrate of the water?

Given: V = 8.5 L
T = 2.3 min

Find: Q

Solution: $Q = \frac{V}{T}$

$= \frac{8.5L}{2.3 \min}$

= 3.7 l/min

6. At a flowrate of 3.7 L/min, how long does it take to fill an 8.5 L jar?

Given: Q = 3.7 l/min
V = 8.5 l

Find: T

Solution: $T = \frac{V}{Q}$

$= \frac{8.5L}{3.7L / \min}$

= 2.3 min

7. A total of 3,105 m^3 passes through a flow meter in 3 days. What is the flowrate in m^3/day and L/day?

Given: V =3,105 m^3
T = 3 days

Find: Q

Solution: $Q = \frac{V}{T}$

$$= \frac{3,105m^3}{3days}$$

= 1,035 m^3/day

m^3/day x 10 000 = L/day

Q = 1,035 m^3/day x 10 000 = 10,035,000 L/day

8. A flowmeter reads 134 567 030 L on Monday morning and 137 672 428 L on Thursday morning the same week. What is the average daily flow?

Given: V_1 =134,567,030 L

V_2 = 137,672,428 L

T_1 = Monday morning

T_2 = Thursday morning

Find: Q

Solution: $Q = \frac{V}{T}$

V = $V_2 - V_1$

= 137,672,428 L - 134,567,030 L

= 3,105,398 L

T = $T_2 - T_1$

= 3 days

$$Q = \frac{3,105,398L}{3days}$$

= 1,035,000 L/day

9. At a flow rate of 1,000 l/min, how many days would it take to fill a reservoir that 50 m long, 20 m wide and 2 m deep?

Given: Q = 1,000 l/min

L = 50 m

W = 20 m

D = 2 m

Find: T

Solution: $T = \frac{V}{Q}$

V = L x W x H

$= (50\text{ m})(20\text{ m})(2\text{ m})$

$= 2{,}000\text{ m}^3$

$\text{m}^3 \times 1\,000 = \text{L}$

$V = 2{,}000\text{ m}^3 \times 1\,000 = 2{,}000{,}000\text{ L}$

$$T = \frac{2{,}000{,}000L}{1{,}000L/\min}$$

$= 2{,}000\text{ min}$

1 day = 24 hrs = 1 440 min

$$\min \times \frac{1day}{1{,}440\min} = \text{days}$$

$$T = 2{,}000\text{ min} \times \frac{1day}{1{,}440\min} = 1.4\text{ days}$$

10. At a flow rate of 16 m^3/hr, how long would it take to fill an upright cylindrical tank with a diameter of 5 m and a height of 3.6 m?

Given: $Q = 16\text{ m}^3/\text{hr}$

$D = 5\text{ m}$

$H = 3.6\text{ m}$

Find: T

Solution: $T = \dfrac{V}{Q}$

$V = \pi R^2 H$

$R = \dfrac{D}{2}$

$= \dfrac{5m}{2}$

$= 2.5\text{ m}$

$V = (3.14)(2.5\text{ m})(2.5\text{ m})(3.6\text{ m})$

$= 70.65\text{ m}^3$

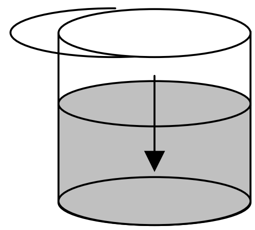
$$= \frac{70.65m^3}{16m^3/hr}$$

$= 4.4\text{ hrs}$

PRESSURE, FORCE AND HEAD

Definitions:

__Force__ **–** The push that is exerted by water, on the surface that is confining or containing it. Force can be expressed in pounds, tons, grams or kilograms. As an example, a certain volume of water may be exerting 22 pounds of force on the bottom of the bucket that is containing it.



__Pressure__ **–** The amount of force per unit of area. Pressure is often measured in pounds per square inch (psi). It may also be expressed in kilopascals (kPa).

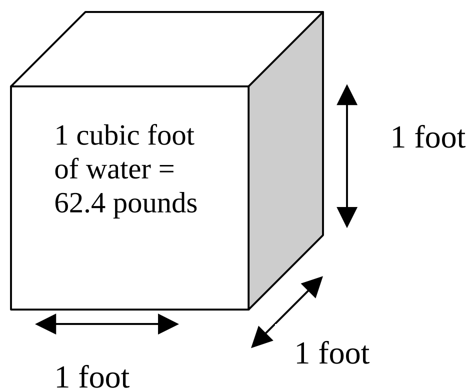
1 psi = 6.89 kPa

It helps to remember that one cubic foot (1 foot^3) of water always weighs 62.4 pounds. A cubic foot of water also contains approximately 7.5 gallons (34L).

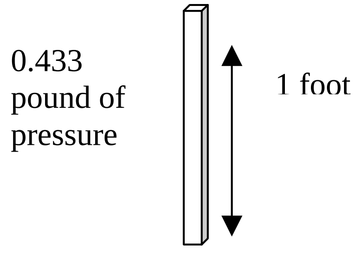
To calculate the pressure of a cubic foot of water,
we need to know that the area the force is
being exerted upon is 12 X 12 inches (144 square inches).

Therefore, 62.4 pounds per 144 inches can be converted to a pressure per square inch (psi).

We can divide 62.4 by 144, and we will find that
The pressure exerted is 0.433 psi.



This means that a column of water that is one square inch and one foot tall exerts a pressure of 0.433 pounds:



<u>Head</u> - Head refers to the height of a column of water above a point of reference (often expressed as feet or metres).

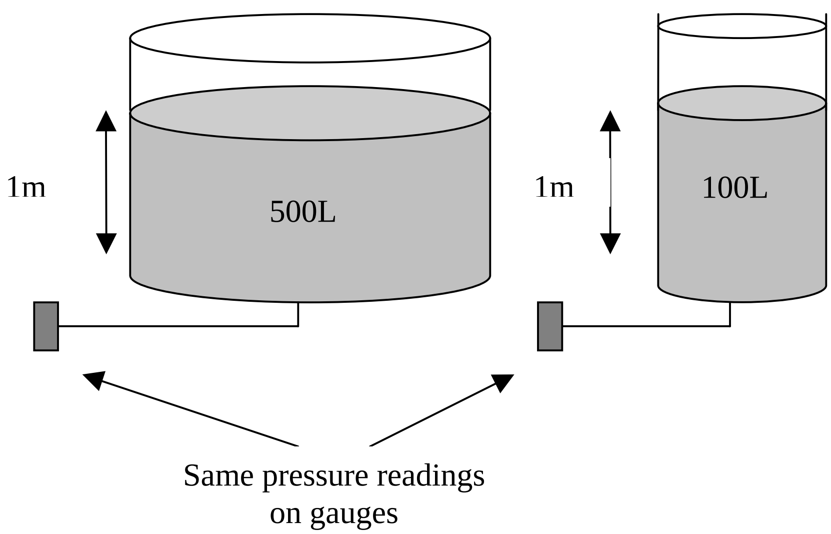
> **Remember:**
> **For every pound per square inch (psi) of pressure, there is 2.31 feet of head – this is standard.**

Head is calculated by the following formula:

Head (in feet) = psi x 2.31

Pressure head refers to the amount of energy in water that is a result of pressure. Pressure Head refers to the height above the top of an open-ended pipe that the water will rise to, due to pressure. If there is more pressure put into the system, there is more pressure head – and the water would rise further above the open end of the pipe due to the increase.

It is important to understand that *pressure head* depends on the elevation of the column of water, and is independent of how large the tank actually is or what volume it holds. For example, if there are two tanks that have different volumes but are both filled to the same level above the bottom of the tank, they will both cause their pressure gauges to have the same reading.

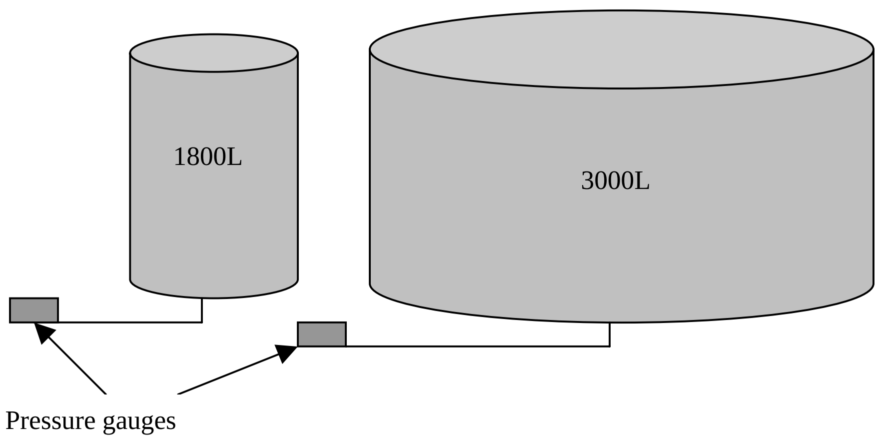


Pressure, Force and Head: Questions

1. Using the conversion factor that you learned for pressure, convert 2.8 psi to kPa.

2. Calculate the head for a volume of water which exerts 12 psi of pressure.

3. If there are two tanks (one holds 3000L and one holds 1800L) that are the *same height*, but have *different diameters*, which will have a greater reading on its' pressure gauge if they are both full to the top? (see figure below)



Pressure, Force and Head: Answers

1. Using the conversion factor that you learned for pressure, convert 2.8 psi to kPa.

1 psi = 6.89 kPa

2.8 psi x 6.89 = 19.29 kPa

2.8 psi is equal to 19.29 kPa.

2. Calculate the head for a volume of water which exerts 12 psi of pressure.

Head (in feet) = psi x 2.31

Head = 12 x 2.31

Head = 27.72 feet

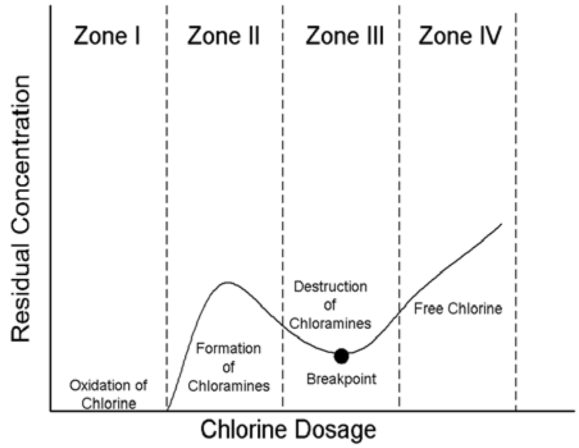
A volume of water that exerts 12 psi of pressure will have 27.72 feet of head.

3. If there are two tanks (one holds 3000L and one holds 1800L) that are the *same height*, but have *different diameters*, which will have a greater pressure head (or reading on its' pressure gauge) if they are both full to the top?

- Both tanks will have exactly the same reading on their pressure gauge.

- Pressure head depends on how high the column of water is, and is independent of how large the tank actually is or what volume it holds.

Chlorine Dosage /Feed Rate



Definition:

Dosage: Amount of chemical applied to water expressed in parts per million (PPM) or milligrams per litre (mg/L).

Feed Rate: amount of chlorine added to treat a volume of water over a course of time.

Residual: amount of active chlorine in water that can be free available chlorine which is uncombined and available to react water properties, combined which is chlorine combined primarily with organics and not very reactive and total residual is the amount of available chlorine and combined chlorine.

Note: It is most important for a plant operator to know how to calculate the dosages of the various chemicals used in water treatment. It is important to be accurate when calculating dosages, as too little chemical may be ineffective and too much may waste money. Exact dosage must be determined through calculation for the purpose of efficient operation and economy.

$$\text{Chlorine Dose} = \frac{C \times 1000}{W}$$

C_D= Chlorine dose

C = Chlorine feed rate in kg/day

W = volume of water to be treated in m^3

$$\text{Feed rate} = \frac{W \times CD}{1000}$$

Residual: Dosage - Demand = Residual

Residual + Demand = Dosage

Example:	Dosage	3.7 mg/L
Subtract	Demand	3.0 mg/L
Equals	Residual	0.7 mg/L

Dosage: Calculations

1. The chlorine dosage of an effluent is 15mg/L. How many kilograms of chlorine will be required to dose a flow of 8 500 m^3/ d?
2. A chlorinator is set to feed a 94.8 kg/d of chlorine. If the average daily flow through the plant is 7 900 m^3/ d, what is the DAILY AVERAGE CHLORINE DOSAGE IN mg/L?
3. Chlorine dosage is 2.5 mg/L and the Flow rate is 87000 m^3/d, what is the feed rate?
4. What should be the chlorine dose of water that has a chlorine demand of 2.3 mg/L if a residual of 0.5 mg/L is desired?

HYPOCHLORINATION

Hypochlorination is the application of hypochlorite (a compound of chlorine and another chemical), usually in the form of a solution, for disinfection purposes.

5. The treated product at a water treatment plant requires a chlorine dosage of 98 kg/d for disinfection purposes. If we are using a solution of hypochlorite containing 60% available chlorine, how many kg/d hypochlorite will be required?

6. A hypochlorite solution contains 5% available chlorine. If 4 kg of available chlorine are needed to disinfect a water main, how much 5% solution would be required?

BATCH CHLORINATION

Batch chlorination is a method of disinfection where by the chlorine is added to a water tank manually. This type of chlorination is used when mechanical methods for chlorination are not available most commonly used to disinfect water in a water truck.

$$C_1V_1 = C_2V_2 \qquad\qquad V_1 = \frac{C_2V_2}{C_1}$$

C_1= Concentration of liquid chlorine solution
V_1= Volume of liquid chlorine solution needed
C_2= Chlorine Dose in Truck
V_2= Volume of water Truck

7. How much liquid bleach (4.5 % chlorine solution) should you add to your 3800L water truck to achieve a chlorine concentration of 0.5 mg/L?

8. What is the concentration of the chlorine solution that was added to 5000L water truck with a chlorine concentration of 1.2 mg/L?

Dosage: Answers

1. In this question, it will be necessary to utilize your knowledge of the metric system.

   $1 mg/L = 1 kg/1000 m^3$
   For every $1000 m^3$ water of flow, we will need to use 15 kg chlorine.

   $$\frac{15 kg\ Cl_2}{1000 m^3} \times 8500 m^3/d = 127.5 kg\ Cl_2/d$$

   Above we express 15 mg/L as $15 kg\ Cl_2 / 1000 m^3$ and multiplied it by the flow to obtain the answer expressed as $127.5 kg\ Cl_2/d$
2. We know that $1 mg/L = 1 kg/1000 m^3$

   We are told we use 94.8 kg chlorine for every $7900 m^3$ water.

   $$\frac{98.8 kg\ Cl_2/d}{7.9 \times 1000 m^3/d} = \frac{12 kg\ Cl_2}{1000 m^3} = 12 mg/L$$

3. $$\text{Feed Rate} = \frac{W \times \text{Dosage}}{1000} = \frac{87000 \times \text{Dose}}{1000} = 217.5 kg/day$$

4. We know that the chlorine dose is equal to the demand plus the residual.

   $$2.3 mg/L + 0.5 mg/L = 2.8 mg/L$$

5. We are told in the problem that 60% of the hypochlorite is available chlorine which is the portion of the solution capable of disinfecting. Solving the equation we have:

   $$\text{kg/d hypochlorite} = \frac{98 \text{ kg/d of chlorine needed}}{0.6 \text{ available chlorine in solution}}$$

   $$= 163.3 \text{ kg/d hypochlorite solution}$$

6. We are told 4 kg of chlorine will do the job of disinfection. By a 5% solution we mean that 5% by mass of the solution is to be made up of chlorine. So 100 kg of 5% hypochlorite solution will contain 5kg of chlorine.
   Using the formula for ratios $\frac{A}{B} = \frac{C}{D}$

   We substitute:

   $$\frac{5 \text{ kg chlorine}}{100 \text{ kg solution}} = \frac{4 \text{kg chlorine required}}{? \text{ kg solution required}}$$

   $$\text{Since } D = \frac{C \times B}{A} = \frac{4 kg \times 100 kg}{5 kg} = 80 \text{ kg solution}$$

7. $V_1 = \frac{C_2 V_2}{C_1}$ $V_1 = \frac{(0.5 mg/L)(3600 L)}{45 mg/L}$ $V_1 = 40 L$

8. $C_1 = \frac{C_2 V_2}{V_1}$ $C_1 = \frac{(0.02 mg/L)(5000 L)}{70 L}$ $C_1 = 4.9. mg/L$

End Suction Lift

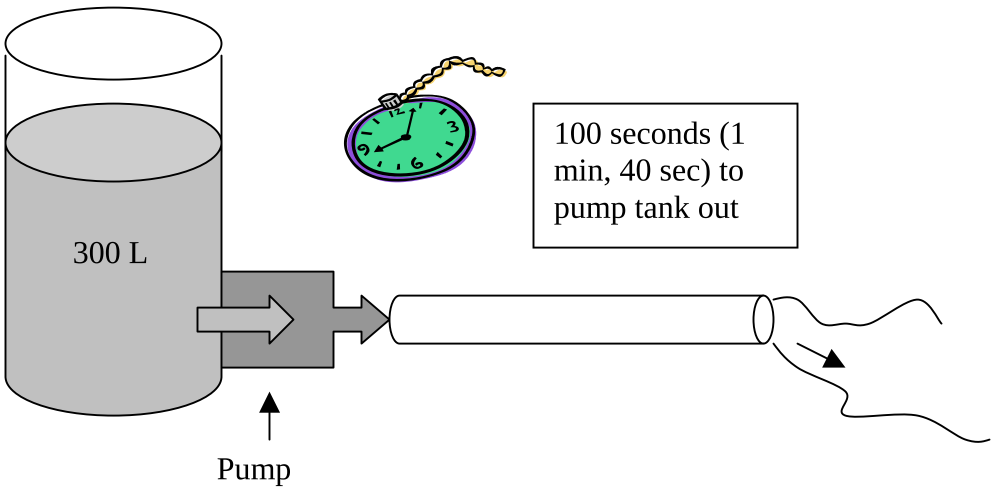
Definition:

PUMPING RATES

Definition:

The volume of wastewater that is moved by a pump over a certain amount of time.

As an example, a pump may have a pumping rate of 3L/second. It is important to understand that in many cases, you may need to convert from one set of units to another. You may be told that a pumping rate is 30L/second but will be asked to calculate how many m^3/day that equals.



Pumping rate is an expression of $\frac{\text{Volume}}{\text{Time}}$

If you know that a pump has a certain rate if discharge (pumping rate) then you can calculate things such as the amount of time it would take to fill or empty a tank or truck.

Pumping Rates: Questions

1. It takes 8 minutes to fill a tank that holds 480L of water. What is the pumping rate of the pump that is moving the water?

2. How many minutes will it take to fill a 4,800 liter truck if the pumping rate is 320 L/minute?

3. It takes an hour to fill a 3,000L tank. What is the pumping rate, expressed as L/second?

Pumping Rates: Answers

1. It takes 8 minutes to fill a tank that holds 480L of water. What is the pumping rate of the pump that is moving the water?

Given: Volume = 480L
Time = 8 minutes

Find: Rate (ratio between these two)

Solution: $Q = \frac{V}{T}$

$= \frac{480L}{8\ min}$

$= \frac{60L}{1\ min}$

Pumping rate is 60L/minute.

2. How many minutes will it take to fill a 4,800 liter truck if the pumping rate is 320 L/minute?

$Q = \frac{V}{T}$

$320L/minute = \frac{4800L}{X\ time}$

(X) 320L/minute = 4800L (now divide both sides by 320)

$X = \frac{4800L}{320L/minute}$

X = 15 minutes

It will take 15 minutes to fill the truck if the pumping rate is 320L/minute.

3. It takes an hour to fill a 3,000L tank. What is the pumping rate, expressed as L/second?

$$\text{Rate} = \frac{3000\text{L}}{60\text{ minutes}}$$

$$\text{Rate} = \frac{3000\text{L}}{3600\text{ seconds}}$$

$$\text{Rate} = 0.83\text{L/second}$$

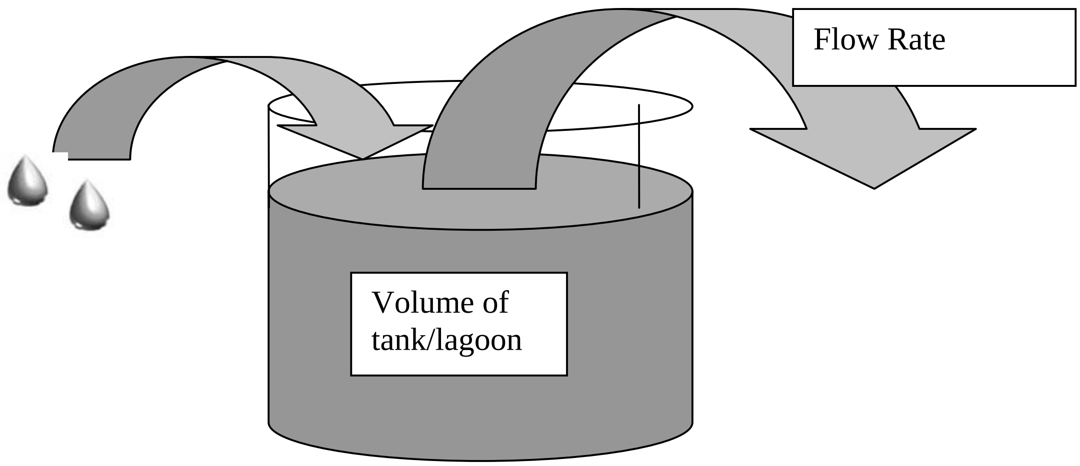
DETENTION TIME

Definition:

The theoretical amount of time that a particular volume of wastewater is held for treatment in a tank or lagoon.

$$\textbf{Detention time} = \frac{\mathbf{V}}{\mathbf{Q}}$$

Where Q is the flow rate
V is the volume of the tank or lagoon



For example, think of detention time as the amount of time (hours, days, months, or which ever unit applies) sewage sits in a tank or lagoon, before the water is released from it. The detention time depends on the *capacity* of the tank/lagoon (in litres, cubic metres, etc), and the *flow rate* out of the tank/lagoon.

Detention time can be expressed in many ways as well, depending on the units used. As an example, the detention time can vary from minutes to hours, weeks, months, or perhaps even years – depending on the volume and flow rate you are dealing with.

Detention Time: Questions

1. What is the detention time (in minutes) of a 400L tank that has an outflow rate of 0.5L/second?

2. Calculate the detention time (in months) for a lagoon that holds 2,000m^3 of wastewater, and that has a discharge rate of 10m^3/day (you can assume 30 days in an average month).

Detention Time: Answers

1. What is the detention time (in minutes) of a 400L tank that has an outflow rate of 0.5L/second?

$$\text{Detention time} = \frac{\text{Volume}}{\text{Flow rate}}$$

$$\text{Detention time} = \frac{400\text{L}}{0.5\text{L/sec}}$$

Detention time = 800 seconds

Detention time = 13 minutes, 20 seconds (approximately 13 minutes)

2. Calculate the detention time (in months) for a lagoon that holds 2,000m^3 of wastewater, and that has a discharge rate of 10m^3/day (assume 30 days in an average month).

$$\text{Detention time} = \frac{\text{Volume}}{\text{Flow rate}}$$

$$\text{Detention time} = \frac{2{,}000\text{m}^3}{10\text{m}^3\text{/day}}$$

Detention time = 200 days

Detention time = 6 months, 20 days (approximately 6 months)

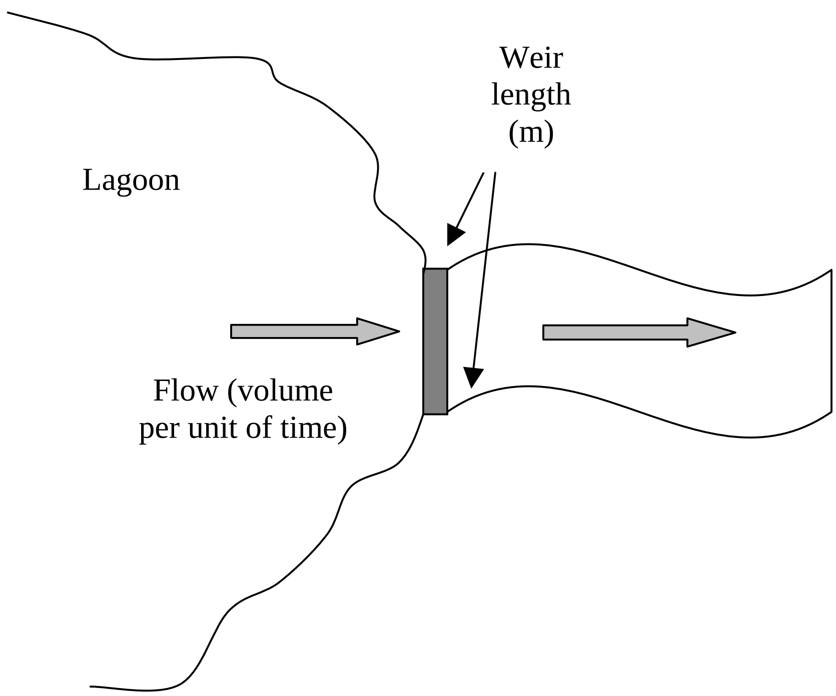
WEIR OVERFLOW RATE

Definition:

The volume of wastewater that flows over each metre of weir length, in a given amount of time.

Weir overflow rate is determined by measuring the length of the weir (outlet) and then calculating flowrate (volume per unit of time) of wastewater over the weir (outlet). It is important to be able to calculate this number. High weir overflow rates can affect the proper treatment of wastewater.

$$\textbf{Weir Overflow Rate} = \frac{\textbf{Flow (volume/time)}}{\textbf{Weir length (m)}}$$



Weir overflow rate is often expressed as liters per second flow over each meter of weir length. For example, a weir overflow rate may be 2.5 L/s/m. In slower systems where discharge is slow and takes place over a long amount of time, weir overflow rate could be expressed as L/day/m, or m^3/day/m, etc.

Weir Overflow Rate: Questions

1. Calculate the weir overflow rate (in liters per hour) for a system that has a weir 2.4m long, where the flow rate is 8.2L/minute.

2. If a weir is 32m long where the water flows out, calculate the weir overflow rate (in m^3/minute) if the flow is 110 L/s.

Weir Overflow Rate: Answers

1. Calculate the weir overflow rate (in liters per hour) for a system that has a weir 2.4m long, where the flow rate is 8.2L/minute.

$$\text{Weir Overflow Rate} = \frac{\text{Flow (volume/time)}}{\text{Weir length (m)}}$$

$$= \frac{8.2 \text{ L/minute}}{2.4 \text{ m}} \quad \text{(now multiply 8.2 by 60 to get L/hour)}$$

$$= \frac{492\text{L/hour}}{2.4 \text{ m}}$$

The weir overflow rate is 492 L/h/m.

2. If a weir is 32m long at the outflow, calculate the weir overflow rate (in m^3/minute) if the flow is 110 L/s.

$$\text{Weir Overflow Rate} = \frac{\text{Flow (volume/time)}}{\text{Weir length (m)}}$$

$$= \frac{110\text{L/second}}{32 \text{ m}} \quad \text{(now multiply 110 by 60 to get L/minute)}$$

$$= \frac{6600\text{L/minute}}{32 \text{ m}} \quad \text{(now divide 6600 by 1000 to get m}^3\text{/minute)}$$

$$= \frac{6.6 \text{ m}^3\text{/minute}}{32 \text{ m}} \quad \text{(now divide 6.6 by 32 to get rate per 1m)}$$

$$= 0.21 \text{ m}^3\text{/minute/m}$$

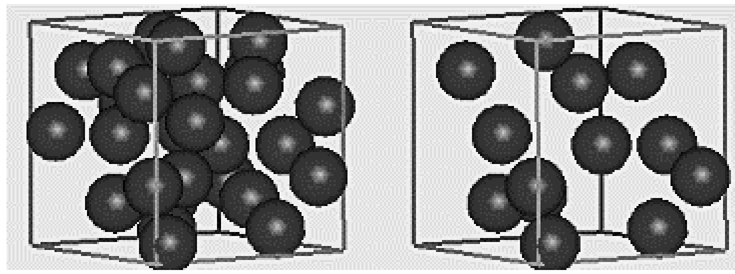
The weir overflow rate is 0.21 m^3/min/m.

DENSITY

Definition:

The amount of mass that is contained in a particular volume.

For example, the mass of waste compacted into a 300L trench, the mass of water in a 1 L jug, or the mass of garbage contained in a 3000 L collection truck.



**Density = ** $= \dfrac{\mathbf{m}}{\mathbf{V}}$

Where m is the mass
V is the volume

In the figure above, the box on the left has a higher density because there is more mass in the same amount of space (volume) than the box on the right.

Density can be expressed in many ways, using many different units. As an example, compacted solid waste is often expressed as kilograms per cubic meter (kg/m^3), tons per cubic meter ($tons/m^3$), etc. You can convert back and forth using the conversion factors for volume and mass listed on the provided tables.

Density: Questions

1. Calculate the density of 40 tons of solid waste that has been compacted to a volume of 80 cubic meters.

2. What is the density (in kg/m^3) of 220kg of solid waste, if it fills an area that is 55cm wide, 80cm long and 60cm deep?

Density: Answers

1. Calculate the density of 40 tons of solid waste that has been compacted to a volume of 80 cubic meters.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density} = \frac{40 \text{ tons}}{80\text{m}^3}$$

$$\text{Density} = 0.5 \text{ tons/m}^3$$

2. What is the density (in kg/m^3) of 220kg of solid waste, if it fills an area that is 55cm wide, 80cm long and 60cm deep?

Part A: Volume

Volume = L x W x H
Volume = 0.80m x 0.55m x 0.60m
Volume = $0.264m^3$

Part B: Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density} = \frac{220\text{kg}}{0.264\text{m}^3}$$ (now divide 220 by 0.264 to get mass per $1m^3$)

$$\text{Density} = 833.3\text{kg/m}^3$$

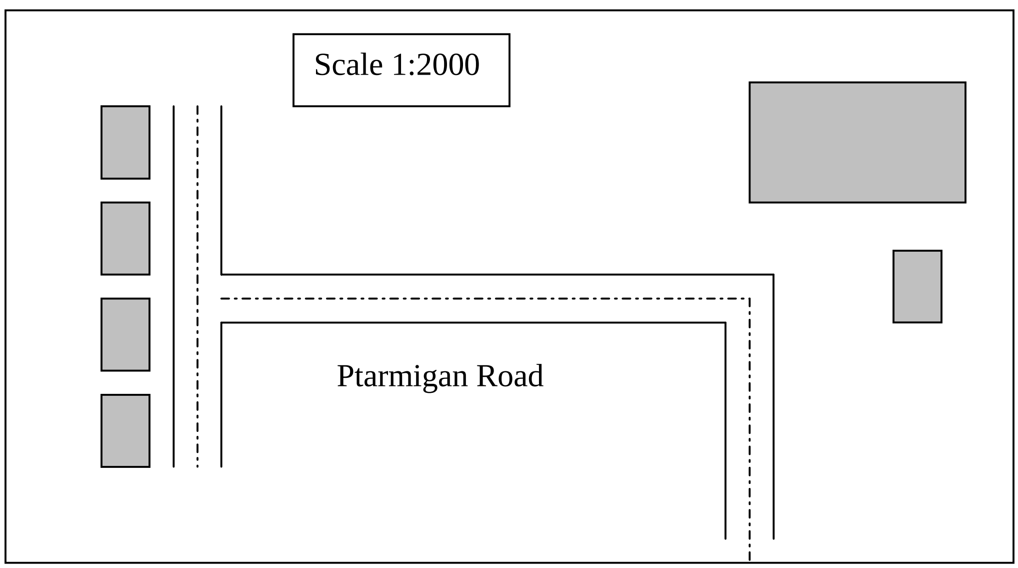
PLAN SCALE

Definition:

A ratio between the dimension (length, height, distance, etc) of a drawing of an object or distance on a drawing or map, to the dimension of the real-life object or distance.

Scale is expressed as a ratio. Take the following example: the scale is 1:20,000. This means that for every 1 unit of measurement on the paper, there are 20,000 of those same units on the real-life object.

The units of measurement do not have to be the same units; for example, one centimeter on a map could equal 3.5 kilometers in reality. As long as you know the conversions between the units, you can calculate. You could express the ratio in the same units, but to tell someone that the building is 15,000 cm long does not provide user-friendly information! Instead, it helps to be able to calculate that the building is 15m long.



On the above drawing, if you were to find that Ptarmigan Road measured five cm (5cm), you could then calculate the length of the road is it is in the town:
1: 2,000 scale
1cm = 2000 cm
5 cm = 10,000 cm
10,000 cm = 100 m

So, Ptarmigan Road is 100 m long in real life.

Plan Scale: Questions

1. How long (in meters) is a building in real life, if it measures 2 cm on a plan or map? (The plan scale is 1:3000)

2. What is the *area* of a fenced area, in square meters, if the plan scale is 1:5000 and the area on the paper plan is 3.5 cm long and 1.8 cm wide?

Plan Scale: Answers

1. How long (in meters) is a building in real life, if it measures 2 cm on a plan or map? (The plan scale is 1:3000)

1:3000, so 1cm on the map is 3000cm in the real world
2cm x 3000cm = 6,000cm in the real world
6,000cm = 60m

The building is 60m long.

2. What is the *area* of a fenced area, in square meters, if the plan scale is 1:5000 and the area on the paper plan is 3.5 cm long and 1.8 cm wide?

Part A: Length
1:5000, so 1cm on the map is 5000cm in the real world
3.5cm x 5000cm = 17,500cm in the real world
17,500cm = 175m

Part B: Width
1:5000, so 1cm on the map is 5000cm in the real world
1.8cm x 5000cm = 9,000cm in the real world
9,000cm = 90m

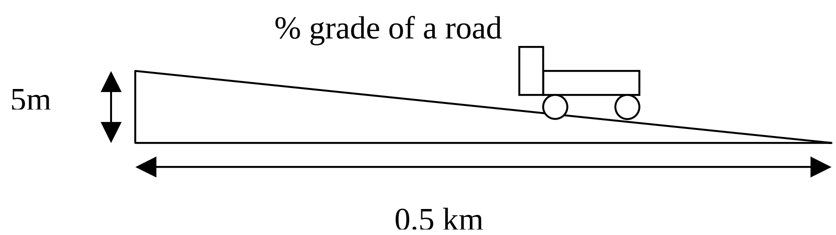
Area = Length x Width, so 175m x 90m

The area of the fenced site is 15,750m^2.

GRADE

Definition:

A measure of the change in elevation between two points, compared to the horizontal (map) distance traveled (not the actual distance traveled).



Grade is usually expressed as a percentage:

$$\textbf{Grade} = \frac{\textbf{Rise}}{\textbf{Run}} \textbf{ x 100 (to convert to \%)}$$

To calculate the percent grade, write a ratio to describe the relationship between the “rise” (height) and the “run” (length). Then divide the rise by the run, and change the decimal to its percent form.

In the example above, grade of the road is equal to $\frac{5m}{0.5km}$ which is $\frac{5m}{500m}$ or 1% grade.

The term *slope* expresses the steepness of the grade, and can also be written as a ratio. You express the ratio as Run : Rise. For example, a 10% slope is the same as a rise of 10m over a run of 100m. The slope ratio for this would be 100 : 10, which can then be reduced to 10 :1.

Grade: Questions

1. What is the percent grade for a road that rises 0.8m over a length of 250m?

2. With a 4% grade, what would the elevation gain be over 600m?

3. If the working face at a landfill is 15m (from toe to top) and the lift height (elevation gain) is 3m, what is the slope ratio?

Grade: Answers

1. What is the percent grade for a road that rises 0.8m over a length of 250m?

$$\% \text{ Grade} = \frac{\text{Rise}}{\text{Run}} \times 100$$

$$= \frac{3.75\text{m}}{250\text{m}} \times 100$$

$$= 1.5\% \text{ Grade}$$

2. With a 4% grade, what would the elevation gain be over 600m?

$$\% \text{ Grade} = \frac{\text{Rise}}{\text{Run}} \times 100$$

$$4 = \frac{\text{Rise}}{600\text{m}} \times 100 \quad \text{(Now, multiply both sides by 600)}$$

$$2400 = \text{Rise} \times 100 \quad \text{(Now, divide both sides by 100)}$$

$$24 = \text{Rise}$$

The rise (elevation gain) is 24m over 600m for a 4% grade.

3. If the working face at a landfill is 15m (from toe to top) and the lift height (elevation gain) is 3m, what is the slope ratio?

Slope ratio = Run : Rise

Slope ratio = 15m : 3m (now reduce this to a ratio to 1 by dividing each by 3)

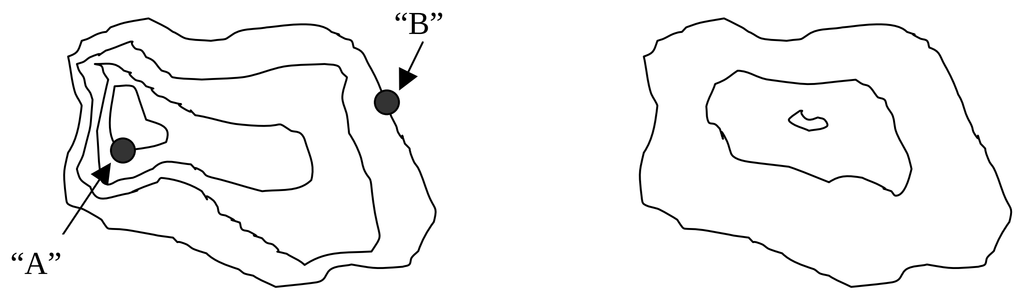
Slope ratio = 5 : 1

CONTOUR LINES

Definition:

Lines drawn on a map connecting points that are of equal elevation on the land.

If you walk along a contour line, you neither gain nor lose elevation. Contour lines are useful because they allow us to show the shape of the land surface (topography) on a map. The closer together contour lines are, the faster the elevation changes and the steeper the slope of land is.



What the contour lines look like

What the height of the land
looks like for each example

The distance between the contour lines is a representation of the *vertical* distance between the lines. The vertical distance between the lines is the "contour interval." If you know that the contour interval is, for example, 10 meters, then you can calculate the difference in elevation between two points on a map.

In the diagram above, if the contour interval were 10 meters, then the difference in elevation between "A" and "B" would be 3 lines, or 30 meters.

Contour Lines: Questions

1. For a map that has a contour interval of 10m, what is the elevation of a point on a hill that is 4 contour lines in from the line that represents an elevation of 440m?

2. Calculate the % grade for 4 contour lines on a map, if the contour interval is 5m and the distance between the two outside contour lines of 6 cm. The plan scale is 1:3000.

Contour Lines: Answers

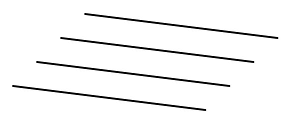
1. For a map that has a contour interval of 10m, what is the elevation of a point on a hill that is 4 contour lines in from the line that represents the top of the hill with an elevation of 440m?

Each contour line is 10m, so there are 40m between the top of the hill and the point in question.

440m – 40m = 400m.

The elevation of the point is 400m.

2. Calculate the % grade for 4 contour lines on a map, if the contour interval is 5m and the distance between the two outside contour lines of 6 cm. The plan scale is 1:3000.



$$\% \text{ Grade} = \frac{\text{Rise}}{\text{Run}} \times 100$$

Rise is calculated by multiplying the interval of 5m by 3 (there are 4 lines, but there are three intervals!) so you get a rise in elevation of 15m.

Run is calculated by knowing that 1:3000 gives you a real-world distance of 6 x 3000cm. This works out to be 18,000cm or 180m.

$$\% \text{ Grade} = \frac{\text{Rise}}{\text{Run}} \times 100$$

$$\% \text{ Grade} = \frac{15\text{m}}{180\text{m}} \times 100$$

% Grade between the four contour lines is 8.33%.

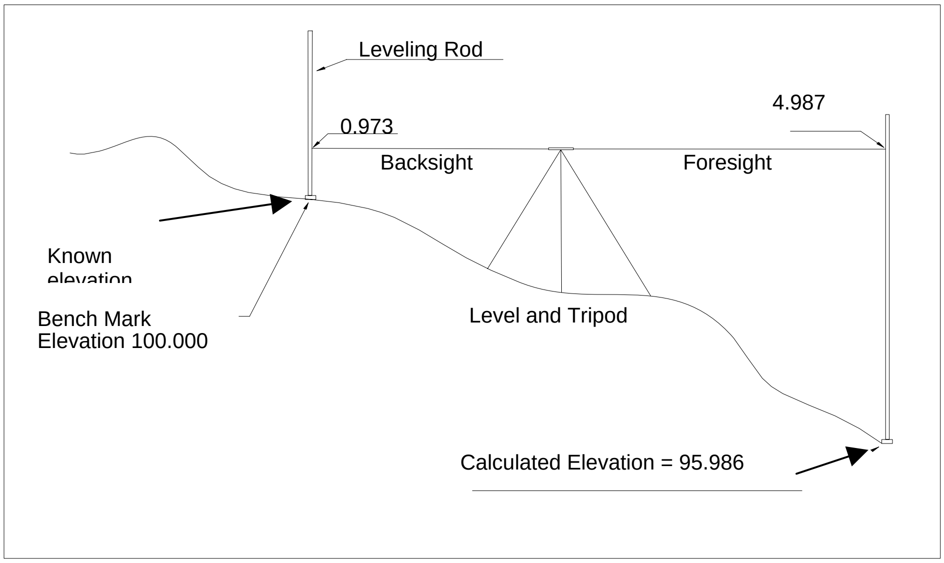
BACKSIGHT AND FORESIGHT

Definitions:

Backsight: In surveying, this is the reading on the rod when held on a known or assumed elevation. Backsights are used to establish the height of instrument.

Forsesight: In surveying, the reading on the rod when held at a location where the elevation needs to be determined. Foresights are used to establish the elevation at another location.

A *Benchmark* is an object or marker with a known elevation that other elevations can be compared to. A *Turning Point* (TP) is a fixed object that you use when determining the elevation of other points.



For finding the elevation of a point, you need to use two simple equations:

Height of Instrument = Known Elevation + Backsight

TP Elevation = Height of Instrument – Foresight

For the example in the figure above:

Height of instrument = Known Elevation + Backsight
= 100.000 + 0.973
= 100.973

TP Elevation = Height of Instrument – Foresight
= 100.973 – 4.987
= 95.986

Forsesight and Backsight: Questions

1. What is the elevation of a point, if the benchmark elevation is 842.477 metres, the backsight is 1.202 meters, and the foresight is 2.145 meters?

2. What is the elevation of a point, if the benchmark elevation is 1204.945 metres, the backsight is 0.112 meters, and the foresight is 4.012 meters?

Forsesight and Backsight: Answers

1. What is the elevation of a point, if the benchmark elevation is 842.477 metres, the backsight is 1.202 meters, and the foresight is 2.145 meters?

Height of instrument	= Known Elevation + Backsight
	= 842.477 + 1.202
	= 843.679
TP Elevation	= Height of Instrument – Foresight
	= 843.679 – 2.145
	= 841.534

The elevation of the point is 841.534 meters.

2. What is the elevation of a point, if the benchmark elevation is 1204.945 metres, the backsight is 0.112 meters, and the foresight is 4.012 meters?

Height of instrument	= Known Elevation + Backsight
	= 1204.945 + 0.112
	= 1205.057
TP Elevation	= Height of Instrument – Foresight
	= 1205.057 – 4.012
	= 1201.045

The elevation of the point is 1201.045 meters.

Conversions: Practice Questions

1. 64mL = _______________ cm^3
2. 4560 mL= _______________L
3. 52300L= ______________ kL
4. 52300 L= _______________m^3
5. 346mL = ______________ L
6. 0.027 L =_______________mL
7. 940000 cm^3=___________L
8. 0.00022 m^3_____________ L
9. 25 m^3= _______________L
10. 62.5 kg (water)=_________L
11. 2 kL= _______________ g
12. 28.4 cm^3= _______________kg