MATHS

HSC EXTENSION 1 MATHEMATICS

BINOMIAL THEOREM

CONTENTS

BINOMIAL THEOREM

THE PASCAL TRIANGLE

When the coefficients in the expansions of $(1+x)^n$ are arranged in a table, the result is known as the Pascal triangle

Draw Pascal's Triangle:

Basic properties of the Pascal triangle:

1. Each row starts and ends with 1
2. Each row is reversible
3. The Sum of each row is 2^n
4. [The addition property] Every number in the triangle, apart from the 1s, is the sum of the number directly above, and the number above and to the left

Exercise 1

1. Using Pascal's triangle of binomial coefficients, give the expansions of each of the following:
   - **a.** $(1+x)^6$
   - **b.** $(1-x)^9$
   - **c.** $(1+2y)^4$
   - **d.** $\left(1-\frac{1}{x}\right)^8$
   - **e.** $\left(1+\frac{2}{x}\right)^5$
2. Find the value of k if in the expansion of $(1+2kx)^6$
   - **a.** The terms x^4 and x^3 have coefficients in the ratio $2:3$
   - **b.** The terms in x^2, x^3, x^4 have coefficients in an arithmetic progression

<u>Further work with the Pascal Triangle</u>

- The terms of $(x+y)^n$: The expansion of $(x+y)^n$ has $n+\ 1$ terms, and in each term the indices of x and y are whole numbers adding to n
- That is, the expression $(x+y)^n$ is homogenous of degree n in x and y together, and so also is its expansion
- The definition of ${}^nC_r$ Define the number ${}^nC_r$ to be the coefficient of x^r in the expansion $(1+x)^n$
- The symbol is usually read as 'n choose r' and the notations ${}^nC_r$ and $\binom{n}{r}$ are both used for these coefficients
- The expansion of (1+ x)n= ${}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \cdots + {}^nC_n\, x^n$
- There are n+ 1 terms, and the general term of the expansion is term in $x^r = {}^nC_r x^r$
- Alternatively, using sigma notation, the expansion can be written as

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

Basic properties of the Pascal's triangle

1. Each row starts and ends with 1, i.e.:

$$\binom{n}{0} = \binom{n}{n} = 1 \text{ for all cardinals } n$$

2. Each is reversible

$$\binom{n}{r} = \binom{n}{n-r} \text{ for all cardinals } n \text{ and } r \text{ with } r \leq n$$

3. The sum of each row is 2^n

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$$

Exercise 2

1. Use Pascal's triangle to expand each of the following

- **a.** $(x+y)^4$
- **b.** $(r-s)^6$
- **c.** $(a-b)^9$
- **d.** $(3x+2y)^4$
- **e.** $\left(\frac{1}{2}r + \frac{1}{3}s\right)^5$
- **f.** $\left(x + \frac{1}{x}\right)^6$

2. Use Pascal's triangle to expand each of the following:

- **a.** $(1-3x^2)^3$
- **b.** $\left(\sqrt{x} + \sqrt{y}\right)^7$
- **c.** $\left(\frac{2}{x} + 3x^2\right)^5$
- **d.** $(x^2+2y^3)^6$

3. Simplify the following without expanding the brackets

- **a.** $y^5 + 5y^4(x-y) + 10y^3(x-y)^2 + 10y^2(x-y)^3 + 5y(x-y)^4 + (x-y)^5$
- **b.** $x^3 + 3x^2(2y-x) + 3x(2y-x)^2 + (2y-x)^3$

4. Find the coefficient of:

- **a.** x^3 in $(2-5x)(x^2-3)^4$
- **b.** x^5 in $(x^2-3x+11)(4+x^3)^3$
- **c.** x^0 in $(3-2x)^2\left(x+\frac{2}{x}\right)^5$

5.

- **a.** Show that $\left(3+\sqrt{5}\right)^6 + \left(3-\sqrt{5}\right)^6 = 20608$
- **b.** Show that $\left(2+\sqrt{7}\right)^4 + \left(2-\sqrt{7}\right)^4$ is rational

6. Expand $(x+2y)^5$ and hence evaluate:

- **a.** $(1.02)^5$ correct to five decimal places
- **b.** $(0.98)^5$ correct to five decimal places
- **c.** $(2.2)^5$ correct to 4 significant figures