MATHS

HSC MATHEMATICS

PHYSICAL APPLICATION OF CALCULUS

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PHYSICAL APPLICATIONS OF CALCULUS

RATES OF CHANGE

A rate of change is the instantaneous rate at which a quantity Q is changing with respect to time. Mathematically, this is written as $\frac{dQ}{dt}$ (although you also might sometimes see $\dot{Q}$). Graphically, the rate of change is the gradient of a graph of Q.

Rates of change questions will usually require you to either differentiate a given function to find the rate, or integrate a given rate to find the original function. Some questions may give you a graph instead of a function.

DIFFERENTIATING TO FIND THE RATE

These sorts of questions will provide you with an equation where a quantity Q is given as a function of time *t*. Differentiating this function will give you a formula for the rate of change of Q.

Example:
Assume that at any given time where t > 0, the volume of a pool in litres is given by the equation $V = 3t^2$ *where t is in hours. At what rate is the volume changing after 3 hours?*

STEP 1: Differentiate
STEP 2: Substitute
STEP 3: Check units

INTEGRATING TO FIND THE FUNCTION

Sometimes, a question will instead give you the rate at which a quantity Q is changing over time t. Integrating this equation will give you the original function for Q. The constant c of integration can be found subbing in the value of Q at a particular time t (this will be provided in the question).

Worked example:

A valve is slowly opened in a pipeline such that the volume flow rate $\frac{dV}{dt} = 1.3t\ \mathrm{m^3s^{-1}}$. *Find an expression for the volume then calculate the total volume of water that flows through the valve in the first 10 seconds.*

STEP 1: Integrate
STEP 2: Find c
STEP 3: Substitute
STEP 4: Check units

QUESTIONS WITH A DIAGRAM OR A GRAPH INSTEAD OF AN EQUATION

In some problems, you will be given a graph of either Q against t or $\frac{dQ}{dt}$ against t. These problems will require careful attention to zeroes, turning points and inflexion points. You will also often be asked to draw another rough sketch (usually Q v t if given $\frac{dQ}{dt}$ v t and vice versa)

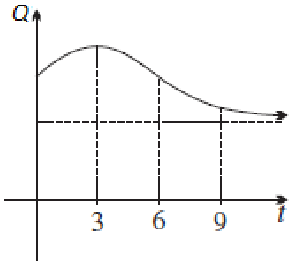
Graph of Q against t:
Important features:

- Zeroes indicate instants when $Q = 0$
- Gradient indicates the rate of change $\frac{dQ}{dt}$
- Turning points (when $\frac{dQ}{dt} = 0$) indicate instants when the quantity is not changing/maxes or mins
- Inflexion points ($\frac{d^2Q}{dt^2} = 0$) indicate instants when the rate of change is a maximum or a minimum

Example:

The graph to the right shows the temperature T of a patient suffering form Sydmond's syndrome at time t hours after her admission to hospital at midnight.

a) When did her temperature reach its maximum?
b) When was her temperature increasing most rapidly, and when was it decreasing most rapidly?
c) What happened to her temperature eventually?
d) Sketch the graph of the rate $\frac{dT}{dt}$ at which the temperature is changing.



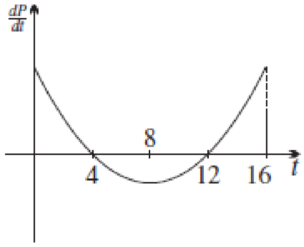
Graph of $\frac{dQ}{dt}$ against t:

Important features:

- Zeroes indicate instants when the quantity is not changing
- When the graph is above the x-axis, the quantity is increasing. When the graph is below the x-axis, the quantity is decreasing
- Gradient indicates the rate at which the rate of change is changing
- Turning points indicate instants when the rate of change stays the same
- The area under the curve can be used to find Q (because integrating $\frac{dQ}{dt}$ gives Q)

Example:

Frog numbers were increasing in the Ranavilla district, but during a long drought, the rate of increase fell and actually became negative for a few years. The rate $\frac{dP}{dt}$ of population growth of the frogs has been graphed as a function of the time t years after careful observations began.



a) When was the frog population neither increasing nor decreasing?

b) When was the frog population decreasing and when was it increasing?

c) When was the frog population decreasing most rapidly?

d) When, during the first 12 years, was the frog population at a maximum?

e) When, during the years $4 \leq t \leq 16$, was the frog population at a minimum?

f) Draw a possible graph of the frog population P against time t

Further examples:

1. The share price $\$P$ of the Eastcom Bank t years after it opened in 1st January 1970 was $P = -0.4t^2 + 4t + 2$

- **a.** What was the initial share price?
- **b.** What was the share price after one year?
- **c.** At what rate was the share price increasing after two years?
- **d.** By letting $\frac{dP}{dt} = 0$, show that the maximum share price was \$12, at the start of 1975
- **e.** The directors decided to close the bank when the share price fell back to its initial value. When did this happen?

2. For a certain brand of medicine, the amount M present in the blood after t hours is given by $M = 3t^2 - t^3$, for $0 \leq t \leq 3$

- **a.** Sketch a graph of M against t, showing any stationary points and points of inflexion.
- **b.** When is the amount of medicine in the blood a maximum?
- **c.** When is the amount of medicine increasing most rapidly?

3. A certain brand of medicine tablet is in the shape of a sphere with diameter 5mm. The rate at which the pill dissolves is $\frac{dr}{dt} = -k$, where r is the radius of the sphere at time t hours, and k is a positive constant.

- **a.** Show that $r = \frac{5}{2} - kt$
- **b.** If the pill dissolves completely in 12 hours, find k