Digital Signal Processing

POSTECH
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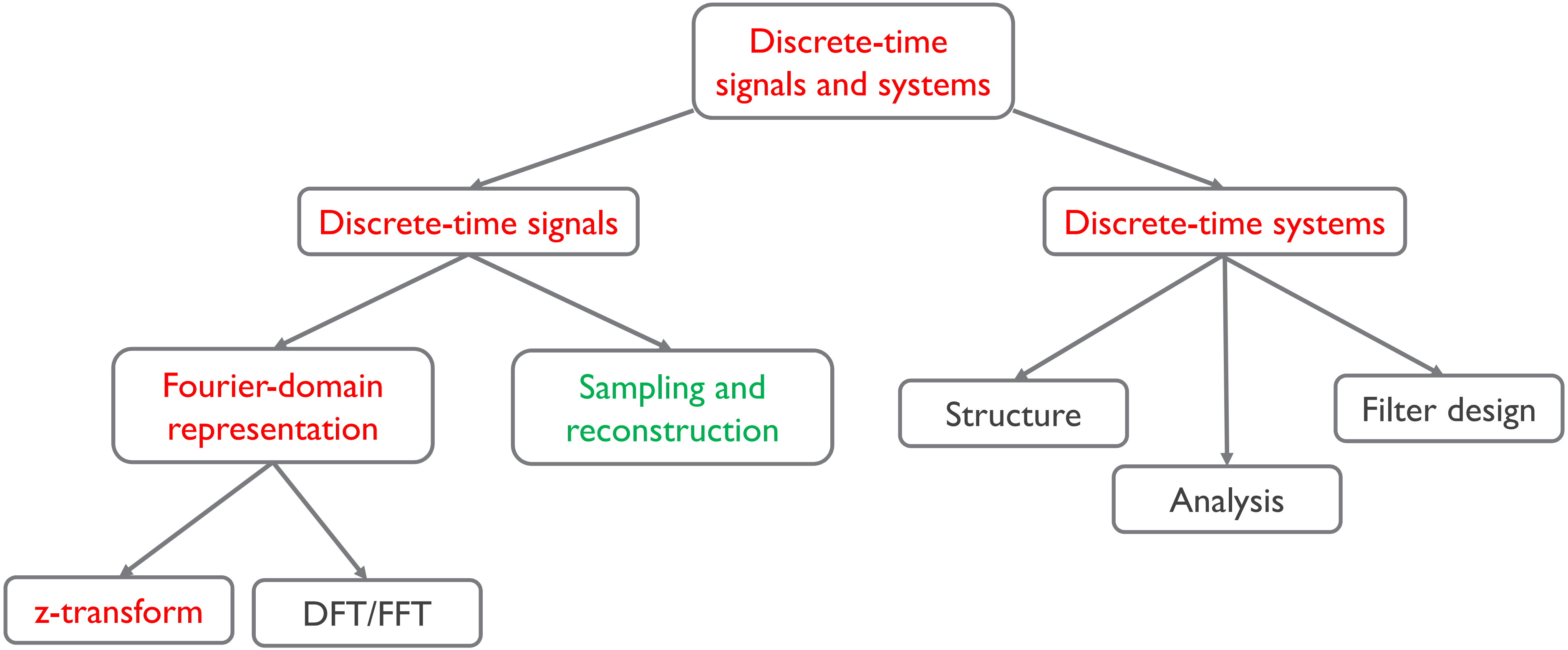
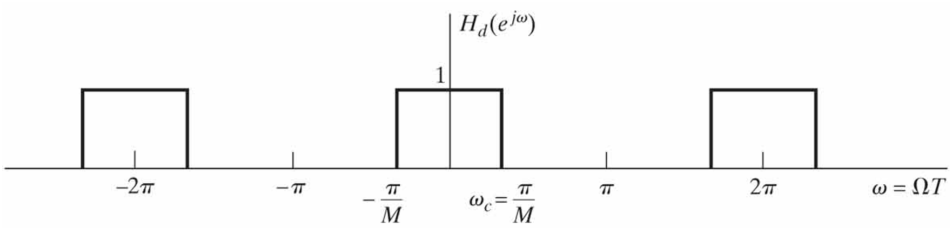
Course at glance

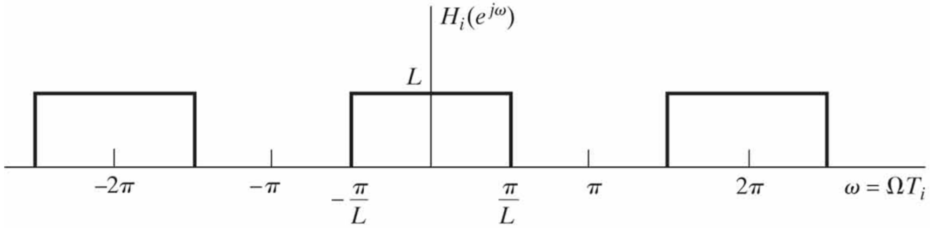


Multirate Signal Processing

Changing sampling rate possible

- Use downsampling/upsampling to reduce/increase the sampling rate
  - Both systems need lowpass filters
- With large changing factors *M* and *L*, we need narrowband lowpass filters

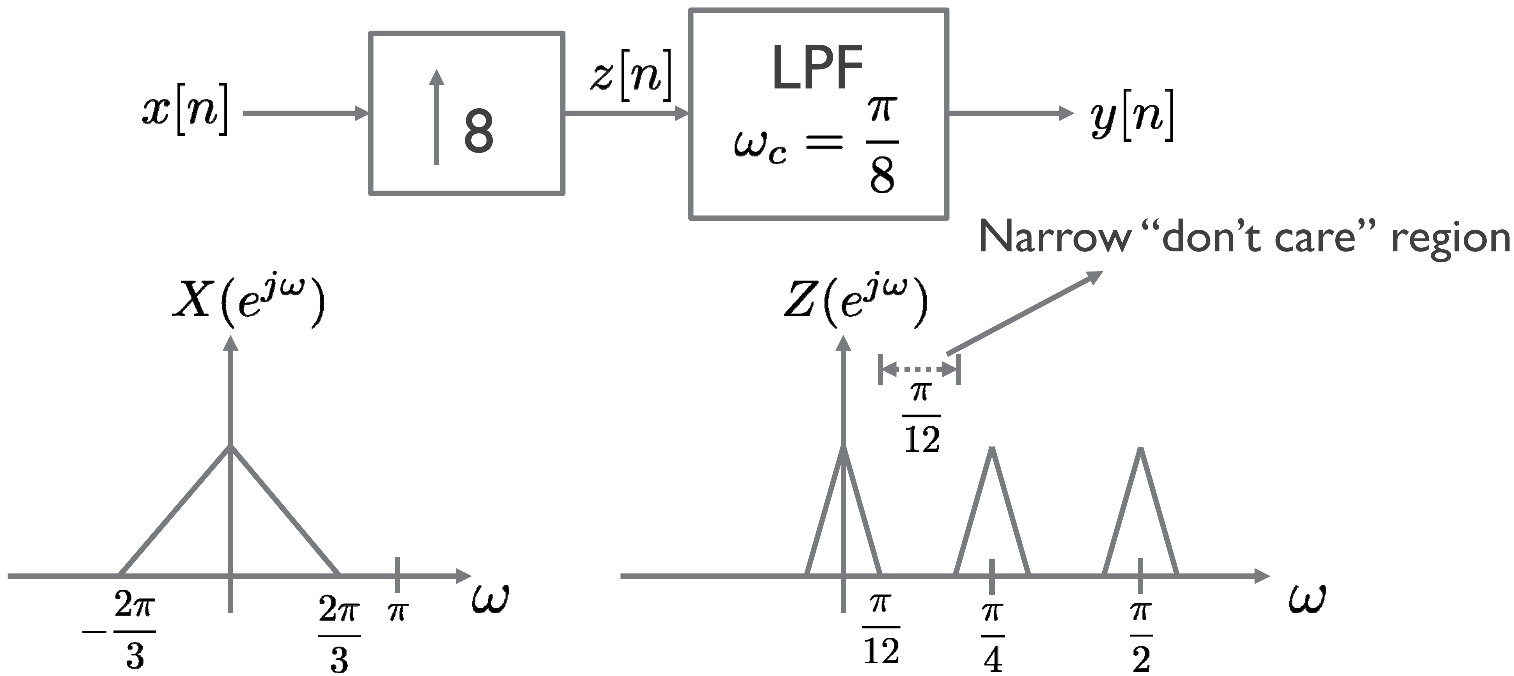




- Hard to implement narrowband lowpass filters with sharp cutoff frequency
  - Requires "long" FIR filter
  - Compare ideal lowpass filter and linear interpolation filter
- How to reduce the complexity of resampling for e.g., *M*=101 and *L*=100?

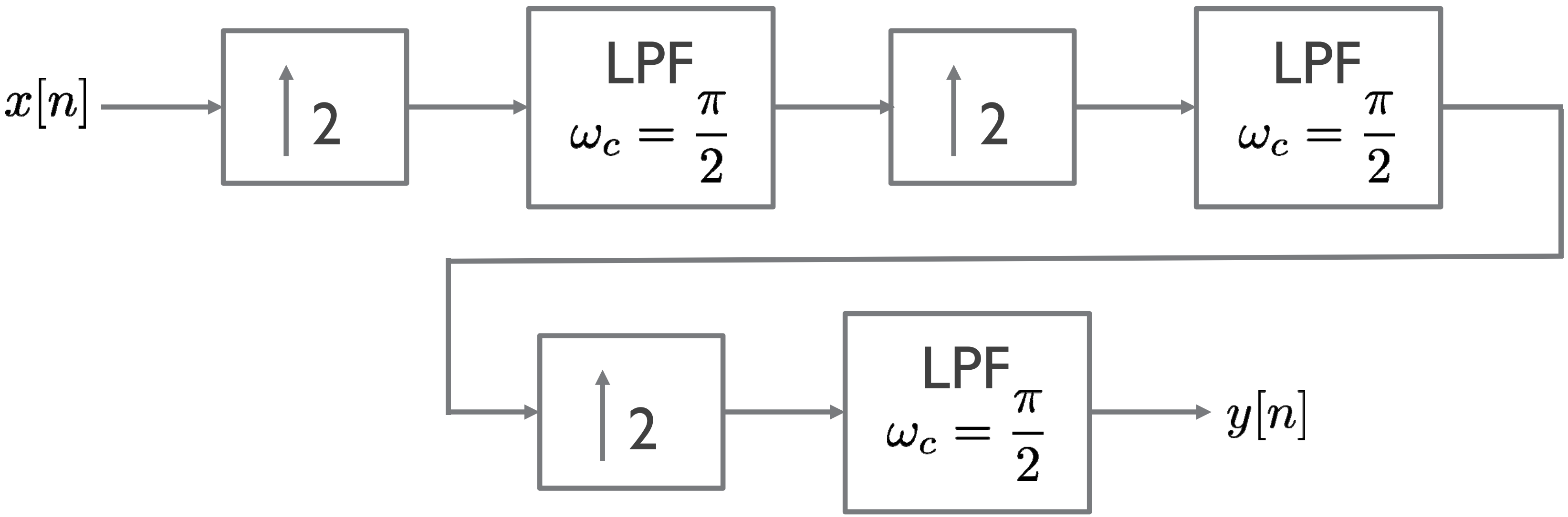
Example of multistage interpolation

- Consider 4kHz bandwidth speech sampled at 12kHz
- Want to implement a system



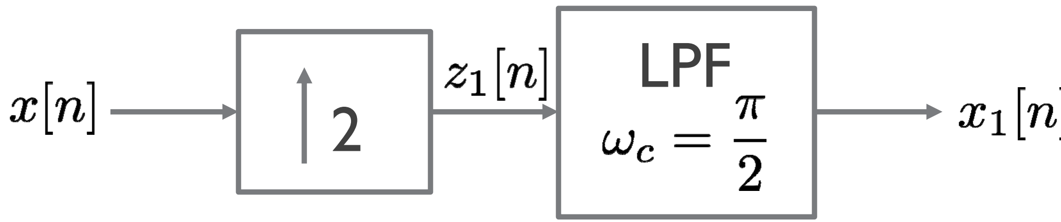
Practical example of multistage interpolation

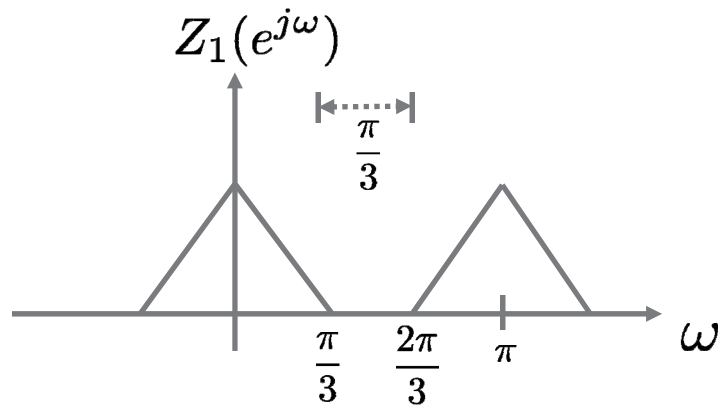
- Instead, consider implementing in 3 stages



Practical example of multistage interpolation

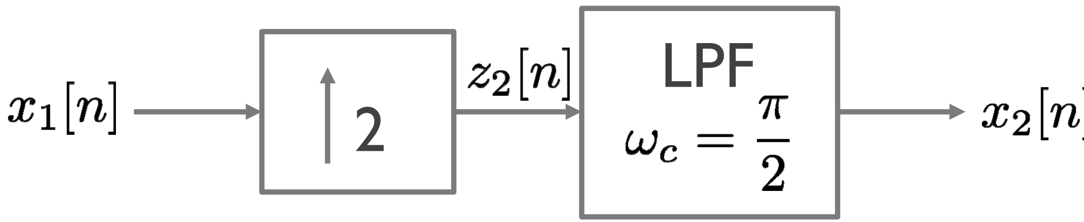
- Stage I

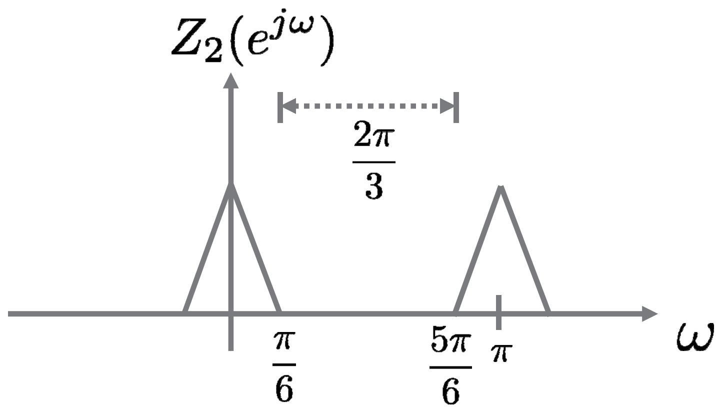




Practical example of multistage interpolation

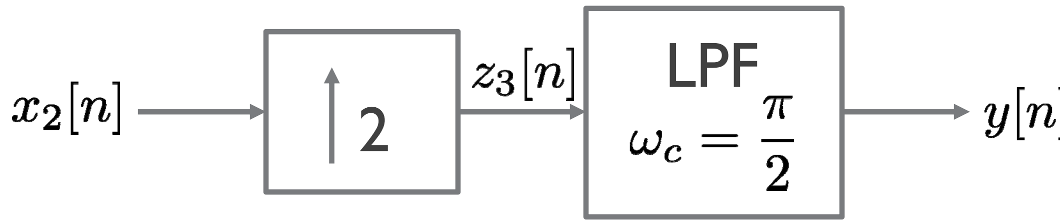
- Stage 2

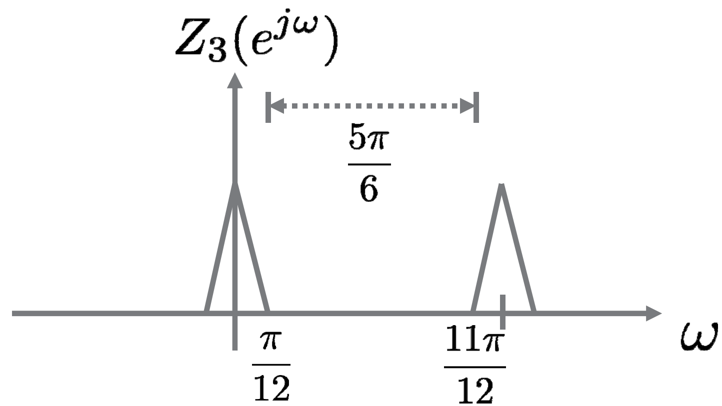




Practical example of multistage interpolation

- Stage 3

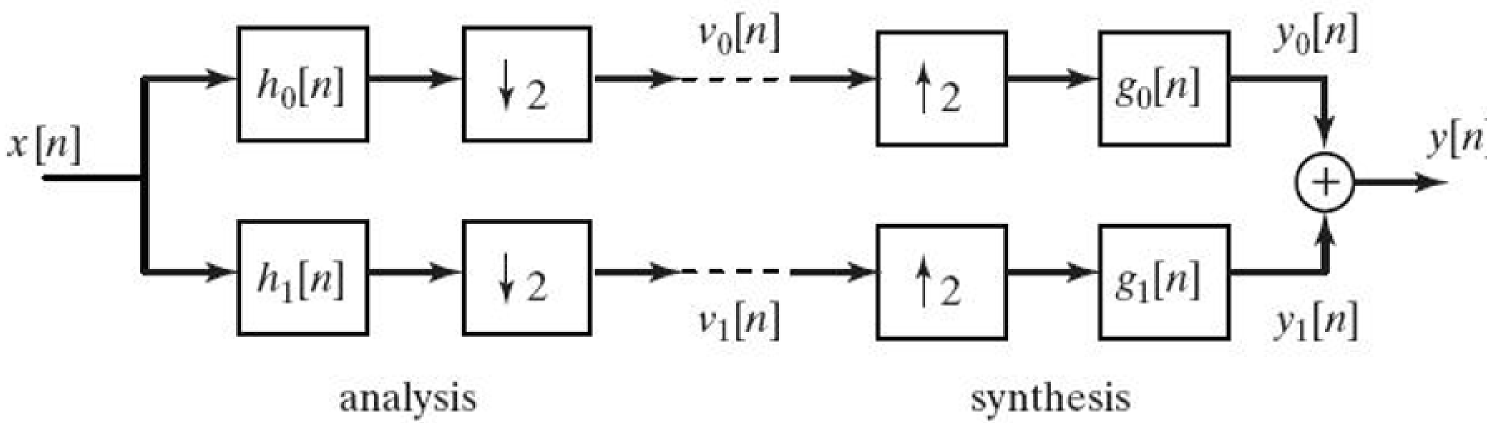




Multirate filter bank

- Split audio/speech signals into different frequency components
  - Low, medium, high frequency components
  - Process (quantize and/or storage) each frequency component separately
  - Reconstruct the signal by synthesizing frequency components
- We can exploit downsampling/upsampling for multirate filter bank

Two-channel perfect reconstruction filter bank

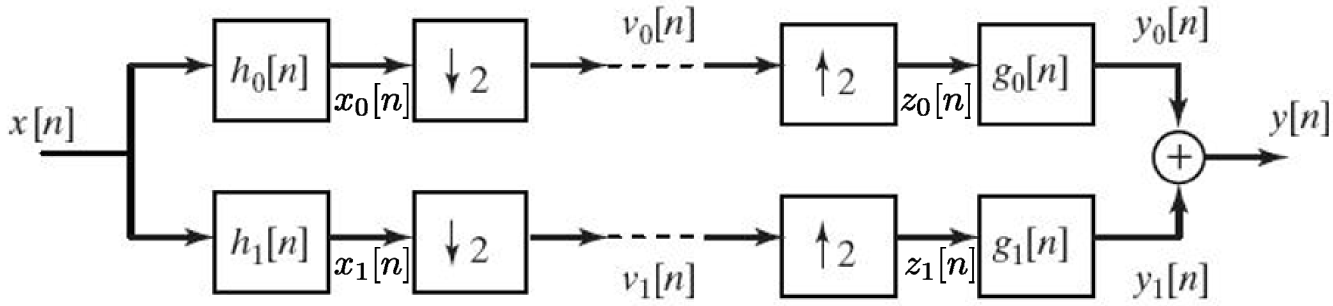


- Analysis part by downsampling / synthesis part by upsampling
- We want to have $x[n] = y[n]$ without further processing on ----
- What are relationships among $h_0[n], h_1[n], g_0[n], g_1[n]$?
  - Decomposition requires $h_0[n], h_1[n]$ to be lowpass and highpass filters

$$H_0(e^{j\omega}) : \text{ lowpass filter with passband } 0 \le |\omega| \le \pi/2$$

$$h_1[n] = e^{j\pi n} h_0[n] \overset{\mathcal{F}}{\longleftrightarrow} H_1(e^{j\omega}) = H_0(e^{j(\omega - \pi)})$$

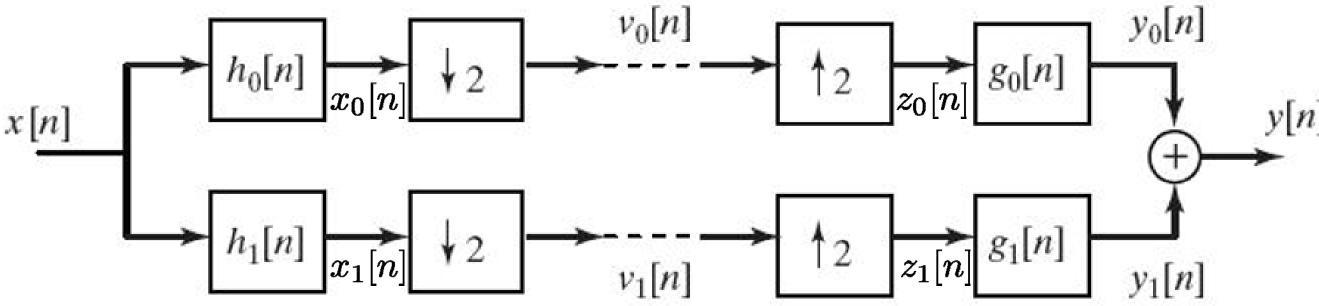
Frequency-domain relationships



- $X_i(e^{j\omega}) = H_i(e^{j\omega})X(e^{j\omega})$

$$\begin{aligned} V_i(e^{j\omega}) &= \frac{1}{2}\left(X_i(e^{j\omega/2}) + X_i(e^{j(\omega-2\pi)/2})\right) \\ &= \frac{1}{2}\left(H_i(e^{j\omega/2})X(e^{j\omega/2}) + H_i(e^{j(\omega-2\pi)/2})X(e^{j(\omega-2\pi)/2})\right) \end{aligned}$$

Frequency-domain relationships



- $$\begin{aligned} Z_i(e^{j\omega}) &= V_i(e^{2j\omega}) \\ &= \frac{1}{2}\left(H_i(e^{j\omega})X(e^{j\omega}) + H_i(e^{j(\omega-\pi)})X(e^{j(\omega-\pi)})\right) \end{aligned}$$

- $$\begin{aligned} Y(e^{j\omega}) &= G_0(e^{j\omega})Z_0(e^{j\omega}) + G_1(e^{j\omega})Z_1(e^{j\omega}) \\ &= \frac{1}{2}\left(H_0(e^{j\omega})G_0(e^{j\omega}) + H_1(e^{j\omega})G_1(e^{j\omega})\right)X(e^{j\omega}) \\ &\quad + \frac{1}{2}\left(H_0(e^{j(\omega-\pi)})G_0(e^{j\omega}) + H_1(e^{j(\omega-\pi)})G_1(e^{j\omega})\right)X(e^{j(\omega-\pi)}) \end{aligned}$$

Potential aliasing distortion

Ideal case

- With ideal filters that exactly split the band $0 \leq |\omega| \leq \pi$ without overlapping, it is easy to show that

$$\left(H_0(e^{j\omega})G_0(e^{j\omega}) + H_1(e^{j\omega})G_1(e^{j\omega})\right) = 2$$

$$\left(H_0(e^{j(\omega-\pi)})G_0(e^{j\omega}) + H_1(e^{j(\omega-\pi)})G_1(e^{j\omega})\right) = 0$$

- What about with non-ideal, practical filters?

Using practical filters

- Alias cancellation condition

$$g_0[n] = 2h_0[n] \overset{\mathcal{F}}{\longleftrightarrow} G_0(e^{j\omega}) = 2H_0(e^{j\omega})$$

$$g_1[n] = -2h_1[n] \overset{\mathcal{F}}{\longleftrightarrow} G_1(e^{j\omega}) = -2H_0(e^{j(\omega-\pi)})$$

- Combined with $h_1[n] = e^{j\pi n}h_0[n] \overset{\mathcal{F}}{\longleftrightarrow} H_1(e^{j\omega}) = H_0(e^{j(\omega-\pi)})$

$$Y(e^{j\omega}) = \left[H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega-\pi)})\right] X(e^{j\omega})$$

- Thus, perfect reconstruction (with possible delay of *M* samples) requires

$$H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega-\pi)}) = e^{-j\omega M}$$

- $h_0[n] = c_0\delta[n - 2n_0] + c_1\delta[n - 2n_1 - 1]$ satisfies this condition

$c_0c_1 = 1/4$ Arbitrary integer

Simple example

- Let $h_0[n] = \frac{1}{2}(\delta[n] + \delta[n-1]) \xleftrightarrow{\mathcal{F}} H_0(e^{j\omega}) = \cos(\omega/2)e^{-j\omega/2}$

- $$\begin{aligned} H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega-\pi)}) &= \cos^2(\omega/2)e^{-j\omega} - \cos^2((\omega-\pi)/2)e^{-j(\omega-\pi)} \\ &= \frac{1+\cos\omega}{2}e^{-j\omega} + \frac{1+\cos(\omega-\pi)}{2}e^{-j\omega} \\ &= \frac{1+\cos\omega}{2}e^{-j\omega} + \frac{1-\cos\omega}{2}e^{-j\omega} \\ &= e^{-j\omega} \end{aligned}$$

$$\cos^2\left(\frac{\omega}{2}\right) = \frac{1+\cos\omega}{2}$$

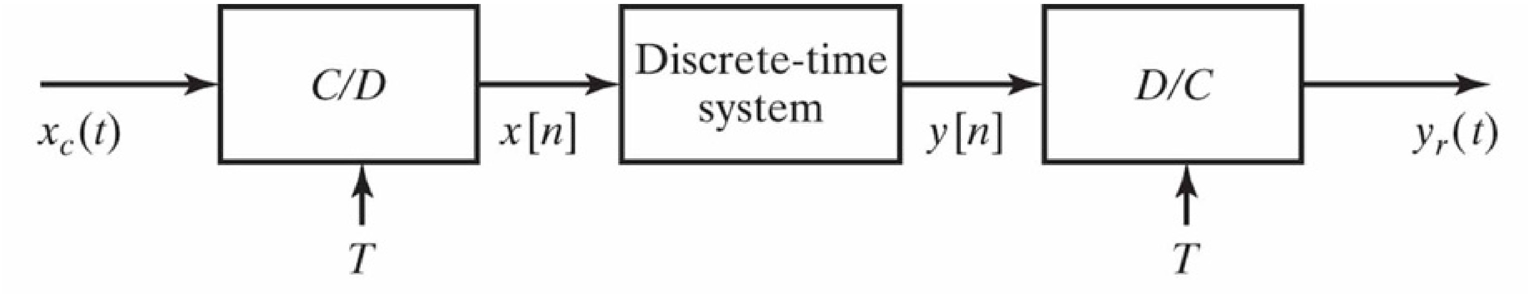
- Therefore, $y[n] = x[n-1]$

- $h_0[n] = c_0\delta[n-2n_0] + c_1\delta[n-2n_1-1]$ is very crucial lowpass filter though

Digital Processing of Analog Signals

Ideal system

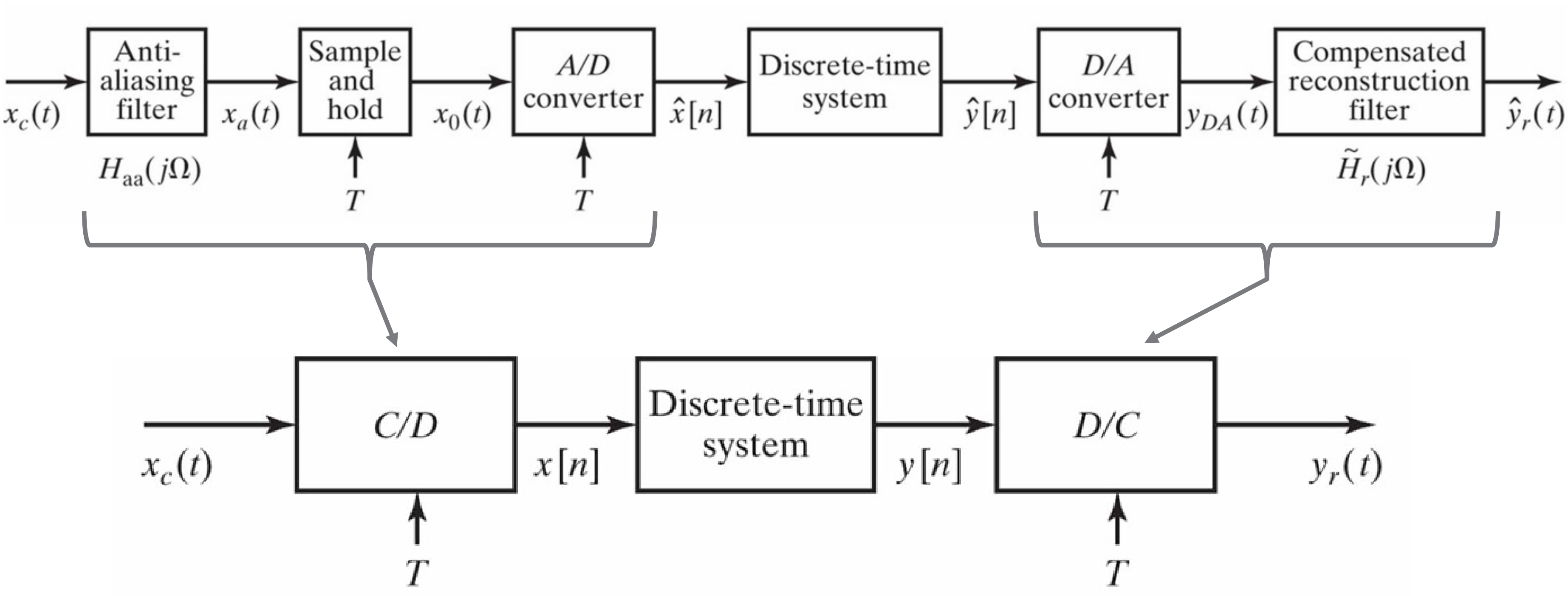
- So far, we assumed ideal components for sampling and reconstruction



- In practice, they are not possible
  - Input signal not band limited
  - Impulse train
  - Ideal lowpass filter
  - Infinite precision
  - Etc…

Practical system

- General diagram of practical system



Antialiasing filter

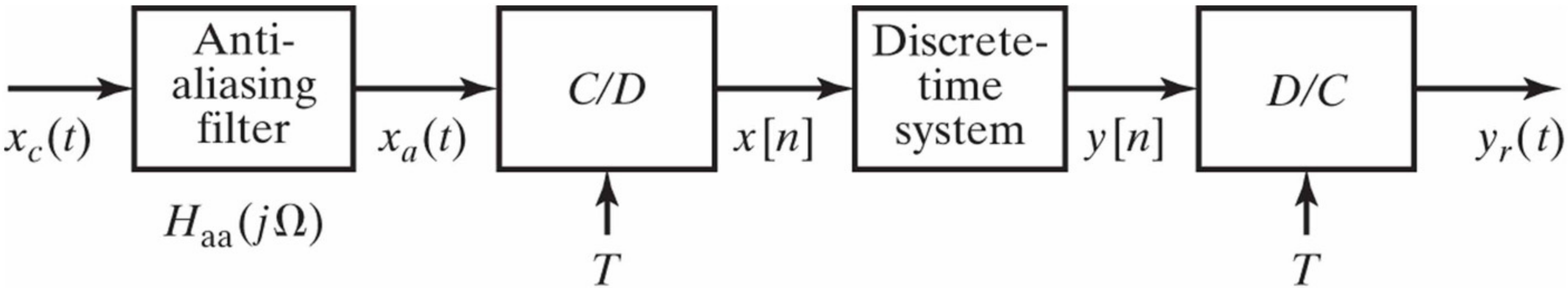
- Input signal not bandlimited in general
  - Even the signal is bandlimited, noise can have high frequency component

- Useful information may be contained only in low-frequencies
  - Speech signal
    - Can have 3Hz~20kHz frequency range
    - 3~4kHz frequency range enough for intelligibility

- High-frequency components (unnecessary information or noise) can be aliased into the low-frequency band.

- Need analog lowpass filter before sampling to avoid aliasing

Ideal antialiasing filter

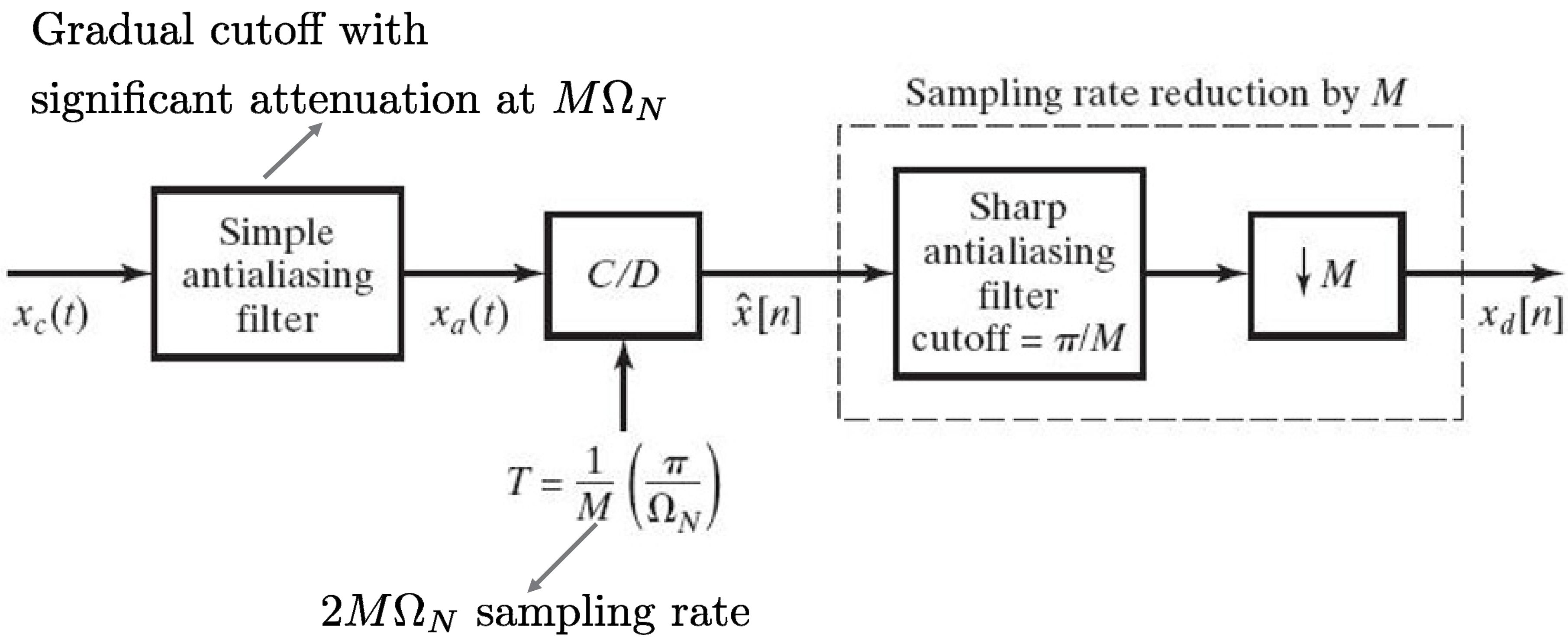


- Frequency response of ideal antialiasing filter would be

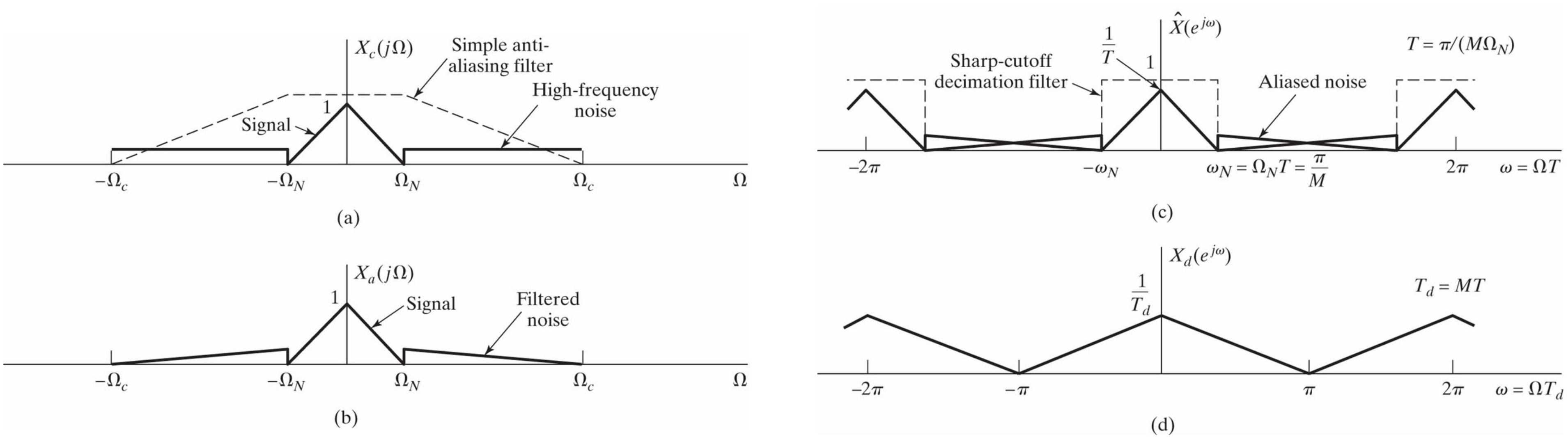
$$H_{aa}(j\Omega) = \begin{cases} 1, & |\Omega| < \Omega_c \le \pi/T \\ 0, & |\Omega| \ge \Omega_c \end{cases}$$

- This requires sharp cutoff frequency
  - Can be implemented using active networks and integrated circuits
  - Much harder & more expensive & less flexible than digital filters

Practical antialiasing filter design



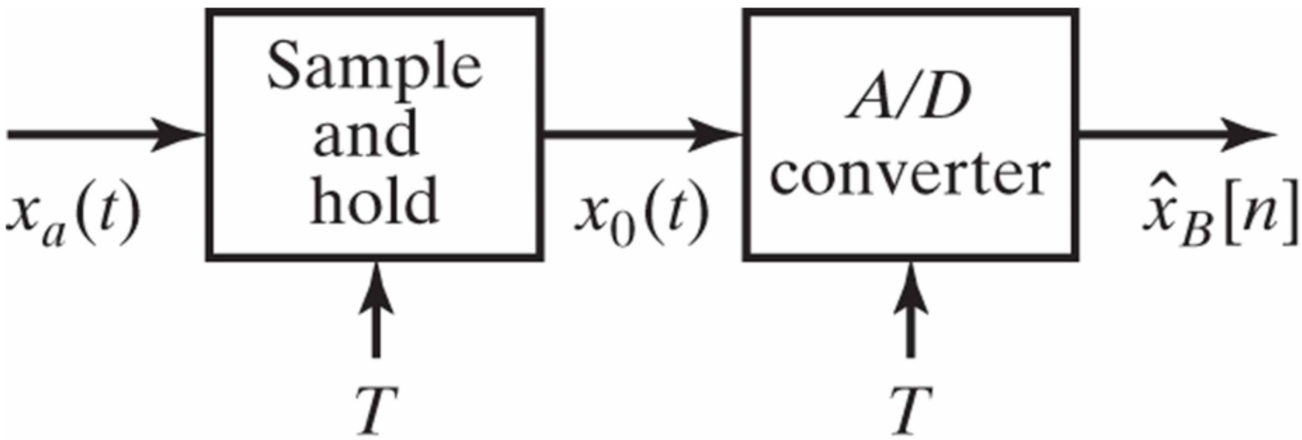
Frequency-domain illustration



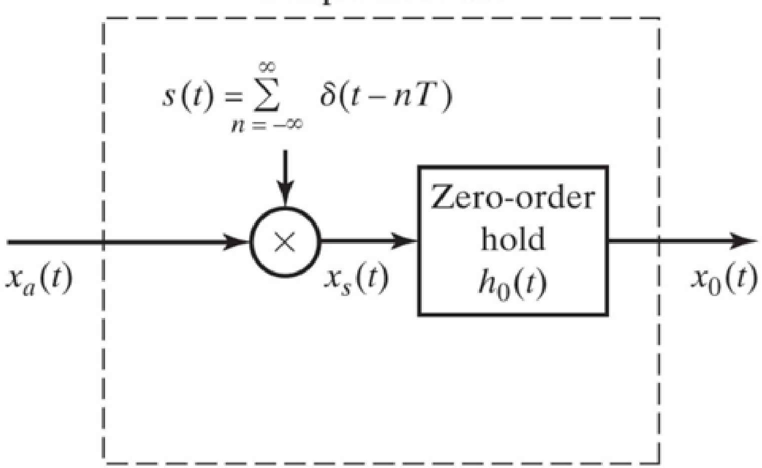
A/D conversion

- Ideal C/D conversion not possible
  - Impulse train, infinite precision of amplitude, …

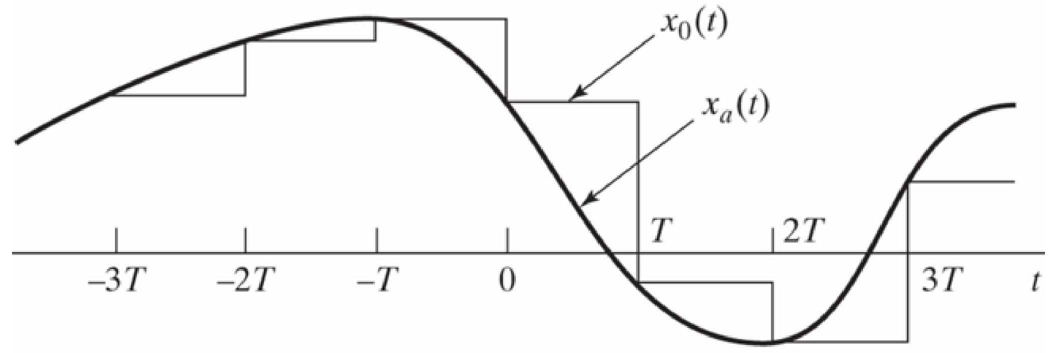
- Practical C/D conversion is A/D conversion



Sample-and-hold block



(a)



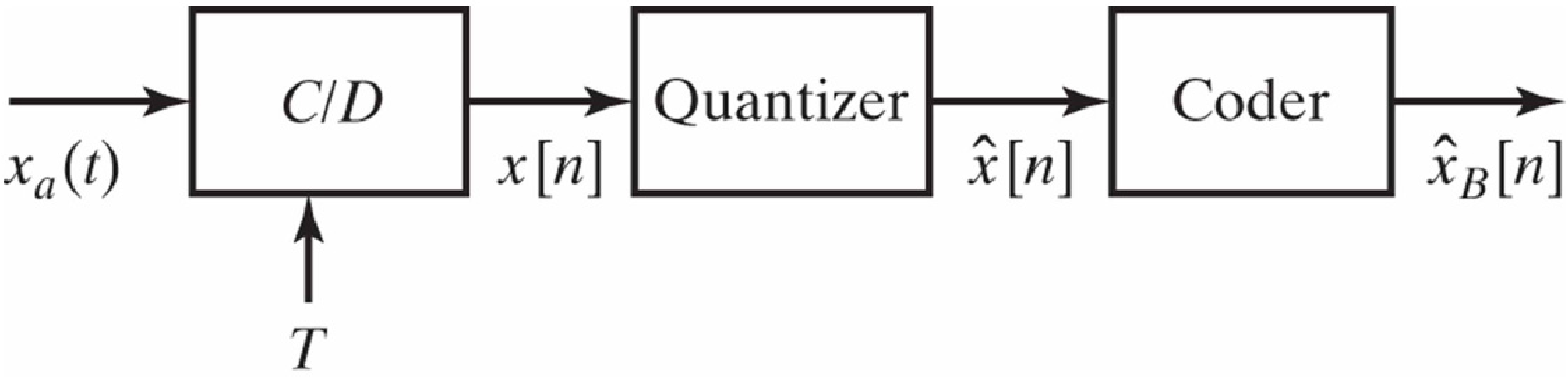
(b)

$$h_0(t) = \begin{cases} 1, & 0 < t < T, \\ 0, & \text{otherwise} \end{cases},$$

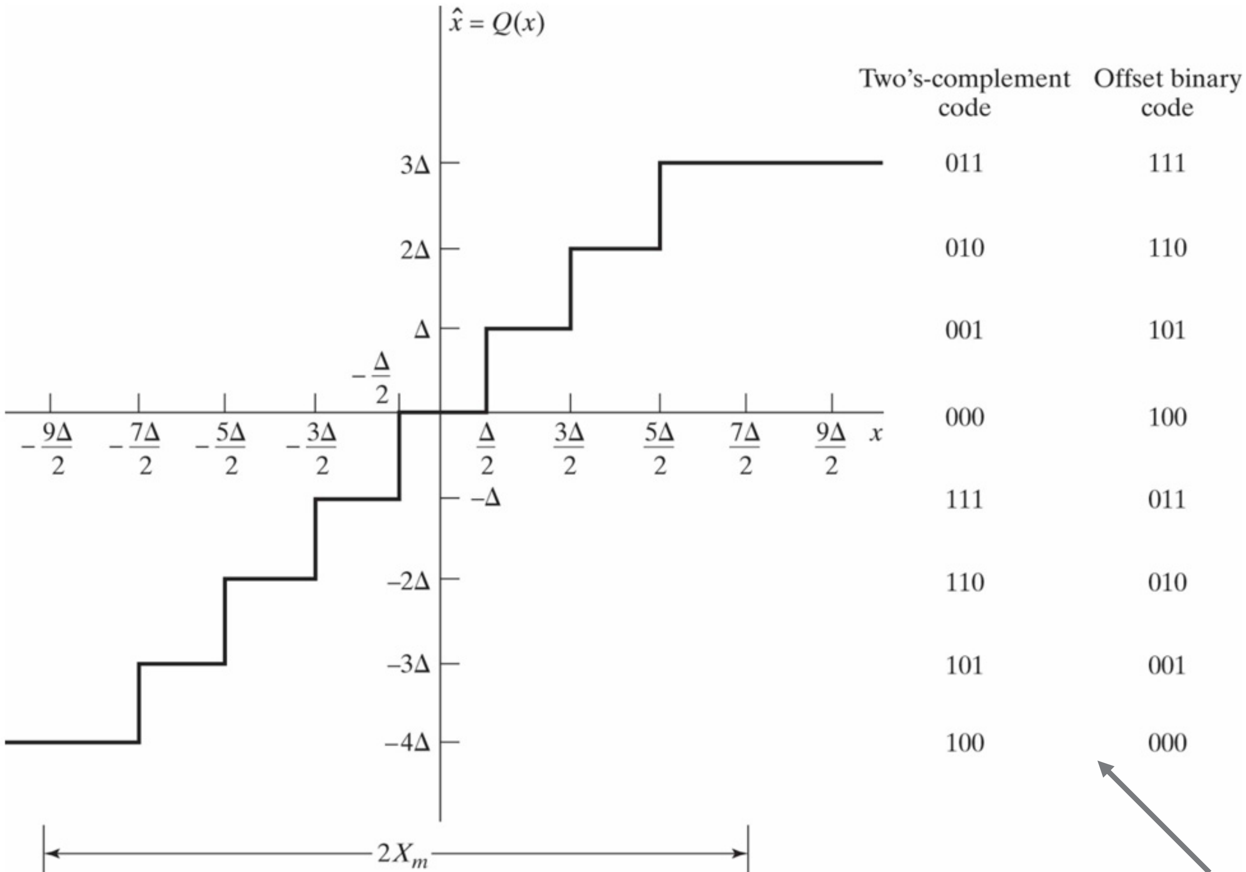
$$x_0(t) = \sum_{n=-\infty}^{\infty} x[n] h_0(t - nT)$$

$$= h_0(t) * \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t - nT)$$

Conceptual illustration of A/D

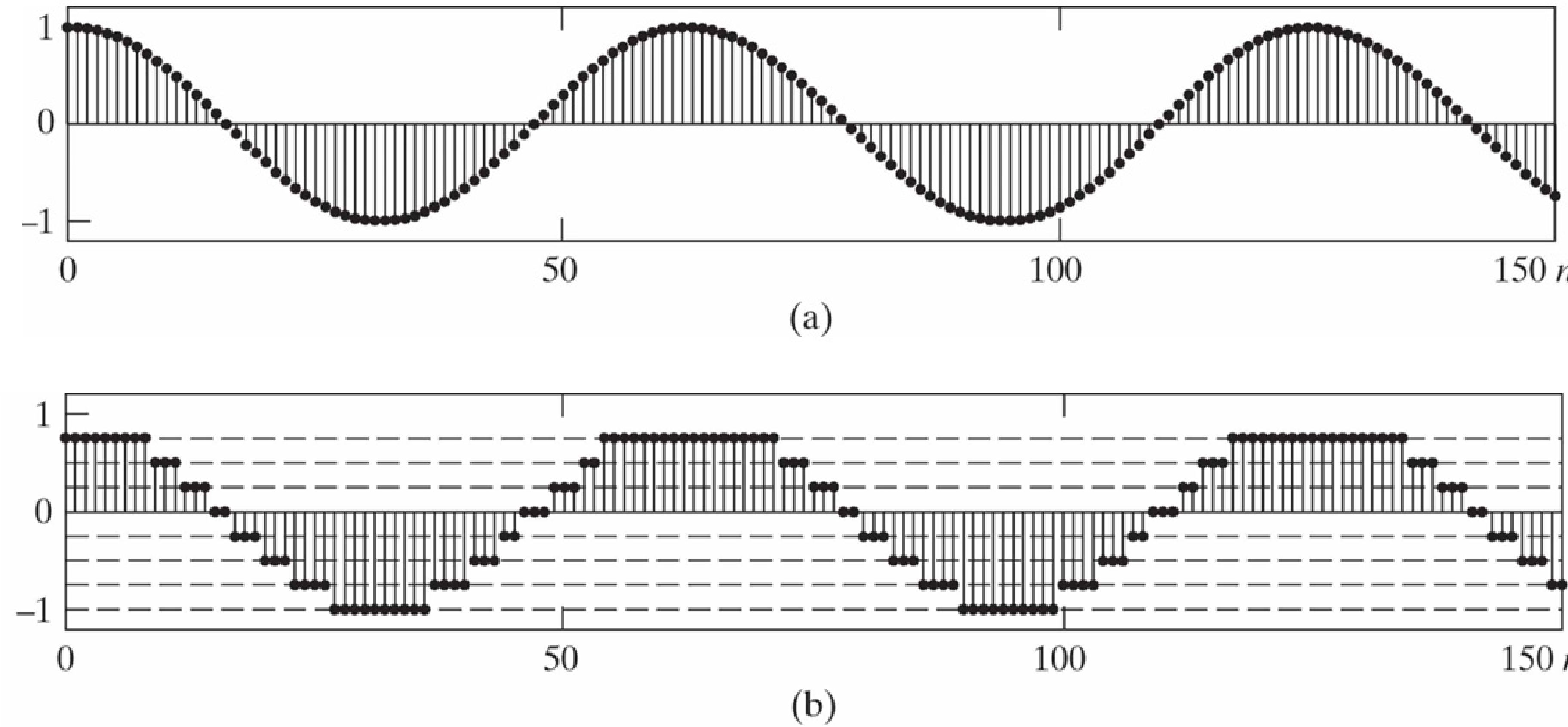


Quantizer and coder



Possible coders

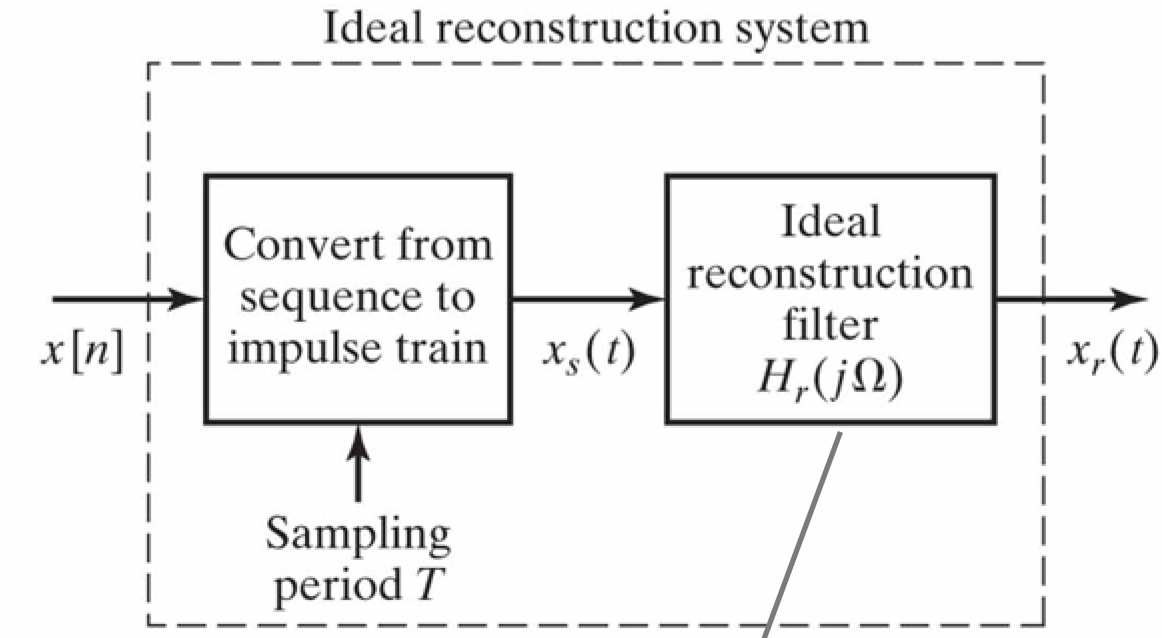
Example



(a)
(b)

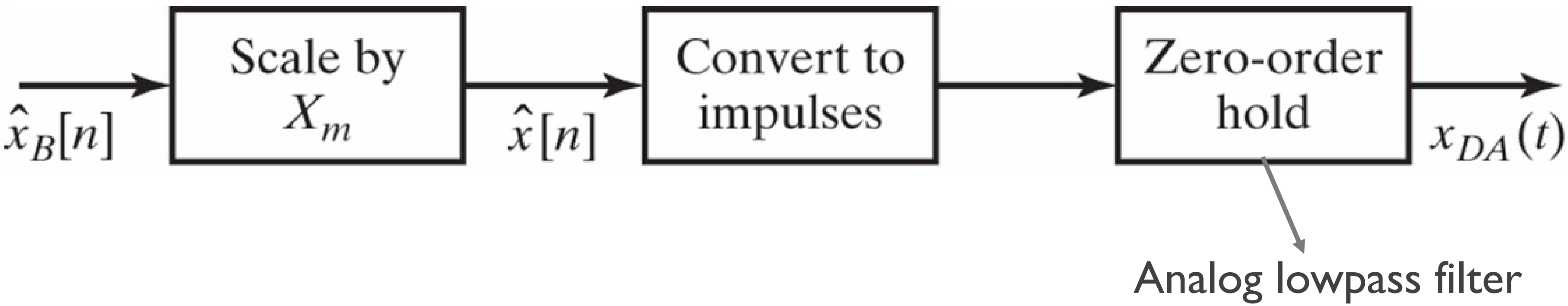
D/A conversion

- Ideal D/C conversion

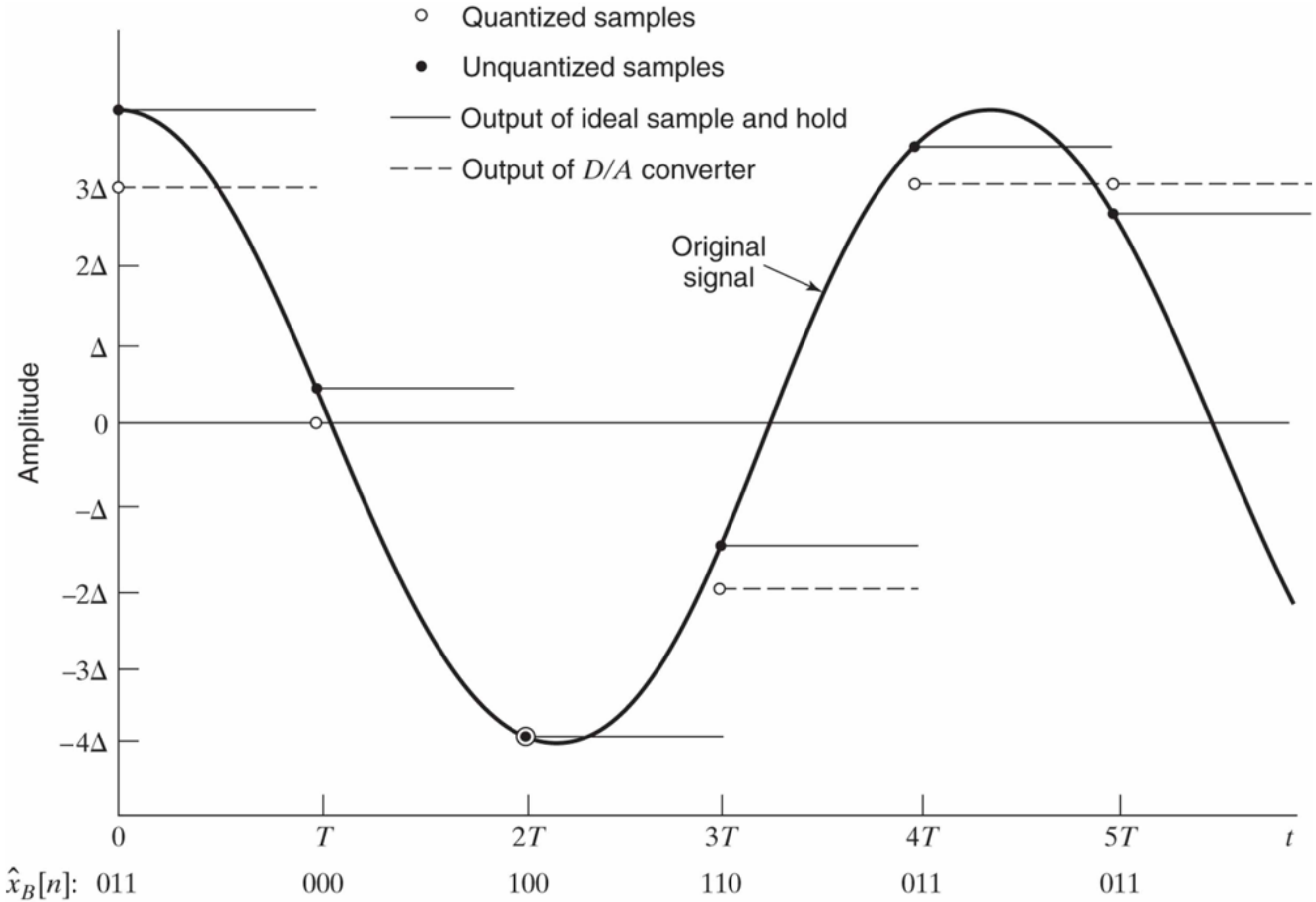


$$H_r(j\Omega) = \begin{cases} T, & |\Omega| < \pi/T, \\ 0, & |\Omega| \geq \pi/T \end{cases}$$

Practical D/A conversion



Time-domain illustration of D/A conversion



Issue

- The frequency response of the zero-order-hold filter

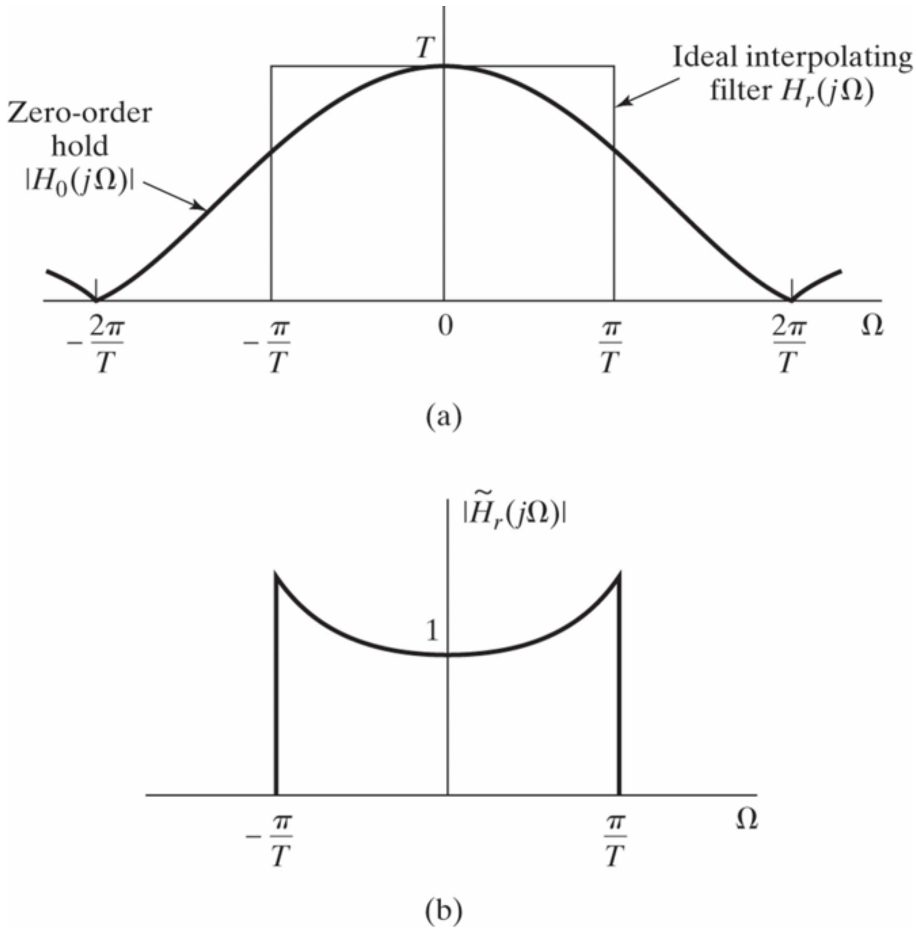
$$H_0(j\Omega) = \frac{2\sin(\Omega T/2)}{\Omega} e^{-j\Omega T/2}$$

- Need a compensated reconstruction filter

$$\tilde{H}_r(j\Omega) = \frac{H_r(j\Omega)}{H_0(j\Omega)}$$

Ideal lowpass filter

Frequency-domain illustration



Homework

- Problems in textbook: 4.23, 4.25, 4.28, 4.31, 4.34
  - Solution uploaded on the webpage