

Simulating Behaviors of Children with Autism Spectrum Disorders Through Reversal of the Autism Diagnosis Process

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Abstract. Children affected by Autism Spectrum Disorders (ASD) exhibit behaviors that may vary drastically from child to child. The goal of achieving accurate computer simulations of behavioral responses to given stimuli for different ASD severities is a difficult one, but it could unlock interesting applications such as informing the algorithms of agents designed to interact with those individuals. This paper demonstrates a novel research direction for high-level simulation of behaviors of children with ASD by exploiting the structure of available ASD diagnosis tools. Building on the observation that the simulation process is in fact the reverse of the diagnosis process, we take advantage of the structure of the Autism Diagnostic Observation Schedule (ADOS), a state-of-the-art standardized tool used by therapists to diagnose ASD, in order to build our ADOS-Based Autism Simulator (ABASim). We first define the ADOS-Based Autism Space (ABAS), a feature space that captures individual behavioral differences. Using this space as a high-level behavioral model, the simulator is able to stochastically generate behavioral responses to given stimuli, consistent with provided child descriptors, namely ASD severity, age and language ability. Our method is informed by and generalizes from real ADOS data collected on 67 children with different ASD severities, whose correlational profile is used as our basis for the generation of the feature vectors used to select behaviors.

Keywords: Behavioral Simulation; Computational Modeling; Autism; Autism Diagnostic Observation Schedule

1 Introduction

Autism Spectrum Disorders (ASD) is a set of developmental conditions that affect 1 in 68 children in the US⁴ and can have varying degrees of impact on social abilities, verbal and non-verbal communication, and motor and cognitive skills. These disorders have been widely studied from a developmental, neuropsychological [17] and genetic [13] point of view, whereby researchers try to explain the underlying mechanisms that

⁴ according to a 2014 report by the Centers for Disease Control and Prevention.

cause or characterize ASD. However, apart from diagnostic procedures, efforts for understanding, classifying, formalizing and predicting the wide range of *behaviors* of individuals with ASD remain as of now limited. In particular in children, which are the focus of this work, the exhibited behaviors may be even more diverse and hard to predict compared to adults. The lack of such accurate behavioral models poses a problem in the design of autonomous agents that are expected to interact socially with children with ASD, such as avatars [12] or robots [14] [6]. Several studies have shown that introducing such agents in ASD therapy session has notable benefits [3], which has encouraged researchers to design such agents in a variety of ways. However, most of the existing agents either lack autonomy or are rigid when it comes to personalizing the interaction to account for different ASD severities or types. We believe one major bottleneck comes from the lack of useful behavioral models and simulation tools to enable the agents to intelligently adapt their social interaction with the child. Without such tools, the effectiveness of the agent’s reasoning, including both planning and learning, is limited.

On the other hand, several tools have been developed to diagnose ASD through observations or questionnaires. This paper specifically builds upon the Autism Diagnostic Observation Schedule (ADOS)⁵, “a semi-structured, standardized assessment of communication, social interaction, play, and imagination designed for use in diagnostic evaluations of individuals referred for a possible Autism Spectrum Disorder (ASD)” [10]. Compared to other diagnostic and evaluation tools (e.g., ADI-R, CARS, SRS, etc.), the ADOS possesses enough structure to be interesting from a computational point of view. It provides us with a detailed, precise and quasi-comprehensive assessment of the characteristics of a child suspected of having an ASD, thanks to its detailed coding scheme and the inherent structure of its activities. For instance, some of these activities are ‘algorithmic’ in nature and use a hierarchy of ‘presses’ (social structures) to help the coding. We take advantage of these points and use the ADOS as a starting point for the design of a high-level behavioral simulator of children with different severities of ASD.

The basis for this paper stems from the observation that the simulation process can be seen as the reverse of the diagnosis process. While the latter maps observed behaviors to a set of coded features, the former maps features to simulated behaviors consistent with those features. Unlike existing simulation procedures for ASD, our simulator, the *ADOS-Based Autism Simulator (ABASim)*, captures the individual differences in behaviors of children with varying ASD severities. We summarize the contributions of this paper as follows:

1. We define the ADOS-based Autism Space (ABAS), which captures behavioral differences among children with ASD, through the use of ADOS-based features,
2. We provide a method for stochastically mapping high-level descriptors of a child with ASD (namely: age, ASD severity, and language ability) to a point in ABAS. To inform the mapping process, our method uses real data collected on children with different ASD severities,
3. We provide a method for mapping a point in ABAS to behaviors occurring in response to a fixed set of stimuli, corresponding to ADOS activities.

⁵ Our work uses version 2 of the tool, namely ADOS-2, but for simplicity we refer to it by ADOS throughout the paper.

2 Background

We start by discussing some related work on simulating/modeling human behaviors. Then, we provide some details on the structure of the ADOS diagnostic tool.

2.1 Simulating/modeling human behaviors

Simulating and modeling human behaviors have been a widespread practice to inform any type of decision-making involving humans. Examples include consumer modeling in market research [1] [5] [16], online recommendation systems [9], and driver modeling [4], to name a few. These models focus on behavior, which is predicted either using an underlying cognitive process (e.g., driver modeling) or through the use of data (e.g., recommendation systems). Moreover, a general purpose “computer based mental simulator” (NL_MAMS) has been developed and used to simulate the underlying mental processes of individuals with ASD [7].

Computational models of ASD include techniques such as neural networks or game theory to model low-level mechanisms of the brain affecting behavior [7]. These methods are good at explaining different observed autistic behaviors, but not as successful in computationally predicting high-level behavior, especially for different types or severities of ASD. Reinforcement Learning methods have been proven to be useful in modeling some high-level behaviors seen in individuals with ASD [2], but they are only able to distinguish between ASD and non-ASD populations. Individual differences, well established in available diagnostic tools, are starting to be studied from a modeling/simulation perspective [15] but the parts of the model accounting for these differences is usually simplistic. To the best of our knowledge, the simulation of high-level autistic behaviors in an *individualized* way and in response to *different types of stimuli* remains unexplored.

2.2 Autism Diagnostic Observation Schedule (ADOS) structure

The ADOS diagnostic tool comprises 5 modules suitable for different language abilities and/or ages. Module 1 (Pre-verbal/Single Words) remains the main module used by therapists as an initial assessment of children 31 months or older, and up to 14 years of age. For this reason, we focus on this particular module in this work. However, our methods can be applied to any of the ADOS modules as they possess a very similar structure.

The ADOS Module 1 is composed of 10 standardized activities, with varying degrees of structuredness, including rather unstructured activities such as ‘Free Play’ (where the child is left to freely play in the room) to very structured activities such as ‘Response to name’ (where the therapists calls the child’s name at different degrees of intensity and observes the child’s response). In a typical session where the ADOS is administered, the therapist performs the activities and records behaviors of interest throughout the session. At the end of the session (i.e., after all 10 activities are over), the therapist codes the behaviors exhibited by the child throughout the whole session.

There are a total of 29 ADOS codes for different, usually exclusive, behavior types. However, of these 29 codes only 14 are used in the algorithm that returns the *total score*

Table 1. Summary of the ADOS module 1 codes (features) used for computing the total score for the two different language abilities

Code name	Label	Few/No Words	Some Words
Frequency of Vocalization Directed to Others	A2	✓	✓
Pointing	A7	✗	✓
Gestures	A8	✓	✓
Unusual Eye Contact	B1	✓	✓
Facial Expressions Directed to Others	B3	✓	✓
Integration of Gaze [etc.] During Social Overtures	B4	✓	✓
Shared Enjoyment in Interaction	B5	✓	✓
Showing	B9	✓	✓
Spontaneous Initiation of Joint Attention	B10	✓	✓
Response to Joint Attention	B11	✓	✗
Quality of Social Overtures	B12	✓	✓
Intonation of Vocalizations or Verbalizations	A3	✓	✗
Stereotyped/Idiosyncratic Use of Words or Phrases	A5	✗	✓
Unusual Sensory Interest in Play Material/Person	D1	✓	✓
Hand and Finger and Other Complex Mannerisms	D2	✓	✓
Unusually Repetitive Interests or Stereotyped Behaviors	D4	✓	✓

used for diagnosis, and those slightly vary depending on the language ability of the child. Table 1 shows the codes used for computing the total score and of interest in this work. Codes are all converted to a 0-2 integer scale before they are summed to produce a total score between 0 and 28. The total score can be further broken down into three subtotals for Communication (A2 to A8), Reciprocal Social Interaction (B1 to B12) and Restricted and Repetitive Behavior (A3 to D4). From the total score, one can compute a *comparison score* (between 1 and 10) which serves as our measure for autism severity. In this paper, we focus only on the codes of Table 1, although using more codes to get more detailed simulated behaviors is possible.

3 ADOS-Based Autism Space (ABAS)

In this section, we formally introduce our domain, based on the structure of the ADOS.

3.1 ABAS definition

Through its use of codes to map observed behaviors to numbers, the ADOS effectively defines a *feature space* for ASD, which we will call the *ADOS-based Autism space (ABAS)*. In this space, the **features** under consideration correspond to the different ADOS codes, and ABAS points represent different individuals with different ASD characteristics. We refer to our features (codes) as $c_{i,L}$ for $i = 1, \dots, 14$ and $L =$ ‘No words’, ‘Some words’, where $c_{i,L} \in \{0, 1, 2\}$. Even though some items may be originally coded by the therapist using values outside the 0-2 range, the codes are remapped to it for the algorithm’s purposes. We refer to a point in ABAS using the feature vector $[c_1, \dots, c_{14}]$ (ignoring the dependence on language ability for simplicity).

3.2 Total score constraint on ABAS

The total score C is defined as $\sum_{i=1}^{14} c_i$ ($C \in \{0, \dots, 28\}$). Except for edge cases, there are many feature vectors (combinations of ADOS codes) that sum up to the same total

score. To evaluate the impact on constraining the L1 norm of the feature vector, we solve the following equation for each of the possible values of C :

$$\sum_{i=1}^{14} c_i = C, \quad c_i \in \{0, 1, 2\} \quad (1)$$

The number of elements in the solution set of Eq. 1 as a function of C is shown in Fig. 1. For some values of the total score, the number of possible feature vectors can be very large (e.g., more than 600,000 for a total score of 14). Although mathematically feasible, we suspect that some of these feature vectors will be unlikely to occur in nature due to inevitable dependencies between the different features. This observation will be tackled in section 4.2.

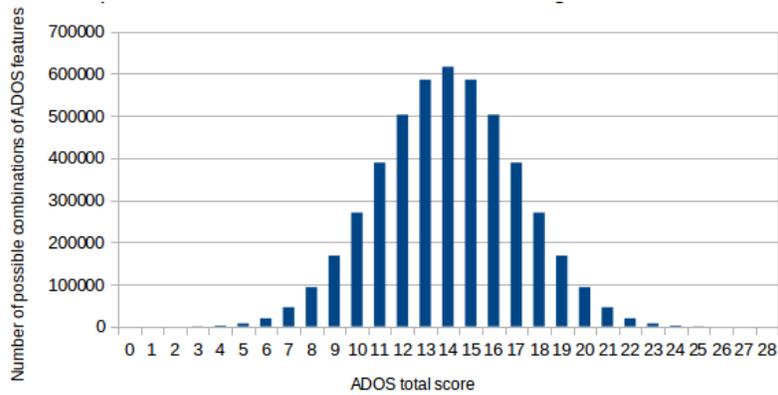


Fig. 1. Number of different possible combinations of ADOS codes (feature vectors) for a fixed total score.

3.3 Descriptors

As for any simulator, it is useful to define a set of high-level variables which could be inputted by a user to create a range of simulations with different characteristics. In particular, the individual features might be too many to input by hand, or as mentioned in the previous subsection, some of the combinations of features may be unlikely to even occur in nature. Also, we may want to be able to stochastically generate different simulation runs for a smaller set of higher-level input variables. For these reasons, we introduce **descriptors**, defined as:

- The child's chronological *age* A ,
- The child's *language ability* L ('no words' or 'some words'),
- The child's *ASD severity* S (on a scale from 1 to 10).

These descriptors are used in the ADOS, which defines a relationship between A , L , S and C in the form of a conversion table. The (A,L,S) triplet will be used as a convenient yet expressive input to our simulator.

4 Behavioral Simulation of Children with Different ASD Severities

In this section, we describe the different components along the pipeline of our ADOS-Based Autism Simulator (ABASim), which generates behavioral responses to input stimuli for specified descriptors.

4.1 ABASim components overview

ABASim enables the generation of a set of behaviors which, according to ADOS, are likely to occur as a response to a given stimulus, for a child with given descriptors. The stimuli we consider in this work correspond to the standardized activities performed during an ADOS session; therefore the set of behaviors ABASim is able to generate are those typically observed during or as a response to these activities.

The pipeline of our simulator is shown in Fig. 2. We use the descriptors (A, L, S) to specify the characteristics of the fictional child to be simulated. The input descriptors first get translated into a total score range, from which a single total score is randomly selected. From this total score, we stochastically generate 14 feature values whose sum matches the specified total score. The sampling process used is informed by data collected on real children suspected of having an ASD. Finally, for each of the 10 activities, we have identified a set of relevant features which dictate what kinds of behaviors are likely to be observed for the given activity. These behaviors are selected from a database of behaviors that we compiled based on the explanation of the coding scheme of the ADOS manual [10]. We now give some more detail on the different simulator components.

4.2 Stochastic generation of feature vectors from descriptors

As mentioned in the simulator overview, the user will be specifying the descriptors as an input to the simulator. We hence need a mapping from descriptors to a feature vector consistent with these descriptor values.

The first step is to map given descriptors to a total score C . This step is trivial since it is directly given by a conversion table present at the end of the ADOS. This table converts a given set of age, language ability, and total score range to a severity value. Reversing this conversion gives us a range for C for a given (A,L,S) triplet (the width of this range is typically between 0 and 7). We then uniformly select one integer value in that range as our total score C .

The second step is to map the obtained total score C to a feature vector $[c_1, \dots, c_{14}]$ in ABAS. In other words, we are looking for a method to sample a feature vector with the property $\sum_{i=1}^{14} c_i = C$. As emphasized in section 3.2, some vectors will be unlikely to occur because we do not expect to have complete independence between the features c_i . We use real data to verify and make use of this hypothesis, as explained in the following subsections.

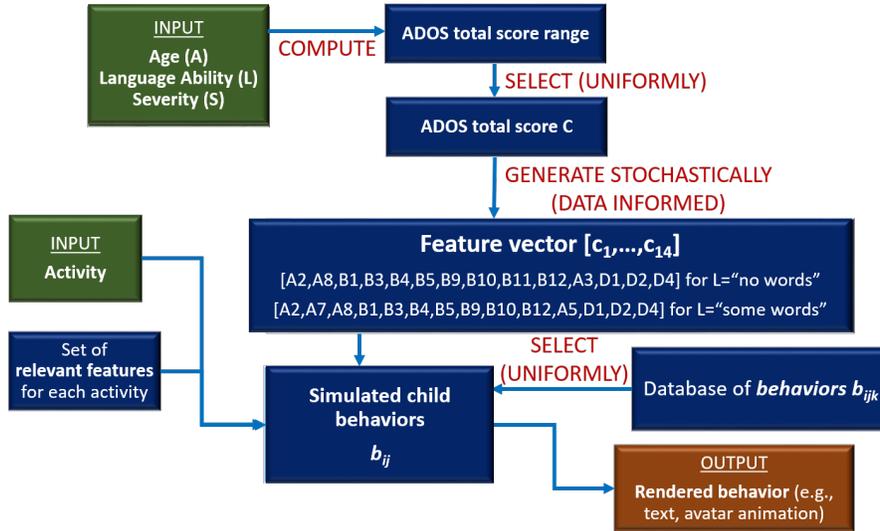


Fig. 2. ABASim pipeline: Generating simulated behaviors for input descriptors and activity.

Dataset description and analysis In order to inform our method for sampling feature vectors, we gathered the Module 1 ADOS scores (feature values) of 67 children with different severities of ASD⁶. Ages ranged between 3 and 7 years and the female-to-male ratio was 11:56. The total scores in the dataset ranged between 0 and 25, with a more or less uniform distribution over total scores (no blatant skewness). We analyzed the correlation between pairs of features, as shown in Fig. 3(a). Since we are dealing with ordinal data, we used Spearman correlation coefficients; we also ignored higher-order correlations. P-values were computed for each correlation coefficient using a t-statistic. Note that, of the 14 features, only the 12 in common between the two L categories were included. Because of our small sample size, breaking down our dataset into two parts (one for each value of L) would have been problematic statistically speaking. As a workaround, we could set the value of the two remaining features randomly.

Most features turn out to be correlated (hence dependent), except for three pairs of features whose p-value is above 0.05, namely (D2,B1) ($p = 0.060$), (D2,B4) ($p = 0.068$), and (D2, B12) ($p = 0.105$). Note that all three pairs contain D2 which corresponds to ‘Hand and Finger and Other Complex Mannerisms’. Furthermore, the computed correlation matrix does not contain any negative correlations, which makes sense given that all features are partial measures of ASD severity, along different dimensions, where higher means more severe.

⁶ The ADOS data used in this research are part of a database for autistic children that the ASD group, at the Child Development Center of the Hospital Garcia de Orta (Lisbon, Portugal), keeps for statistical purposes. All data was anonymous; only age and gender were collected from the sample for biographical characterization.

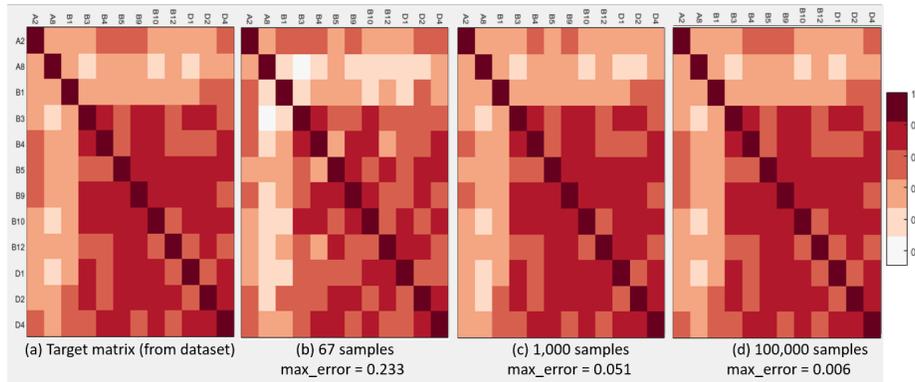


Fig. 3. Spearman correlation matrices of features for: (a) the real dataset (67 feature vectors), (b-d) synthetic datasets of size 67, 1,000, and 100,000 respectively, along with maximum absolute errors as compared to (a). Target feature marginals were set to uniform distributions.

Generating consistent (unconstrained) feature vectors The ABAS is a very large space which inevitably makes most reasonably sized datasets sparse. In particular, with our limited dataset (67 data points), it is important to enable the simulator to generalize from data to generate synthetic feature vectors that are consistent with our limited data points. In this work, we aim to generate feature vectors according to the correlational structure between features, as obtained from our dataset. In other words, we would like to generate correlated discrete (ordinal) data according to the dataset’s correlational profile. Several methods exist to achieve this, including the Gaussian copula [11], binary conversion, and mean mapping [8] methods. We use the mean mapping method, which gave best results for our application. The method takes as input the target marginal distributions and target correlation matrix of the features, and generates a set of feature vectors with (asymptotically) identical marginals and correlation matrix. The ordinal data is first mapped to the continuous space by computing a corresponding multivariate normal correlation matrix (achieved through function fitting). Multivariate normal data is then drawn according to this correlation matrix and reconverted to ordinal data. We used an R implementation of this method through the `orddata`⁷ package to generate the synthetic feature vectors analyzed in Fig. 3. The figure shows the obtained correlation matrices, which are consistent with the target correlation matrix (from the real dataset) and converge to it as the number of samples increases. We report the maximum absolute errors, defined as the maximum absolute difference between the sample matrices and the target matrix over all matrix entries. In order not to bias the generated total scores, target marginals were set to the uniform distribution.

Incorporating the constraint to achieve the mapping In our feature vector generation scheme, we have so far ensured that the sampling was consistent with real data,

⁷ <https://r-forge.r-project.org/R/?group.id=708>

but we have ignored the constraint on the L1 norm of the feature vector (Eq. 1). To incorporate the constraint, we iteratively generate unconstrained feature vectors until the total score constraint is satisfied. Unlike the bell-shaped distribution of Fig. 1, the statistical distribution of our generated (unconstrained) feature vectors is almost uniform, as shown in Fig. 4, which we attribute primarily to the target uniform marginals we enforced. This result suggests that the amount of computation needed to generate a feature vector for a given total score does not significantly rely on the value of that score as one might expect from the different subspace sparsities emphasized in Fig. 1.

4.3 Mapping feature vectors to behaviors

The problem of mapping a feature vector in ABAS to a set of behaviors in response to stimuli is a tricky one because there are usually more than a single behavior that fall under the same feature value and the degree of specificity in describing the different behaviors in the ADOS manual varies greatly. Also, there is always some level of subjectivity in the ADOS coding process which makes the generation and rendering of the different behaviors a sensitive task which would need to be backed up by extensive empirical studies if deployed in an actual system (beyond the scope of this paper).

As a first step in the mapping process, we defined a set of relevant features for each of the ADOS activities, which we summarize in Table 2. Relevant features capture the types of behaviors that are expected to be exhibited in - or are of special importance for - a particular activity. For example, in the activity ‘Response to joint attention’, the feature ‘Spontaneous Initiation of Joint Attention’ (B10) is relevant but the feature ‘Unusual Sensory Interest in Play Material/Person’ (D1) is not. These relevant features were chosen based on the nature of the activity as well as the observational guidelines included in the ADOS manual.

On the other hand, for each feature, we extracted from the ADOS manual one or more corresponding behavior(s) b_{ijk} , where $i \in \{1, \dots, 14\}$ is the feature index, $j \in \{0, 1, 2\}$ is the feature value, and $k \in \{1, \dots, K_{ij}\}$ is the behavior index. The ADOS manual describes the coding process by listing more or less specific behaviors that would fall under a given value for each feature. The database of behaviors b_{ijk} was manually

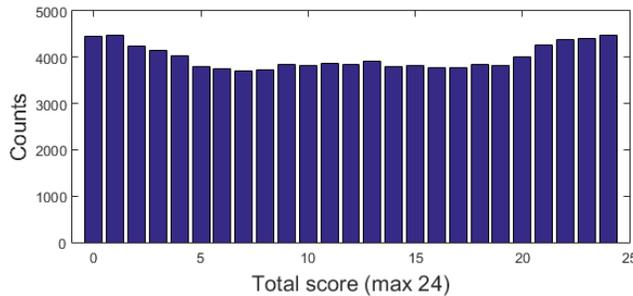


Fig. 4. Histogram of total scores for 100,000 generated feature vectors (only 12 features used).

Table 2. Relevant features for each activity considered (for both L='no words' and L='some words' combined).

Activity	Relevant features
Free play	A2,A7,A8,B5,B9,B10,B3,D1,D2,D4
Response to name	B1,B4,A2
Response to joint attention	B11,B1,A2,B5,B10,B12
Bubble play	B3,B10,B5,D1,D2,D4
Anticipation of a routine with objects	B10,B5,D2,D4
Responsive social smile	B3
Anticipation of a social routine	B4,B1,B3,A2,A8,B5
Functional and symbolic imitation	B4,B5
Birthday party	D4,B5,B12,B1,B3,B4
Snack	B1,A8,B3,A3,B12

compiled to include all those described behaviors. The number of behaviors that fall under the same value for a given feature ranges from a single behavior to up to 8 behaviors with varying degrees of similarity.

Sample behavior database entry for feature A7 (c_2):

- $c_2 = 0$: $\{b_{201}$:“Child points with index finger to show visually directed referencing” $\}$,
- $c_2 = 1$: $\{b_{211}$:“Child produces an approximation of pointing”, $b_{212} - b_{217}$:“Child (gazes)/(vocalizes) while (touching object)/(pointing to a person)/(pointing to himself/herself)” (all combinations) $\}$,
- $c_2 = 2$: $\{b_{221} - b_{222}$:“Child points when (close to)/(touching) object only, and with no gaze or vocalization”, b_{223} :“Child doesn't point” $\}$.

The behavior generation for a particular activity is done by sampling a behavior for each of the features relevant to the activity according to the given value of the feature. Because of the small number of behaviors that fall under the same (feature,value) pair, we opt for a simple uniform selection rule. Note that when selecting behaviors, the algorithm has to use the L value since the corresponding features slightly differ. We render generated behaviors as text, but one could imagine other ways of rendering them, such as for instance using an animated virtual avatar.

We implemented the pipeline of Fig. 2 in Python with a GUI to control the different simulator inputs. Fig. 5 shows an illustrative example of the different steps computed by ABASim for three different sets of input descriptors and activities.

5 Conclusion and Future Work

We presented ABASim, a method for simulating the behaviors of children with Autism Spectrum Disorders (ASD) of different severities, as a response to a range of stimuli. While the Autism Diagnostic Observation Schedule (ADOS), a standardized tool for diagnosing ASD, maps child behaviors to a score, our method aims at mapping a score (along with the age and language ability of the child) to a set of behaviors consistent

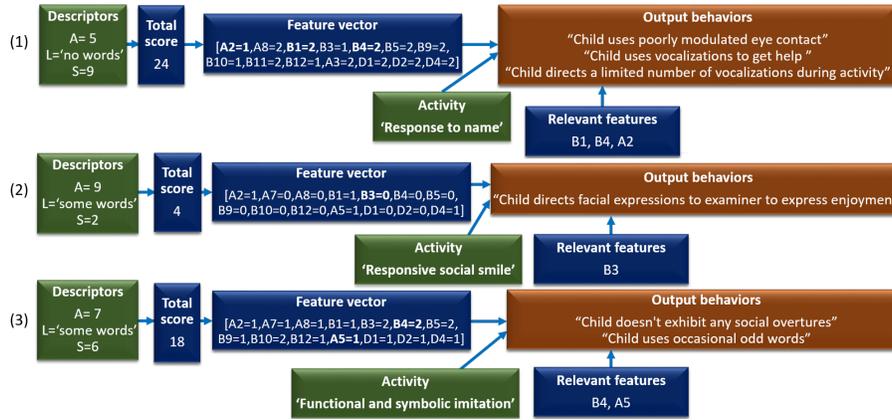


Fig. 5. Illustrative examples showing ABASim steps for three different inputs.

with these descriptors. We first defined the Autism-Based Autism Space (ABAS) where features correspond to ADOS codes. We then contributed a pipeline enabling us to generate behaviors from descriptors. In particular, our stochastic mapping from descriptors to a point in ABAS ensures a correlational structure between features that is consistent with actual ADOS data from 67 children suspected of having an ASD.

Our method could enable agents designed to enhance ASD therapy to reason better about interactions with children with ASD, accounting for individual behavioral differences. Another possible application that we foresee is for therapist training. Simulating the behavioral responses of virtual kids could provide new tools for therapists by enabling them to virtually interact with hypothetical children, which they cannot do only by observing videos. The simulator could also expose the therapists in training to a wide range of hypothetical cases of ASD that goes beyond the sample they physically interact with in their real professional life.

In the future, we would like to use a larger dataset to better inform our sampling process. Also, it would be useful to have some way of evaluating our generated feature vectors as well as behaviors to ensure they accurately reflect actual behavioral patterns. Finally, we are interested in integrating these simulation methods as part of the reasoning of an agent interacting with children with ASD, such as a mobile robot for therapy.

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