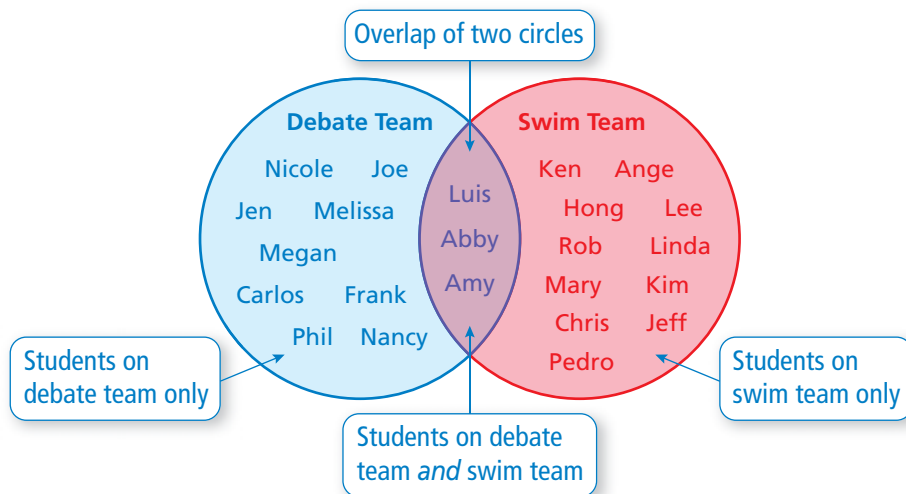


1.5 Greatest Common Factor

Essential Question

How can you find the greatest common factor of two numbers?

A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the names of students enrolled in two activities. Students enrolled in both activities are represented by the overlap of the two circles.

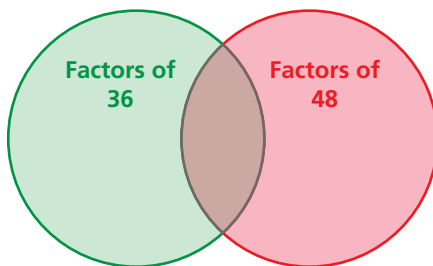


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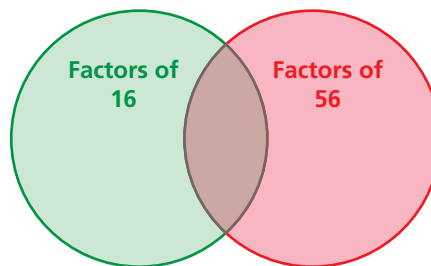
ACTIVITY: Identifying Common Factors

Work with a partner. Copy and complete the Venn diagram. Identify the *common factors* of the two numbers.

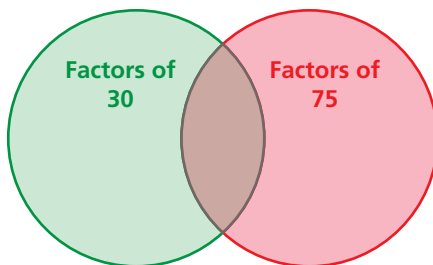
a. 36 and 48



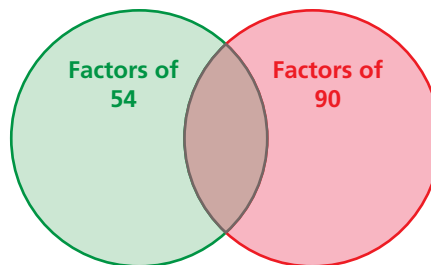
b. 16 and 56



c. 30 and 75



d. 54 and 90



Common Factors

In this lesson, you will

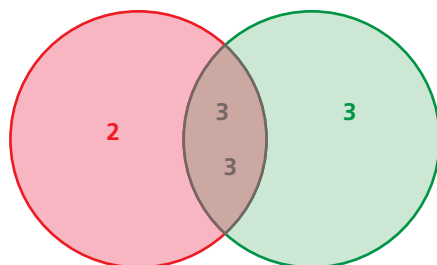
- use diagrams to identify common factors.
- find greatest common factors.

e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *greatest common factor* of each pair of numbers. Then circle it in each diagram.

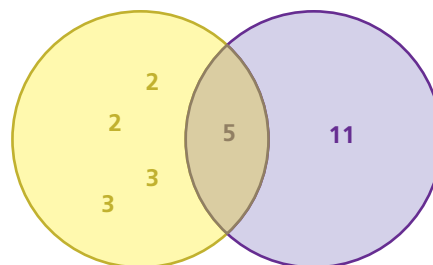
2 ACTIVITY: Interpreting a Venn Diagram of Prime Factors

Work with a partner. The Venn diagram represents the prime factorization of two numbers. Identify the two numbers. Explain your reasoning.

a.



b.



3 ACTIVITY: Identifying Common Prime Factors

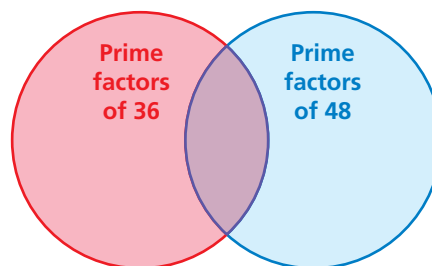
Math Practice

Interpret a Solution

What does the diagram of the resulting prime factorization mean?

Work with a partner.

- a. Write the prime factorizations of 36 and 48. Use the results to complete the Venn diagram.



- b. Repeat part (a) for the remaining number pairs in Activity 1.
- c. **STRUCTURE** Compare the numbers in the overlap of the Venn diagrams to your results in Activity 1. What conjecture can you make about the relationship between these numbers and your results in Activity 1?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the greatest common factor of two numbers? Give examples to support your explanation.
5. Can you think of another way to find the greatest common factor of two numbers? Explain.

Practice

Use what you learned about greatest common factors to complete Exercises 4–6 on page 34.

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor (GCF)**. One way to find the GCF of two or more numbers is by listing factors.

EXAMPLE 1 Finding the GCF Using Lists of Factors

Key Vocabulary

Venn diagram, p. 30
common factors,
p. 32
greatest common
factor, p. 32

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Circle the common factors.

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

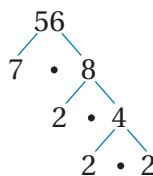
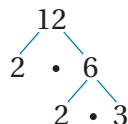
So, the GCF of 24 and 40 is 8.

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Make a factor tree for each number.



Write the prime factorization of each number.

$$12 = 2 \cdot 2 \cdot 3$$

Circle the common prime factors.

$$56 = 2 \cdot 2 \cdot 2 \cdot 7$$

$$2 \cdot 2 = 4$$

Find the product of the common prime factors.

So, the GCF of 12 and 56 is 4.

On Your Own

Now You're Ready
Exercises 7–18

Find the GCF of the numbers using lists of factors.

1. 8, 36

2. 18, 72

3. 14, 28, 49

Find the GCF of the numbers using prime factorizations.

4. 20, 45

5. 32, 90

6. 45, 75, 120

EXAMPLE 3 Finding Two Numbers with a Given GCF

Which pair of numbers has a GCF of 15?

- (A) 10, 15 (B) 30, 60 (C) 21, 45 (D) 45, 75

The number 15 cannot be a factor of the lesser number 10. So, you can eliminate Statement A.

The number 15 cannot be a factor of a number that does not have a 0 or 5 in the ones place. So, you can eliminate Statement C.

List the factors for Statements B and D. Then identify the GCF for each.

Choice B: Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

The GCF of 30 and 60 is 30.

Choice D: Factors of 45: 1, 3, 5, 9, 15, 45

Factors of 75: 1, 3, 5, 15, 25, 75

The GCF of 45 and 75 is 15.

∴ The correct answer is (D).

EXAMPLE 4 Real-Life Application

- * 18 bottles of nail polish
- * 24 pairs of earrings
- * 42 lollipops



You are filling piñatas for your sister's birthday party. The list shows the gifts you are putting into the piñatas. You want identical groups of gifts in each piñata with no gifts left over. What is the greatest number of piñatas you can make?

The GCF of the numbers of gifts represents the greatest number of identical groups of gifts you can make with no gifts left over. So, to find the number of piñatas, find the GCF.

$$\begin{aligned} 18 &= 2 \cdot 3 \cdot 3 \\ 24 &= 2 \cdot 3 \cdot 2 \cdot 2 \\ 42 &= 2 \cdot 3 \cdot 7 \end{aligned}$$

$$2 \cdot 3 = 6$$

Find the product of the common prime factors.

The GCF of 18, 24, and 42 is 6.

∴ So, you can make at most 6 piñatas.



Now You're Ready
Exercises 23–25

On Your Own

- Write a pair of numbers whose greatest common factor is 10.
- WHAT IF?** In Example 4, you add 6 more pairs of earrings. Does this change your answer? Explain your reasoning.



Vocabulary and Concept Check

- VOCABULARY** What is the greatest common factor (GCF) of two numbers?
- WRITING** Describe how to find the GCF of two numbers by using prime factorization.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is the greatest common factor of 24 and 32?

What is the greatest common divisor of 24 and 32?

What is the greatest common prime factor of 24 and 32?

What is the product of the common prime factors of 24 and 32?



Practice and Problem Solving

Use a Venn diagram to find the greatest common factor of the numbers.

4. 12, 30

5. 32, 54

6. 24, 108

Find the GCF of the numbers using lists of factors.

1 7. 6, 15

8. 14, 84

9. 45, 76

10. 39, 65

11. 51, 85

12. 40, 63

Find the GCF of the numbers using prime factorizations.

2 13. 45, 60

14. 27, 63

15. 36, 81

16. 72, 84

17. 61, 73

18. 189, 200

ERROR ANALYSIS Describe and correct the error in finding the GCF.

19.



$$42 = 2 \cdot 3 \cdot 7$$

$$154 = 2 \cdot 7 \cdot 11$$

The GCF is 7.

20.

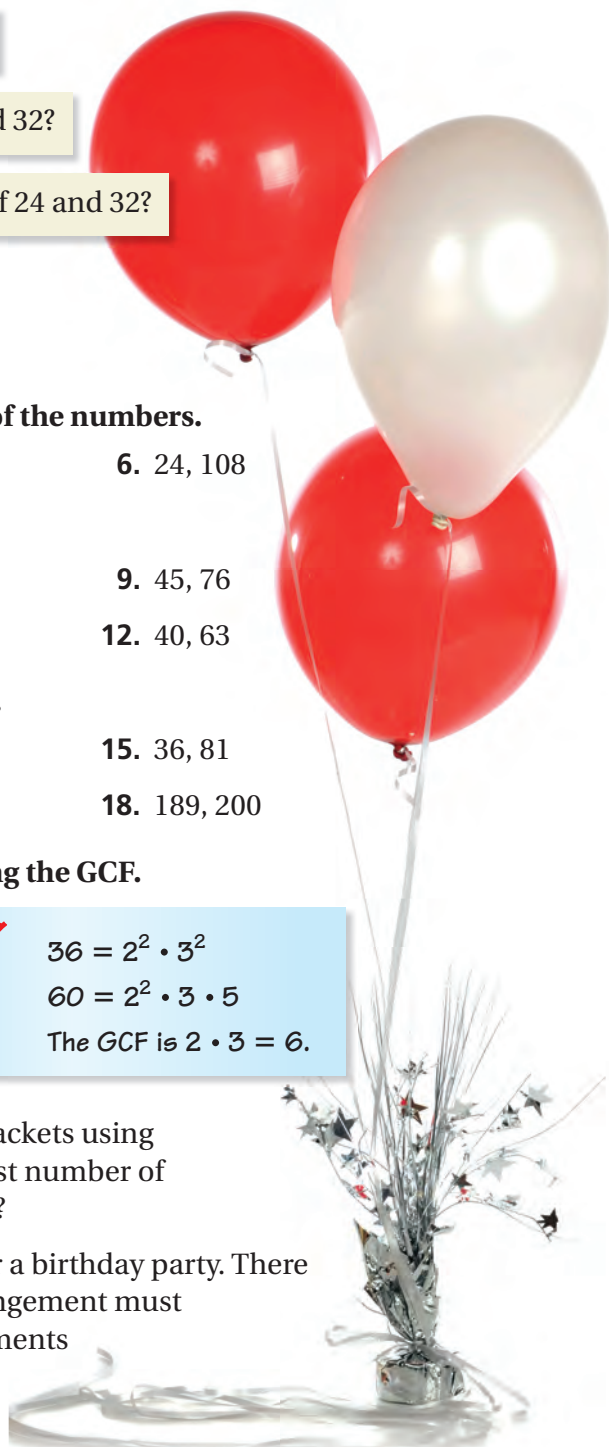


$$36 = 2^2 \cdot 3^2$$

$$60 = 2^2 \cdot 3 \cdot 5$$

The GCF is $2 \cdot 3 = 6$.

- CLASSROOM** A teacher is making identical activity packets using 92 crayons and 23 sheets of paper. What is the greatest number of packets the teacher can make with no items left over?
- BALLOONS** You are making balloon arrangements for a birthday party. There are 16 white balloons and 24 red balloons. Each arrangement must be identical. What is the greatest number of arrangements you can make using every balloon?



Find the GCF of the numbers.

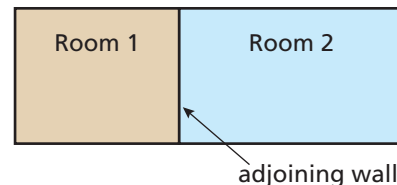
- 4 23. 35, 56, 63 24. 30, 60, 78 25. 42, 70, 84
26. **OPEN-ENDED** Write a set of three numbers that have a GCF of 16. What procedure did you use to find your answer?
27. **REASONING** You need to find the GCF of 256 and 400. Would you rather list their factors or use their prime factorizations? Explain.

CRITICAL THINKING Tell whether the statement is *always*, *sometimes*, or *never* true.

28. The GCF of two even numbers is 2.
29. The GCF of two prime numbers is 1.
30. When one number is a multiple of another, the GCF of the numbers is the greater of the numbers.
31. **BOUQUETS** A florist is making identical bouquets using 72 red roses, 60 pink roses, and 48 yellow roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?



32. **VENN DIAGRAM** Consider the numbers 252, 270, and 300.
- Create a Venn diagram using the prime factors of the numbers.
 - Use the Venn diagram to find the GCF of 252, 270, and 300.
 - What is the GCF of 252 and 270? 252 and 300? Explain how you found your answer.
33. **FRUIT BASKETS** You are making fruit baskets using 54 apples, 36 oranges, and 73 bananas.
- Explain why you cannot make identical fruit baskets without leftover fruit.
 - What is the greatest number of identical fruit baskets you can make with the least amount of fruit left over? Explain how you found your answer.
34. **Problem Solving** Two rectangular, adjacent rooms share a wall. One-foot-by-one-foot tiles cover the floor of each room. Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room. Draw a diagram that represents one possibility.



Fair Game Review What you learned in previous grades & lessons

Tell which property is being illustrated. (*Skills Review Handbook*)

35. $13 + (29 + 7) = 13 + (7 + 29)$ 36. $13 + (7 + 29) = (13 + 7) + 29$
37. $(6 \times 37) \times 5 = (37 \times 6) \times 5$ 38. $(37 \times 6) \times 5 = 37 \times (6 \times 5)$
39. **MULTIPLE CHOICE** In what order should you perform the operations in the expression $4 \times 3 - 12 \div 2 + 5$? (*Section 1.3*)
- (A) $\times, -, \div, +$ (B) $\times, \div, -, +$ (C) $\times, \div, +, -$ (D) $\times, +, -, \div$