2018

CFA® EXAM REVIEW

COVERS ALL TOPICS IN LEVEL I

LEVEL I CFA®

FORMULA SHEETS

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Quantitative Methods

The Time Value of Money

Effective Annual Rates

$$\text{EAR} = (1 + \text{Periodic interest rate})^{N} - 1$$

The Future Value of a Single Cash Flow

$$FV_N = PV\,(1+r)^N$$

The Present Value of a Single Cash Flow

$$PV = \frac{FV}{(1+r)^N}$$

The Present and Future Value of an Ordinary Annuity

$PV_{Annuity}$: # periods N; % interest per period I/Y; amount FV or amount $PMT \rightarrow PV$
$FV_{Annuity}$: # periods N; % interest per period I/Y; amount FV or amount $PMT \rightarrow FV$

The Present and Future Value of an Annuity Due

$$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1+r)$$
$$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1+r)$$

Present Value of a Perpetuity

$$PV_{Perpetuity} = \frac{PMT}{I/Y}$$

Continuous Compounding and Future Values

$$FV_N = PVe^{r_s \cdot N}$$

Discounted Cash Flow Applications

Net Present Value

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

where:
CF_t = the expected net cash flow at time t
N = the investment's projected life
r = the discount rate or appropriate cost of capital

Internal Rate of Return

$$NPV = 0 = \sum_{t=0}^{n} \frac{CF_t}{(1+IRR)^t}$$

Solved as an iterative process using calculator TVM functions.

Bank Discount Yield

$$r_{BD} = \frac{D}{F} \times \frac{360}{t}$$

where:
r_{BD} = the annualized yield on a bank discount basis
D = the dollar discount (face value – purchase price)
F = the face value of the bill
t = number of days remaining until maturity

Holding Period Yield

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

where:
P_0 = initial price of the investment.
P_1 = price received from the instrument at maturity/sale.
D_1 = interest or dividend received from the investment.

Money-weighted rate of return

$$r_{mw} = IRR_{CF}$$

Time-weighted rate of return

$$r_{tw} = \left[(1+HPY_1)(1+HPY_2)\ldots(1+HPY_n)\right]^{1/n} - 1$$
$$= \left[\prod_{t=1}^{n}(1+HPY_t)\right]^{1/n} - 1$$

Effective Annual Yield

$$EAY = (1 + HPY)^{365/t} - 1$$

where:
HPY = holding period yield
t = numbers of days remaining till maturity

$$HPY = (1 + EAY)^{t/365} - 1$$

Money Market Yield

$$R_{MM} = \frac{360 \times r_{BD}}{360 - (t \times r_{BD})}$$

$$R_{MM} = HPY \times (360/t)$$

Bond Equivalent Yield

$$BEY = [(1 + EAY)^{0.5} - 1] \times 2$$

Statistical Concepts

Population Mean

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

where:
x_i = is the *i*th observation.

Sample Mean

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Geometric Mean

$$1 + R_G = \sqrt[T]{(1+R_1)\times(1+R_2)\times\ldots\times(1+R_T)} \quad \text{OR} \quad G = \sqrt[n]{X_1X_2X_3\ldots X_n}$$

with $X_i \geq 0$ for $i = 1, 2, \ldots, n$.

$$R_G = \left[\prod_{t=1}^{T}(1+R_t)\right]^{\frac{1}{T}} - 1$$

Harmonic Mean

Harmonic mean: $\overline{X}_H = \frac{N}{\sum_{i=1}^{N}\frac{1}{x_i}}$ with $X_i > 0$ for $i = 1,2,\ldots,N$.

Percentiles

$$L_y = \frac{(n+1)y}{100}$$

where:
y = percentage point at which we are dividing the distribution
L_y = location (L) of the percentile (P_y) in the data set sorted in ascending order

Range

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

Mean Absolute Deviation

$$\text{MAD} = \frac{\sum_{i=1}^{n} \left| X_i - \bar{X} \right|}{n}$$

where:
n = number of items in the data set
$\bar{X}$ = the arithmetic mean of the sample

Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

where:
X_i = observation i
μ = population mean
N = size of the population

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

Sample Variance

$$\text{Sample variance} = s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

where:
n = sample size.

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}$$

Coefficient of Variation

$$\text{Coefficient of variation} = \frac{s}{\bar{X}}$$

where:
s = sample standard deviation
$\bar{X}$ = the sample mean.

Sharpe Ratio

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{s_p}$$

where:
$\bar{r}_p$ = mean portfolio return
r_f = risk-free return
s_p = standard deviation of portfolio returns

Sample skewness, also known as sample relative skewness, is calculated as:

$$S_K = \left[\frac{n}{(n-1)(n-2)}\right]\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3}$$

As n becomes large, the expression reduces to the mean cubed deviation.

$$S_K \approx \frac{1}{n}\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3}$$

where:
s = sample standard deviation

Sample Kurtosis uses standard deviations to the fourth power. Sample excess kurtosis is calculated as:

$$K_E = \left(\frac{n(n+1)}{(n-1)(n-2)(n-3)}\frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{s^4}\right) - \frac{3(n-1)^2}{(n-2)(n-3)}$$

As n becomes large the equation simplifies to:

$$K_E \approx \frac{1}{n}\frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{s^4} - 3$$

where:
s = sample standard deviation

For a sample size greater than 100, a sample excess kurtosis of greater than 1.0 would be considered unusually high. Most equity return series have been found to be leptokurtic.

Probability Concepts

Odds for an Event

$$P(E) = \frac{a}{(a+b)}$$

Where the odds for are given as "a to b", then:

Odds for an Event

$$P(E) = \frac{b}{(a+b)}$$

Where the odds *against* are given as "a to b", then:

Conditional Probabilities

$$P(A|B) = \frac{P(AB)}{P(B)} \text{ given that } P(B) \neq 0$$

Multiplication Rule for Probabilities

$$P(AB) = P(A|B) \times P(B)$$

Addition Rule for Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

For Independent Events

$$P(A|B) = P(A), \text{ or equivalently, } P(B|A) = P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

The Total Probability Rule

$$P(A) = P(AS) + P(AS^c)$$

$$P(A) = P(A|S) \times P(S) + P(A|S^c) \times P(S^c)$$

The Total Probability Rule for *n* Possible Scenarios

$$P(A) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + \cdots + P(A|S_n) \times P(S_n)$$

where the set of events $\{S_1, S_2, \ldots, S_n\}$ is mutually exclusive and exhaustive.

Expected Value

$$E(X) = P(X_1)X_1 + P(X_2)X_2 + \ \dots \ P(X_n)X_n$$

$$E(X) = \sum_{i=1}^{n} P(X_i)X_i$$

where:
X_i = one of *n* possible outcomes.

Variance and Standard Deviation

$$\sigma^2(X) = E\{[X - E(X)]^2\}$$

$$\sigma^2(X) = \sum_{i=1}^{n} P(X_i)\ [X_i - E(X)]^2$$

The Total Probability Rule for Expected Value

1. $E(X) = E(X|S)P(S) + E(X|S^c)P(S^c)$
2. $E(X) = E(X|S_1) \times P(S_1) + E(X|S_2) \times P(S_2) + \ldots + E(X|S_n) \times P(S_n)$

where:
$E(X)$ = the unconditional expected value of X
$E(X|S_1)$ = the expected value of X given Scenario 1
$P(S_1)$ = the probability of Scenario 1 occurring
The set of events $\{S_1, S_2, \ldots, S_n\}$ is mutually exclusive and exhaustive.

Covariance

$$Cov(XY) = E\{[X - E(X)][Y - E(Y)]\}$$
$$Cov(R_A, R_B) = E\{[R_A - E(R_A)][R_B - E(R_B)]\}$$

Correlation Coefficient

$$Corr(R_A, R_B) = \rho(R_A, R_B) = \frac{Cov(R_A, R_B)}{(\sigma_A)(\sigma_B)}$$

Expected Return on a Portfolio

$$E(R_p)=\sum_{i=1}^{N}w_iE(R_i)=w_1E(R_1)+w_2E(R_2)+\cdots+w_NE(R_N)$$

where:

$$\text{Weight of asset i}=\frac{\text{Market value of investment i}}{\text{Market value of portfolio}}$$

Portfolio Variance

$$Var(R_p)=\sum_{i=1}^{N}\sum_{j=1}^{N}w_iw_jCov(R_i,R_j)$$

Variance of a 2 Asset Portfolio

$$Var(R_p)=w_A^2\sigma^2(R_A)+w_B^2\sigma^2(R_B)+2w_Aw_BCov(R_A,R_B)$$

$$Var(R_p)=w_A^2\sigma^2(R_A)+w_B^2\sigma^2(R_B)+2w_Aw_B\rho(R_A,R_B)\sigma(R_A)\sigma(R_B)$$

Variance of a 3 Asset Portfolio

$$\begin{aligned}Var(R_p)&=w_A^2\sigma^2(R_A)+w_B^2\sigma^2(R_B)+w_C^2\sigma^2(R_C)\\&\quad+2w_Aw_BCov(R_A,R_B)+2w_Bw_CCov(R_B,R_C)+2w_Cw_ACov(R_C,R_A)\end{aligned}$$

Bayes' Formula

$$P(\text{Event}\,|\,\text{Information})=\frac{P\ (\text{Information}\,|\,\text{Event})\times P\ (\text{Event})}{P\ (\text{Information})}$$

Counting Rules

The number of different ways that the k tasks can be done equals $n_1\times n_2\times n_3\times\ldots n_k$.

Combinations

$${}_nC_r=\binom{n}{r}=\frac{n!}{(n-r)!(r!)}$$

Remember: The combination formula is used when the order in which the items are assigned the labels is NOT important.

Permutations

$${}_nP_r=\frac{n!}{(n-r)!}$$

Common Probability Distributions

Discrete Uniform Distribution

$$F(x) = n \times p(x) \text{ for the } n\text{th observation.}$$

Binomial Distribution

$$P(X{=}x) = {}_nC_x\,(p)^x\,(1-p)^{n-x}$$

where:
p = probability of success
1 − p = probability of failure
${}_nC_x$ = number of possible combinations of having *x* successes in *n* trials. Stated differently, it is the number of ways to choose *x* from *n* when the order does not matter.

Mean of a Binomial Random Variable

$$\overline{B(n,p)} = np$$

Variance of a Binomial Random Variable

$$\sigma_x^2 = n \times p \times (1-p)$$

The Continuous Uniform Distribution

$$P(X < a),\ P\,(X > b) = 0$$

$$P\,(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

Confidence Intervals

For a random variable X that follows the normal distribution:
The 90% confidence interval is $\bar{x} - 1.65s$ to $\bar{x} + 1.65s$
The 95% confidence interval is $\bar{x} - 1.96s$ to $\bar{x} + 1.96s$
The 99% confidence interval is $\bar{x} - 2.58s$ to $\bar{x} + 2.58s$

The following probability statements can be made about normal distributions

- Approximately 50% of all observations lie in the interval $\mu \pm (2/3)\sigma$
- Approximately 68% of all observations lie in the interval $\mu \pm 1\sigma$
- Approximately 95% of all observations lie in the interval $\mu \pm 2\sigma$
- Approximately 99% of all observations lie in the interval $\mu \pm 3\sigma$

z-Score

$$z = (\text{observed value} - \text{population mean})/\text{standard deviation} = (x - \mu)/\sigma$$

Roy's Safety-First Criterion

Minimize $P(R_P < R_T)$

where:
R_P = portfolio return
R_T = target return

Shortfall Ratio

$$\text{Shortfall ratio (SF Ratio)} = \frac{E(R_P) - R_T}{\sigma_P}$$

Continuously Compounded Returns

$$EAR = e^{r_{cc}} - 1 \qquad r_{cc} = \text{continuously compounded annual rate}$$

$$HPR_t = e^{r_{cc} \times t} - 1$$

Sampling and Estimation

Sampling Error

Sampling error of the mean = Sample mean − Population mean = $\bar{x} - \mu$

Standard Error of Sample Mean when Population Variance is known

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

where:
$\sigma_{\bar{x}}$ = the standard error of the sample mean
σ = the population standard deviation
n = the sample size

Standard Error of Sample Mean when Population Variance is not known

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where:
$s_{\bar{x}}$ = standard error of sample mean
s = sample standard deviation.

Confidence Intervals

Point estimate ± (reliability factor × standard error)

where:
Point estimate = value of the sample statistic that is used to estimate the population parameter
Reliability factor = a number based on the assumed distribution of the point estimate and the level of confidence for the interval $(1 - \alpha)$.
Standard error = the standard error of the sample statistic (point estimate)

$$\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

where:

$\overline{x}$ = The sample mean (point estimate of population mean)
$z_{\alpha/2}$ = The standard normal random variable for which the probability of an observation lying in either tail is σ / 2 (reliability factor).
$\frac{\sigma}{\sqrt{n}}$ = The standard error of the sample mean.

$$\overline{x} \pm t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}$$

where:
$\overline{x}$ = sample mean (the point estimate of the population mean)
$t_{\frac{\alpha}{2}}$ = the t-reliability factor
$\frac{s}{\sqrt{n}}$ = standard error of the sample mean
s = sample standard deviation

Hypothesis Testing

Test Statistic

$$\text{Test statistic} = \frac{\text{Sample statistic} - \text{Hypothesized value}}{\text{Standard error of sample statistic}}$$

Power of a Test

$$\text{Power of a test} = 1 - P(\text{Type II error})$$

Decision Rules for Hypothesis Tests

Decision	H_0 is True	H_0 is False
Do not reject H_0	Correct decision	Incorrect decision **Type II error**
Reject H_0	Incorrect decision **Type I error** Significance level = P(Type I error)	Correct decision Power of the test = 1 − P(Type II error)

Confidence Interval

$$\left[\begin{pmatrix}\text{sample}\\\text{statistic}\end{pmatrix} - \begin{pmatrix}\text{critical}\\\text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\\text{error}\end{pmatrix}\right] \le \begin{pmatrix}\text{population}\\\text{parameter}\end{pmatrix} \le \left[\begin{pmatrix}\text{sample}\\\text{statistic}\end{pmatrix} + \begin{pmatrix}\text{critical}\\\text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\\text{error}\end{pmatrix}\right]$$

$$\bar{x} - (z_{\alpha/2})(s/\sqrt{n}) \le \mu_0 \le \bar{x} + (z_{\alpha/2})(s/\sqrt{n})$$

Summary

Type of test	Null hypothesis	Alternate hypothesis	Reject null if	Fail to reject null if	P-value represents
One tailed (upper tail) test	$H_0 : \mu \le \mu_0$	$H_a : \mu > \mu_0$	Test statistic > critical value	Test statistic ≤ critical value	Probability that lies above the computed test statistic.
One tailed (lower tail) test	$H_0 : \mu \ge \mu_0$	$H_a : \mu < \mu_0$	Test statistic < critical value	Test statistic ≥ critical value	Probability that lies below the computed test statistic.
Two-tailed	$H_0 : \mu = \mu_0$	$H_a : \mu \ne \mu_0$	Test statistic < lower critical value Test statistic > upper critical value	Lower critical value ≤ test statistic ≤ upper critical value	Probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic.

t-Statistic

$$\text{t-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where:
x = sample mean
μ_0 = hypothesized population mean
s = standard deviation of the sample
n = sample size

z-Statistic

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

where:
x = sample mean
μ = hypothesized population mean
σ = standard deviation of the population
n = sample size

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where:
x = sample mean
μ = hypothesized population mean
s = standard deviation of the sample
n = sample size

Tests for Means when Population Variances are Assumed Equal

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{1/2}}$$

where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

degrees of freedom = $n_1 + n_2 - 2$

Tests for Means when Population Variances are Assumed Unequal

$$\text{t-stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^{1/2}}$$

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(s_1^2/n_1\right)^2}{n_1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2}}$$

where:

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

Paired Comparisons Test

$$t = \frac{\bar{d} - \mu_{dz}}{s_{\bar{d}}}$$

where:
$\bar{d}$ = sample mean difference
$s_{\bar{d}}$ = standard error of the mean difference = $\dfrac{s_d}{\sqrt{n}}$
s_d = sample standard deviation
n = the number of paired observations

Hypothesis Tests Concerning the Mean of Two Populations - Appropriate Tests

Population distribution	Relationship between samples	Assumption regarding variance	Type of test
Normal	Independent	Equal	t-test pooled variance
Normal	Independent	Unequal	t-test with variance not pooled
Normal	Dependent	N/A	t-test with paired comparisons

Chi Squared Test-Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

where:
n = sample size
s^2 = sample variance
σ_0^2 = hypothesized value for population variance

Test-Statistic for the F-Test

$$F = \frac{s_1^2}{s_2^2}$$

where:
s_1^2 = Variance of sample drawn from Population 1
s_2^2 = Variance of sample drawn from Population 2

Hypothesis tests concerning the variance

Hypothesis Test Concerning	Appropriate Test Statistic
Variance of a single, normally distributed population	Chi-square stat
Equality of variance of two independent, normally distributed populations	F-stat

TECHNICAL ANALYSIS

Setting Price Targets with Head and Shoulders Patterns

$$\text{Price target} = \text{Neckline} - (\text{Head} - \text{Neckline})$$

Setting Price Targets for Inverse Head and Shoulders Patterns

$$\text{Price target} = \text{Neckline} + (\text{Neckline} - \text{Head})$$

Momentum or Rate of Change Oscillator

$$M = (V - Vx) \times 100$$

where:
M = momentum oscillator value
V = last closing price
Vx = closing price x days ago, typically 10 days

Relative Strength Index

$$RSI = 100 - \frac{100}{1 + RS}$$

where:

$$RS = \frac{\Sigma(\text{Up changes for the period under consideration})}{\Sigma(|\text{Down changes for the period under consideration}|)}$$

Stochastic Oscillator

$$\%K = 100\left(\frac{C - L14}{H14 - L14}\right)$$

where:
C = last closing price
L14 = lowest price in last 14 days
H14 = highest price in last 14 days

%D (signal line) = Average of the last three %K values calculated daily.

Short Interest ratio

$$\text{Short interest ratio} = \frac{\text{Short interest}}{\text{Average daily trading volume}}$$

Arms Index

$$\text{Arms index} = \frac{\text{Number of advancing issues / Number of declining issues}}{\text{Volume of advancing issues / Volume of declining issues}}$$

Economics

Topics in Demand and Supply Analysis

The demand function captures the effect of all these factors on demand for a good.

$$\text{Demand function: } QD_x = f(P_x, I, P_y, \ldots) \quad \ldots \text{ (Equation 1)}$$

Equation 1 is read as "the quantity demanded of Good X (QD_X) depends on the price of Good X (P_X), consumers' incomes (I) and the price of Good Y (P_Y), etc."

The own-price elasticity of demand is calculated as:

$$ED_{Px} = \frac{\%\Delta QD_x}{\%\Delta P_x} \quad \ldots \text{ (Equation 6)}$$

If we express the percentage change in X as the change in X divided by the value of X, Equation 6 can be expanded to the following form:

Slope of demand function.

Coefficient on own-price in market demand function

$$ED_{Px} = \frac{\%\Delta QD_x}{\%\Delta P_x} = \frac{\Delta QD_x / QD_x}{\Delta P_x / P_x} = \left(\frac{\Delta QD_x}{\Delta P_x}\right)\left(\frac{P_x}{QD_x}\right) \quad \ldots \text{ (Equation 7)}$$

Arc elasticity is calculated as:

$$E_P = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}} = \frac{\%\,\Delta\,Q_d}{\%\,\Delta\,P} = \frac{\dfrac{(Q_0 - Q_1)}{(Q_0 + Q_1)/2} \times 100}{\dfrac{(P_0 - P_1)}{(P_0 + P_1)/2} \times 100}$$

Income Elasticity of Demand

Income elasticity of demand measures the responsiveness of demand for a particular good to a change in income, holding all other things constant.

$$ED_I = \frac{\%\Delta QD_x}{\%\Delta I} = \frac{\Delta QD_x / QD_x}{\Delta I / I} = \left(\frac{\Delta QD_x}{\Delta I}\right)\left(\frac{I}{QD_x}\right) \quad \ldots \text{(Equation 8)}$$

Same as coefficient on I in market demand function (Equation 11)

$$E_I = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in income}}$$

Cross-Price Elasticity of Demand

Cross elasticity of demand measures the responsiveness of demand for a particular good to a change in price of *another* good, holding all other things constant.

$$ED_{Py} = \frac{\%\Delta QD_x}{\%\Delta P_y} = \frac{\Delta QD_x / QD_x}{\Delta P_y / P_y} = \left(\frac{\Delta QD_x}{\Delta P_y}\right)\left(\frac{P_y}{QD_x}\right) \quad \ldots \text{(Equation 9)}$$

Same as coefficient on P_Y in market demand function (Equation 11)

$$E_C = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price of substitute or complement}}$$

Accounting Profit

Accounting profit (loss) = Total revenue – Total explicit (accounting) costs.

Economic Profit

Also known as abnormal profit or supernormal profit:

Economic profit = Total revenue – Total economic costs
= Total revenue – (Explicit costs + Implicit costs)
= Accounting profit – Implicit costs

Under perfect competition, economic profit = 0

Normal Profit

Normal profit = Accounting profit – Economic profit
= Implicit costs

Total, Average, and Marginal Revenue

Table: Summary of Revenue Terms

Revenue	Calculation
Total revenue (TR)	Price times quantity ($P \times Q$), or the sum of individual units sold times their respective prices; $\Sigma(P_i \times Q_i)$
Average revenue (AR)	Total revenue divided by quantity; (TR / Q)
Marginal revenue (MR)	Change in total revenue divided by change in quantity; ($\Delta TR / \Delta Q$)

Total, Average, Marginal, Fixed, and Variable Costs

Table: Summary of Cost Terms

Costs	Calculation
Total fixed cost (TFC)	Sum of all fixed expenses; here defined to include all opportunity costs
Total variable cost (TVC)	Sum of all variable expenses, or per unit variable cost times quantity; (per unit VC × Q)
Total costs (TC)	Total fixed cost plus total variable cost; (TFC + TVC)
Average fixed cost (AFC)	Total fixed cost divided by quantity; (TFC / Q)
Average variable cost (AVC)	Total variable cost divided by quantity; (TVC / Q)
Average total cost (ATC)	Total cost divided by quantity; (TC / Q) or (AFC + AVC)
Marginal cost (MC)	Change in total cost divided by change in quantity; (ΔTC / ΔQ)

Breakeven, Shutdown, and Exit Points

Revenue/ Cost Relationship	Short-run Decision	Long-run Decision
TR = TC	Continue operating	Continue operating
TR > TVC, but < TC	Continue operating	Exit market
TR < TVC	Shut down production	Exit market

The Firm And Market Structures

Relationship of Marginal Revenue to Price Elasticity

The relationship between MR and price elasticity can be expressed as:

$$MR = P[1-(1/E_p)]$$

In a monopoly, MC = MR so:

$$P[1-(1/E_p)] = MC$$

QSMax where MC = MR in any market, and MC = MR = ATC in perfectly competitive markets.

Income Elasticity

$$E_Y = \frac{\%\Delta Q_D}{\%\Delta Y}$$

Inferior goods: $E_Y < 0$
Normal goods: $0 < E_Y < 1$
Luxury goods: $E_Y > 1$

Changes shift entire demand curve rather than cause movement along it.

Cross-Price Elasticity

$$E_{P_Y} = \frac{\%\Delta Q_{DX}}{\%\Delta P_Y}$$

Substitutes: EPY > 0 (i.e., direct relationship; P_Y increases, QD_X increases)
Complements: EPY < 0 (i.e., inverse relationship; P_Y increases, QD_X decreases)
Changes shift entire demand curve rather than cause movement along it.

Concentration Measures

N-firm concentration ratio: Aggregate market share of N largest firms in the industry. The ratio will equal 0 for perfect competition and 100 for a monopoly.

Herfindahl-Hirschman Index (HHI): Adds up the squares of the market shares of each of the largest N companies in the market. The HHI equals 1 for a monopoly. If there are M firms in the industry with equal market shares, the HHI will equal 1/M.

Aggregate Output, Price, And Economic Growth

Nominal GDP refers to the value of goods and services included in GDP measured at **current prices**.

$$\text{Nominal GDP} = \text{Quantity produced in Year t} \times \text{Prices in Year t}$$

Real GDP refers to the value of goods and services included in GDP measured at **base-year prices**.

$$\text{Real GDP} = \text{Quantity produced in Year t} \times \text{Base-year prices}$$

GDP Deflator

$$\text{GDP deflator} = \frac{\text{Value of current year output at current year prices}}{\text{Value of current year output at base year prices}} \times 100$$

$$\text{GDP deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$$

The Components of GDP

Based on the expenditure approach, GDP may be calculated as:

$$GDP = C + I + G + (X - M)$$

C = Consumer spending on final goods and services
I = Gross private domestic investment, which includes business investment in capital goods (e.g. plant and equipment) and changes in inventory (inventory investment)
G = Government spending on final goods and services
X = Exports
M = Imports

Expenditure Approach

Under the expenditure approach, GDP at market prices may be calculated as:

$$\begin{aligned} \text{GDP} = & \text{ Consumer spending on goods and services} \\ & + \text{Business gross fixed investment} \\ & + \text{Change in inventories} \\ & + \text{Government spending on goods and services} \\ & + \text{Government gross fixed investment} \\ & + \text{Exports} - \text{Imports} \\ & + \text{Statistical discrepancy} \end{aligned}$$

This equation is just a breakdown of the expression for GDP we stated in the previous LOS, i.e. GDP = C + I + G + (X − M).

Income Approach

Under the income approach, GDP at market prices may be calculated as:

GDP = National income + Capital consumption allowance
+ Statistical discrepancy ... (Equation 1)

National income equals the sum of incomes received by all factors of production used to generate final output. It includes:

- Employee compensation
- Corporate and government enterprise profits before taxes, which includes:
  - Dividends paid to households
  - Corporate profits retained by businesses
  - Corporate taxes paid to the government
- Interest income
- Rent and unincorporated business net income (proprietor's income): Amounts earned by unincorporated proprietors and farm operators, who run their own businesses.
- Indirect business taxes less subsidies: This amount reflects taxes and subsidies that are included in the final price of a good or service, and therefore represents the portion of national income that is directly paid to the government.

The **capital consumption allowance (CCA)** accounts for the wear and tear or depreciation that occurs in capital stock during the production process. It represents the amount that must be reinvested by the company in the business to maintain current productivity levels. You should think of profits + CCA as the amount earned by capital.

Personal income = National income
− Indirect business taxes
− Corporate income taxes
− Undistributed corporate profits
+ Transfer payments ... (Equation 2)

Personal disposable income = Personal income − Personal taxes ... (Equation 3)

Personal disposable income = Household consumption + Household saving
... (Equation 4)

Household saving = Personal disposable income − Consumption expenditures − Interest paid by consumers to businesses − Personal transfer payments to foreigners

… (Equation 5)

Business sector saving = Undistributed corporate profits + Capital consumption allowance

… (Equation 6)

GDP = Household consumption + Total private sector saving + Net taxes

The equality of expenditure and income

$$\mathbf{S = I + (G - T) + (X - M)}$$ … (Equation 7)

The IS Curve (Relationship between Income and the Real Interest Rate)

Disposable income = GDP − Business saving − Net taxes

$$\mathbf{S - I = (G - T) + (X - M)}$$ **… (Equation 7)**

The LM Curve

Quantity theory of money: $MV = PY$

The quantity theory equation can also be written as:

$$M/P \text{ and } M_D/P = kY$$

where:
$k = I/V$
M = Nominal money supply
M_D = Nominal money demand
M_D/P is referred to as real money demand and M/P is real money supply.

Equilibrium in the money market requires that money supply and money demand be equal.

Money market equilibrium: $M/P = RM_D$

Solow (neoclassical) growth model

$$Y = AF(L,K)$$

where:
Y = Aggregate output
L = Quantity of labor
K = Quantity of capital
A = Technological knowledge or total factor productivity (TFP)

Growth Accounting Equation

Growth in potential GDP = Growth in technology + W_L (Growth in labor) + W_K (Growth in capital)

Growth in per capita potential GDP = Growth in technology + W_K (Growth in capital-labor ratio)

Measures of Sustainable Growth

Labor productivity = Real GDP/Aggregate hours

Potential GDP = Aggregate hours × Labor productivity

This equation can be expressed in terms of growth rates as:

Potential GDP growth rate = Long-term growth rate of labor force + Long-term labor productivity growth rate

Understanding Business Cycles

Unit labor cost (ULC) is calculated as:

$$ULC = W/O$$

where:
O = Output per hour per worker
W = Total labor compensation per hour per worker

Monetary And Fiscal Policy

Required reserve ratio = Required reserves / Total deposits

Money multiplier = 1/ (Reserve requirement)

Quantity Theory of Money

$$M = PY / V$$

Where:
M = Money supply
V = velocity of transactions
P = price level
Y = real output

Quantity Equation of Exchange

$$MV = PY$$

The Fischer effect states that the nominal interest rate (R_N) reflects the real interest rate (R_R) and the expected rate of inflation (II^e).

$$R_N = R_R + \Pi^e$$

The Fiscal Multiplier

Ignoring taxes, the multiplier can also be calculated as:

- $$\frac{1}{(1 - MPC)} = \frac{1}{(1 - 0.9)} = 10$$

Assuming taxes, the multiplier can also be calculated as:

$$\frac{1}{[1 - MPC(1 - t)]}$$

International Trade And Capital Flows

Balance of Payment Components

A country's balance of payments is composed of three main accounts:

- The current account balance largely reflects trade in goods and services.
- The capital account balance mainly consists of capital transfers and net sales of non-produced, non-financial assets.
- The financial account measures net capital flows based on sales and purchases of domestic and foreign financial assets.

Current Account

$$CA = X - M = Y - (C + I + G)$$

Currency Exchange Rates

Real exchange rate

$$\text{Real exchange rate}_{DC/FC} = S_{DC/FC} \times (P_{FC}/P_{DC})$$

where:
$S_{DC/FC}$ = Nominal spot exchange rate
P_{FC} = Foreign price level quoted in terms of the foreign currency
P_{DC} = Domestic price level quoted in terms of the domestic currency

Relative Currency Movement

Where P is the *price currency* (or *quote currency*) and B is the *base currency*:

$$E(\%\Delta S_{P/B}) = \frac{E(S_{P/B})}{S_{P/B}} - 1$$

Premium when E(S) > S; expect *depreciation* of price currency.
Discount when E(S) < S; expect *appreciation* of price currency.

Currency Cross Rates

$$\frac{P_1}{B} \div \frac{P_2}{B} = \frac{P_1}{\cancel{B}} \times \frac{\cancel{B}}{P_2} = \frac{P_1}{P_2}$$

Arbitrage Relationship

Forward pricing is based on a sum of money invested domestically in the base currency at the domestic interest rate, r_B, as equivalent to the same sum of money converted at the spot rate for P units of price (foreign) currency, invested at the foreign rate r_P for the same time, and converted back to domestic currency at a forward price set at the beginning of the term.

$$(1 + r_B) = S_{P/B}(1 + r_P)\left(\frac{1}{F_{P/B}}\right)$$

$$F_{P/B} = S_{P/B}\frac{(1 + r_P)}{(1 + r_B)}$$

The last term $1/F_{P/B}$ in the first equation can also be written $F_{B/P}$ or $F_{d/f}$.

Forward Premium/(Discount)

Using the previous relationship between forward and spot rates, the forward premium/(discount) approximately equates to the price currency (foreign) interest rate versus base currency (domestic) interest rate.

$$\frac{F_{P/B}}{S_{P/B}} = \frac{(1+r_P)}{(1+r_B)}$$

$$\frac{F_{P/B}}{S_{P/B}} - 1 = \frac{(1+r_P)}{(1+r_B)} - 1$$

$$= \frac{(1+r_P)}{(1+r_B)} - \frac{(1+r_B)}{(1+r_B)} = \frac{r_P - r_B}{(1+r_B)}$$

A higher base currency (domestic) interest rate results in a forward discount of approximately the interest differential percentage, leading to base currency (domestic) appreciation of that percentage.

This relationship must hold or arbitrage will take place to realign spot and forward prices with the ratedifferential. However, the *expected* spot exchange rate may differ from the forward exchange rate.

The forward rate may be calculated as:

$$F_{DC/FC} = \frac{1}{S_{FC/DC}} \times \frac{(1+r_{DC})}{(1+r_{FC})} \text{ or } F_{DC/FC} = S_{DC/FC} \times \frac{(1+r_{DC})}{(1+r_{FC})}$$

This version of the formula is perhaps easiest to remember because it contains the DC term in numerator for all three components: $F_{DC/FC}$, $S_{DC/FC}$ and $(1 + r_{DC})$

Forward rates are sometimes interpreted as expected future spot rates.

$$F_t = S_{t+1}$$

$$\frac{(S_{t+1})}{S} - 1 = \Delta\%S(DC/FC)_{t+1} = \frac{(r_{DC} - r_{FC})}{(1+r_{FC})}$$

Exchange Rates and the Trade Balance

$$\text{Marshall-Lerner condition: } \omega_x\varepsilon_x + \omega_M(\varepsilon_M - 1) > 0$$

where:
ω_x = Share of exports in total trade
ω_M = Share of imports in total trade
ε_x = Price elasticity of demand for exports
ε_M = Price elasticity of demand for imports

Financial Reporting Mechanics

Exhibit 1: Accruals [2]

Cash movement in the same period as accounting recognition

↓

Settled transaction- no accrual entry needed

Cash Movement prior to Accounting Recognition

Unearned (deferred) Revenue

Originating Entry
Record cash receipt and establish a liability
Adjusting Entry
Reduce the liability and recognize revenue

Prepaid Expense

Originating Entry
Record cash payment and establish an asset
Adjusting Entry
Reduce the asset and recognize the expense

Cash Movement after Accounting Recognition

Unbilled (accrued) Revenue

Originating Entry
Record revenue and establish an asset
Adjusting Entry
When billing occurs, reduce unbilled revenue and increase accounts receivable. When cash is collected, eliminate the receivable

Accrued Expenses

Originating Entry
Establish a liability and record an expense
Adjusting Entry
Reduce the liability as cash is paid

Income and Comprehensive Income

Net Income = Revenue + Other income – Expenses
= Income – Expenses

Net income + Other comprehensive income = Comprehensive income

Comprehensive income = Profit after tax
+ Exchange differences
+ Actuarial gains/losses
+ Cash flow hedges
+ Available for sale financial assets (marketable securities)
+ Deferred taxes
+ Share of profits/losses in equity-accounted investments

[2] Exhibit 10, Vol 3, CFA Program Curriculum 2018

Cash Flows

Change in cash and equivalents =	Cash from operating activities
	+ Cash from investing activities
	+ Cash from financing activities

Balance Sheet

Assets (basic equation) =	Liabilities + Owner's equity
Owner's equity =	Assets − Liabilities
=	Contributed capital + Retained earnings
Ending retained earnings =	Beginning retained earnings
	+ Revenues − Expenses − Dividends
Assets (expanded equation) =	Liabilities + Contributed capital
	+ Beginning retained earnings + Revenue −
	Expenses − Dividends

Understanding Income Statements

Basic EPS

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

Diluted EPS

$$\text{Diluted EPS} = \frac{\left[\text{Net income} - \begin{array}{c}\text{Preferred}\\ \text{dividends}\end{array}\right] + \begin{array}{c}\text{Convertible}\\ \text{preferred}\\ \text{dividends}\end{array} + \left[\begin{array}{c}\text{Convertible}\\ \text{debt}\\ \text{interest}\end{array} \times (1-t)\right]}{\begin{array}{c}\text{Weighted}\\ \text{average}\\ \text{shares}\end{array} + \begin{array}{c}\text{Shares from}\\ \text{conversion of}\\ \text{convertible}\\ \text{preferred shares}\end{array} + \begin{array}{c}\text{Shares from}\\ \text{conversion of}\\ \text{convertible}\\ \text{debt}\end{array} + \begin{array}{c}\text{Shares}\\ \text{issuable from}\\ \text{stock options}^{1}\end{array}}$$

Comprehensive Income

Net income + Other comprehensive income = Comprehensive income

Ending Shareholders' Equity

Ending shareholders' equity = Beginning shareholders' equity + Net income + Other comprehensive income – Dividends declared

[1]Option warrant conversion uses treasury stock method; i.e., as if company used option value to repurchase shares at average market price during the period. Convertible preferred and debt shares are 'if-converted' at beginning of period.

Understanding the Balance Sheets

Gains and Losses on Marketable Securities

	Held-to-Maturity Securities	Available-for-Sale Securities	Trading Securities
Balance Sheet	Reported at cost or amortized cost.	Reported at fair value. Unrealized gains or losses due to changes in market values are reported in other comprehensive income within owners' equity.	Reported at fair value.
Items recognized on the income statement	Interest income. Realized gains and losses.	Dividend income. Interest income. Realized gains and losses.	Dividend income. Interest income. Realized gains and losses. Unrealized gains and losses due to changes in market values.

Liquidity Ratios

Liquidity ratios indicate a company's ability to meet current obligations.

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio (acid test)} = \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}}$$

Solvency Ratios

Solvency ratios indicate a company's financial leverage and financial risk.

$$\text{L-T debt-to-equity} = \frac{\text{Total L-T debt}}{\text{Total equity}}$$

$$\text{Debt-to-equity} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Total debt} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Financial leverage} = \frac{\text{Total assets}}{\text{Total equity}}$$

Understanding Cash Flow Statements

Cash Flow Classification under U.S. GAAP

CFO

Inflows	Outflows
Cash collected from customers. Interest and dividends received. Proceeds from sale of securities held for trading.	Cash paid to employees. Cash paid to suppliers. Cash paid for other expenses. Cash used to purchase trading securities. Interest paid. Taxes paid.

CFI

Inflows	Outflows
Sale proceeds from fixed assets. Sale proceeds from long-term investments.	Purchase of fixed assets. Cash used to acquire LT investment securities.

CFF

Inflows	Outflows
Proceeds from debt issuance. Proceeds from issuance of equity instruments.	Repayment of LT debt. Payments made to repurchase stock. Dividends payments.

Cash Flow Statements under IFRS and U.S. GAAP

	IFRS	U.S. GAAP
Classification of Cash Flows		
Interest and dividends received	CFO or CFI	CFO
Interest paid	CFO or CFF	CFO
Dividend paid	CFO or CFF	CFF
Dividends received	CFO or CFI	CFO
Taxes paid	CFO, but part of the tax can be categorized as CFI or CFF if it is clear that the tax arose from investing or financing activities.	CFO
Bank overdraft	Included as a part of cash equivalents.	Not considered a part of cash equivalents and included in CFF.
Presentation Format		
CFO (No difference in CFI and CFF presentation)	Direct or indirect method. The former is preferred.	Direct or indirect method. The former is preferred. However, if the direct method is used, a reconciliation of net income and CFO must be included.
Disclosures		
	Taxes paid should be presented separately on the cash flow statement.	If taxes and interest paid are not explicitly stated on the cash flow statement, details can be provided in footnotes.

Free Cash Flow to the Firm

$$FCFF = NI + NCC + [Int * (1 - tax\ rate)] - FCInv - WCInv$$

$$FCFF = CFO + [Int * (1 - tax\ rate)] - FCInv$$

Free Cash Flow to Equity

$$FCFE = CFO - FCInv + Net\ borrowing$$

Performance Ratios

$$\text{CF to revenue} = \frac{\text{CFO}}{\text{Net Revenue}}$$

$$\text{Cash return on assets} = \frac{\text{CFO}}{\text{Average total assets}}$$

$$\text{Cash return on equity} = \frac{\text{CFO}}{\text{Average shareholders' equity}} \div$$

$$\text{Cash to income} = \frac{\text{CFO}}{\text{Operating income}}$$

$$\text{CF per share} = \frac{(\text{CFO} - \text{Preferred dividends})}{\text{Number of common shares outstanding}}$$

Coverage Ratios

$$\text{Debt coverage} = \frac{\text{CFO}}{\text{Total debt}}$$

$$\text{Debt coverage} = \frac{\text{CFO} + \text{Interest paid} + \text{Taxes paid}}{\text{Interest paid}}$$

$$\text{Reinvestment} = \frac{\text{CFO}}{\text{Cash paid for L-T assets}}$$

$$\text{Debt payment} = \frac{\text{CFO}}{\text{Cash paid for L-T debt repayment}}$$

$$\text{Dividend payment} = \frac{\text{CFO}}{\text{Cash paid for dividends}}$$

$$\text{Investing/financing} = \frac{CFO}{\text{Cash outflows for investing/financing}}$$

Financial Analysis Techniques

Activity Ratios

$$\textbf{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\textbf{Days of inventory on hand (DOH)} = \frac{365}{\text{Inventory turnover}}$$

$$\textbf{Receivables turnover} = \frac{\text{Revenue}}{\text{Average receivables}}$$

$$\textbf{Days of sales outstanding (DSO)} = \frac{365}{\text{Receivables turnover}}$$

$$\textbf{Payables turnover} = \frac{\text{Purchases}}{\text{Average trade payables}}$$

$$\textbf{Number of days of payables} = \frac{365}{\text{Payables turnover}}$$

$$\textbf{Working capital turnover} = \frac{\text{Revenue}}{\text{Average working capital}}$$

$$\textbf{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average fixed assets}}$$

$$\textbf{Total asset turnover} = \frac{\text{Revenue}}{\text{Average total assets}}$$

Liquidity Ratios

$$\textbf{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\textbf{Quick ratio} = \frac{\text{Cash} + \text{Short-term marketable investments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\textbf{Cash ratio} = \frac{\text{Cash} + \text{Short-term marketable investments}}{\text{Current liabilities}}$$

$$\textbf{Defensive interval ratio} = \frac{\text{Cash} + \text{Short-term marketable investments} + \text{Receivables}}{\text{Daily cash expenditures}}$$

$$\textbf{Cash conversion cycle} = \text{DSO} + \text{DOH} - \text{Number of days of payables}$$

Solvency Ratios

$$\textbf{Debt-to-assets ratio} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\textbf{Debt-to-capital ratio} = \frac{\text{Total debt}}{\text{Total debt} + \text{Shareholders' equity}}$$

$$\textbf{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Shareholders' equity}}$$

$$\textbf{Financial leverage ratio} = \frac{\text{Average total assets}}{\text{Average total equity}}$$

$$\textbf{Interest coverage ratio} = \frac{\text{EBIT}}{\text{Interest payments}}$$

$$\textbf{Fixed charge coverage ratio} = \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$$

Profitability Ratios

$$\textbf{Gross profit margin} = \frac{\text{Gross profit}}{\text{Revenue}}$$

$$\textbf{Operating profit margin} = \frac{\text{Operating profit}}{\text{Revenue}}$$

$$\textbf{Pretax margin} = \frac{\text{EBT (earnings before tax, but after interest)}}{\text{Revenue}}$$

$$\textbf{Net profit margin} = \frac{\text{Net profit}}{\text{Revenue}}$$

Return on Investment Ratios

$$\textbf{ROA} = \frac{\text{Net income}}{\text{Average total assets}}$$

$$\textbf{Adjusted ROA} = \frac{\text{Net income} + \text{Interest expense } (1 - \text{Tax rate})}{\text{Average total assets}}$$

$$\textbf{Operating ROA} = \frac{\text{Operating income or EBIT}}{\text{Average total assets}}$$

$$\textbf{Return on total capital} = \frac{\text{EBIT}}{\text{Short-term debt} + \text{Long-term debt} + \text{Equity}}$$

$$\textbf{Return on equity} = \frac{\text{Net income}}{\text{Average total equity}}$$

$$\textbf{Return on common equity} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Average common equity}}$$

DuPont Decomposition of ROE

$$\textbf{ROE} = \frac{\text{Net income}}{\text{Average shareholders' equity}}$$

2-Way Dupont Decomposition

$$\textbf{ROE} = \underbrace{\frac{\text{Net income}}{\text{Average total assets}}}_{\text{ROA}} \times \underbrace{\frac{\text{Average total assets}}{\text{Average shareholders' equity}}}_{\text{Leverage}}$$

3-Way Dupont Decomposition

$$\textbf{ROE} = \underbrace{\frac{\text{Net income}}{\text{Revenue}}}_{\text{Net profit margin}} \times \underbrace{\frac{\text{Revenue}}{\text{Average total assets}}}_{\text{Asset turnover}} \times \underbrace{\frac{\text{Average total assets}}{\text{Average shareholders' equity}}}_{\text{Leverage}}$$

5-Way Dupont Decomposition

$$\textbf{ROE} = \underbrace{\frac{\text{Net income}}{\text{EBT}}}_{\text{Tax burden}} \times \overbrace{\frac{\text{EBT}}{\text{EBIT}}}^{\text{Interest burden}} \times \underbrace{\frac{\text{EBIT}}{\text{Revenue}}}_{\text{EBIT margin}} \times \overbrace{\frac{\text{Revenue}}{\text{Average total assets}}}^{\text{Asset turnover}} \times \underbrace{\frac{\text{Average total assets}}{\text{Avg. shareholders' equity}}}_{\text{Leverage}}$$

Valuation Ratios

$$\textbf{P/E} = \frac{\text{Price per share}}{\text{Earnings per share}}$$

$$\textbf{P/CE} = \frac{\text{Price per share}}{\text{Cash flow per share}}$$

$$\textbf{P/S} = \frac{\text{Price per share}}{\text{Sales per share}}$$

$$\textbf{P/BV} = \frac{\text{Price per share}}{\text{Book value per share}}$$

Per Share Ratios

$$\textbf{Cash flow per share} = \frac{\text{Cash flow from operations}}{\text{Average number of shares outstanding}}$$

$$\textbf{EBITDA per share} = \frac{\text{EBITDA}}{\text{Average number of shares outstanding}}$$

$$\textbf{Dividends per share} = \frac{\text{Common dividends declared}}{\text{Weighted average number of ordinary shares}}$$

Dividend-Related Measures

$$\textbf{Dividend payout ratio} = \frac{\text{Common share dividends}}{\text{Net income attributable to common shares}}$$

$$\textbf{Retention Rate} = \frac{\text{Net income attributable to common shares} - \text{Common share dividends}}{\text{Net income attributable to common shares}}$$

$$\textbf{Sustainable growth rate} = \text{Retention rate} \times \text{ROE}$$

Credit Analysis Ratios

$$\text{EBIT interest coverage} = \frac{\text{EBIT}}{\text{Gross interest}}$$

$$\text{EBITDA interest coverage} = \frac{\text{EBIT}}{\text{Gross interest}}$$

$$\text{FFO}^1 \text{ interest coverage} = \frac{\text{FFO} + \text{interest paid} - \text{operating lease adj}}{\text{Gross interest}}$$

$$\text{Return on capital} = \frac{\text{EBIT}}{\text{Avg (equity} + \text{non-current deferred taxes} + \text{debt)}}$$

$$\text{FFO}^1 \text{ to debt} = \frac{\text{FFO}}{\text{Total debt}}$$

$$\text{Free Operating CF to debt} = \frac{\textit{Adj}\ \text{CFO} - \text{capex}}{\text{Total debt}}$$

$$\text{Discretionary CF to debt} = \frac{\text{CFO} - \text{capex} - \text{dividends paid}}{\text{Total debt}}$$

$$\text{Net CF to capex} = \frac{\text{FFO} - \text{dividends}}{\text{capex}}$$

$$\text{Debt to EBITDA} = \frac{\text{Total debt}}{\text{EBITDA}}$$

[1]FFO (free funds to operations) = net income + non-cash charges

INVENTORIES

LIFO versus FIFO (with rising prices and stable inventory levels.)

LIFO versus FIFO when Prices are Rising		
	LIFO	**FIFO**
COGS	Higher	Lower
Income before taxes	Lower	Higher
Income taxes	Lower	Higher
Net income	Lower	Higher
Cash flow	Higher	Lower
EI	Lower	Higher
Working capital	Lower	Higher

Type of Ratio	Effect on Numerator	Effect on Denominator	Effect on Ratio
Profitability ratios NP and GP margins	Income is lower under LIFO because COGS is higher	Sales are the same under both	Lower under LIFO
Debt-to-equity	Same debt levels	Lower equity under LIFO	Higher under LIFO
Current ratio	Current assets are lower under LIFO because EI is lower	Current liabilities are the same	Lower under LIFO
Quick ratio	Assets are higher as a result of lower taxes paid	Current liabilities are the same	Higher under LIFO
Inventory turnover	COGS is higher under LIFO	Average inventory is lower under LIFO	Higher under LIFO
Total asset turnover	Sales are the same	Lower total assets under LIFO	Higher under LIFO

The LIFO Method and the LIFO Reserve:

$$EI_{FIFO} = EI_{LIFO} + LR$$

where
LR = LIFO Reserve

$$COGS_{FIFO} = COGS_{LIFO} - (\text{Change in LR during the year})$$

Net income after tax under FIFO will be greater than LIFO net income after tax by:

$$\text{Change in LIFO Reserve} \times (1 - \text{Tax rate})$$

When converting from LIFO to FIFO assuming rising prices:

Equity (retained earnings) increase by:

LIFO Reserve × (1 – Tax rate)

Liabilities (deferred taxes) increase by:

LIFO Reserve × (Tax rate)

Current assets (inventory) increase by:

LIFO Reserve

LONG-LIVED ASSETS

Financial Statement Effects of Capitalizing versus Expensing

	Effect on Financial Statements
Initially when the cost is capitalized	• Noncurrent assets *increase.* • Cash flow from investing activities *decreases*.
In future periods when the asset is depreciated or amortized	• Noncurrent assets *decrease.* • Net income *decreases.* • Retained earnings *decrease.* • Equity *decreases.*
When the cost is expensed	• Net income *decreases* by the entire after-tax amount of the cost. • No related asset is recorded on the balance sheet and therefore, no depreciation or amortization expense is charged in future periods. • Operating cash flow *decreases.* • Expensed costs have no financial statement impact in future years.

	Capitalizing	Expensing
Net income (first year)	Higher	Lower
Net income (future years)	Lower	Higher
Total assets	Higher	Lower
Shareholders' equity	Higher	Lower
Cash flow from operations	Higher	Lower
Cash flow from investing	Lower	Higher
Income variability	Lower	Higher
Debt-to-equity	Lower	Higher

$$\text{Depreciation expense} = \frac{\text{Original cost} - \text{Salvage value}}{\text{Depreciable life}}$$

$$\text{DDB depreciation in Year X} = \frac{2}{\text{Depreciable life}} \times \text{Book value at the beginning of Year X}$$

Depreciation Components

$$\text{Estimated useful life} = \frac{\text{Gross investment in fixed assets}}{\text{Annual depreciation expense}}$$

$$\text{Average age of asset} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}}$$

$$\text{Remaining useful life} = \frac{\text{Net investment in fixed assets}}{\text{Annual depreciation expense}}$$

$$\frac{\text{Gross investment in fixed assets}}{\text{Annual depreciation expense}} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}} + \frac{\text{Net investment in fixed assets}}{\text{Annual depreciation expense}}$$

Estimated useful or depreciable life

The historical cost of an asset divided by its useful life equals annual depreciation expense under the straight line method. Therefore, the historical cost divided by annual depreciation expense equals the estimated useful life.

Average age of asset

Annual depreciation expense times the number of years that the asset has been in use equals accumulated depreciation. Therefore, accumulated depreciation divided by annual depreciation equals the average age of the asset.

Remaining useful life

The book value of the asset divided by annual depreciation expense equals the number of years the asset has remaining in its useful life.

Income Taxes

Tax Base

Originally booked asset value

$$\text{less} = \frac{\text{previous tax depreciation/amortization}}{\text{Amount deductible in future periods (Tax base)}}$$

$$\text{Carrying amount} - \text{Tax base} = \text{Temporary difference}$$

Effective Tax rate

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pretax income}}$$

Income Tax Expense

$$\text{Income tax expense} = \text{Taxes Payable} + \text{Change in DTL} - \text{Change in DTA}$$

Treatment of Temporary Differences

Balance Sheet Item	Carrying Value versus Tax Base	Results in…
Asset	Carrying amount is greater.	DTL
Asset	Tax base is greater.	DTA
Liability	Carrying amount is greater.	DTA
Liability	Tax base is greater.	DTL

Income Tax Accounting under IFRS versus U.S. GAAP

	IFRS	U.S. GAAP
ISSUE SPECIFIC TREATMENTS		
Revaluation of fixed assets and intangible assets.	Recognized in equity as deferred taxes.	Revaluation is prohibited.
Treatment of undistributed profit from investment in subsidiaries.	Recognized as deferred taxes except when the parent company is able to control the distribution of profits and it is probable that temporary differences will not reverse in future.	No recognition of deferred taxes for foreign subsidiaries that fulfill indefinite reversal criteria. No recognition of deferred taxes for domestic subsidiaries when amounts are tax-free.
Treatment of undistributed profit from investments in joint ventures.	Recognized as deferred taxes except when the investor controls the sharing of profits and it is probable that there will be no reversal of temporary differences in future.	No recognition of deferred taxes for foreign corporate joint ventures that fulfill indefinite reversal criteria.

	IFRS	U.S. GAAP
Treatment of undistributed profit from investments in associates.	Recognized as deferred taxes except when the investor controls the sharing of profits and it is probable that there will be no reversal of temporary differences in future.	Deferred taxes are recognized from temporary differences.
DEFERRED TAX MEASUREMENT		
Tax rates.	Tax rates and tax laws enacted or substantively enacted.	Only enacted tax rates and tax laws are used.
Deferred tax asset recognition.	Recognized if it is probable that sufficient taxable profit will be available in the future.	Deferred tax assets are recognized in full and then reduced by a valuation allowance if it is likely that they will not be realized.
DEFERRED TAX PRESENTATION		
Offsetting of deferred tax assets and liabilities.	Offsetting allowed only if the entity has right to legally enforce it and the balance is related to a tax levied by the same authority.	Same as in IFRS.
Balance sheet classification.	Classified on balance sheet as net noncurrent with supplementary disclosures.	Classified as either current or noncurrent based on classification of underlying asset and liability.

NON-CURRENT (LONG-TERM) LIABILITIES

Income Statement Effects of Lease Classification

Income Statement Item	Finance Lease	Operating Lease
Operating expenses	Lower	Higher
Nonoperating expenses	Higher	Lower
EBIT (operating income)	Higher	Lower
Total expenses- early years	Higher	Lower
Total expenses- later years	Lower	Higher
Net income- early years	Lower	Higher
Net income- later years	Higher	Lower

Balance Sheet Effects of Lease Classification

Balance Sheet Item	Capital Lease	Operating Lease
Assets	Higher	Lower
Current liabilities	Higher	Lower
Long term liabilities	Higher	Lower
Total cash	Same	Same

Cash Flow Effects of Lease Classification

CF Item	Capital Lease	Operating Lease
CFO	Higher	Lower
CFF	Lower	Higher
Total cash flow	Same	Same

Impact of Lease Classification on Financial Ratios

Ratio	Numerator under Finance Lease	Denominator under Finance Lease	Effect on Ratio	Ratio Better or Worse under Finance Lease
Asset turnover	Sales- same	Assets- higher	Lower	Worse
Return on assets*	Net income lower in early years	Assets- higher	Lower	Worse
Current ratio	Current assets- same	Current liabilities- higher	Lower	Worse
Leverage ratios (D/E and D/A)	Debt- higher	Equity same Assets higher	Higher	Worse
Return on equity*	Net income lower in early years	Equity same	Lower	Worse

* In early years of the lease agreement.

Financial Statement Effects of Lease Classification from Lessor's Perspective

	Financing Lease	Operating Lease
Total net income	Same	Same
Net income (early years)	Higher	Lower
Taxes (early years)	Higher	Lower
Total CFO	Lower	Higher
Total CFI	Higher	Lower
Total cash flow	Same	Same

Definitions of Commonly Used Solvency Ratios

Solvency Ratios	Description	Numerator	Denominator
Leverage Ratios			
Debt-to-assets ratio	Expresses the percentage of total assets financed by debt	Total debt	Total assets
Debt-to-capital ratio	Measures the percentage of a company's total capital (debt + equity) financed by debt.	Total debt	Total debt + Total shareholders' equity
Debt-to-equity ratio	Measures the amount of debt financing relative to equity financing	Total debt	Total shareholders' equity
Financial leverage ratio	Measures the amount of total assets supported by one money unit of equity	Average total assets	Average shareholders' equity
Coverage Ratios			
Interest coverage ratio	Measures the number of times a company's EBIT could cover its interest payments.	EBIT	Interest payments
Fixed charge coverage ratio	Measures the number of times a company's earnings (before interest, taxes and lease payments) can cover the company's interest and lease payments.	EBIT + Lease payments	Interest payments + Lease payments

Financial Reporting Quality

Relationship between Financial Reporting Quality and Earnings Quality

<table>
<tr><th></th><th></th><th colspan="2">Financial Reporting Quality</th></tr>
<tr><th></th><th></th><th>Low</th><th>High</th></tr>
<tr><td rowspan="2">Earnings (Results) Quality</td><td>High</td><td rowspan="2">LOW financial reporting quality impedes assessment of earnings quality and impedes valuation.</td><td>HIGH financial reporting quality enables assessment.
HIGH earnings quality increases company value.</td></tr>
<tr><td>Low</td><td>HIGH financial reporting quality enables assessment.
LOW earnings quality decreases company value.</td></tr>
</table>

Capital Budgeting

Net Present Value (NPV)

$$\text{NPV} = \sum_{t=1}^{n} \frac{\text{CF}_t}{(1+r)^t} - \text{Outlay}$$

where:
CFt = after-tax cash flow at time, t.
r = required rate of return for the investment. This is the firm's cost of capital adjusted for the risk inherent in the project.
Outlay = investment cash outflow at t = 0.

Internal Rate of Return (IRR)

$$\sum_{t=1}^{n} \frac{\text{CF}_t}{(1+\text{IRR})^t} = \text{Outlay} \qquad \sum_{t=1}^{n} \frac{\text{CF}_t}{(1+\text{IRR})^t} - \text{Outlay} = 0$$

Payback Period

Payback period and discounted payback period have the same formula, but discounted payback uses cash flows discounted by the appropriate rate:

$$\text{Payback period} = n + \frac{\sum_{t=0}^{n} CF_{n+1} - \sum_{t=0}^{n} CF_n}{\sum_{t=0}^{n} CF_{n+1}}$$

where $n = \#$ of periods that $\sum_{t=0}^{n} CF_n < 0$ and $\dfrac{\sum_{t=0}^{n} CF_{n+1} - \sum_{t=0}^{n} CF_n}{\sum_{t=0}^{n} CF_{n+1}}$ is the partial year (in year n+1)

CF required to recover the remaining investment amount.

Profitability Index

$$\text{PI} = \frac{\text{PV of future cash flows}}{\text{Initial investment}} = 1 + \frac{\text{NPV}}{\text{Initial investment}}$$

Cost of Capital

Weighted Average Cost of Capital

$$\text{WACC} = (w_d)(r_d)(1-t) + (w_p)(r_p) + (w_e)(r_e)$$

where:
w_d = Proportion of debt that the company uses when it raises new funds
r_d = Before-tax marginal cost of debt
t = Company's marginal tax rate
w_p = Proportion of preferred stock that the company uses when it raises new funds
r_p = Marginal cost of preferred stock
w_e = Proportion of equity that the company uses when it raises new funds
r_e = Marginal cost of equity

To Transform Debt-to-equity Ratio into a Component's Weight

$$\frac{D/E}{1 + D/E} = \frac{D}{D+E} = w_d$$

$$w_d + w_e = 1$$

Valuation of Bonds

$$P_0 = \left[\sum_{t=1}^{n} \frac{PMT_t}{\left(1+\frac{r_d}{2}\right)^t}\right] + \frac{M}{\left(1+\frac{r_d}{2}\right)^n}$$

where:
P_0 = current market price of the bond.
PMT_t = interest payment in period t.
r_d = yield to maturity on BEY basis.
n = number of periods remaining to maturity.
M = Par or maturity value of the bond.

Valuation of Preferred Stock

$$V_p = D_p / r_p$$

where:
V_p = current value (price) of preferred stock.
D_p = preferred stock dividend per share.
r_p = cost of preferred stock.

Required Return on a Stock

Capital Asset Pricing Model

$$r_e = R_F + \beta_i [E(R_M) - R_F]$$

where:
$[E(R_M) - Rf]$ = Equity risk premium.
R_M = Expected return on the market.
β_i = Beta of stock. Beta measures the sensitivity of the stock's returns to changes in market returns.
R_F = Risk-free rate.
r_e = Expected return on stock (cost of equity).

Dividend Discount Model

$$P_0 = \frac{D_1}{r_e - g}$$

where:
P_0 = current market value of the security.
D_1 = next year's dividend.
r_e = required rate of return on common equity.
g = the firm's expected constant growth rate of dividends.

Rearranging the above equation gives us a formula to calculate the required return on equity:

$$r_e = \frac{D_1}{P_0} + g$$

Sustainable Growth Rate

$$g = \left(1 - \frac{D}{EPS}\right) \times (ROE)$$

Where (1 – (D/EPS)) = Earnings retention rate

Bond Yield plus Risk Premium Approach

$r_e = r_d$ + risk premium, where r_d is the required return on debt.

Unlevered beta for a comparable asset

$$\beta_{ASSET} = \beta_{EQUITY}\left[\frac{1}{1+\left((1-t)\frac{D}{E}\right)}\right]$$

Beta for a project using a comparable asset re-levered for target company

$$\beta_{PROJECT} = \beta_{ASSET}\left[1+\left((1-t)\frac{D}{E}\right)\right]$$

Country Risk Premium

$$r_e = R_F + \beta\ [E(R_M) - R_F + CRP]$$

$$\text{Country equity risk premium} = \text{Sovereign yield spread} \times \frac{\text{Annualized standard deviation of equity index}}{\text{Annualized standard deviation of sovereign bond market in terms of the developed market currency}}$$

$$\text{Break point} = \frac{\text{Amount of capital at which a component's cost of capital changes}}{\text{Proportion of new capital raised from the component}}$$

Measures of Leverage

Degree of Operating Leverage

$$\text{DOL} = \frac{\text{Percentage change in operating income}}{\text{Percentage change in units sold}}$$

$$\text{DOL} = \frac{Q \times (P - V)}{Q \times (P - V) - F}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
Q × (P – V) = Contribution margin (the amount that units sold contribute to covering fixed costs)
(P – V) = Contribution margin per unit

Degree of Financial Leverage

$$\text{DFL} = \frac{\text{Percentage change in net income}}{\text{Percentage change in operating income}}$$

$$\text{DFL} = \frac{[Q(P - V) - F](1 - t)}{[Q(P - V) - F - C](1 - t)} = \frac{[Q(P - V) - F]}{[Q(P - V) - F - C]}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost
t = Tax rate

Degree of Total Leverage

$$\text{DTL} = \frac{\text{Percentage change in net income}}{\text{Percentage change in the number of units sold}}$$

$$\text{DTL} = \text{DOL} \times \text{DFL}$$

$$\text{DTL} = \frac{Q \times (P - V)}{[Q(P - V) - F - C]}$$

where:
Q = Number of units produced and sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost

Breakeven point

$$PQ_{BE} = VQ_{BE} + F + C$$

where:
P = Price per unit
Q = Number of units produced and sold
V = Variable cost per unit
F = Fixed operating costs
C = Fixed financial cost

The breakeven number of units can be calculated as:

$$Q_{BE} = \frac{F + C}{P - V}$$

Note that taxes are not considered in breakeven analysis because there is no taxable income.

Operating breakeven point

$$PQ_{OBE} = VQ_{OBE} + F$$

$$Q_{OBE} = \frac{F}{P - V}$$

Net income at various levels of sales

$$\text{Net income} = [Q(P - V) - F - C](1 - t)$$

Note that this formula considers taxes, unlike the variant formula for breakeven point (which has no taxable income).

Working Capital Management

Liquidity Measures

$$\text{Current Ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick Ratio} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Accounts receivable turnover} = \frac{\text{Credit sales}}{\text{Average receivables}}$$

$$\text{Number of days of receivables} = \frac{\text{Accounts receivable}}{\text{Average days sales on credit}} = \frac{\text{Accounts receivable}}{\text{Sales on credit} / 365}$$

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average day's cost of goods sold}} = \frac{\text{Inventory}}{\text{Cost of goods sold} / 365}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average trade payables}}$$

$$\text{Number of days of payables} = \frac{\text{Accounts payables}}{\text{Average day's purchases}} = \frac{\text{Accounts payables}}{\text{Purchases} / 365}$$

$$\text{Purchases} = \text{Ending inventory} + \text{COGS} - \text{Beginning inventory}$$

$$\text{Operating cycle} = \text{Number of days of inventory} + \text{Number of days of receivables}$$

$$\text{Net operating cycle} = \text{Number of days of inventory} + \text{Number of days of receivables} - \text{Number of days of payables}$$

Short-term Investment Returns

$$\text{Money market yield} = \left(\frac{\text{Face value} - \text{price}}{\text{Price}}\right) \times \left(\frac{360}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{360}{\text{Days}}\right)$$

$$\text{Bond equivalent yield} = \left(\frac{\text{Face value} - \text{price}}{\text{Price}}\right) \times \left(\frac{365}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{365}{\text{Days}}\right)$$

$$\text{Discount basis yield} = \left(\frac{\text{Face value} - \text{price}}{\text{Face value}}\right) \times \left(\frac{360}{\text{Days}}\right) = \%\ \text{discount} \times \left(\frac{360}{\text{Days}}\right)$$

$$\%\ \text{Discount} = \frac{\text{Face value} - \text{Price}}{\text{Price}}$$

Working Capital Effectiveness Measure

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average days cost of goods sold}} = \frac{\text{Inventory}}{\text{Cost of goods sold} / 365} = \frac{365}{\text{Inventory turnover}}$$

$$\text{Implicit rate} = \text{Cost of trade credit} = \left(1 + \frac{\text{Discount}}{1 - \text{Discount}}\right)^{\left(365 / \text{Number of days beyond discount period}\right)} - 1$$

$$\text{Number of days of payables} = \frac{\text{Accounts payable}}{\text{Average day's purchases}}$$

$$\frac{\text{Accounts payable}}{\text{Purchases} / 365} = \frac{365}{\text{Payables turnover}}$$

Short-term Funding Cost Measures

$$\text{Line of credit cost} = \frac{\text{Interest} + \text{Commitment fee}}{\text{Loan amount}}$$

$$\text{Banker's acceptance cost} = \frac{\text{Interest}}{\text{Net proceeds}} = \frac{\text{Interest}}{\text{Loan amount} - \text{Interest}}$$

$$\frac{\text{Interest} + \text{Dealer's commission} + \text{Backup costs}}{\text{Loan amount} - \text{Interest}}$$

Portfolio Management: An Overview

The Portfolio Management Process

Planning
- Understanding client needs
- Preparing an investment policy statement (IPS)

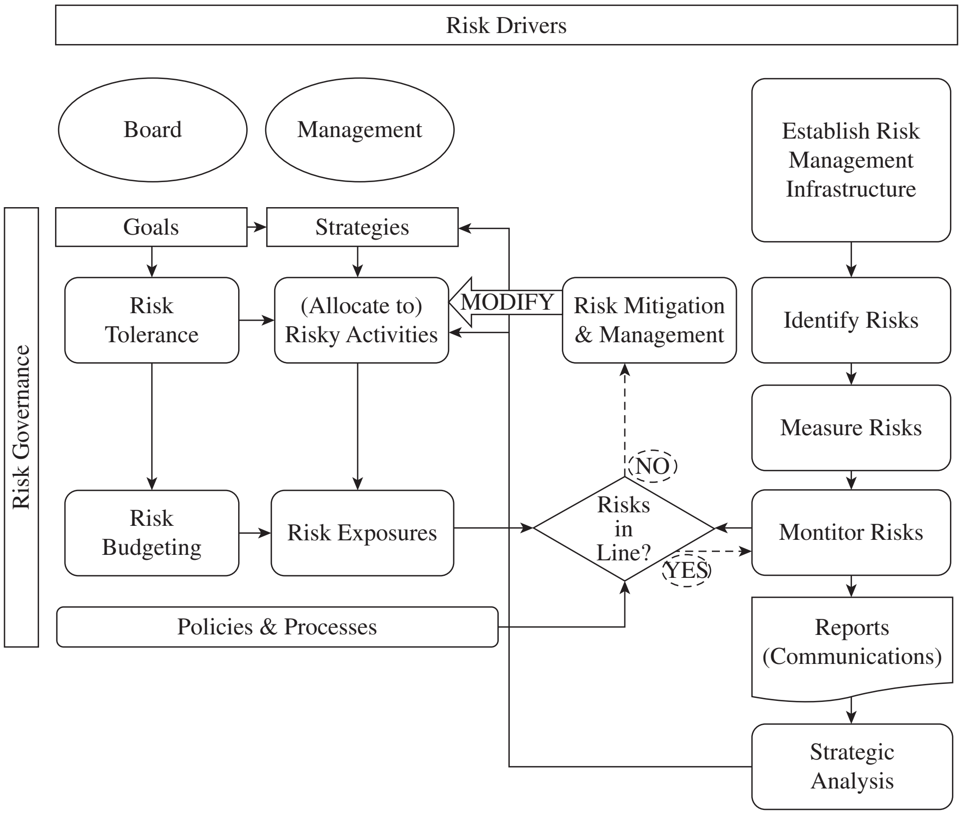
Execution
- Asset allocation
- Security analysis
- Portfolio construction

Feedback
- Portfolio monitoring and rebalancing
- Performance measurement and reporting

Portfolio Management

Risk Management: An Introduction

Figure 1-1: The Risk Management Framework in an Enterprise



Portfolio Risk and Return: Part I

Holding Period Return

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}} = \text{Capital gain} + \text{Dividend yield}$$
$$= \frac{P_T + D_T}{P_0} - 1$$

where:
P_t = Price at the end of the period
P_{t-1} = Price at the beginning of the period
D_t = Dividend for the period

Holding Period Returns for more than One Period

$$R = [(1+R_1)\times(1+R_2)\times\ldots\times(1+R_n)]-1$$

where:
$R_1, R_2, \ldots, R_n$ are sub-period returns

Geometric Mean Return

$$R = \{[(1+R_1)\times(1+R_2)\times\ldots\times(1+R_n)]^{1/n}\}-1$$

Annualized Return

$$r_{annual} = (1+r_{period})^n - 1$$

where:
r = Return on investment
n = Number of periods in a year

Portfolio Return

$$R_p = w_1R_1 + w_2R_2$$

where:
R_p = Portfolio return
w_1 = Weight of Asset 1
w_2 = Weight of Asset 2
R_1 = Return of Asset 1
R_2 = Return of Asset 2

Variance of a Single Asset

$$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{T}$$

where:
R_t = Return for the period t
T = Total number of periods
μ = Mean of T returns

Variance of a Representative Sample of the Population

$$s^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1}$$

where:
R = mean return of the sample observations
s^2 = sample variance

Standard Deviation of an Asset

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T}(R_t - \mu)^2}{T}} \qquad s = \sqrt{\frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1}}$$

Variance of a Portfolio of Assets

$$\sigma_P^2 = \sum_{i,j=1}^{N} w_i w_j \text{Cov}(R_i, R_j)$$

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \text{Var}(R_i) + \sum_{i,j=1,\, i \neq j}^{N} w_i w_j \text{Cov}(R_i, R_j)$$

Standard Deviation of a Portfolio of Two Risky Assets

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \text{ or } \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}}$$

Utility Function

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:
U = Utility of an investment
E(R) = Expected return
σ^2 = Variance of returns
A = Additional return required by the investor to accept an additional unit of risk.

Portfolio Risk and Return: Part II

Capital Allocation Line

The CAL has an intercept of RFR and a constant slope that equals:

$$\frac{[E(R_i)-RFR]}{\sigma_i}$$

Expected Return on portfolios that lie on CML

$$E(R_p)=w_1R_f+(1-w_1)\ E(R_m)$$

Variance of portfolios that lie on CML

$$\sigma^2=\sqrt{w_1^2\sigma_f^2+(1-w_1)^2\sigma_m^2+2w_1(1-w_1)Cov(R_f,R_m)}$$

Equation of CML

$$E(R_p)=R_f+\frac{E(R_m)-R_f}{\sigma_m}\times\sigma_p$$

where:
y-intercept = R_f = risk-free rate

$$\text{slope}=\frac{E(R_m)-R_f}{\sigma_m}=\text{market price of risk.}$$

Systematic and Nonsystematic Risk

Total Risk = Systematic risk + Unsystematic risk

Return-Generating Models

$$E(R_i)-R_f=\sum_{j=1}^{k}\beta_{ij}\ E(F_j)=\beta_{i1}[E(R_m)-R_f]+\sum_{j=2}^{k}\beta_{ij}\ E(F_j)$$

The Market Model

$$R_i=\alpha_i+\beta_iR_m+e_i$$

Calculation of Beta

$$\beta_i=\frac{Cov(R_i,R_m)}{\sigma_m^2}=\frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2}=\frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

The Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Sharpe ratio

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

Treynor ratio

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

M-squared (M^2)

$$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

Jensen's alpha

$$\alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$$

Security Characteristic Line

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

EQUITY

Market Organization and Structure

Leveraged Position Metrics

$$\text{Leverage ratio} = \frac{\text{Position size}}{\text{Equity size}} = \frac{P_0Q_0}{P_0Q_0M_0} = \frac{1}{M_0}$$

$$\text{\% Equity margin} = M = \frac{\text{Equity per share}}{\text{Price per share}}$$

$$\text{Return using margin} = \frac{\text{Equity after disposal}}{\text{Initial Equity}}$$

$$= \frac{P_t + D - C - P_0(1 - M_0)(1 + r_{CM})}{P_0M_0 + C}$$

Where:
P_0 = initial share price
P_t = share disposal price
D = dividend per share during the period
C = commission per share
r_{CM} = call money rate
M_0 = initial margin

Security Price at which the Investor Would Receive a Margin Call

$$P_0 \times \frac{(1 - \text{Initial margin})}{(1 - \text{Maintenance margin})}$$

Security Market Indices

The value of a price return index is calculated as follows:

$$V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where:
V_{PRI} = Value of the price return index
n_i = Number of units of constituent security i held in the index portfolio
N = Number of constituent securities in the index
P_i = Unit price of constituent security i
D = Value of the divisor

Price Return

The price return of an index can be calculated as:

$$PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the price return index at the end of the period
V_{PRI0} = Value of the price return index at the beginning of the period

The price return of each constituent security is calculated as:

$$PR_i = \frac{P_{i1} - P_{i0}}{P_{i0}}$$

where:
PR_i = Price return of constituent security i (as a decimal number)
P_{i1} = Price of the constituent security i at the end of the period
P_{i0} = Price of the constituent security i at the beginning of the period

The price return of the index equals the weighted average price return of the constituent securities. It is calculated as:

$$PR_I = w_1 PR_1 + w_2 PR_2 + \ldots + w_N PR_N$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
PR_i = Price return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Total Return

The total return of an index can be calculated as:

$$TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the total return index at the end of the period
V_{PRI0} = Value of the total return index at the beginning of the period
Inc_I = Total income from all securities in the index held over the period

The total return of each constituent security is calculated as:

$$TR_i = \frac{P_{1i} - P_{0i} + Inc_i}{P_{0i}}$$

where:
TR_i = Total return of constituent security i (as a decimal number)
P_{1i} = Price of constituent security i at the end of the period
P_{0i} = Price of constituent security i at the beginning of the period
Inc_i = Total income from security i over the period

The total return of the index equals the weighted average total return of the constituent securities. It is calculated as:

$$TR_I = w_1 TR_1 + w_2 TR_2 + \ldots + w_N TR_N$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
TR_i = Total return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Calculation of Index Returns over Multiple Time Periods

Given a series of price returns for an index, the value of a price return index can be calculated as:

$$V_{PRIT} = V_{PRI0}\,(1 + PR_{I1})\,(1 + PR_{I2}) \ldots (1 + PR_{IT})$$

where:
V_{PRI0} = Value of the price return index at inception
V_{PRIT} = Value of the price return index at time t
PR_{IT} = Price return (as a decimal number) on the index over the period

Similarly, the value of a total return index may be calculated as:

$$V_{TRIT} = V_{TRI0}\ (1+TR_{I1})\ (1+TR_{I2})\ldots(1+TR_{IT})$$

where:
V_{TRI0} = Value of the index at inception
V_{TRIT} = Value of the index at time t
TR_{IT} = Total return (as a decimal number) on the index over the period

Price Weighting

$$w_i^P = \frac{P_i}{\sum_{i=1}^{N} P_i}$$

Equal Weighting

$$w_i^E = \frac{1}{N}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
N = Number of securities in the index

Market-Capitalization Weighting

$$w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

The float-adjusted market-capitalization weight of each constituent security is calculated as:

$$w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where:
f_i = Fraction of shares outstanding in the market float
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

Fundamental Weighting

$$w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where:
F_i = A given fundamental size measure of company i

Overview of Equity Securities

Return Characteristics of Equity Securities

$$\text{Total Return, } R_t = (P_t - P_{t-1} + D_t) / P_{t-1}$$

where:
P_{t-1} = Purchase price at time t – 1
P_t = Selling price at time t
D_t = Dividends paid by the company during the period

Accounting Return on Equity

$$ROE_t = \frac{NI_t}{\text{Average } BVE_t} = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Equity Valuation: Concepts and Basic Tools

Dividend Discount Model (DDM)

$$V_0 = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_\infty}{(1+k_e)^\infty}$$

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t}$$

One year holding period:

$$V_0 = \frac{\text{dividend to be received}}{(1+k_e)^1} + \frac{\text{year-end price}}{(1+k_e)^1}$$

Multiple-Year Holding Period DDM

$$V_0 = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{P_n}{(1+k_e)^n}$$

where:
P_n = Price at the end of n years.

Infinite Period DDM (Gordon Growth Model)

$$V_0 = \frac{D_0(1+g_c)^1}{(1+k_e)^1} + \frac{D_0(1+g_c)^2}{(1+k_e)^2} + \frac{D_0(1+g_c)^3}{(1+k_e)^3} + \cdots + \frac{D_0(1+g_c)^\infty}{(1+k_e)^\infty}$$

This equation simplifies to:

$$V_0 = \frac{D_0(1+g_c)^1}{(k_e - g_c)^1} = \frac{D_1}{k_e - g_c}$$

The long-term (constant) growth rate is usually calculated as:

$$g_c = RR \times ROE$$

Where RR is the reinvestment rate, or 1 – dividend payout rate.

Multi-Stage Dividend Discount Model

$$V_0 = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_n}{(1+k_e)^n} + \frac{P_n}{(1+k_e)^n}$$

where:

$$P_n = \frac{D_{n+1}}{k_e - g_c}$$

D_n = Last dividend of the supernormal growth period
D_{n+1} = First dividend of the constant growth period

The Free-Cash-Flow-to-Equity (FCFE) Model

$$FCFE = CFO - FC\ Inv + Net\ borrowing$$

Analysts may calculate the intrinsic value of the company's stock by discounting their projections of future FCFE at the required rate of return on equity.

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+k_e)^t}$$

Value of a Preferred Stock

When preferred stock is non-callable, non-convertible, has no maturity date and pays dividends at a fixed rate, the value of the preferred stock can be calculated using the perpetuity formula:

$$V_0 = \frac{D_0}{r}$$

For a non-callable, non-convertible preferred stock with maturity at time, n, the value of the stock can be calculated using the following formula:

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{F}{(1+r)^n}$$

where:
V_0 = value of preferred stock today (t = 0)
D_t = expected dividend in year t, assumed to be paid at the end of the year
r = required rate of return on the stock
F = par value of preferred stock

Price Multiples

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r-g}$$

$$\text{Price to cash flow ratio} = \frac{\text{Market price of share}}{\text{Cash flow per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market price per share}}{\text{Net sales per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market value of equity}}{\text{Total net sales}}$$

$$\text{P/BV} = \frac{\text{Current market price of share}}{\text{Book value per share}}$$

$$\text{P/BV} = \frac{\text{Market value of common shareholders' equity}}{\text{Book value of common shareholders' equity}}$$

where:
Book value of common shareholders' equity =
(Total assets – Total liabilities) – Preferred stock

Enterprise Value Multiples

EV/EBITDA

where:
EV = Enterprise value and is calculated as the market value of the company's common stock plus the market value of outstanding preferred stock if any, plus the market value of debt, less cash and short term investments (cash equivalents).

FIXED INCOME

Fixed-Income Securities: Defining Elements

Bond Coupon

$$\text{Coupon} = \text{Coupon rate} \times \text{Par value}$$

Coupon Rate (Floating)

$$\text{Coupon Rate} = \text{Reference rate} + \text{Quoted margin}$$

Coupon Rate (Inverse Floaters)

$$\text{Coupon rate} = K - L \times (\text{Reference rate})$$

Callable Bonds

$$\text{Value of callable bond} = \text{Value of non-callable bond} - \text{Value of embedded call option}$$

$$\text{Value of embedded call option} = \text{Value of non-callable bond} - \text{Value of callable bond}$$

Putable Bonds

$$\text{Value of putable bond} = \text{Value of non-putable bond} + \text{Value of embedded put option}$$

$$\text{Value of embedded put option} = \text{Value of putable bond} - \text{Value of non-putable bond}$$

Traditional Analysis of Convertible Securities

$$\text{Conversion value} = \text{Market price of common stock} \times \text{Conversion ratio}$$

$$\text{Market conversion price} = \frac{\text{Market price of convertible security}}{\text{Conversion ratio}}$$

$$\text{Market conversion premium per share} = \text{Market conversion price} - \text{Current market price}$$

$$\text{Market conversion premium ratio} = \frac{\text{Market conversion premium per share}}{\text{Market price of common stock}}$$

$$\text{Premium payback period} = \frac{\text{Market conversion premium per share}}{\text{Favorable income differential per share}}$$

$$\text{Favorable income differential per share} = \frac{\text{Coupon interest} - (\text{Conversion ratio} \times \text{Common stock dividend per share})}{\text{Conversion ratio}}$$

$$\text{Premium over straight value} = \frac{\text{Market price of convertible bond}}{\text{Straight value}} - 1$$

Fixed Income Markets: Issuance, Trading, and Funding

Discount Interest

The borrower receives less than the full amount that must eventually be repaid:

$$Y_D = \frac{(V_M - V_B)}{V_M}\left(\frac{360}{t}\right) = \frac{D}{V_M}\left(\frac{360}{t}\right)$$

$$D = V_M Y_D \left(\frac{t}{360}\right)$$

$$V_B = V_M\left[1 - Y_D\left(\frac{t}{360}\right)\right]$$

$$i_d = \frac{D}{V_B}\left(\frac{360}{t}\right), \text{ and } V_B < V_M \therefore i_d < Y_D$$

Where
Y_D = Discount rate (or discount yield)
V_M = Value repaid at maturity (i.e., amount of the loan)
V_B = Borrower's proceeds
D = Discount from value repaid at maturity (i.e., $V_M - V_B$)
I_d = borrower's interest rate, which is greater than the discount yield.

Add-On Interest

The borrower borrows one amount and repays another amount that includes both the borrowed amount and interest on the borrowed amount:

$$V_M = V_B\left[1 + i_d\left(\frac{t}{360}\right)\right]$$

Introduction to Fixed-Income Valuation

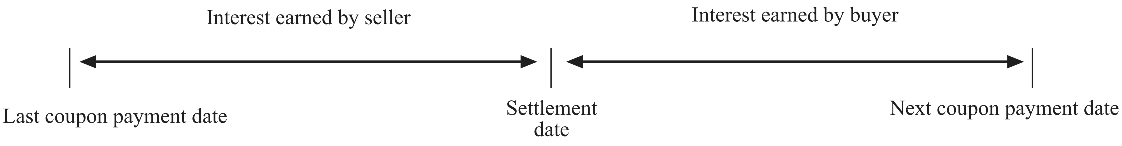
Pricing Bonds with Spot Rates

$$PV = \frac{PMT}{(1+z_1)^1} + \frac{PMT}{(1+z_2)^2} + \ldots + \frac{PMT + FV}{(1+z_N)^N}$$

z_1 = Spot rate for Period 1
z_2 = Spot rate for Period 2
z_N = Spot rate for Period N

Flat Price, Accrued Interest and the Full Price

Figure: Valuing a Bond between Coupon-Payment Dates



$$PV^{Full} = PV^{Flat} + AI$$

$$AI = t/T \times PMT$$

$$PV^{Full} = PV \times (1+r)^{t/T}$$

Semiannual bond basis yield or semiannual bond equivalent yield

$$\left(1+\frac{SAR_M}{M}\right)^M = \left(1+\frac{SAR_N}{N}\right)^N$$

Important: What we refer to as stated annual rate (SAR) is referred to in the curriculum as **APR or annual percentage rate**. We stick to SAR to keep your focus on a stated annual rate versus the effective annual rate. Just remember that if you see an annual percentage rate on the exam, it refers to the stated annual rate.

Current yield

$$\text{Current yield} = \frac{\text{Annual cash coupon payment}}{\text{Bond price}}$$

Option-adjusted price

Value of non-callable bond (option-adjusted price) = Flat price of callable bond + Value of embedded call option

Money Market Pricing, Discount-rate basis

$$PV = FV \times \left(1 - \frac{Days}{year} \times DR\right)$$

$$DR = \left(\frac{Year}{Days}\right) \times \left(\frac{FV - PV}{FV}\right)$$

Money Market Pricing, Add-on Rate Basis

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)}$$

$$AOR = \left(\frac{Year}{Days}\right) \times \left(\frac{FV - PV}{PV}\right)$$

Yield Spreads over the Benchmark Yield Curve

$$PV = \frac{PMT}{(1 + z_1 + Z)^1} + \frac{PMT}{(1 + z_2 + Z)^2} + \ldots + \frac{PMT + FV}{(1 + z_N + Z)^N}$$

- The benchmark spot rates z_1, z_2, z_N are derived from the government yield curve (or from fixed rates on interest rate swaps).
- Z refers to the z-spread per period. It is constant for all time periods.

Option-adjusted Spread (OAS)

OAS = z-spread − Option value (bps per year)

Implied Forward Rates

$$(1 + z_A)^A (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$$

$$IFR_{A,B-A} = \left[\sqrt[(B-A)]{\frac{(1 + z_B)^B}{(1 + z_A)^A}}\right] - 1$$

Introduction to Asset-Backed Securities

Parties to the Securitization

Party	Description	Party in Illustration
Seller	Originates the loans and sells loans to the SPV	ABC Company
Issuer/Trust	The SPV that buys the loans from the seller and issues the asset-backed securities	SPV
Servicer	Services the loans	Servicer

$$SMM_t = \frac{\text{Prepayment in month } t}{\text{Beginning mortgage balance for month } t - \text{Scheduled principal payment in month } t}$$

Understanding Fixed-Income Risk and Return

Macaulay Duration

$$\text{MacDur} = \left\{\frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r}\right\} - (t/T)$$

c = Coupon rate per period (PMT/FV)

Modified Duration

$$\text{ModDur} = \frac{\text{MacDur}}{1+r}$$

Modified duration has a very important application in risk management. It can be used to estimate the percentage price change for a bond in response to a change in its yield-to-maturity.

$$\%\Delta PV^{\text{Full}} \approx -\text{AnnModDur} \times \Delta\text{Yield}$$

If Macaulay duration is not already known, annual modified duration can be estimated using the following formula:

$$\text{ApproxModDur} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\text{Yield}) \times (PV_0)}$$

We can also use the approximate modified duration (ApproxModDur) to estimate Macaulay duration (ApproxMacDur) by applying the following formula:

$$\text{ApproxMacDur} = \text{ApproxModDur} \times (1+r)$$

Effective Duration

$$\text{EffDur} = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta\text{Curve}) \times (PV_0)}$$

Duration of a Bond Portfolio

$$\text{Portfolio duration} = w_1 D_1 + w_2 D_2 + \ldots + w_N D_N$$

$$\text{Annual ModDur} = \frac{\text{Annual MacDur}}{1+r}$$

Money Duration

$$\text{MoneyDur} = \text{AnnModDur} \times \text{PV}^{\text{Full}}$$

The estimated (dollar) change in the price of the bond is calculated as:

$$\Delta\text{PV}^{\text{Full}} = -\text{MoneyDur} \times \Delta\text{Yield}$$

Price Value of a Basis Point

$$\text{PVBP} = \frac{(\text{PV}_{-}) - (\text{PV}_{+})}{2}$$

Basis Point Value (BPV)

$$\text{BPV} = \text{MoneyDur} \times 0.0001 \text{ (1 bps expressed as a decimal)}$$

Annual Convexity

$$\text{ApproxCon} = \frac{(\text{PV}_{-}) + (\text{PV}_{+}) - [2 \times (\text{PV}_0)]}{(\Delta\text{Yield})^2 \times (\text{PV}_0)}$$

Price Effects Based on Duration and Convexity

$$\%\Delta\text{PV}^{\text{Full}} \approx (-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right]$$

Money convexity

$$\Delta\text{PV}^{\text{Full}} \approx (-\text{MoneyDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{MoneyCon} \times (\Delta\text{Yield})^2\right]$$

Effective convexity

$$\text{EffCon} = \frac{[(\text{PV}_{-}) + (\text{PV}_{+})] - [2 \times (\text{PV}_0)]}{(\Delta\text{Curve})^2 \times (\text{PV}_0)}$$

Yield Volatility

$$\%\Delta\text{PV}^{\text{Full}} \approx (-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right]$$

Duration Gap

$$\text{Duration gap} = \text{Macaulay duration} - \text{Investment horizon}$$

Fundamentals of Credit Analysis

Expected Loss

$$\text{Expected loss} = \text{Default probability} \times \text{Loss severity given default}$$

Yield on a corporate bond:

$$\text{Yield on a corporate bond} = \text{Real risk-free interest rate} + \text{Expected inflation rate} + \text{Maturity premium} + \text{Liquidity premium} + \text{Credit spread}$$

Yield Spread:

$$\text{Yield spread} = \text{Liquidity premium} + \text{Credit spread}$$

For small, instantaneous changes in the yield spread, the return impact (i.e. the percentage change in price, including accrued interest) can be estimated using the following formula:

$$\text{Return impact} \approx -\text{Modified duration} \times \Delta\text{Spread}$$

For larger changes in the yield spread, we must also incorporate the (positive) impact of convexity into our estimate of the return impact:

$$\text{Return impact} \approx -(\text{MDur} \times \Delta\text{Spread}) + (1/2 \times \text{Convexity} \times \Delta\text{Spread}^2)$$

Derivatives

Basics of Derivative Pricing and Valuation

Fundamental Value of an Asset

$$S_0 = \left[\frac{E(S_T)}{(1+r+\lambda)^T}\right] - \theta + \gamma$$

Where:
r = risk free rate
λ = asset risk premium
θ = Storage, insurance, and other costs at present value
γ = Convenience and other benefits at present value

Arbitrage and Replication:

Asset + Derivative = Risk-free asset

Asset − Risk-free asset = −Derivative

Derivative − Risk-free asset = −Asset

Forward Contract Payoffs:

	$S_T > F(0,T)$	$S_T < F(0,T)$
Long position	$S_T - F(0,T)$ (Positive payoff)	$S_T - F(0,T)$ (Negative payoff)
Short position	$-[S_T - F(0,T)]$ (Negative payoff)	$-[S_T - F(0,T)]$ (Positive payoff)

Forward price:

$$F(0,T) = S_0(1+r)^T$$

$$F(0,T) = (S_0 - \gamma + \theta)(1+r)^T \text{ or } F(0,T) = S_0(1+r)^T - (\gamma - \theta)(1+r)^T$$

*Note that benefits (γ) and costs (θ) are expressed in terms of present value.

Value of a forward contract:

$$V_t(0,T) = S_t - [F(0,T)/(1+r)^{T-t}]$$

$$V_t(0,T) = S_t - (\gamma - \theta)(1+r)^t - [F(0,T)/(1+r)^{T-t}]$$

Time	Long Position Value	Short Position Value
At initiation	Zero, as the contract is priced to prevent arbitrage	Zero, as the contract is priced to prevent arbitrage
During life of the contract	$S_t - \left[\frac{F(0,T)}{(1+r)^{T-t}}\right]$	$\left[\frac{F(0,T)}{(1+r)^{T-t}}\right] - S_t$
At expiration	$S_T - F(0,T)$	$F(0,T) - S_T$

Net payment made (received) by the fixed-rate payer on a swap:

$$\text{Net fixed rate payment}_t = [\text{Swap fixed rate} - (\text{LIBOR}_{t-1} + \text{spread})]^* (\text{No. of days}/360)^* \text{NP}$$

Call Option Payoffs

Option Position	Descriptions	Payoff	
		$S_T > X$	$S_T < X$
		Option holder exercises the option	Option holder does not exercise the option
Call option holder	Choice to buy the underlying asset for X	$S_T - X$	0
Call option writer	Obligation to sell the underlying asset for X if the option holder chooses to exercise the option	$-(S_T - X)$	0

Moneyness and Exercise Value of a Call Option

Moneyness	Current Market Price (S_t) versus Exercise Price (X)	Intrinsic Value Max [0, (S_t – X)]
In-the-money	S_t is greater than X	$S_t - X$
At-the-money	S_t equals X	0
Out-of-the-money	S_t is less than X	0

Put Option Payoffs

Option Position	Descriptions	Payoff	
		$S_T < X$	$S_T > X$
		Option holder exercises the option	Option holder does not exercise the option
Put option holder	Choice to sell the underlying asset for X	$X - S_T$	0
Put option writer	Obligation to buy the underlying asset for X if the option holder chooses to exercise the option	$-(X - S_T)$	0

Moneyness and Exercise Value of a Put Option

Moneyness	Current Market Price (S_t) versus Exercise Price (X)	Intrinsic Value Max [0, (X – S_t)]
In-the-money	S_t is less than X	$X - S_t$
At-the-money	S_t equals X	0
Out-of-the-money	S_t is greater than X	0

Fiduciary Call and Protective Put Payoffs

Security	Value if $S_T > X$	Value if $S_T < X$
Call option	$S_T - X$	Zero
Zero coupon bond	X	X
Fiduciary call payoff	S_T	X
Put option	Zero	$X - S_T$
Stock	S_T	S_T
Protective put payoff	S_T	X

Put-Call Parity

$$c_0 + \frac{X}{(1+R_F)^T} = p_0 + S_0$$

Combining Portfolios to Make Synthetic Securities

Strategy	Consisting of	Value	Equals	Strategy	Consisting of	Value
fiduciary call	long call + long bond	$c_0 + \frac{X}{(1+R_F)^T}$	=	Protective put	long put + long underlying asset	$p_0 + S_0$
long call	long call	c_0	=	Synthetic call	long put + long underlying asset + short bond	$p_0 + S_0 - \frac{X}{(1+R_F)^T}$
long put	long put	p_0	=	Synthetic put	long call + short underlying asset + long bond	$c_0 - S_0 + \frac{X}{(1+R_F)^T}$
long underlying asset	long underlying asset	S_0	=	Synthetic underlying asset	long call + long bond + short put	$c_0 + \frac{X}{(1+R_F)^T} - p_0$
long bond	long bond	$\frac{X}{(1+R_F)^T}$	=	Synthetic bond	long put + long underlying asset + short call	$p_0 + S_0 - c_0$

Lowest Prices of European Calls and Puts

$$c_0 \geq \text{Max}[0, S_0 - \frac{X}{(1+R_F)^T}]$$

$$p_0 \geq \text{Max}[0, \frac{X}{(1+R_F)^T} - S_0]$$

Put-Call Forward Parity

$$p_0 - c_0 = \frac{[X - F(0,T)]}{(1+R_F)^T}$$

Binomial Option Pricing

$$c = \frac{\pi c^+ + (1-\pi)c^-}{(1+r)}$$

$$\pi = \frac{(1+r-d)}{(u-d)}$$

Where $u = \frac{S_1^+}{S_0}$ and $d = \frac{S_1^-}{S_0}$

Hedge ratio

$$n = \frac{c^+ - c^-}{S^+ - S^-}$$

Lowest Prices of American Calls and Puts

$$C_0 \geq \text{Max}[0,\ S_0 - X/(1+\text{RFR})^T]$$

$$P_0 \geq \text{Max}[0,\ (X - S_0)]$$

Summary of Options Strategies

	Call	**Put**
Holder	$C_T = \max(0, S_T - X)$ Value at expiration $= C_T$ Profit: $\prod = C_T - C_0$ Maximum profit $= \infty$ Maximum loss $= C_0$ Breakeven: $S_T^* = X + C_0$	$P_T = \max(0, X - S_T)$ Value at expiration $= P_T$ Profit: $\prod = P_T - P_0$ Maximum profit $= X - P_0$ Maximum loss $= P_0$ Breakeven: $S_T^* = X - P_0$
Writer	$C_T = \max(0, S_T - X)$ Value at expiration $= -C_T$ Profit: $\prod = -C_T - C_0$ Maximum profit $= C_0$ Maximum loss $= \infty$ Breakeven: $S_T^* = X + C_0$	$P_T = \max(0, X - S_T)$ Value at expiration $= -P_T$ Profit: $\prod = -P_T - P_0$ Maximum profit $= P_0$ Maximum loss $= X - P_0$ Breakeven: $S_T^* = X - P_0$

Where:
C_0, C_T = price of the call option at time 0 and time T
P_0, P_T = price of the put option at time 0 and time T
X = exercise price
S_0, S_T = price of the underlying at time 0 and time T
V_0, V_T = value of the position at time 0 and time T
$\prod$ = profit from the transaction: $V_T - V_0$
r = risk-free rate

Covered Call

Value at expiration: $V_T = S_T - \max(0, S_T - X)$
Profit: $\prod = V_T - S_0 + C_0$
Maximum profit $= X - S_0 + C_0$
Maximum loss $= S_0 - C_0$
Breakeven: $S_T^* = S_0 - C_0$

Protective Put

Value at expiration: $V_T = S_T + \max(0, X - S_T)$
Profit: $\prod = V_T - S_0 - P_0$
Maximum profit $= \infty$
Maximum loss $= S_0 + P_0 - X$
Breakeven: $S_T^* = S_0 + P_0$

Alternative Investments

Introduction to Alternative Investments

Hedge Fund Fee Structure

When quoted as 2 and 20, 1 and 10, etc., the first number is the percent management fee and the second number is the percent performance fee. Where management fees and performance fees are calculated independently, it means that management fees are charged against AUM gross of fees and performance fees are charged against profit calculated net of fees.

Hedge Fund Management Fee

$$\text{Management fee} = F_M \times AUM_{t-1}$$

AUM_{t-1} = beginning assets under management (end of last period)
FM = percent management fee

Note that hedge funds may apply a management fee to ending AUM, but this is not as common because performance fees already realize a share of investor profits during the period.

Hedge Fund Performance Fee

In general, performance fees equal a percentage fee multiplied by investor profit. Profit may be described as gross of fees (i.e., independent of management fees) or net of fees.

A performance fee may be subject to a high-water mark based on the highest end-of-period AUM value (to avoid additional fees against a profit that only improves an investor position to a previous high water mark):

$$\begin{gathered}\text{IF } AUM_{t-1} < HWM < AUM_t\text{, then}\\ \text{Performance fee} = F_P \times (AUM_t - HWM)\text{, ELSE}\\ \text{Performance fee} = F_P \times (AUM_t - AUM_{t-1})\end{gathered}$$

Where
AUM_t = ending assets under management
HWM = high-water mark
F_P = percent performance fee

Note that an investor usually must make a profit before performance fees are applied.

Hurdle Rate

The percentage performance fee applied may be subject to a hurdle rate (e.g., risk-free rate, S&P500 index return, etc.):

$$F_P = \left(\frac{AUM_t}{AUM_{t-1} \text{ or } HWM} - 1\right) - r_H$$
$$= r_A - r_H$$

Where
r_A = percent return of the asset (i.e., fund)
r_H = percent hurdle rate (i.e., benchmark rate)

Hedge Fund Net-of-Fee Returns

$$\text{net-of-fees return} = r_I = \frac{AUM_t}{AUM_{t-1}} - 1$$

Where
r_I = investor return

Note that AUM will be actual, net-of-fee returns for this measure.