Semiconductor Materials and Devices
(반도체 재료 및 소자)

Chapter 6. Nonequilibrium excess carriers in semiconductors

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In Equilibrium

Generation : the process whereby electrons and holes are created

Recombination : the process whereby electrons and holes are annihilated

Any deviation from thermal equilibrium (temp. change or external excitation)

→ Change electrons and hole concentrations

→ New equilibrium electron and hole concentration

$$G_{n0} = G_{p0} \qquad (\#/\text{cm}^3\text{-s})$$

$$R_{n0} = R_{p0}$$

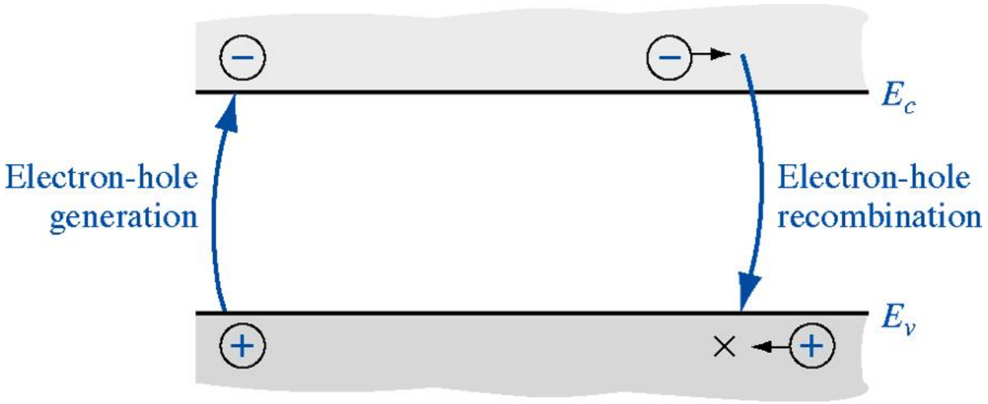
$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

G_{n0} : thermal generation rates of electrons

G_{p0} : thermal generation rates of electrons

R_{n0} : recombination rates of electrons

R_{p0} : recombination rates of holes



Direct Band-to-Band Generation and Recombination

Excess Carrier Generation and Recombination

For the direct band-to-band generation,

$$g'_n = g'_p$$

When excess electrons and holes are created,

$$n = n_0 + \delta n$$

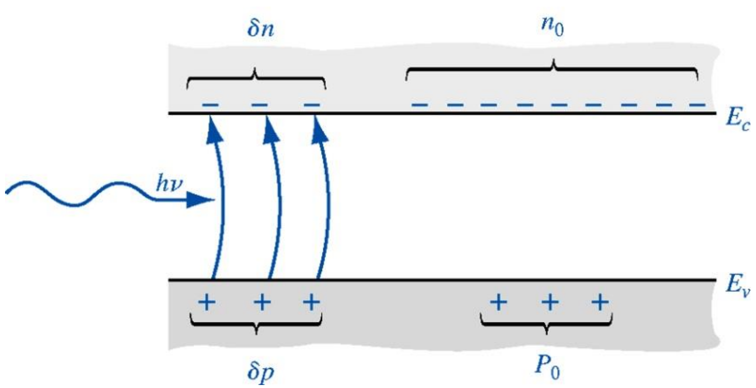
$$p = p_0 + \delta p$$

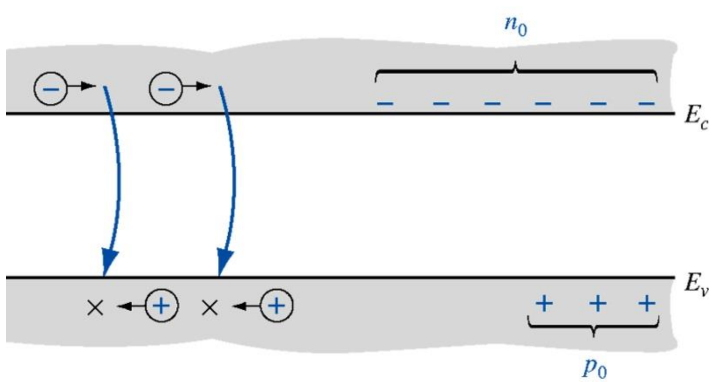
For spontaneous recombination,

$$R'_n = R'_p$$

Table 6.1 Relevant notations used in Chapter 6

$n_0,\ p_0$	Thermal equilibrium electron and hole concentrations (independent of time and also usually positions).
$n,\ p$	Total electron and hole concentrations (may be functions of time and/or position).
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or positions).
$g'_n,\ g'_p$	Excess electron and hole generation rates.
$R'_n,\ R'_p$	Excess electron and hole recombination rates.
$\tau_{n0},\ \tau_{p0}$	Excess minority carrier electron and hole lifetimes.





Excess Carrier Generation and Recombination

The net rate of change in the electron concentration :

$$\frac{dn(t)}{dt} = \alpha_r\left[n_i^2 - n(t)p(t)\right]$$

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

$$\frac{dn(t)}{dt} = \frac{d(\delta n(t))}{dt} = \alpha_r\left[n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t))\right]$$

$$= -\alpha_r \delta n(t)[(n_0 + p_0) + \delta n(t)]$$

($\delta n(t) = \delta p(t)$)

For low-level injection, the magnitude of the excess carrier concentration is small compared with the thermal equilibrium majority carrier concentrations,

For a p-type material ($p_0 \gg n_0$) and $\delta n(t) \ll p_0$,

$$\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t)$$

$\tau_{n0} = (\alpha_r p_0)^{-1}$: Excess minority carrier lifetime

$$\Rightarrow \quad \delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_{n0}}$$

Recombination rate (positive quantity) :

$$R_n' = \frac{-d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}}$$

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{n0}}$$

(In case of direct band-to-band recombination)

Similarly, for a n-type material ($n_0 \gg p_0$) and $\delta p(t) \ll n_0$,

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{p0}}$$

$\tau_{p0} = (\alpha_r n_0)^{-1}$

Continuity Equations

$$F_{px}^{+}(x+dx) = F_{px}^{+}(x) + \frac{\partial F_{px}^{+}}{\partial x} \cdot dx$$

$F_{px}{}^{+}$: hole flux (#/cm^2-s)

The net increase in the number of holes per unit time within the differential volume due to the x-component of hole flux :

$$\frac{\partial p}{\partial t}\,dx\,dy\,dz = [F_{px}^{+}(x) - F_{px}^{+}(x+dx)]\,dy\,dz = -\frac{\partial F_{px}^{+}}{\partial x}\,dx\,dy\,dz$$

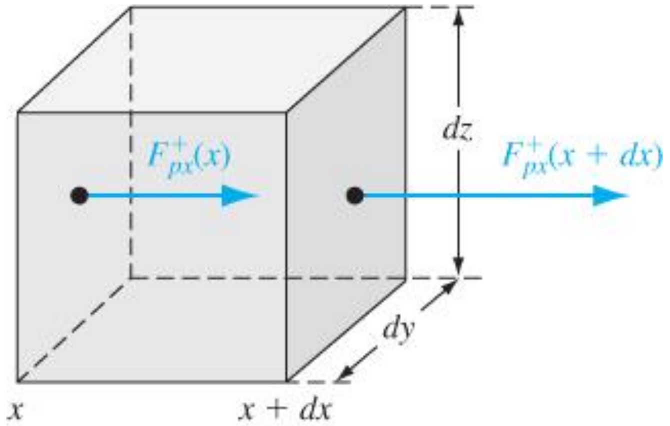


Figure 6.4 | Differential volume showing x component of the hole-particle flux.

The generation and recombination rate of holes within the differential volume will also affect the hole concentration :

$$\frac{\partial p}{\partial t}\,dx\,dy\,dz = -\frac{\partial F_{p}^{+}}{\partial x}\,dx\,dy\,dz + g_p\,dx\,dy\,dz - \frac{p}{\tau_{pt}}\,dx\,dy\,dz \quad \Longrightarrow \quad \frac{\partial p}{\partial t} = -\frac{\partial F_{p}^{+}}{\partial x} + g_p - \frac{p}{\tau_{pt}}$$

hole flux + generation - recombination

$$\left(\frac{p}{\tau_{pt}} = \frac{p_0}{\tau_p} + \frac{\delta p}{\tau_{p0}}\right)$$

thermal equilibrium carrier lifetime

excess carrier lifetime

$$\frac{\partial n}{\partial t} = -\frac{\partial F_{n}^{-}}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

: for electron

Time-Dependent Diffusion Equations

From continuity equations

$$J_p = e\mu_p p\mathrm{E} - eD_p\frac{\partial p}{\partial x} \qquad \frac{J_p}{(+e)} = F_p^+ = \mu_p p\mathrm{E} - D_p\frac{\partial p}{\partial x} \qquad \frac{\partial p}{\partial t} = -\mu_p\frac{\partial(p\mathrm{E})}{\partial x} + D_p\frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$J_n = e\mu_n n\mathrm{E} + eD_n\frac{\partial n}{\partial x} \qquad \frac{J_n}{(-e)} = F_n^- = -\mu_n n\mathrm{E} - D_n\frac{\partial n}{\partial x} \qquad \frac{\partial n}{\partial t} = +\mu_n\frac{\partial(n\mathrm{E})}{\partial x} + D_n\frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

$$D_p\frac{\partial^2 p}{\partial x^2} - \mu_p\left(\mathrm{E}\frac{\partial p}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial p}{\partial t} \qquad D_p\frac{\partial^2(\delta p)}{\partial x^2} - \mu_p\left(\mathrm{E}\frac{\partial(\delta p)}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

$$D_n\frac{\partial^2 n}{\partial x^2} + \mu_n\left(\mathrm{E}\frac{\partial n}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial n}{\partial t} \qquad D_n\frac{\partial^2(\delta n)}{\partial x^2} + \mu_n\left(\mathrm{E}\frac{\partial(\delta n)}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

: determine the space and time behavior of the excess carriers !!

For homogeneous semiconductor, n_0 and p_0 are independent of the space.

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

$$\mathrm{E} = \mathrm{E}_{\mathrm{app}} + \mathrm{E}_{\mathrm{int}}$$

Since the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and excess holes together. → The electrons and holes will drift or diffuse together with a single effective mobility or diffusion coefficient : Ambipolar transport !

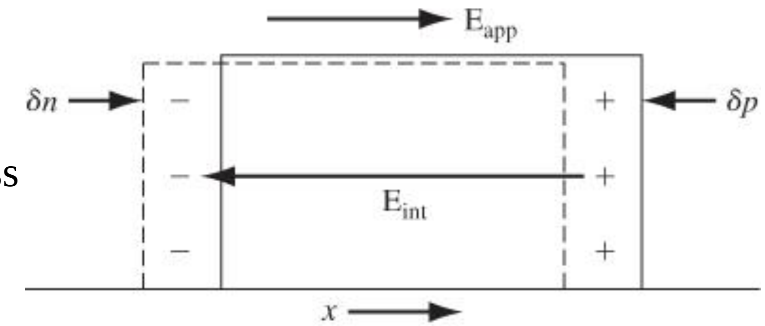


Figure 6.5 | The creation of an internal electric field as excess electrons and holes *tend* to separate.

Derivation of the Ambipolar Transport Equation

$$g_n = g_p \equiv g$$

$$R_n = \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}} \equiv R$$

$$\delta n \approx \delta p$$

$$D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_p \left(\mathrm{E} \frac{\partial(\delta n)}{\partial x} + p \frac{\partial \mathrm{E}}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t} \qquad \times \mu_n n$$

$$\oplus$$

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(\mathrm{E} \frac{\partial(\delta n)}{\partial x} + n \frac{\partial \mathrm{E}}{\partial x} \right) + g - R = \frac{\partial(\delta n)}{\partial t} \qquad \times \mu_p p$$

$$\Rightarrow \quad (\mu_n n D_p + \mu_p p D_n) \frac{\partial^2(\delta n)}{\partial x^2} + (\mu_n \mu_p)(p - n)\mathrm{E} \frac{\partial(\delta n)}{\partial x} + (\mu_n n + \mu_p p)(g - R) = (\mu_n n + \mu_p p) \frac{\partial(\delta n)}{\partial t}$$

$$\div (\mu_n n + \mu_p p)$$

Ambipolar Transport Equation !

$$\Rightarrow \quad D' \frac{\partial^2(\delta n)}{\partial x^2} + \mu' \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g - R = \frac{\partial(\delta n)}{\partial t}$$

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \qquad D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

For extrinsic semiconductor and by considering low-level injection :

For a p-type material ($p_0 \gg n_0$) and $\delta n \ll p_0$,

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p} = \frac{D_n D_p [(n_0 + \delta n) + (p_0 + \delta n)]}{D_n (n_0 + \delta n) + D_p (p_0 + \delta n)} = D_n$$

Ambipolar diffusion coefficient and mobility → minority carrier parameters (constants)

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} = \mu_n$$

$$g - R = g_n - R_n = (G_{n0} + g_n') - (R_{n0} + R_n')$$

$$G_{n0} = R_{n0}$$

$$g - R = g_n' - R_n' = g_n' - \frac{\delta n}{\tau_n}$$

⇨ $$D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial (\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial (\delta n)}{\partial t}$$

Similarly,

$$D_p \frac{\partial^2 (\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial (\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial (\delta p)}{\partial t}$$

: describe the drift, diffusion, and recombination of excess minority carriers as a function of spatial coordinates and as a function of time.

Example 6.1 : For an infinite and homogeneous n-type semiconductor, a uniform concentration of excess carriers at t =0 and g'=0 for t >0. Low level injection and no E-field.

$$\partial^2(\delta p)/\partial x^2 = \partial(\delta p)/\partial x = 0$$

The ambipolar transport equation for the minority carrier holes,

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t} \Longrightarrow \frac{d(\delta p)}{dt} = -\frac{\delta p}{\tau_{p0}} \Longrightarrow \delta p(t) = \delta p(0) e^{-t/\tau_{p0}}$$

Example 6.2 : For an infinite and homogeneous n-type semiconductor, a uniform generation rate at $t \geq 0$. Low level injection and no E-field.

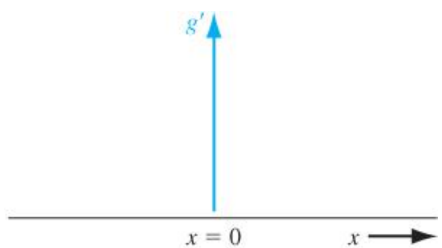
$$g' - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt} \Longrightarrow \delta p(t) = g'\tau_{p0}(1 - e^{-t/\tau_{p0}}) \quad \delta p(t) = (5 \times 10^{21})(10^{-7})[1 - e^{-t/10^{-7}}] = 5 \times 10^{14}\,[1 - e^{-t/10^{-7}}]\ \mathrm{cm}^{-3}$$

Example 6.3 : For an infinite and homogeneous p-type semiconductor, the excess carriers are generated only at x =0 position. Low level injection and no E-field. Steady-state excess carrier concentration ?

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t} \Longrightarrow D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0$$

$$\Longrightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{D_n \tau_{n0}} = \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \Longrightarrow \delta n(x) = A e^{-x/L_n} + B e^{x/L_n}$$

$$\Longrightarrow \begin{cases} \delta n(x) = \delta n(0) e^{+x/L_n} & x \leq 0 \\ \delta n(x) = \delta n(0) e^{-x/L_n} & x \geq 0 \end{cases}$$



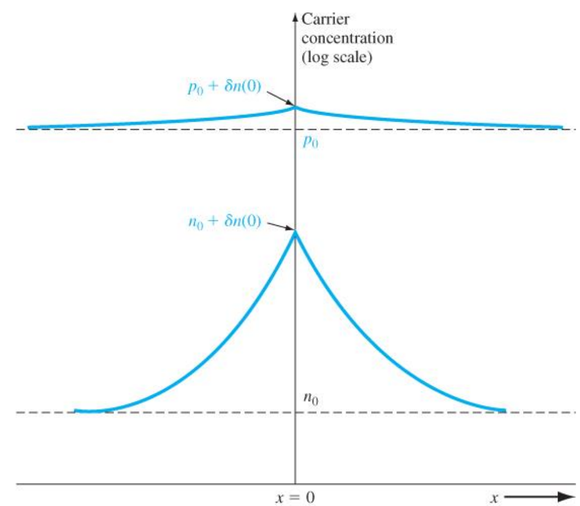


Figure 6.7 | Steady-state electron and hole concentrations for the case when excess electrons and holes are generated at $x = 0$.

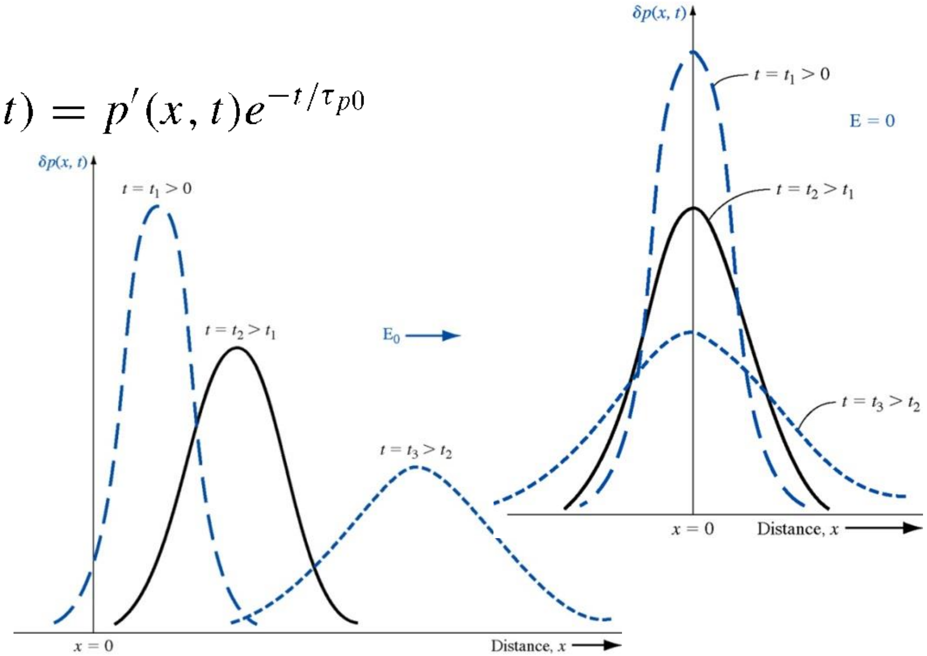
Example 6.4 : For an infinite and homogeneous n-type semiconductor, a finite number of electron-hole pairs is generated instantaneously at time t =0 and at x =0, but g'=0 for t >0. Low level injection and constant E-field E_0 in the +x direction.

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E_0 \frac{\partial(\delta p)}{\partial x} - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t} \Rightarrow \delta p(x,t) = p'(x,t)e^{-t/\tau_{p0}}$$

$$\Rightarrow D_p \frac{\partial^2 p'(x,t)}{\partial x^2} - \mu_p E_0 \frac{\partial p'(x,t)}{\partial x} = \frac{\partial p'(x,t)}{\partial t}$$

$$\Rightarrow p'(x,t) = \frac{1}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$

$$\Rightarrow \delta p(x,t) = \frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$



Dielectric Relaxation Time Constant

δp is suddenly injected into a portion of semiconductor → How is charge neutrality achieved and how fast ?

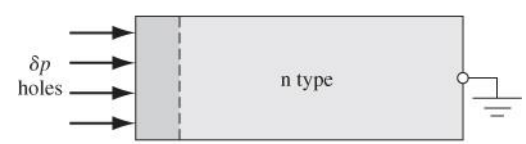


Figure 6.10 | The injection of a concentration of holes into a small region at the surface of an n-type semiconductor.

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\epsilon}$$

$$J = \sigma \mathrm{E}$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot J = \sigma \nabla \cdot E = \frac{\sigma\rho}{\epsilon}$$

$$\Rightarrow \frac{\sigma\rho}{\epsilon} = -\frac{\partial\rho}{\partial t} = -\frac{d\rho}{dt}$$

$$\frac{d\rho}{dt} + \left(\frac{\sigma}{\epsilon}\right)\rho = 0$$

$$\rho(t) = \rho(0)e^{-(t/\tau_d)}$$

$$\tau_d = \frac{\epsilon}{\sigma}$$

: dielectric Relaxation Time Constant

Haynes-Shockley Experiment

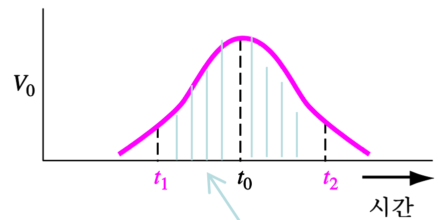
Experimental measurement of **mobility, diffusion coefficient**, and **minority carrier lifetime**.

$$\delta p(x,t) = \frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4 D_p t}\right]$$

from Example 6.4

$$x - \mu_p E_0 t = 0 \Longrightarrow \mu_p = \frac{d}{E_0 t_0}$$

Output V_0 vs. time



: area S

t_1, t_2 : e^{-1} of peak and not too long

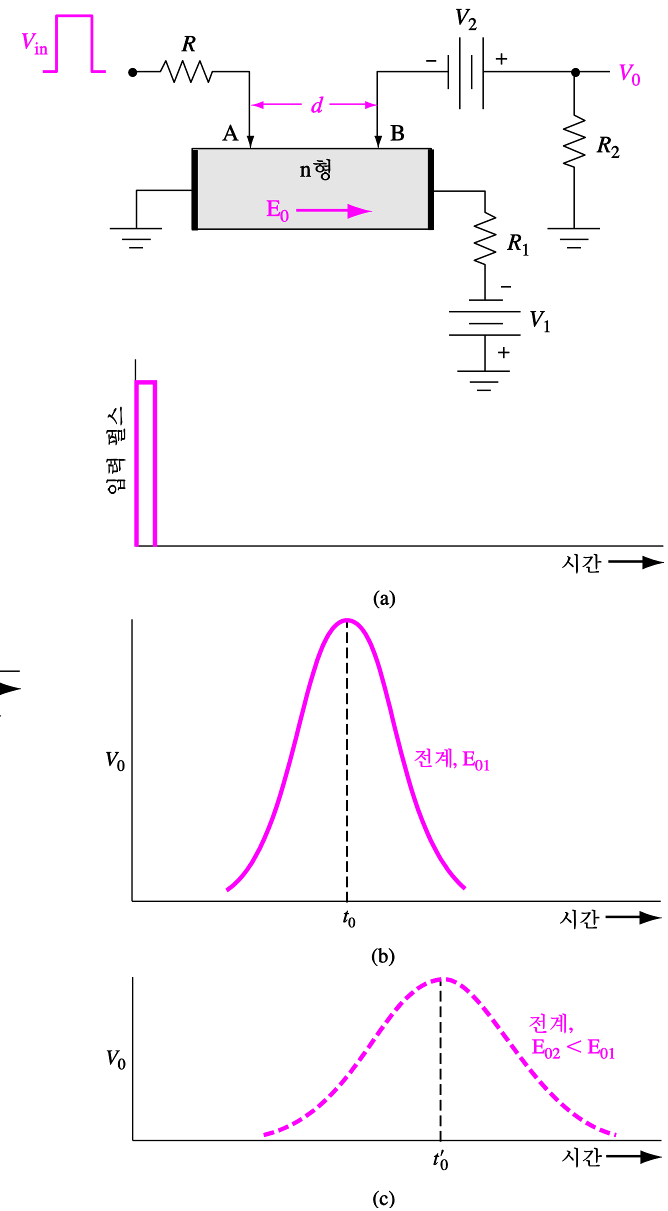
$$(d - \mu_p E_0 t)^2 = 4 D_p t \quad at\ t = t_1, t_2$$

$$\Longrightarrow D_p = \frac{(\mu_p E_0)^2 (\Delta t)^2}{16 t_0} \qquad \Delta t = t_2 - t_1$$

The area S is proportional to the number of excess holes that not recombined with majority electrons :

$$S = K \exp\left(\frac{-t_0}{\tau_{p0}}\right) = K \exp\left(\frac{-d}{\mu_p E_0 \tau_{p0}}\right)$$

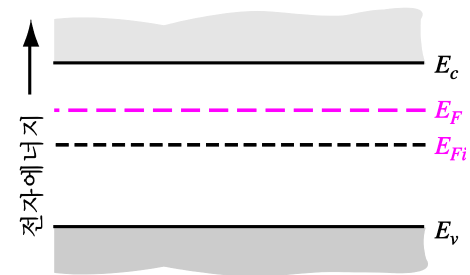
τ_{p0} can be obtained from the plot of ln$S$ versus ($d/\mu_p E_0$).



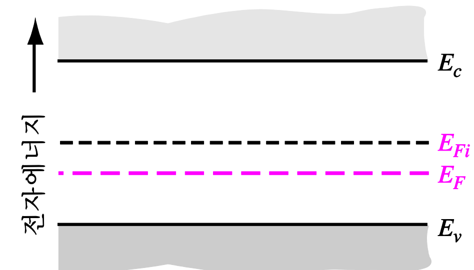
Quasi-Fermi Energy Levels

If excess carriers are created in a semiconductor, we are no longer in thermal equilibrium and the Fermi energy level is strictly no longer defined.

→ We may define a quasi-Fermi levels for electron and holes that apply for nonequilibrium.



(a)



(b)

At Equilibrium

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

At Nonequilibrium

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

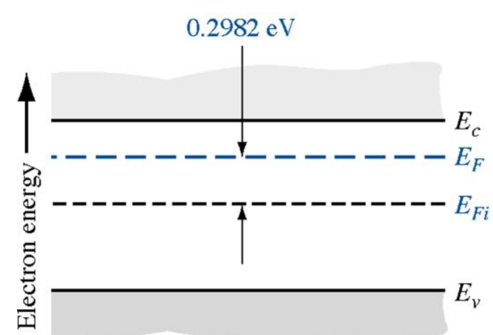
$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Example 6.6 : An n-type semiconductor at T =300K with n_0 =10^{15}cm^{-3}, n_i =10^{10}cm^{-3}, and p_0 =10^5cm^{-3}. In nonequilibrium, assume that the excess carrier concentrations are $\delta n = \delta p$ =10^{13}cm^{-3}.

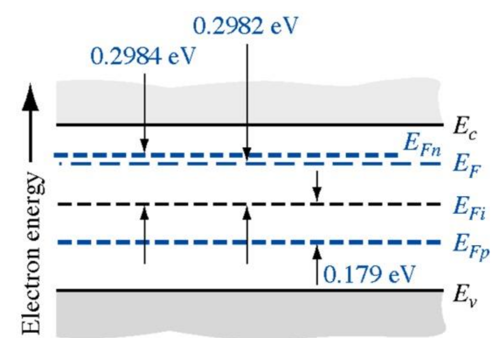
$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982 \text{ eV}$$

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_0 + \delta n}{n_i}\right) = 0.2984 \text{ eV}$$

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_0 + \delta p}{n_i}\right) = 0.179 \text{ eV}$$



(a)



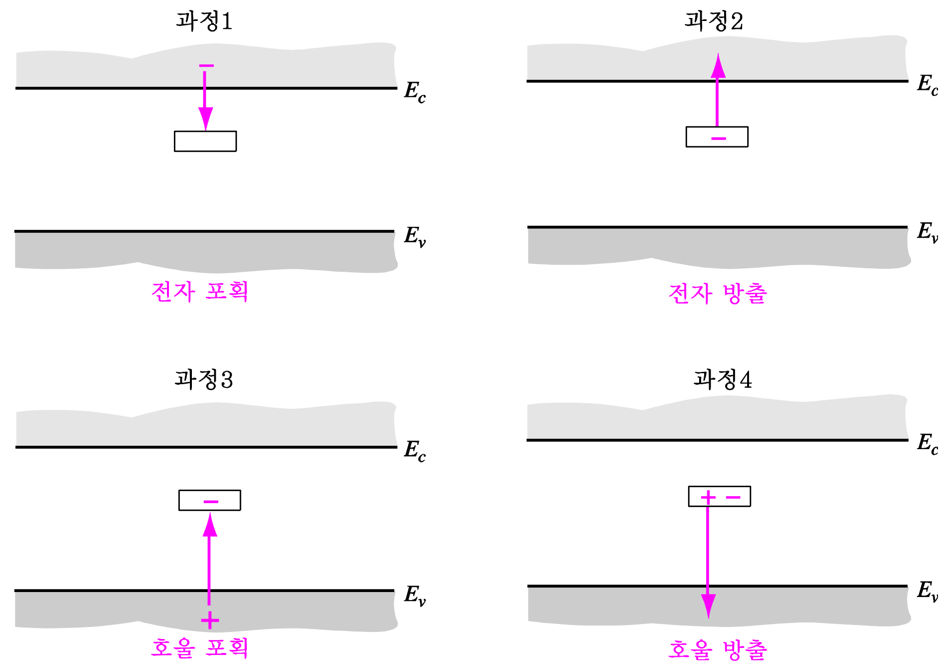
(b)

Shockley-Read-Hall Theory of Recombination

Trap : An allowed energy state within the forbidden bandgap caused by defects or imperfection of periodic potential.

→Acts as a *recombination center* , capturing both electrons and holes with almost equal probability.

→ four basic processes: Electron Capture, Electron Emission, Hole Capture, and Hall Emission



$$R_{cn} = C_n N_t (1 - f_F(E_t))n$$

R_{cn} = capture rate (#/cm^3-s)

C_n = constant proportional to electron-capture cross section

N_t = total concentration of trapping centers

n = electron concentration

$f_F(E_t)$ = Fermi function at the trap energy

$$f_F(E_t) = \frac{1}{1 + \exp\left[\dfrac{E_t - E_F}{kT}\right]}$$

$$R_{en} = E_n N_t f_F(E_t)$$

R_{en} = emission rate (#/cm^3-s)

E_n = constant proportional to emission cross section

$f_F(E_t)$ = probability that the trap is occupied by electron

In thermal equilibrium, $R_{en} = R_{cn}$

$$\Rightarrow E_n N_t f_{F0}(E_t) = C_n N_t (1 - f_{F0}(E_t)) n_0$$

$f_{F0}(E_t)$ = Thermal equilibrium Fermi function

n_0 = electron concentration at equilibrium

$$\frac{E_n}{C_n} = n' = \frac{n_0 (1 - f_{F0}(E_t))}{f_{F0}(E_t)} = N_C \exp\left(\frac{E_F - E_C}{kT}\right) \exp\left(\frac{E_t - E_F}{kT}\right) = N_C \exp\left(\frac{-(E_C - E_t)}{kT}\right)$$

In nonequilibrium, the net rate at which electrons are captured from the conduction band is given by :

$$R_n = R_{cn} - R_{en} = [C_n N_t (1 - f_F(E_t))n] - [E_n N_t f_F(E_t)]$$
$$= C_n N_t [n(1 - f_F(E_t)) - n' f_F(E_t)]$$

n includes the excess electron concentration.

$E_F \rightarrow E_{Fn}$

Similarly, the net rate at which holes are captured from the valence band is given by :

$$R_p = C_p N_t [p f_F(E_t) - p'(1 - f_F(E_t))] \qquad \text{where} \qquad p' = N_v \exp\left[\frac{-(E_t - E_v)}{kT}\right]$$

In a semiconductor in which the trap density is not too large, the excess electron and hole concentrations are equal and the recombination rates are equal :

Recombination rate due to the trap at $E = E_t$ in case that excess carriers exist

$$R_n = R_p \Rightarrow \left. \begin{aligned} f_F(E_t) &= \frac{C_n n + C_p p'}{C_n(n + n') + C_p(p + p')} \\ n' p' &= n_i^2 \end{aligned} \right\}$$

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \equiv R = \frac{\delta n}{\tau}$$

For an n-type semiconductor under low injection, $n_0 \gg p_0$, $n_0 \gg \delta p$, $n_0 \gg n'$, $n_0 \gg p'$:

$$R \cong \frac{C_n C_p N_t (\delta p \cdot n_0 + \delta n \cdot p_0 + \delta p \cdot \delta n)}{C_n \cdot n_0} = C_p N_t \delta p \equiv \frac{\delta p}{\tau_{p0}} \qquad \text{where} \qquad \tau_{p0} = \frac{1}{C_p N_t}$$

: function of minority carrier parameters