

Theory of Semiconductor Devices (반도체 소자 이론)

Lecture 13

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Mass Action Law

- When we multiply n times p in the classical approximation,

$$np = N_c N_v \exp\left(\frac{\mathcal{E}_v - \mathcal{E}_c}{kT}\right) \quad (4.83)$$

$$np = N_c N_v \exp\left(\frac{-\mathcal{E}_g}{kT}\right)$$

we find that the product is independent of \mathcal{E}_f and depends only on \mathcal{E}_g and the temperature T . N_c and N_v are also functions of temperature. Since the product is independent of \mathcal{E}_f , this means that in the classical approximation (4.83) is valid for **any pair of values of n and p** , including the intrinsic case.

- We thus have

$$np = n_i^2(T) \quad (4.84)$$

where for a given material n_i^2 is a function of **temperature only**. If we recall the **quasi-chemical equation** (3.38) for the formation of an electron and a hole,



we see that (4.83) or (4.84) indicates a **relationship between the concentrations of the products of the quasi-chemical reaction for electrons and holes**.

This relationship between concentrations is **exponentially dependent on the energy** (per particle) required to produce the reaction, \mathcal{E}_g , and is called a **mass-action law**.

Mass Action Law

- Considering the **limitations** on the classical equations (4.73) and (4.79), the mass-action law for electrons and holes given in (4.84) is **valid** for

$$\mathcal{E}_v + 4kT < \mathcal{E}_f < \mathcal{E}_c - 4kT \quad (4.86)$$

- Under these conditions the semiconductor is said to be **nondegenerate**.

- When \mathcal{E}_f is greater than $\mathcal{E}_c - 4kT$ or less than $\mathcal{E}_v + 4kT$, (4.84) can **no longer be used**. Under these conditions the semiconductor is said to be **degenerate**, and (4.68) and (4.78) must be used to obtain the mass-action law.

$$np = N_c N_v F_{1/2} \left(\frac{\mathcal{E}_f - \mathcal{E}_c}{kT} \right) F_{1/2} \left(\frac{\mathcal{E}_v - \mathcal{E}_f}{kT} \right) \quad (4.87)$$

$$np \neq n_i^2$$

Thus, under these conditions the **np product depends on the Fermi energy**.

- Extrinsic case -

- Let us next look at the relationship between \mathcal{E}_f and the electron occupancy of discrete donor levels. From (4.25) we obtain

$$\frac{N_d^0}{N_d^0 + N_d^+} = \frac{1}{1 + (1/g_d) \exp [(\mathcal{E}_d - \mathcal{E}_f)/kT]} \quad (4.88)$$

which can be put in the form

Mass Action Law

$$\frac{N_d^+}{N_d^0} = \frac{1}{g_d} \exp\left(\frac{\mathcal{E}_d - \mathcal{E}_f}{kT}\right) \quad (4.89)$$

- Thus the *ratio* of the concentration of *ionized* to *neutral* donors depends exponentially on the separation of the Fermi energy from the donor energy level. If we multiply (4.89) by (4.73), we obtain

$$\frac{nN_d^+}{N_d^0} = \frac{N_c}{g_d} \exp\left(\frac{\mathcal{E}_d - \mathcal{E}_c}{kT}\right) \quad (4.90)$$

which, using (3.33), is simply

$$\frac{nN_d^+}{N_d^0} = \frac{N_c}{g_d} \exp\left(\frac{-\Delta\mathcal{E}_d}{kT}\right) \quad (4.91)$$

This equation applies *only for* a *nondegenerate* semiconductor. Notice that (4.91) is independent of the concentrations (or Fermi energy) and depends only on temperature and the donor ionization energy, $\Delta\mathcal{E}_d$. Equation (4.91) is the *mass-action law for the donor ionization reaction* (3.37),



For *acceptors*

$$\frac{pN_a^-}{N_a^0} = \frac{N_v}{g_a} \exp(-\Delta\mathcal{E}_a/kT) \quad (4.95)$$

$$A^0 = A^- + e^+ : \Delta\mathcal{E}_a \quad (4.96)$$

Just as with the donor mass-action law, (4.95) is *valid only for a nondegenerate* semiconductor.

Mass Action Law

- Let us now consider the **electron and hole concentrations** for an **extrinsic semiconductor**.
- In general, we have a concentration of N_d donors, of which N_d^+ are ionized, and N_a acceptors per unit volume, of which N_a^- are ionized.
- For a nondegenerate semiconductor the electron and hole concentrations are given by (4.73) and (4.79), respectively.
- For a **degenerate** semiconductor n and p are given by (4.68) and (4.78). From (4.88) and (4.89) the **ratio of ionized to total donor concentrations** is

$$\frac{N_d^+}{N_d} = \frac{1}{1 + g_d \exp [(\mathcal{E}_f - \mathcal{E}_d)/kT]} \quad (4.97)$$

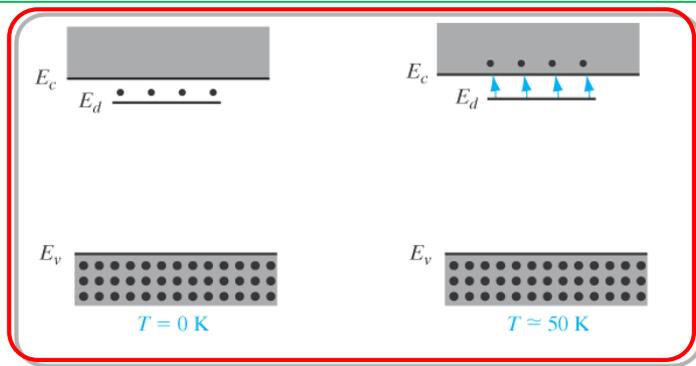
and from (4.93) the ratio of ionized to total acceptor concentrations is

$$\frac{N_a^-}{N_a} = \frac{1}{1 + g_a \exp [(\mathcal{E}_a - \mathcal{E}_f)/kT]} \quad (4.98)$$

- A relationship among these four charge concentrations n , p , N_d^+ , and N_a^- can be obtained by invoking **charge neutrality**.

$$N_d^+ - N_a^- + p - n = 0 \quad (4.101)$$

Band & Bond Models of Dopants



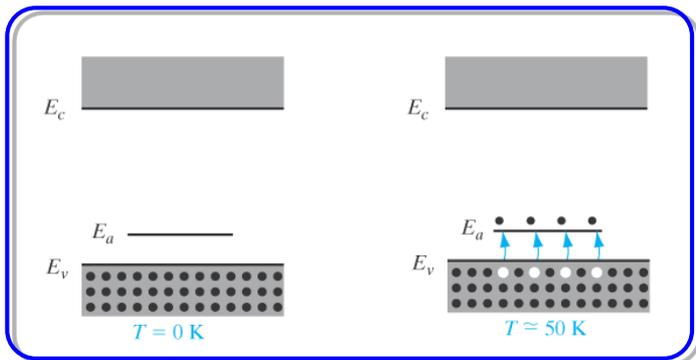
neutral

ionized

(a)

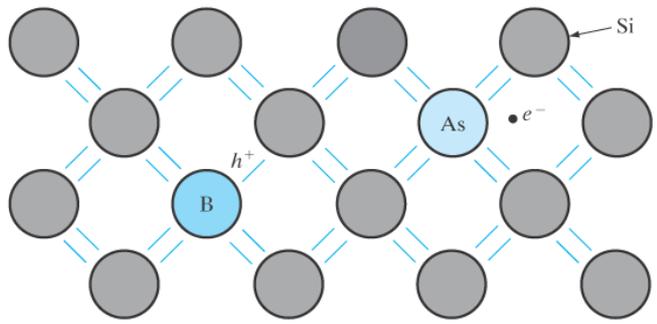
Energy band model and chemical bond model of **dopants** in semiconductors:

(a) donation of electrons from **donor** level to conduction band;



(b)

(b) Acceptance of valence band electrons by an **acceptor** level, and the resulting creation of holes;

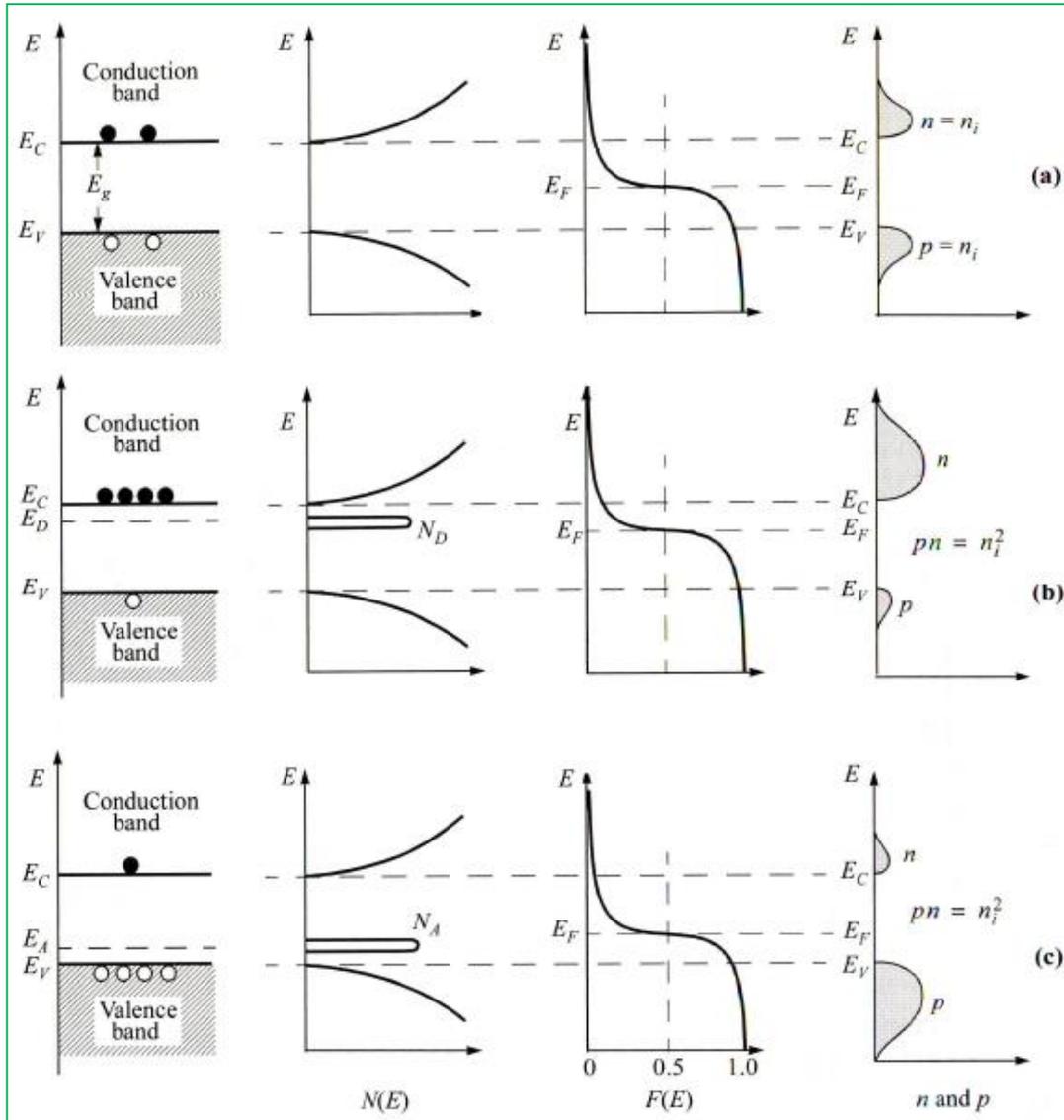


(c)

(c) donor and acceptor atoms in the covalent **bonding** model of a Si crystal.

From Solid State Electronic Devices, Sixth Edition, by Ben G. Streetman and Sanjay Kumar Banerjee.

DOS, FD Dist. & Carrier Conc.



Schematic band diagram, density of states, Fermi-Dirac distribution, and carrier concentrations for

(a) *intrinsic*

(b) *n-type*, and

(c) *p-type* semiconductors

at thermal equilibrium.

Note that $pn = n_i^2$ for all three cases.

From Sze, "Physics of Semiconductor Devices"

Fermi Energy Variation

- With (4.73) or (4.68), (4.79) or (4.78), (4.97), (4.98), and (4.101), we can in principle solve for the five unknown variables n , p , N_d^+ , N_a^- , and \mathcal{E}_f .

$$n = N_c \exp\left(\frac{\mathcal{E}_f - \mathcal{E}_c}{kT}\right) \quad (4.73)$$

$$p = N_v \exp\left(\frac{\mathcal{E}_v - \mathcal{E}_f}{kT}\right) \quad (4.79)$$

$$\frac{N_d^+}{N_d} = \frac{1}{1 + g_d \exp[(\mathcal{E}_f - \mathcal{E}_d)/kT]} \quad (4.97)$$

$$\frac{N_a^-}{N_a} = \frac{1}{1 + g_a \exp[(\mathcal{E}_a - \mathcal{E}_f)/kT]} \quad (4.98)$$

$$N_d^+ - N_a^- + p - n = 0 \quad (4.101)$$

Fermi Energy Variation

- With (4.73) or (4.68), (4.79) or (4.78), (4.97), (4.98), and (4.101), we can in principle solve for the five unknown variables n , p , N_d^+ , N_a^- , and \mathcal{E}_f .
- In general, however, this is quite difficult. For a *nondegenerate* semiconductor the equation to be solved for \mathcal{E}_f is

$$\frac{N_d}{1 + g_d \exp [(\mathcal{E}_f - \mathcal{E}_d)/kT]} = \frac{N_a}{1 + g_a \exp [(\mathcal{E}_a - \mathcal{E}_f)/kT]}$$

$$= N_c \exp \left(\frac{\mathcal{E}_f - \mathcal{E}_c}{kT} \right) - N_v \exp \left(\frac{\mathcal{E}_v - \mathcal{E}_f}{kT} \right) \quad (4.102)$$

- To see how \mathcal{E}_f varies with $N_d - N_a$, it is necessary to make certain simplifying *assumptions*.
- Let us first *assume* that essentially all the donors and acceptors are ionized. This is equivalent to the condition $\mathcal{E}_a + 4kT < \mathcal{E}_f < \mathcal{E}_d - 4kT$, so that (4.102) reduces to

$$N_d - N_a \approx N_c \exp \left(\frac{\mathcal{E}_f - \mathcal{E}_c}{kT} \right) - N_v \exp \left(\frac{\mathcal{E}_v - \mathcal{E}_f}{kT} \right) \quad (4.103)$$

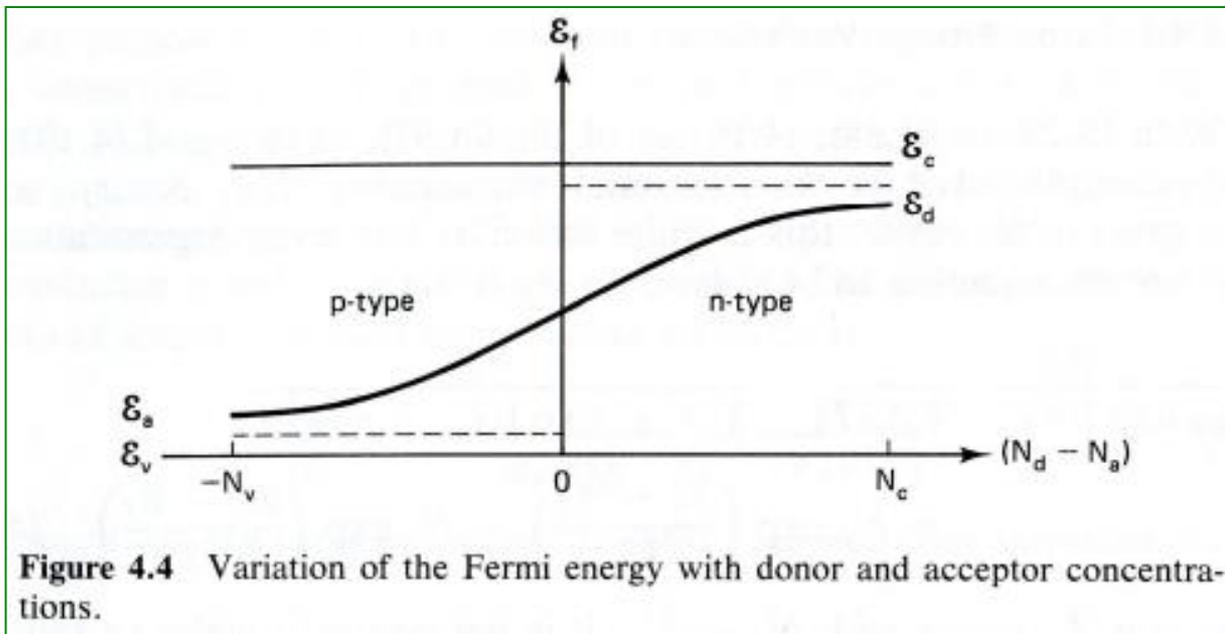
- When \mathcal{E}_f is in the *upper half of the energy gap*, $\mathcal{E}_v + \mathcal{E}_g/2 < \mathcal{E}_f < \mathcal{E}_d - 4kT$, the hole concentration will be small, and from (4.103) we obtain

$$\mathcal{E}_f \approx \mathcal{E}_c - kT \ln \frac{N_c}{N_d - N_a} \quad (4.104)$$

Fermi Energy Variation

- When \mathcal{E}_f is in the *lower half of the energy gap* $\mathcal{E}_a + 4kT < \mathcal{E}_f < \mathcal{E}_c - \mathcal{E}_g/2$, the electron concentration will be small and (4.103) gives us

$$\mathcal{E}_f \approx \mathcal{E}_v + kT \ln \frac{N_v}{N_a - N_d} \quad (4.105)$$



From (4.104), (4.105), and (4.82) the variation of the Fermi energy with $N_d - N_a$ appears as shown in Fig. 4.4.

Fermi Energy Variation

- For *sufficiently low temperatures* the assumption that the Fermi energy is $4kT$ removed from the donor or acceptor energies, \mathcal{E}_d or \mathcal{E}_a , is no longer valid.
 - In *n-type material*, \mathcal{E}_f moves toward \mathcal{E}_d as the temperature is lowered and electrons in the conduction band begin "freezing out" on the donor levels.
 - Under these conditions the concentration of *neutral donors* can **no longer be neglected**.
 - The acceptors, however, are still all ionized and the hole concentration can be neglected.
- Assuming that the electron concentration in the conduction band is small, (4.102) gives us, for $\mathcal{E}_v + \mathcal{E}_g/2 < \mathcal{E}_f$,

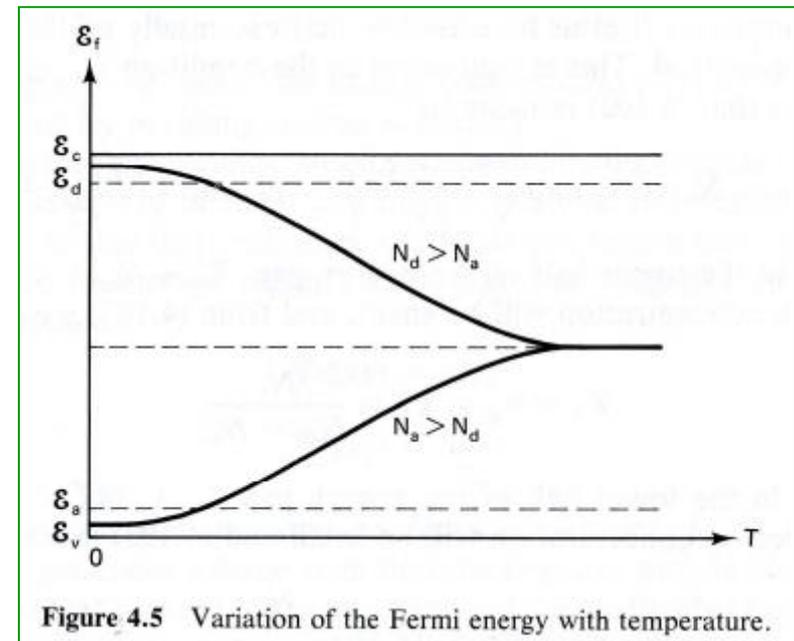
$$\mathcal{E}_f = \mathcal{E}_d + kT \ln \frac{N_d - N_a}{g_d N_a} \quad (4.106)$$

Thus the Fermi energy lies *above* the donor energy.

- In *p-type material* under similar conditions,

$$\mathcal{E}_f = \mathcal{E}_a - kT \ln \frac{N_a - N_d}{g_a N_d} \quad (4.107)$$

From (4.102) the temperature dependence of the Fermi energy under conditions such that either N_d is much greater than N_a or N_a is much greater than N_d is shown in Fig. 4.5.



Transport Properties

- When **electrons** are in **thermal equilibrium** with the lattice, they are distributed among possible **energy levels** in a manner given by the **Fermi-Dirac function** of (4.41).
- Under these conditions **no net transport** of charge or energy occurs since the probability that a state with wave vector **k** is occupied is the same as that for a state with wavevector **-k**. That is, the **equilibrium distribution function**, f_0 , is **symmetrical** about the origin in **k-space**.
- When **external forces** or **temperature gradients** are applied to the material, however, this is no longer true. Under these conditions we can, in principle, **determine** the **nonequilibrium distribution function**, f , in a manner similar to that used for f_0 in Section 4.1.
- In the nonequilibrium case, however, we would have to maximize (4.10) for the most probable distribution subject to the additional constraints that a steady flow of charge and energy be maintained. That is,

$$W = \frac{g_d^{N_d^0} N_d!}{N_d^0! (N_d - N_d^0)!} \frac{g_a^{(N_a - N_a^-)} N_a!}{N_a^-! (N_a - N_a^-)!} \prod_k \frac{g_k!}{n_k! (g_k - n_k)!} \quad (4.10)$$

where **J** is the **electrical current density**, **W** is the **heat flow density**, and e_k is the heat content per electron. These additional **constraints** produce an **asymmetry** in the nonequilibrium distribution function which **shifts its center away from the origin** in **k-space**. In this chapter we examine this **nonequilibrium distribution** of electrons and use it to determine the **transport of charge and energy** in semiconductors.

Transport Properties

- When **electrons** are in **thermal equilibrium** with the lattice, they are distributed among possible **energy levels** in a manner given by the **Fermi-Dirac function** of (4.41).
- Under these conditions **no net transport** of charge or energy occurs since the probability that a state with wave vector \mathbf{k} is occupied is the same as that for a state with wavevector $-\mathbf{k}$. That is, the **equilibrium distribution function**, f_0 , is **symmetrical** about the origin in \mathbf{k} -space.
- When **external forces** or **temperature gradients** are applied to the material, however, this is no longer true. Under these conditions we can, in principle, **determine** the **nonequilibrium distribution function**, f , in a manner similar to that used for f_0 in Section 4.1.
- In the nonequilibrium case, however, we would have to maximize (4.10) for the most probable distribution subject to the additional constraints that a steady flow of charge and energy be maintained. That is,

$$\mathbf{J} = -q \sum_k n_k \mathbf{v}_k \quad (5.1)$$

$$\mathbf{W} = \sum_k e_k n_k \mathbf{v}_k \quad (5.2)$$

where \mathbf{J} is the **electrical current density**, \mathbf{W} is the **heat flow density**, and e_k is the heat content per electron. These additional **constraints** produce an **asymmetry** in the nonequilibrium distribution function which **shifts its center away from the origin** in \mathbf{k} -space. In this chapter we examine this **nonequilibrium distribution** of electrons and use it to determine the **transport of charge and energy** in semiconductors.

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Transport Properties

- 5.1 **Boltzmann's Equation**
- 5.2 Distribution Function
- 5.3 Charge Transport
- 5.4 Charge and Energy Transport
- 5.5 High Frequency Transport
- 5.6 High Electric Field Effects

Boltzmann's Equation

- The approach we take in *determining* the transport properties of semiconductors is to first **construct an electron wave packet** from plane wave solutions to the time-dependent Schrodinger equation. This is the same method as that used in Section 2.8. Then, from the *correspondence principle*, we can use a *semiclassical* approach.

- On this basis, let $f(\mathbf{k}, \mathbf{r}, t)$ be the **probability** that a state with wavevector \mathbf{k} is occupied by an electron with position \mathbf{r} at time t . The electrons are *continually changing their position* according to (2.109)

$$\mathbf{v} = \frac{1}{\hbar} \nabla_{\mathbf{k}} \mathcal{E} = \frac{1}{\hbar} \frac{\partial \mathcal{E}}{\partial \mathbf{k}} \quad (2.109)$$

and, under the influence of forces \mathbf{F}_t (other than the periodic crystal forces), are continually changing their wavevector according to (2.111).

$$\hbar \frac{d\mathbf{k}}{dt} = \mathbf{F} \quad (2.111)$$

- \mathbf{F}_t includes applied forces \mathbf{F} and forces \mathbf{F}_c due to electron collisions with lattice vibrations and other imperfections in the crystal.

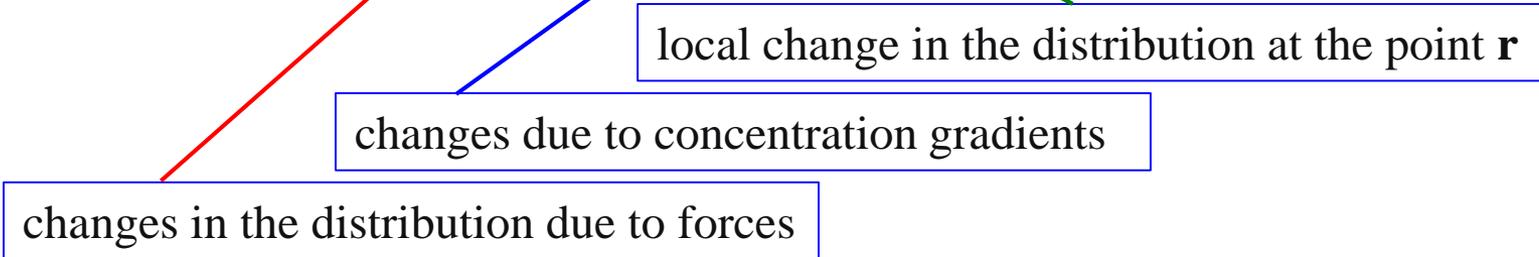
- Therefore, at time $t + dt$ the **probability** that a state with wavevector $\mathbf{k} + d\mathbf{k}$ is occupied by an electron with position $\mathbf{r} + d\mathbf{r}$ is given by

$$f \left(\mathbf{k} + \frac{1}{\hbar} \mathbf{F}_t dt, \mathbf{r} + \mathbf{v} dt, t + dt \right)$$

Boltzmann's Equation

- The **total rate of change** in the distribution function in the region of the point \mathbf{r} is then

$$\frac{df}{dt} = \left[\frac{1}{\hbar} \mathbf{F}_r \cdot \nabla_k f \right] + \left[\mathbf{v} \cdot \nabla_r f \right] + \left[\frac{\partial f}{\partial t} \right] \quad (5.3)$$



- Equation (5.3) is referred to as **Boltzmann's transport equation**.

- Since the **total number of states in the crystal is constant**, the total rate of change of the distribution function must be **zero** (Liouville's theorem), and

$$\frac{\partial f}{\partial t} = - \frac{1}{\hbar} \mathbf{F}_r \cdot \nabla_k f - \mathbf{v} \cdot \nabla_r f = \frac{\partial f}{\partial t} \Big|_c - \frac{1}{\hbar} \mathbf{F} \cdot \nabla_k f - \mathbf{v} \cdot \nabla_r f \quad (5.4)$$

- Because of the difficulty of finding a value for \mathbf{F}_c , we **separate** the collision forces from the applied forces by **defining** a **local change in the distribution due to collisions only** as

$$\frac{\partial f}{\partial t} \Big|_c \equiv - \frac{1}{\hbar} \mathbf{F}_c \cdot \nabla_k f \quad (5.5)$$

Boltzmann's Equation

- Let us examine this **collision term**. The action of **applied forces** and **gradients** tends to disturb the distribution function f from its equilibrium value f_0 . **If this disturbance is removed, the scattering processes will tend to restore equilibrium.** When the change in the distribution is **not large** compared to its initial value, it is reasonable to **assume** that

$$\left. \frac{\partial f}{\partial t} \right|_c = \frac{-(f - f_0)}{\tau_m} \quad (5.6)$$

where τ_m is a **constant of proportionality** called the **momentum relaxation time**.

- In general, τ_m **depends on the electron energy** and is different for different scattering mechanisms. Integrating (5.6), we find that

$$f(t) - f_0 = [f(0) - f_0] \exp\left(-\frac{t}{\tau_m}\right) \quad (5.7)$$

That is, the momentum relaxation time τ_m characterizes an **exponential relaxation of the distribution function f to its equilibrium value f_0 .**

In the **steady state** $\partial f / \partial t = 0$ and using this and (5.6) in (5.4), we obtain the **steady-state Boltzmann equation** in the **relaxation time approximation**,

$$f = f_0 - \frac{\tau_m}{\hbar} \mathbf{F} \cdot \nabla_k f - \tau_m \mathbf{v} \cdot \nabla_r f \quad (5.8)$$

Boltzmann's Equation

- Since

$$\nabla_k f = \frac{\partial f}{\partial \mathcal{E}} \nabla_k \mathcal{E} \quad (5.9)$$

the Boltzmann equation can be put in the form

$$f = f_0 - \frac{\tau_m}{\hbar} \frac{\partial f}{\partial \mathcal{E}} \mathbf{F} \cdot \nabla_k \mathcal{E} - \tau_m \mathbf{v} \cdot \nabla_r f \quad (5.10)$$

From (2.109) $\mathbf{v} = \frac{1}{\hbar} \nabla_k \mathcal{E}$

so that (5.10) is finally

$$f = f_0 - \tau_m \mathbf{v} \cdot \left(\frac{\partial f}{\partial \mathcal{E}} \mathbf{F} + \nabla_r f \right) \quad (5.11)$$

or

$$f = f_0 - \frac{\tau_m}{\hbar} \nabla_k \mathcal{E} \cdot \left(\frac{\partial f}{\partial \mathcal{E}} \mathbf{F} + \nabla_r f \right) \quad (5.12)$$

The Boltzmann equation in the form of (5.12) tells us that the nonequilibrium distribution of electrons depends on the scattering processes through the term τ_m , on the band structure through $\nabla_k \mathcal{E}$, on applied forces through $(\partial f / \partial \mathcal{E}) \mathbf{F}$, and on concentration gradients through $\nabla_r f$. Therefore, we have, in general, a rather *difficult* partial differential equation to solve for f , the nonequilibrium distribution function .

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5.5 High Frequency Transport

5.6 High Electric Field Effects

Distribution Function

- Before looking at a more general solution for f , let us look at the *simplest* possible case.
- We will *assume* that the only applied force is a **small electric field \mathbf{E}** and that there are **no concentration or temperature gradients**. Under these conditions (5.11) becomes

$$f = f_0 - \tau_m \mathbf{v} \cdot \left(\frac{\partial f}{\partial \mathcal{E}} \mathbf{F} + \nabla_{\mathbf{r}} f \right) \quad \mathbf{F} = -q\mathbf{E} \quad (5.11)$$

$$f = f_0 + q\tau_m \frac{\partial f}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{E} \quad (5.13)$$

- Equation (5.13) *can be integrated* to obtain an *analytical* expression for f **provided that the energy dependence of τ_m is known**. The solution, however, is **nonlinear** in \mathbf{E} .
- Under the *relaxation time assumption* that the change in distribution function is *not large*, we can also make the *approximation* that

$$\frac{\partial f}{\partial \mathcal{E}} \simeq \frac{\partial f_0}{\partial \mathcal{E}} \quad (5.14)$$

so that

$$f = f_0 + q\tau_m \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{E} \quad (5.15)$$

This retains **only a linear term** in \mathbf{E} , which is consistent with our initial assumption of a **small electric field**.

Distribution Function

- Let us now look at a more *general situation*, where we include a *small electric field \mathbf{E}* and an *arbitrary magnetic field \mathbf{B}* in the force term and retain the term for *concentration and temperature gradients*. Under these conditions (5.8) is

$$\frac{f - f_0}{\tau_m} = \frac{+q}{\hbar} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_k f - \mathbf{v} \cdot \nabla_r f \quad (5.16)$$

We will assume that the solution for (5.16) has the form of (5.15),

$$f = f_0 + \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{G} \quad (5.17)$$

and then solve for the *unknown vector \mathbf{G}* . Inserting (5.17) into (5.16) the term on the left-hand side of (5.16) is simply

$$\frac{f - f_0}{\tau_m} = \frac{1}{\tau_m} \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{G} \quad (5.18)$$



- We obtain the result after a series of calculations,

$$\frac{-q}{\hbar} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_k f = -q \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{E} - \frac{q}{\hbar^2} \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot [\mathbf{B} \times (\mathbf{G} \cdot \nabla_k) \nabla_k \mathcal{E}] \quad (5.23)$$

This is the desired form for the *first term* on the right in (5.16).

Distribution Function

- Let us now examine the second term on the right in (5.16),

$$f = f_0 + \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{G}$$

$$\mathbf{v} \cdot \nabla_r f \leftarrow \mathbf{v} \cdot \nabla_r f_0 + \mathbf{v} \cdot \nabla_r \left(\frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{G} \right) \quad (5.24)$$

- For our purposes we can *assume* that the *spatial* dependence of \mathbf{G} is *small* and consider only the first term in (5.24). We then have

$$\begin{aligned} \mathbf{v} \cdot \nabla_r f &= \frac{\partial f_0}{\partial ((\mathcal{E} - \mu)/kT)} \mathbf{v} \cdot \nabla_r \left(\frac{\mathcal{E} - \mu}{kT} \right) \\ &= kT \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \nabla_r \left(\frac{\mathcal{E} - \mu}{kT} \right) \end{aligned} \quad (5.25)$$

where μ is the *chemical potential*.

- Using (5.18), (5.23), and (5.25), **Boltzmann's equation** is now

$$\begin{aligned} \frac{1}{\tau_m} \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{G} &= +q \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \mathbf{E} + \frac{q}{\hbar^2} \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot [\mathbf{B} \times (\mathbf{G} \cdot \nabla_k) \nabla_k \mathcal{E}] \\ &\quad - kT \frac{\partial f_0}{\partial \mathcal{E}} \mathbf{v} \cdot \nabla_r \left(\frac{\mathcal{E} - \mu}{kT} \right) \end{aligned} \quad (5.26)$$

Distribution Function

- Since each term in (5.26) has a common factor $(\partial f_0 / \partial \mathcal{E}) \mathbf{v}$ on the left, it can be eliminated to obtain

$$\frac{1}{\tau_m} \mathbf{G} = +q\mathbf{E} - kT \nabla_r \left(\frac{\mathcal{E} - \mu}{kT} \right) + \frac{q}{\hbar^2} [\mathbf{B} \times (\mathbf{G} \cdot \nabla_k) \nabla_k \mathcal{E}] \quad (5.27)$$

Defining an **electrothermal field** for electrons, \mathcal{F} , by

$$q\mathcal{F} = +q\mathbf{E} - T \nabla_r \left(\frac{\mathcal{E} - \mu}{T} \right) \quad (5.28)$$

(5.27) has the form

$$\mathbf{G} = q\tau_m \mathcal{F} + \frac{q\tau_m}{\hbar^2} [\mathbf{B} \times (\mathbf{G} \cdot \nabla_k) \nabla_k \mathcal{E}] \quad (5.29)$$

- Equation (5.29) *can be solved for G* by using an *explicit* expression for the conduction band minima. For this purpose we will *assume ellipsoidal minima* with a quadratic dispersion relationship as given by (2.117)

- In vector notation we have

$$\mathcal{E} = \mathcal{E}_c + \frac{1}{2} \hbar^2 \mathbf{k} \cdot \mathbf{M} \cdot \mathbf{k} \quad (5.30)$$

where

$$\mathbf{M} = \begin{bmatrix} \frac{1}{m_1^*} & 0 & 0 \\ 0 & \frac{1}{m_2^*} & 0 \\ 0 & 0 & \frac{1}{m_3^*} \end{bmatrix} \quad (5.31)$$

effective mass tensor

Distribution Function

- Reducing this equation to its components and solving for G , we have, *finally*,

$$\mathbf{G} = q\tau_m \left[\frac{\mathcal{F} - q\tau_m \mathbf{M} \cdot (\mathcal{F} \times \mathbf{B}) + (q\tau_m)^2 (\det \mathbf{M}) (\mathcal{F} \cdot \mathbf{B}) (\mathbf{M}^{-1} \cdot \mathbf{B})}{1 + (q\tau_m)^2 (\det \mathbf{M}) (\mathbf{M}^{-1} \cdot \mathbf{B}) \cdot \mathbf{B}} \right] \quad (5.34)$$

- The **nonequilibrium distribution function** for electrons in ellipsoidal conduction band minima is obtained by using \mathbf{G} from (5.34) in (5.17).

- For spherical minima the distribution function is

$$f = f_0 + \frac{\partial f_0}{\partial \mathcal{E}} q\tau_m \mathbf{v} \cdot \left[\frac{\mathcal{F} - (q\tau_m/m^*) (\mathcal{F} \times \mathbf{B}) + (q\tau_m/m^*)^2 (\mathcal{F} \cdot \mathbf{B}) \mathbf{B}}{1 + (q\tau_m/m^*)^2 \mathbf{B} \cdot \mathbf{B}} \right] \quad (5.35)$$

- We can see that there are **four components** to the distribution function.

1. The first is simply the **equilibrium function** f_0 given by (4.41), which **does not contribute to the transport of charge and energy**.

2. The term that involves \mathcal{F} is the **ohmic contribution to the transport properties**. This term accounts for **electrical and thermal conductivity** as well as the **Seebeck, Peltier, and Thomson effects**.

3. The term with $\mathcal{F} \times \mathbf{B}$ is the **Hall contribution** to transport and accounts for the **Hall, Ettinghausen, Nernst, and Righi-Leduc effects**.

4. The \mathbf{B}^2 terms in the numerator and denominator of (5.35) account for **magnetoresistive effects**.

Distribution Function

-The distribution function derived above for **electrons** can also be used for **holes** when the appropriate parameters of q , m^* , and τ_m are substituted in the equations.

- From (5.1) and (5.2) we can determine the **current density \mathbf{J}** and **heat flow density \mathbf{W}** by summing (or integrating) $n_k \mathbf{v}_k$ and $e_k n_k \mathbf{v}_k$, respectively, over the first Brillouin zone. In (4.68), however, we have *already* obtained an expression for n , the **number of electrons in the conduction band minima**. For this reason we can approach the problem from a **different point of view**. That is, the **current density can be determined by**

$$\mathbf{J} = -qn\langle \mathbf{v} \rangle \quad (5.36)$$

where $\langle \mathbf{v} \rangle$ is the **average velocity** of the n electrons in the *nonequilibrium* distribution.

- In a similar manner the **heat flow density can be obtained from**

$$\mathbf{W} = n\langle e\mathbf{v} \rangle \quad (5.37)$$

where the **heat content per electron and the velocity are averaged over the distribution**. The problem is to determine *how* this averaging should be performed.