ABSTRACT
The effects of roughness in an incompressible turbulent boundary layer include increased production of turbulence kinetic energy and increased drag. By appropriate averaging, the kinetic energy can be decomposed into the KE of the mean flow (\(KE_m\)), the KE of the steady roughness wakes (\(KE_w\)) and the KE of the residual or turbulent flow (\(KE_t\)). By formulating the exact Reynolds averaged Navier-Stokes and the turbulence kinetic energy equations in a manner that includes an arbitrary roughness, the averaged terms representing the roughness production of TKE and the roughness drag can be written explicitly. The KE production terms are calculated from a collection of direct numerical simulations (DNS) wherein the roughness geometry is represented using the immersed boundary methodology. The roughness geometry is limited to a simple array of vertically oriented cylinders in these simulations. The results include an examination of the partitioning of the production of wake KE and TKE into shear production and production by roughness, and the partitioning of drag into form drag and viscous shear drag.

INTRODUCTION
The development of smooth wall turbulence models has benefited for data derived from direct numerical simulations (DNS) by allowing the detailed examination of the processes to be modeled. A set of DNS results covering a range of roughness parameter range would be useful in examining the role played by the roughness elements in the generation of form drag, turbulence kinetic energy and Reynolds shear stress.

SUMMARY OF RESEARCH
To obtain DNS data, an existing pseudo-spectral channel flow algorithm has been extended to include an immersed boundary forcing term. The approach adopted follows that of Goldstein et al. [1], in which an immersed boundary force with a slow term proportional to the time-integrated (mean) velocity and a fast term proportional to the instantaneous velocity is used to enforce the no-slip surface condition of a virtual surface.

The driving pressure gradient is held constant for all simulations, resulting in a friction velocity Reynolds number, \(Re_f = 316\). The friction velocity is determined from the pressure gradient. Roughness is applied on both boundaries of the simple channel flow geometry.

In the seven DNS channel flow calculations the top and bottom boundaries were roughened by an array of vertically oriented stout cylinders, ranging in height from 0.0 to 49.5 \(l^+\). The 12x8 array of cylindrical roughness elements is laid out in staggered arrangement. The actual shape of the roughness elements is constrained by the need to approximate them on a coarse grid, relative to their size. The mean velocity gradients for the range of roughness heights are shown below. The flow to be discussed below has a roughness height of 49.5 \(l^+\).

FIGURE 1. Mean velocity for a range of roughness heights.
transferring energy from the mean velocity to the wake and roughness elements, while the terms for the kinetic energy are the standard approach. Formulation of the averaged equations follows directly from the spatial average and the residual velocity fluctuations. This well represents the roughness or the force can be considered as known part of the solved flow field. Either the immersed boundary approach is the easiest to implement here, with the spatial averaging occurring naturally as a result of averaging two phases. One approach is to decompose the velocity into a normal component, and a wake component. The red curves are contributions to the turbulent kinetic energy, representing the roughness or the liquid interface [2]. The former approach is applied to experimental roughness element. Since the immersed boundary force is explicitly known the two-stage averaging process, in which a time average is followed by a spatial average of surfaces of constant wall normal distance can be used to decompose the velocity into a spatial and time mean velocity, a wake component with a zero spatial average and the residual velocity fluctuations. This well described in Pokrajac, et al. [3], where it was applied to experimental roughness element.

To exploit DNS data for model development, a set of Reynolds-averaged governing equations need to be developed. Two approaches are available for developing transport equations for the kinetic energy. Either the immersed boundary force can be considered as known part of the solved flow field representing the roughness or the problem can be treated as a two-phase problem with the mean surface force due to the roughness elements occurring naturally as a result of averaging through the solid-liquid interface [2]. The former approach is the easiest to implement here, with the spatial averaging occurring across the immersed boundary.

A two stage averaging process, in which a time average is followed by a spatial average of surfaces of constant wall normal distance can be used to decompose the velocity into a spatial and time mean velocity, a wake component with a zero spatial average and the residual velocity fluctuations. This well described in Pokrajac, et al. [3], where it was applied to experimental roughness element. Since the immersed boundary force is explicitly known the boundary drag is available during the averaging process. Formulation of the averaged equations follows directly from the standard approach. Schematically, the transport equations for the kinetic energy are

\[
\begin{align*}
\frac{DKE_m}{Dt} &= -P_{w-r} - P_{t-m} - P_{w-m} + \text{other terms} \\
\frac{DKE_{w}}{Dt} &= P_{w-r} - P_{t-w} + P_{w-m} + \text{other terms} \\
\frac{DKE_{t}}{Dt} &= P_{t-w} + P_{t-m} + \text{other terms}.
\end{align*}
\]

The terms \( P_{w-r} \) are wake production terms resulting from the roughness elements, while \( P_{w-m} \) and \( P_{t-m} \) are production terms transferring energy from the mean velocity to the wake and turbulent flows, respectively. The last term, \( P_{t-w} \) is the exchange of energy between the wake and turbulence flows mediated by the velocity gradient of the wake flow. In the limit of smooth walls, these averaged equations reduce to their smooth wall counterparts, with the kinetic energy of the wake becoming zero.

**RESULTS**

The production terms for channel flow over 491 roughness elements are shown in Figure 2. The curves marked \( P_{turbulent} \) and \( P_{wake} \) are the sum of the production terms contributing to those flow components.

As would be anticipated, the wake flows are primarily due to roughness elements, with a significant reverse contribution to the wake flow from the turbulent flow near the tops of the elements. This maybe an example of turbulent stress-gradient induced secondary flow. Within the roughness elements, the wake energy is being transferred to the turbulent flow via velocity gradients of the wake flow.

The production of turbulence by the mean velocity shear is limited below the roughness elements, but dominates in the mean velocity gradient above the roughness. This is consistent with the experimentally observed collapse of the turbulent stress profiles above rough walls to smooth wall results. Within the roughness elements, turbulent energy is being extracted from the wake flow.

**CONCLUSIONS**

The primary objective of this work is to examine the decomposed kinetic energy production terms for the eventual purpose of supporting turbulent stress model development. Since the mean velocity will not be able to represent the wake flow the modeling will have to encompass both the wake and turbulent contributions.

Although the other terms in the balance equations are not assumed to be unimportant, examining the production terms has shown an unexpected coupling between the turbulent and wake flows implying the production of secondary wake flows at the crests of the roughness elements.

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**REFERENCES**


**FIGURE 2.** Production of the kinetic energy of the turbulent and wake components. The red curves are the contributions to the wake flow. The blue curves are contributions to the turbulent flow. The blue curve with square symbols is an exchange from to the turbulence from the wake flow. The solid green curve is the production from a smooth wall channel flow. The dashed vertical line marks the height of the roughness elements.