Proof that NFL is a Markov chain:

Let State space E={1,2,3,4,….,32}, where each state represents a team such that state 1 is Indianapolis and state 32 is Arizona Cardinals.

$P[X_{n+m} = k | X_0 = i] = \sum_j P[X_{n+m} = k, X_n = j \mid X_0 = i]$

$= \sum_j P[X_{n+m} = k \mid X_n = j , X_0 = i] * P[X_n = j \mid X_0 = i]$

$= \sum_j P[X_{n+m} = k \mid X_n = j] * P [X_n = j \mid X_0 = i]$

$= \sum_j P[X_m = k \mid X_0 = j]$

Therefore, $P[X_{n+m} = k | X_0 = i] = \sum_j P[X_m = k \mid X_0 = j]$

The given chain is a Markov chain because let us say after each match, the losing team casts votes for the winning team. So, the votes go on increasing or stay same after each match and it is a process similar to that of counting the number of heads after ‘n’ flips. Suppose there are 4 Heads after 10 flips, now on the 11^{th} flip, there can either be 5 or 4 heads, the probability of heads >5 or the probability of heads <4 is 0 and that means that the probability of the number of heads only depends on the present data and it does not depend on the previous all data from the past.

The transition matrix for the Markov Chain is a square matrix of size 32x32 with
State Space= {1,2, 3….,31,32},where each state represents a team.
Let 'F' be the transition matrix and 'x' be the limiting matrix which has converged with the stationary probabilities corresponding to the rankings of the team after n time steps.

Properties of the transition matrix:
1.Normalization: $\sum F_{i,j}=1$ for i={1,2,…,32} , where i= rows & j=columns
2.Non Negativity: $F_{i,j} \geq 0$
3.F is a Square matrix.
Note:(The above characteristics apply to both the methods)

Method 1: Justification
We make the transition matrix by keeping into account that the loosing team casts votes for the winning team proportional to the score difference that is the points scored by the winning team minus the points scored by the loosing team that is the Margin of Victory. The probabilities in the transition matrix are found out by taking the Margin of Victory and dividing it by the total difference of goals for that team.
For eg. If Oakland Raiders have scored 10 goals against Kansas State Chiefs and Kansas State in turn scored 12 goals, then Oakland Raiders will cast votes for Kansas State with
Probability= (Goals Scored-Goals Against)/Total Goal Difference
= (12-10)/3152
=0.01316
Hence, $F_{27,26}=0.01316$, where $F_{27,26}$ is the value in the transition matrix corresponding to the 27^{th}row and 26^{th} column.
Now if a team has played 2 games against a single team and if it has won both the games then the individual probabilities are calculated as shown above and added to get the final value of probability in the transition matrix that is the losing team will cast votes both the times and if a team has lost and won games simultaneously then the probability will only be the number of votes casted by the losing team because in the game which they lost, the winning team won't cast votes for them.
Now if a team has lost both the games then the corresponding value in the transition matrix will be 0 because there will be no votes casted for them as votes are only cast by the losing team in favor of the winning team.
The diagonal elements of the transition matrix $F_{i,i}=0$ for all i's except i=8 because when i=8,the team New England Patriots is undefeated and hence let $F_{8,j}=1/n$ where n is the dimension of the square matrix that is $F_{8,j}=1/32=0.03125$ because that is equivalent to the meaning that the team which has won all its games in the tournament can lose to any other team on a special day with equal probability and hence $F_{8,j}=1/32=0.03125$ since there are 32 teams and is 0 otherwise.
Similarly, all other values in the transition matrix are found out such that the sum of values in rows is equal to 1.
The computational part is done in 'Matlab' to find the corresponding limiting matrix 'x' with values equal to the stationary probabilities corresponding to the Rankings, which indicates higher the stationary probability higher is its ranking meaninig better ranked the team is.

Algorithm:
(Note: Both the transition matrix and the final limiting matrix are computed using the following code in 'Matlab')

```
%% Import the data
[~, ~, raw] = xlsread('\\blender\homes\s\m\smithkrshah\nt\nfldata12007.xls','Worksheet');
raw(cellfun(@(x) ~isempty(x) && isnumeric(x) && isnan(x),raw)) = {''};
stringVectors = string(raw(:,[1,2]));
stringVectors(ismissing(stringVectors)) = '';
raw = raw(:,[3,4]);
%% Create output variable
data = reshape([raw{:}],size(raw));

%% Allocate imported array to column variable names
Winnertie1 = stringVectors(:,1);
Losertie1 = stringVectors(:,2);
PtsW1 = data(:,1);
PtsL1 = data(:,2);

%% Clear temporary variables
clearvars data raw stringVectors R;

%% Transition matrix
Losertie(1:16,1);
Winnertie(1:16,1);
C = [Winnertie(1:16,1);Losertie(1:16,1)];
PtsW;
PtsL;
D = zeros(32,32);
a = 0;
b = 0;
for k = 1:256
  for j = 1:32
    if Losertie(k)== C(j)
    a = j;
    end
    if Winnertie(k)== C(j)
    b = j;
    end
  end
D(a,b) = D(a,b)+PtsW(k)-PtsL(k);
end
```

```
%% Normalization
E = sum(D,2);
F = D./E;
for t=1:32
   F(8,t) = 0.03125;
end
 %% Solving for the limiting matrix
mc = dtmc(F);
x = asymptotics(mc);
```

Rankings:

Initial State	Team Name	Stationary Probabilities	Rankings
1	"Indianapolis Colts"	0.0551	"3"
2	"Carolina Panthers"	0.0163	"23"
3	"Minnesota Vikings"	0.0377	"11"
4	"Denver Broncos"	0.0242	"19"
5	"Green Bay Packers"	0.0504	"5"
6	"Washington Redskins"	0.0546	"4"
7	"Pittsburgh Steelers"	0.0385	"10"
8	"New England Patriots"	0.1386	"1"
9	"Tennessee Titans"	0.0447	"8"
10	"Houston Texans"	0.0293	"13"
11	"Detroit Lions"	0.0264	"17"
12	"San Diego Chargers"	0.049	"7"
13	"Seattle Seahawks"	0.0281	"15"
14	"Dallas Cowboys"	0.0597	"2"
15	"Cincinnati Bengals"	0.0271	"16"
16	"San Francisco 49ers"	0.0099	"28"
17	"New Orleans Saints"	0.0306	"12"
18	"St. Louis Rams"	0.0074	"31"
19	"Atlanta Falcons"	0.0101	"27"
20	"Buffalo Bills"	0.0106	"26"
21	"Philadelphia Eagles"	0.029	"14"
22	"Miami Dolphins"	0.0048	"32"
23	"Cleveland Browns"	0.0145	"24"
24	"New York Jets"	0.008	"29"
25	"Jacksonville Jaguars"	0.0406	"9"
26	"Kansas City Chiefs"	0.0179	"22"
27	"Oakland Raiders"	0.0078	"30"
28	"Chicago Bears"	0.0499	"6"
29	"Tampa BayBuccaneers"	0.0237	"20"
30	"New York Giants"	0.0254	"18"
31	"Baltimore Ravens"	0.0114	"25"
32	"Arizona Cardinals"	0.0187	"21"

Rankings in Ascending Order:

Team Name	Stationary Probabilities	Rankings
"New England Patriots"	0.1386	"1"
"Dallas Cowboys"	0.0597	"2"
"Indianapolis Colts"	0.0551	"3"
"Washington Redskins"	0.0546	"4"
"Green Bay Packers"	0.0504	"5"
"Chicago Bears"	0.0499	"6"
"San Diego Chargers"	0.049	"7"
"Tennessee Titans"	0.0447	"8"
"Jacksonville Jaguars"	0.0406	"9"
"Pittsburgh Steelers"	0.0385	"10"
"Minnesota Vikings"	0.0377	"11"
"New Orleans Saints"	0.0306	"12"
"Houston Texans"	0.0293	"13"
"Philadelphia Eagles"	0.029	"14"
"Seattle Seahawks"	0.0281	"15"
"Cincinnati Bengals"	0.0271	"16"
"Detroit Lions"	0.0264	"17"
"New York Giants"	0.0254	"18"
"Denver Broncos"	0.0242	"19"
"Tampa BayBuccaneers"	0.0237	"20"
"Arizona Cardinals"	0.0187	"21"
"Kansas City Chiefs"	0.0179	"22"
"Carolina Panthers"	0.0163	"23"
"Cleveland Browns"	0.0145	"24"
"Baltimore Ravens"	0.0114	"25"
"Buffalo Bills"	0.0106	"26"
"Atlanta Falcons"	0.0101	"27"
"San Francisco 49ers"	0.0099	"28"
"New York Jets"	0.008	"29"
"Oakland Raiders"	0.0078	"30"
"St. Louis Rams"	0.0074	"31"
"Miami Dolphins"	0.0048	"32"

Method2: Justification

In this method, we make the transition matrix in a way that not only the losing team cast votes for the winning team depending on the Margin of Victory but both the teams playing cast votes for each other depending upon the goals scored by them and then the corresponding probabilities are found out by normalizing the matrix as usual.

For e.g. If there is a match between Indianapolis and New Orleans Saints in which Indianapolis scored 41 goals whereas New Orleans Saints scored 10 goals then the votes casted by Indianapolis for New Orleans Saints would be in harmony with the goals scored by New Orleans Saints that is 10 and similarly the votes casted by New Orleans for Indianapolis will be in harmony with the goals scored by Indianapolis against them that is 41. So, indicating that both the teams will vote against each other depending upon the goals scored by them and the transition probability for that is then found out by normalizing.

That is $F_{1,17}$ = 41/(Total goals scored by Indianapolis in all games)

=41/450

=0.0911, where $F_{1,17}$ means the value in the transition matrix corresponding to the 1^{st} row and 17^{th} column.

And $F_{17,1}$ = 10/(Total goals scored by New Orleans Saints in all games)

= 10/379

=0.02639, where $F_{17,1}$ means the value in the transition matrix corresponding to the 17^{th} row and 1^{st} column.

Similarly, if the teams have played 2 games against each other then the goals scored by both the teams will be added and then the corresponding transition probabilities are calculated as shown above and if the teams have not played against each other then the corresponding value in the transition matrix will be 0.

Similarly, all other values in the transition matrix are found out such that the sum of values in rows is equal to 1.

The computational part is done in 'Matlab' to find the corresponding limiting matrix 'x' with values equal to the stationary probabilities corresponding to the Rankings, which indicates higher the stationary probability higher is its ranking meaning better ranked the team is.

Algorithm:
(Note: Both the transition matrix and the final limiting matrix are computed using the following code in 'Matlab')

```
%% Import the data
[~, ~, raw] = xlsread('\\blender\homes\s\m\smithkrshah\nt\nfldata12007.xls','Worksheet');
raw(cellfun(@(x) ~isempty(x) && isnumeric(x) && isnan(x),raw)) = {''};
stringVectors = string(raw(:,[1,2]));
stringVectors(ismissing(stringVectors)) = '';
raw = raw(:,[3,4]);
%% Create output variable
data = reshape([raw{:}],size(raw));

%% Allocate imported array to column variable names
Winnertie1 = stringVectors(:,1);
Losertie1 = stringVectors(:,2);
PtsW1 = data(:,1);
PtsL1 = data(:,2);

%% Clear temporary variables
clearvars data raw stringVectors R;

%% Transition matrix
Losertie(1:17,1);
Winnertie(1:17,1);
C = [Winnertie(1:17,1);Losertie(1:17,1)];
PtsW;
PtsL;
D = zeros(32,32);
a = 0;
b = 0;
for k = 1:256
  for j = 1:32
    if Losertie(k)== C(j)
    a = j;
    end
    if Winnertie(k)== C(j)
    b = j;
    end
  end
D(a,b) = D(a,b)+PtsW(k);
D(b,a) = D(b,a)+PtsL(k);
end
```

```
%% Normalization
E = sum(D,2);
F = D./E;

%% Solving for the limiting matrix
mc = dtmc(F);
x = asymptotics(mc);
```

Rankings:

Initial State	Team Name	Stationary Probabilities	Rankings
1	"Indianapolis Colts"	0.0411	"3"
2	"Carolina Panthers"	0.0218	"29"
3	"Minnesota Vikings"	0.0336	"12"
4	"Denver Broncos"	0.0299	"18"
5	"Green Bay Packers"	0.0392	"4"
6	"Washington Redskins"	0.0346	"10"
7	"Pittsburgh Steelers"	0.0305	"17"
8	"New England Patriots"	0.0558	"1"
9	"Tennessee Titans"	0.0285	"20"
10	"Houston Texans"	0.0354	"8"
11	"Detroit Lions"	0.034	"11"
12	"San Diego Chargers"	0.0376	"6"
13	"Seattle Seahawks"	0.0282	"21"
14	"Dallas Cowboys"	0.0451	"2"
15	"Cincinnati Bengals"	0.0308	"16"
16	"San Francisco 49ers"	0.0171	"32"
17	"New Orleans Saints"	0.0309	"15"
18	"St. Louis Rams"	0.0213	"31"
19	"Atlanta Falcons"	0.0217	"30"
20	"Buffalo Bills"	0.0244	"27"
21	"Philadelphia Eagles"	0.035	"9"
22	"Miami Dolphins"	0.0271	"22"
23	"Cleveland Browns"	0.0327	"14"
24	"New York Jets"	0.0253	"26"
25	"Jacksonville Jaguars"	0.0368	"7"
26	"Kansas City Chiefs"	0.0225	"28"
27	"Oakland Raiders"	0.0259	"24"
28	"Chicago Bears"	0.0332	"13"
29	"Tampa BayBuccaneers"	0.0256	"25"
30	"New York Giants"	0.0382	"5"
31	"Baltimore Ravens"	0.027	"23"
32	"Arizona Cardinals"	0.0292	"19"

Rankings in Ascending Order:

Team Name	Stationary Probabilities	Rankings
"New England Patriots"	0.0558	"1"
"Dallas Cowboys"	0.0451	"2"
"Indianapolis Colts"	0.0411	"3"
"Green Bay Packers"	0.0392	"4"
"New York Giants"	0.0382	"5"
"San Diego Chargers"	0.0376	"6"
"Jacksonville Jaguars"	0.0368	"7"
"Houston Texans"	0.0354	"8"
"Philadelphia Eagles"	0.035	"9"
"Washington Redskins"	0.0346	"10"
"Detroit Lions"	0.034	"11"
"Minnesota Vikings"	0.0336	"12"
"Chicago Bears"	0.0332	"13"
"Cleveland Browns"	0.0327	"14"
"New Orleans Saints"	0.0309	"15"
"Cincinnati Bengals"	0.0308	"16"
"Pittsburgh Steelers"	0.0305	"17"
"Denver Broncos"	0.0299	"18"
"Arizona Cardinals"	0.0292	"19"
"Tennessee Titans"	0.0285	"20"
"Seattle Seahawks"	0.0282	"21"
"Miami Dolphins"	0.0271	"22"
"Baltimore Ravens"	0.027	"23"
"Oakland Raiders"	0.0259	"24"
"Tampa BayBuccaneers"	0.0256	"25"
"New York Jets"	0.0253	"26"
"Buffalo Bills"	0.0244	"27"
"Kansas City Chiefs"	0.0225	"28"
"Carolina Panthers"	0.0218	"29"
"Atlanta Falcons"	0.0217	"30"
"St. Louis Rams"	0.0213	"31"
"San Francisco 49ers"	0.0171	"32"