●Using a random number generator of uniform numbers on [0,1], generate n = 10^6 samples of an exponential random variable with rate 1/2. For this part, note that -ln U is unit-rate exponentially distributed when U is uniform on [0,1].

CDF, $F(x) = P(X<=x) = 1 – e^\wedge \lambda^x$

$x = F^{-1}(y)$

$= - \ln (1 – y) / \lambda$

$= - \ln (y) / \lambda$ (Since there is no change in the distribution if we start with 1-y or y)

Code:

```
%Number of samples
n=1000000;
%Rate
lambda=0.5;
%Generate Uniform distribution
uni_r=rand(1,n);

%Generate Exponential distribution
exp_r_u=-(log(uni_r)/lambda);
```

●Using the n samples (from the previous step) as interarrival times, generate sample paths of a Poisson process {N(t), t ≥ 0} on the interval [0, T], where T = 5. Compute the expected number of sample paths that can be generated in this way. Compare the computed number with the actual number.

```
%Generating sample paths and computing the expected number of paths
sum(1)=0;
for i=1:n
   sum(i+1)=sum(i)+exp_r_u(i);
end
t=sum(1,2:length(sum));
j=1;
f=5;
count=0;
i=1;
while i<=n
   if f-t(i)>=0
      count=count+1;
      if i==n
         ul(j)=count;
         break;
      end
      i=i+1;
   elseif f-t(i)<0
      ul(j)=count;
      j=j+1;
     f=f+5;
      count=0;
   end
end
X=['Expected number of paths that can be generated are ',num2str(j)];
disp(X)
```

Output:
Expected number of paths that can be generated are 399920.(the variable j represents the expected number of paths that can be generated)

Calculation of Actual number of Sample paths:

Expected number of arrivals in a time T units $= \lambda T$

$= 0.5 * 5$ (since $\lambda=0.5$ and T=5units)

$= 2.5$

Actual number of sample paths = Total samples of inter-arrival times/ expected number of arrivals in a time T units

= 1000000 / 2.5

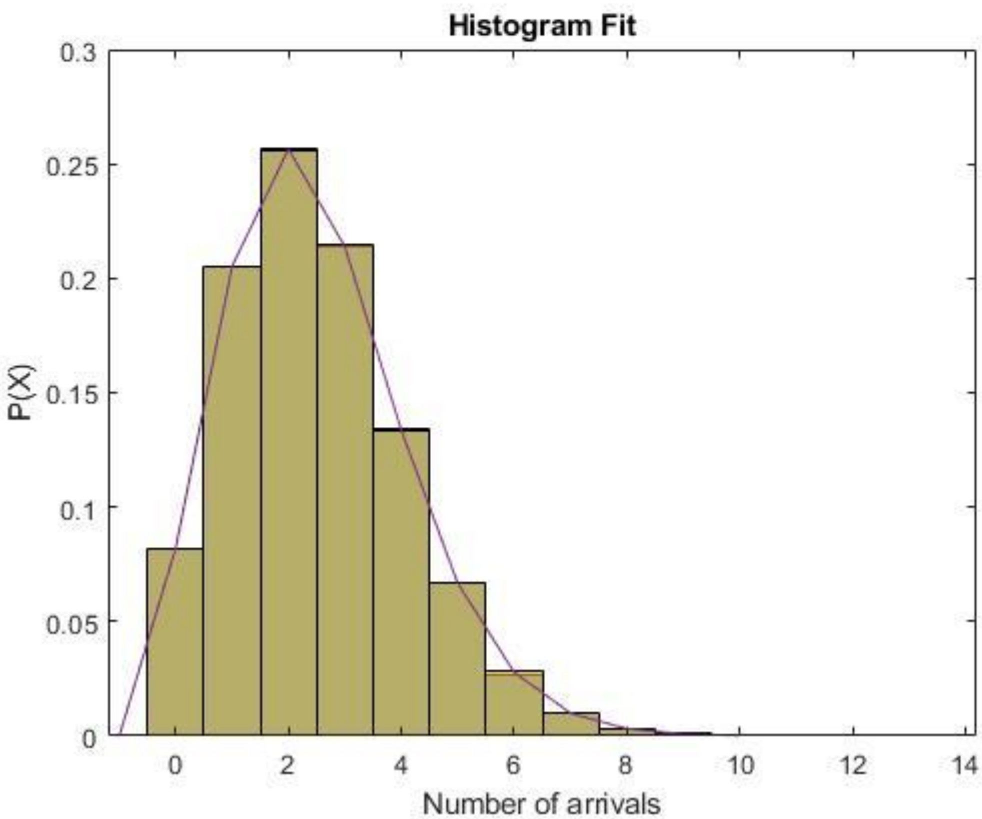
Therefore, Actual number of sample paths = 400000

From the matlab code, the Expected number of sample paths that can be generated turns out to be 399920 which is very close to the actual value of 4000000 sample paths.

●Using the generated sample paths of {N(t), t belongs to [0, T]}, create a histogram of N(T). On the same graph, plot a scaled probability mass function for a Poisson random variable (with an appropriate mean) to verify that N(T) is Poisson distributed.

```
%Plot histogram for the generated sample paths and the scaled probability mass function for poisson random variable

histogram(ul,'normalization','probability')
hold on
P=-1:10;
P1=poisspdf(P,2.5);
plot(P,P1);
title('Histogram Fit')
xlabel('Number of arrivals')
ylabel('P(X)')
```



1. The above graph in green shows the histogram of the number of sample paths generated.
2. The scaled probability mass function is plotted on the histogram for the Poisson variable and since it fits the distribution as shown, it verifies that N(T) Poisson distributed.

● Provide evidence that conditional arrivals are uniformly distributed on [0, T]. To this end, plot the empirical cumulative distribution function (CDF) of arrival times $F^{\sim}$, and compare it with the uniform CDF F. In particular, compute $\sup_{x \text{ belongs to } [0, T]} |F(x) - F(x)^{\sim}|$.

Let the arrival times be ∼exp(λ). Denote T_1 to be the time of the first arrival, and N_t to be the number of arrivals upto time t.

$P(T_1 \leq x \mid N_t = 1) \quad = P(T_1 \leq x, N_t = 1)/P(N_t = 1)$

{i.e Given one arrival what is the probability that it happens before time 'x'}

$= P(N_x = 1, N_t – N_t = 0)/P(N_t = 1)$

$= P(N_x = 1).P(N_t – N_x = 0)/P(N_t = 1)$

$= P(N_x = 1)P(N_{t-x} = 0)/P(N_t = 1)$

$= \lambda x e^{-\alpha x} \cdot e^{\wedge}-\lambda(t-x)/ \lambda t e^{\wedge}-\lambda t$

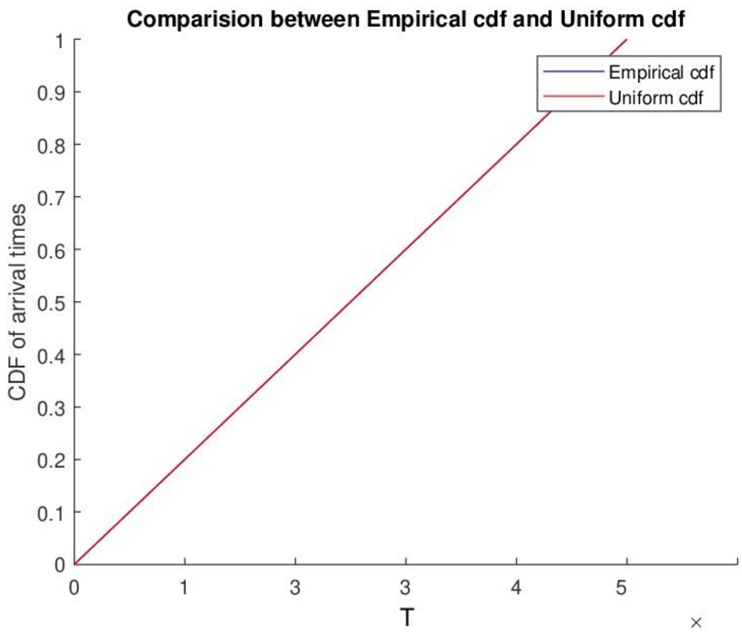
$= x/t$

T_1 is uniform on [0,t] given N_t =1 that is if we have one arrival in [0,t], it can be anywhere.

Meaning that the arrival time in poisson process has uniform distribution.

```
%Initializing a vector
h=0;
d=zeros(1,1000000);
for m=1:1000000
   h= h + exp_r_u(m);
   while h>5
      h=h-5;
   end
   d(m)=h;
end
%ecdf function computes the empirical cumulative distribution function
ecdf(d)
ecdf(uni_o)
compPlot = figure('Name', 'Comparison of Stuff');
ax1 = axes('Parent', compPlot);
hold(ax1, 'on');
plot(ax1, ecdf(d), 'Color', 'blue');
plot(ax1, ecdf(uni_o), 'Color', 'red');
title(ax1, 'Comparision between Empirical cdf and Uniform cdf');
legend('Empirical cdf','Uniform cdf');
hold(ax1, 'off');
```

```
xlabel('T')
ylabel('CDF of arrival times')
```




```
%To find the maximum distance between Empirical CDF and Uniform CDF

%Generate uniform random variables between 0 and 5
w=makedist('uniform','lower',0,'upper',5);
uni_o=random(w,1,n);

kstest2(d,uni_o);
q=['The maximum difference between empirical CDF and Uniform CDF is',num2str(kstest2(d,uni_o)]
```

Output:
The maximum difference between empirical CDF and Uniform CDF is 0.00125.

Here, kstest2 is used to find supremum of set of distances.
After computing the maximum difference between empirical CDF and Uniform CDF is 0.00125,that is $\sup_{x \text{ belongs to } [0, T]} |F(x) - F(x)^{\sim}| = 0.00125$ and as n goes to infinity, this difference becomes zero. Since, the supremum value is very less, we can say that conditional arrivals are uniformly distributed.