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## STATIC OPTIMIZATION SOLUTION TO THE FORCE SHARING PROBLEM

<sup>1</sup>Yunus Ziya Arslan, <sup>2</sup>Motoshi Kaya, <sup>3</sup>Azim Jinha, <sup>3</sup>Walter Herzog

<sup>1</sup>Department of Mechanical Engineering, Faculty of Engineering, Istanbul University, Istanbul, Turkey

<sup>2</sup>Department of Physics, Graduate School of Science, University of Tokyo, Tokyo, Japan

<sup>3</sup> Human Performance Laboratory, The University of Calgary, Calgary, AB, Canada

email: yzarслан@istanbul.edu.tr

### SUMMARY

The force sharing problem is one of the basic problems in biomechanics. One common approach to this problem is to model the musculoskeletal system accurately and apply static optimization theory to predict the variable individual muscle forces during human movement. In this study, we introduce a novel static optimization strategy. Unlike previously proposed models, the new approach incorporates the instantaneous contractile conditions and physiological properties of muscle in the objective function. The proposed strategy was validated using direct muscle force measurements in the cat hind limb. Using the proposed cost function, it was possible to predict experimentally observed force-sharing features among synergistic muscles that cannot be predicted using classical static optimization approaches.

### INTRODUCTION

Solving the force sharing, or the so-called force distribution, problem is an active area of research in biomechanics [1]. Theoretical calculations of individual muscle forces depend on solving the redundancy problem which arises because the number of muscles crossing a joint exceeds the number of rotational degrees-of-freedom. Since it is relatively simple to formulate and computationally inexpensive, static optimization is one of the preferred methods to this problem [2]. In the literature, typical objective functions take the form of a weighted sum of muscle forces raised to some arbitrary power [3]. The weighting is done using constant values either for the maximal isometric force that a muscle can exert or for the physiological cross-sectional area (PCSA) [2]. These cost functions do not take the instantaneous contractile conditions into account. However, during force generation and movement, the contractile conditions change continuously and affect the force capacity of muscles.

Here, we propose a novel cost function that incorporates the physiological properties of muscles, that is the PCSA, fibre type composition, and the contractile conditions represented by the instantaneous force and speed of contraction. The predicted forces were then compared to those predicted by the optimization model proposed by Crowninshield and Brand [3] and were also compared to the experimentally measured forces in five cats.

### METHODS

The optimization strategies in this study were tested using a musculoskeletal model of the cat hind limb (Figure 1). The synergistic soleus (SO) and medial gastrocnemius (MG) were modeled as straight line muscles.

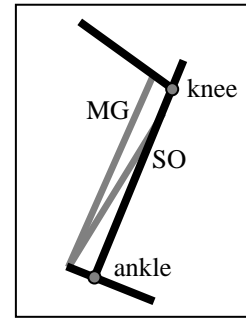


Figure 1: Musculoskeletal model of the cat hind limb.

The proposed cost function was formulated as seen below (Cost Function I).

$$\text{Minimize } \left\{ \sum_i^n \frac{(f_i v_i)^2 + \omega (f_i l_i)^2}{F_i(v_i) F_i(l_i) S_i} \right\} \quad (1)$$

subject to

$$h - \sum_{i=1}^n \mathbf{d} \mathbf{f}^T = 0, \quad f_i \geq 0, \quad i = 1, 2. \quad (2)$$

where  $f_i$ ,  $PCSA_i$ ,  $h$ , and  $\mathbf{d}$  denote the unknown muscle forces of SO and MG, the physiological cross-sectional areas of these muscles, the ankle joint moment, and the moment arm vector at the ankle, respectively.  $F_i(v_i)$ ,  $F_i(l_i)$  and  $S_i$  are the maximal muscle forces at the instantaneous muscle contraction velocity  $v_i$ , maximal muscle force at the instantaneous muscle length  $l_i$  and the percentage of slow-twitch muscle fibers, respectively.  $\omega$  was assumed constant and equal to 1 (in unit of  $1/s^2$ ).

The objective function proposed by Crowninshield and Brand [5] is (Cost Function II)

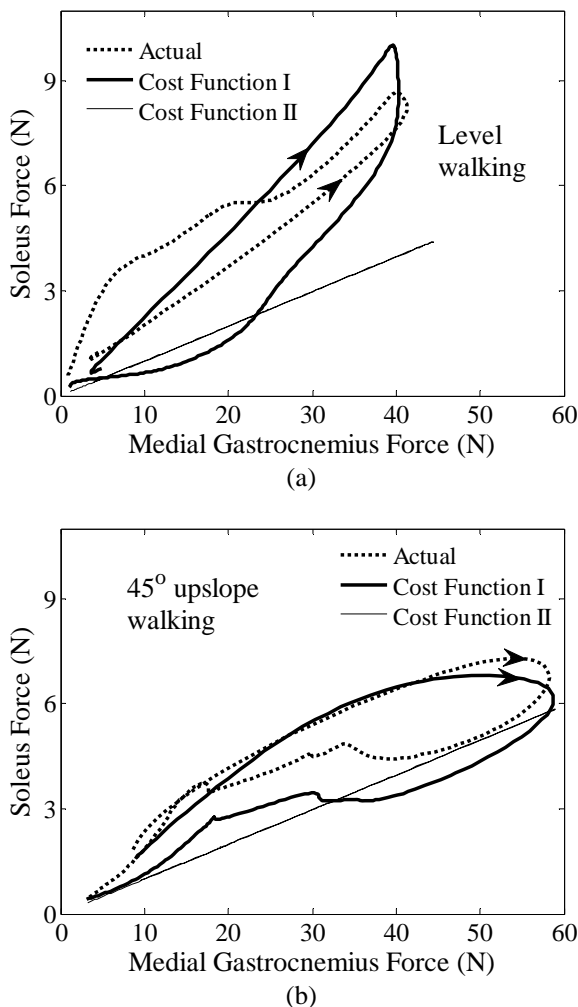
$$\text{Minimize } \left\{ \sum_i^n \left( \frac{f_i}{PCSA_i} \right)^3 \right\} \quad (3)$$

subject to equation (2).

In order to test the performance of the cost functions, theoretically determined muscle forces were compared with experimentally obtained forces for a wide range of locomotor conditions [4]. In order to record experimental data, five cats were trained to perform level and upslope walking. In these experiments, the instantaneous contractile conditions, such as muscle length and contraction velocity, and forces of the cat SO and MG muscles were measured. All experimental procedures were approved by the Life Sciences Animal Ethics Committee of the University of Calgary.

## RESULTS AND DISCUSSION

Representative force sharing loops for the level and upslope walking trials are shown in Figure 2. Comparisons of the predicted forces from both cost functions with actual muscle forces are given for level (Figure 2a) and upslope walking (Figure 2b).



**Figure 2:** Comparison of the theoretically predicted and measured muscle forces obtained for (a) level and (b) 45° upslope walking. Arrows indicate the direction of loop formation.

Muscle force patterns obtained from the newly proposed cost function are closer to the measured forces than those obtained by Cost Function II in terms of magnitude and shape. It was also observed that the force sharing patterns

among SO and MG is task-dependent and differs substantially for different contractile conditions.

The newly proposed cost function that includes the instantaneous contractile conditions of the muscles and the physiological properties of muscular contraction gave promising results in predicting cat SOL and MG force-time histories and their associated force-sharing loops. Since traditional cost functions cannot predict loop-type force-sharing behavior for the one-degree of freedom system studied here, and cannot predict decreasing SOL forces with increasing MG forces, as observed experimentally in this study, it appears that classical cost functions of the type introduced first by Crowninshield and Brand [3] cannot be used to make accurate force predictions for individual muscles during unrestrained movements. It appears that more accurate muscle force predictions are possible when accounting for the instantaneous contractile conditions and the corresponding force-length and force-velocity properties.

The primary limitation of the proposed optimization strategy is the difficulty of obtaining the required experimental data that need to be included in the cost function, especially if one wanted to use this approach to predict muscle forces in human movements. In order to measure the instantaneous length and velocity, invasive methods may be required, especially for deep muscles, although some amazing results have recently been obtained using ultrasound imaging in human movement. The main reason for the extensive use of the classical cost functions may be more based on its ease of implementation rather than accuracy in force prediction.

## CONCLUSION

The newly proposed cost function helped predict experimentally observed features of force-sharing among synergistic muscles that cannot be predicted using classical static optimization approaches. The new approach allows for predictions of force-sharing loops of agonistic muscles in one degree-of-freedom systems and for simultaneous increases in force in one muscle and decreases in a corresponding agonist. It was concluded from the results that the incorporation of the contractile conditions in the weighting of cost functions provides a natural way to incorporate observed force-sharing features in synergistic muscles.

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