Design of 1-bit and 2-bit magnitude comparators using electro-optic effect in Mach–Zehnder interferometers

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ABSTRACT

The Mach–Zehnder interferometer (MZI) structures collectively show powerful capability in switching an input optical signal to a desired output port from a collection of output ports. Hence, it is possible to construct complex optical combinational digital circuits using the electro-optic effect constituting MZI structure as a basic building block. Optical switches have been designed for 1-bit and 2-bit magnitude comparators based on electro-optic effect using Mach–Zehnder interferometers. The paper constitutes a mathematical description of the proposed device and thereafter simulation using MATLAB. Analysis of some factors influencing the performances of proposed device has been discussed properly. The study is verified using beam propagation method.

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1. Introduction

The last few decades have seen an increasing interest in optical signal processing due to its key features of large bandwidth [1], electromagnetic interference (EMI) immunity, massive parallel interconnectivity [1], low power consumption [2,3] and high speed [1,4,5]. Several combinational and sequential circuits have been proposed utilizing various technologies such as periodically poled LiNbO3 (PPLN) [6], Erbium doped optical amplifier [7], semiconductor optical amplifier (SOA) [8–14], Fabry perot laser diodes (FP-LDs) [15,16]. Erbium doped optical amplifier operates exclusively at low speed upto 1 Gb/s [7]. PPLN based logic circuits require numerous light sources and so, they are costly [6,17]. The most extensively utilized and appealing SOA based logic circuits need interferometric structure that requires numerous devices with matching characteristics and proper control. Utilizing non-interferometric techniques require two extra beams and more correlated components for accomplishing logic units [17]. In addition, optical design utilizing SOAs requires high driving current and suffers from gain saturation [8]. Logic circuits implemented with FP-LDs are complex in the sense that more number of FP-LDs and some external devices are required to implement even a simple logic circuit [15,16]. Mach–Zehnder interferometer based on electro-optic effect in LiNbO3 seems to be a promising solution, because of its characteristic features of compact size, thermal stability [18], re-configurability [19], integration potential [18], low latency [20,21] and low power consumption. Owing to which various authors have shown their keen interest in implementing a number of combinational and sequential circuits [21–29] using electro-optic effect in LiNbO3 based MZI. The basic switching mechanism taking place in LiNbO3 based MZI has been explained [23], in which authors have also proposed the design for a 1 × 4 router. A digital comparator is widely used for decision making and threshold circuits. Till date, several optical digital comparator circuits have been proposed using SOA [30] and FP-LDs [31]. But due to the above mentioned drawbacks inherent in devices utilizing SOA and FP-LDs, the performance of devices get hindered.

In this work, authors have proposed a novel scheme to design 1-bit and 2-bit magnitude comparators using electro-optic effect in LiNbO3 based MZI. The proposed devices are simulated using Beam propagation method (BPM) and the results are verified using MATLAB. The basic working principle, mathematical formulation and MATLAB simulation of the proposed structures is presented in Section 2. The
result obtained through BPM and their discussions are well presented in Section 3. Section 4 presents the analysis of some factors influencing the performances of proposed device. Finally, Section 5 comprises the conclusion.

2. Design of magnitude comparators using the MZIs

The application of magnitude comparator is essential in order to improve the flexibility in the complex digital circuits. Many researchers have shown their interest to implement the magnitude comparators using optical devices.

2.1. Design of 1-bit magnitude comparator

Fig. 1 shows a schematic diagram of 1-bit magnitude comparator using MZIs. An optical signal is given to the first input port of MZI1. The first output port of MZI1 is connected to the first input port of MZI2. The second output port of MZI2 is connected to the first output port of MZI3. The first output port of MZI3 gives the results of \( A < B \) and is named as OUT2. The second output port of MZI3 gives the results of \( A > B \) and is named as OUT3.

2.2. Mathematical formulation of normalized power at various output ports

By using the configuration shown in Fig. 1, the 1-bit magnitude comparator can be made. For all possible minterms at the first output port of MZI2 (OUT1 in Fig. 1) to perform \( A = B \) operation, the normalized power is calculated as follows;

Using the relation for single stage MZI structure in Eqs. (12) and (13) [24], we can write,

\[
\text{OUT}_{\text{MZI2}} = \left\{ \left\{ \text{je}^{-j\phi MZI1}\cos\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\cos\left(\frac{\Delta\phi MZI2}{2}\right)\right\} + \left\{ \text{je}^{-j\phi MZI1}\sin\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\sin\left(\frac{\Delta\phi MZI2}{2}\right)\right\} \right\} E_{in}
\]

\[
\frac{\text{OUT}_{\text{MZI2}}}{E_{in}} = \left\{ \left\{ \text{je}^{-j\phi MZI1}\cos\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\cos\left(\frac{\Delta\phi MZI2}{2}\right)\right\} + \left\{ \text{je}^{-j\phi MZI1}\sin\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\sin\left(\frac{\Delta\phi MZI2}{2}\right)\right\} \right\}
\]

\[
\text{OUT1} = \left[\frac{\text{OUT}_{\text{MZI2}}^2}{E_{in}}\right] = \left\{ \text{cos}^2\left(\frac{\Delta\phi MZI1}{2}\right)\text{cos}^2\left(\frac{\Delta\phi MZI2}{2}\right)\right\} + \left\{ \text{sin}^2\left(\frac{\Delta\phi MZI1}{2}\right)\text{sin}^2\left(\frac{\Delta\phi MZI2}{2}\right)\right\}
\]

In the same manner, to perform \( A < B \) operation, the normalized power at first output port of MZI3 (OUT2 in Fig. 1) is calculated as follows;

\[
\text{OUT}_{\text{MZI3}} = \left\{ \text{je}^{-j\phi MZI1}\cos\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI3}\sin\left(\frac{\Delta\phi MZI3}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\sin\left(\frac{\Delta\phi MZI2}{2}\right)\right\} E_{in}
\]

\[
\frac{\text{OUT}_{\text{MZI3}}}{E_{in}} = \left\{ \text{je}^{-j\phi MZI1}\cos\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI3}\sin\left(\frac{\Delta\phi MZI3}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\sin\left(\frac{\Delta\phi MZI2}{2}\right)\right\}
\]

\[
\text{OUT2} = \left[\frac{\text{OUT}_{\text{MZI3}}^2}{E_{in}}\right] = \text{cos}^2\left(\frac{\Delta\phi MZI1}{2}\right)\text{sin}^2\left(\frac{\Delta\phi MZI3}{2}\right)\text{sin}^2\left(\frac{\Delta\phi MZI2}{2}\right)
\]

Similarly, to perform \( A > B \) operation, the normalized power at the second output port of MZI3 (OUT3 in Fig. 1) is calculated as follows;

\[
\text{OUT}_{\text{MZI3}} = \left\{ \text{je}^{-j\phi MZI1}\sin\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI3}\cos\left(\frac{\Delta\phi MZI3}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\cos\left(\frac{\Delta\phi MZI2}{2}\right)\right\} E_{in}
\]

\[
\frac{\text{OUT}_{\text{MZI3}}}{E_{in}} = \left\{ \text{je}^{-j\phi MZI1}\sin\left(\frac{\Delta\phi MZI1}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI3}\cos\left(\frac{\Delta\phi MZI3}{2}\right)\right\}\left\{ \text{je}^{-j\phi MZI2}\cos\left(\frac{\Delta\phi MZI2}{2}\right)\right\}
\]

Fig. 1. Schematic diagram of 1-bit comparator using the MZIs.
\[ \text{OUT1} = \left| \frac{\text{OUT1}_{MZ1}}{E_{in}} \right|^2 = \sin^2\left( \frac{\Delta \phi_{MZ1}}{2} \right) \cos^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \]  

For calculation of Eqs. (1)–(9), we have assumed,

\[ \begin{align*} 
\psi_{0MZ1} &= \frac{\phi_{1MZ1} + \phi_{2MZ1}}{2} \\
\psi_{0MZ2} &= \frac{\phi_{1MZ2} + \phi_{2MZ2}}{2} \\
\psi_{0MZ3} &= \frac{\phi_{1MZ3} + \phi_{2MZ3}}{2} 
\end{align*} \]

\[ \begin{align*} 
\Delta \phi_{MZ1} &= \phi_{1MZ1} - \psi_{0MZ1} = \frac{V_A}{V_r} \\
\Delta \phi_{MZ2} &= \phi_{1MZ2} - \psi_{0MZ2} = \frac{V_B}{V_r} \\
\Delta \phi_{MZ3} &= \phi_{1MZ3} - \psi_{0MZ3} = \frac{V_B}{V_r} 
\end{align*} \]  

\[ \phi_{0MZ1}, \phi_{0MZ2}, \phi_{0MZ3} \] are the phase angle generated at the arm of MZ1, MZI2 and MZI3 respectively.  

\[ \phi_{1MZ1}, \phi_{1MZ2}, \phi_{1MZ3} \] are the phase angle generated at the lower arm of MZ1, MZI2 and MZI3 respectively. The MZI1 is controlled by signal A (the voltage applied at the second electrode, keeping other two electrodes at the ground potential). Similarly, MZI2 and MZI3 are controlled by control signal B. Basically, the control signals are 0 (0.00 V) and 1 (6.75 V) at the second electrodes of each MZI. Hence the expression for 1-bit magnitude comparator can be obtained by the following Eqs. (11)–(13),

For case 1 \((A = B)\):

\[ \text{OUT1} = \left| \frac{\text{OUT}_{1MZ1}}{E_{in}} \right|^2 = \cos^2\left( \frac{\Delta \phi_{MZ1}}{2} \right) \cos^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) + \sin^2\left( \frac{\Delta \phi_{MZ1}}{2} \right) \sin^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \]  

For case 2 \((A < B)\):

\[ \text{OUT2} = \left| \frac{\text{OUT}_{2MZ2}}{E_{in}} \right|^2 = \cos^2\left( \frac{\Delta \phi_{MZ1}}{2} \right) \sin^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \sin^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \]  

For case 3 \((A > B)\):

\[ \text{OUT3} = \left| \frac{\text{OUT}_{3MZ2}}{E_{in}} \right|^2 = \sin^2\left( \frac{\Delta \phi_{MZ1}}{2} \right) \cos^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \cos^2\left( \frac{\Delta \phi_{MZ2}}{2} \right) \]  

Fig. 2 shows the MATLAB simulation results of 1-bit magnitude comparator, where first row represents the first bit, which acts as a control signal applied at the second electrode of MZI1. The second row shows the second bit, which acts as a control signal at the second electrodes of MZI2 and MZI3. The outputs of 1-bit magnitude comparator are achieved at output ports 1, 2 and 3, whose results are shown in third, fourth and fifth row of Fig. 2. When both the control signals are low (0), then the output at port \(A = B\) is high and there is no optical signal available at other output ports. When \(A = 0V\) (low logic) and \(B\) is increasing from 0 V (low logic) to 6.75 V (high logic) then output at port \(A = B\) is decreasing from high to low and output at \(A < B\) is increasing from low to high. There is no optical signal at port \(A > B\) at this time. Next, When \(A\) is increasing from 0 V (low logic) to 6.75 V (high logic) and \(B = 0 V\) (low logic) then output at port \(A = B\) is decreasing from high to low and output at \(A > B\) is increasing from low to high. There is no optical signal at port \(A < B\) at this time.
When A is increasing from 0 V (low logic) to 6.75 V (high logic) and B is increasing from 0 V (low logic) to 6.75 V (high logic) then output signal at $A = B = 0$ V getting at port $A = B$. As control signal is increasing the equality nature is going to loss and getting some optical signal at all ports, finally at control signal $A = B = 6.75$ V, the optical signal at port $A = B$ is high. There is some optical signal at port $A < B$ and $A > B$ at this time but output signal at port $A = B$ is more than others. The result at $A = 6.75$ V (high logic) and $B = 6.75$ V (high logic) is of our interest.

2.3. Design of 2-bit magnitude comparator

Twelve MZIs are being used to implement the 2-bit comparator as shown in Fig. 3. The optical signal is provided to first input port of MZI1. The outputs of MZI1 are directly connected with the inputs of MZI2. The second output of MZI2 is connected to the first input port of MZI3 and first output of MZI3 is connected to the first input ports of MZI4 and MZI7. The combination of output of MZI4, MZI7 and MZI10 gives the result of $A > B$ at output port 1. The second output of MZI13 is connected to the first input ports of MZI5 and MZI8. The combination of MZI5, MZI8 and MZI10 gives the result of $A < B$ at output port 3. The second input optical signal is given to second input port of MZI6 and outputs of this MZI are provided to the inputs of MZI9. The first output port of MZI9 is connected to first input port of MZI10. The outputs of MZI10 are connected to the output port 1 and 3.

The combination of MZI1, MZI2, MZI6, MZI9, MZI11 and MZI12 gives the result of $A = B$ at output port 2, where output of first output port of MZI9 is directly fed to one PIN photo-detector, which converts the optical signal into electrical signal. But the amount of this signal is very small. So, with the help of amplifier, this signal can be amplified to an appropriate voltage and given as the control signal to the second electrode of MZI11. Similarly, the output from second output port of MZI19 is applied to another PIN photo-detector, which converts the optical signal into electrical signal and again amount of this signal is very small so with the help of amplifier, which amplifies this signal to an appropriate voltage level and given as control signal to second electrode of MZI12. MZI11 and MZI12 works as the AND gate [25]. The first output port of MZI12 gives the result of $A = B$ at output port 2.

2.4. Mathematical formulation of normalized power at various output ports

For all possible minterms at the output ports 1, 2 and 3 (to perform a 2-bit magnitude comparator), the normalized power is calculated in Appendix A.

Fig. 4(a)–(d) shows MATLAB simulation results of 2-bit magnitude comparator, where first column represents $B_0$, Similarly, second, third and fourth columns represent bits $B_0$, $A_1$ and $A_0$ respectively. Outputs $A = B$, $A > B$ and $A < B$ are shown in fifth, sixth and seventh columns respectively.

3. Design of magnitude comparators using the BPM

OptiBPM is used to analyze the proposed structures. OptiBPM basically works on the principle of FD-BPM and provides the complete information of the optical waveguide depending upon its refractive index profile, structure, and material used for the construction of the optical waveguide.

3.1. Design of 1-bit magnitude comparator

The layout of 1-bit comparator consists of three MZIs as shown in the Fig. 5. Here, an optical signal is given to first input port of MZI1. The first output port of MZI1 is connected to the first input port of MZI2. The first output port of MZI2 is considered as $A = B$. The second output port of MZI2 is connected to the first input port of MZI3. The first output port of MZI3 is taken as output port 2 of 1-bit comparator, which gives the result of $A < B$. The second output port of MZI3 is taken as output port 3 of 1-bit comparator, which gives the results of $A > B$. First bit is given as control signal $A$ to the second electrode of MZI1 and second bit is given as control signal $B$ to second electrodes of MZI2 and MZI3.

The different combination of control signals are applied to the proposed device as shown in Fig. 5 and its corresponding responses can be discussed as follows:

Case 1: $A = 0$, $B = 0$

The light from the continuous wave optical source is incident on the first input port of the MZI1. As the control signal $A$ of MZI1 is OFF, the light emerges from the second output port of MZI1 and goes to the second input port of MZI2. As it is given that, $B = 0$ (the control...
signal of MZI2), the signal appears at the first output port of MZI2 (Port 1 in Fig. 5) which shows that the magnitude of A is equal to B (as shown in Fig. 6).

Case 2: $A = 0, B = 1$

In the absence of the signal A of MZI1, the incident optical signal of first input port of MZI1 appears at the second output port of the MZI1. Due to the variation of the electrode voltage B (the control signal of MZI2 and MZI3) from 0 V to 6.75 V, the incident optical signal of second input port of MZI2 is transferred to the second output port of MZI2 and it acts as input to the MZI3. Finally, it appears at first output port of MZI3 (Port 2 in Fig. 6). Hence, it (Port 2 in Fig. 5) can be treated as number B is greater than A i.e. ($A < B$).

Case 3: $A = 1, B = 0$

Due to the variation of the electrode voltage A of MZI1 from 0 V to 6.75 V, the incident optical signal of first input port of MZI1 is transferred to the first output port of MZI1 and it acts as input to the MZI2 but in the absence of B, it appears at the second output port of MZI2. Finally, due to the absence of the signal B of MZI3 it appears at the second output port of MZI3 (Port 2 in Fig. 6). Hence, second

Fig. 4. MATLAB simulation results of 2-bit comparator, when magnitude of B is (a) 0 (b) 1 (c) 2 (d) 3 and magnitude of A changes from 0 to 3.
output port of MZI3 (Port 2 in Fig. 5) verifies the inequality i.e. $A > B$.

Case 4: $A = 1, B = 1$

Similarly, due to the variation of the control voltage $A$ and $B$ from 0 V to 6.75 V, the optical signal incident at first input port of MZI1 is transferred to the first output port of MZI1. Finally, due to the appearance of electrode voltage $B$ (the control signal of MZI2) it appears at the first output port of MZI2 (Port 2 in Fig. 6). Hence, first output port of MZI2 (Port 1 in Fig. 5) again verifies the $(A = B)$ logic. Table 1 shows all possible combinations of the control signal and the output optical signal obtained at the different output ports of the structure as shown in Fig. 6.

The proper selection of control signals provides the suitable result of 1-bit magnitude comparator, which is shown in Fig. 6. Fig. 6 represents the different combinations of control signals (electrode voltages) and the corresponding results of the 1-bit comparator obtained through beam propagation method. The results obtained from the proposed structure can be verified with the observation in Table 1.

![Fig. 4. (continued)](image-url)
3.2. Design of 2-bit comparator

The schematic layout diagram for comparing two 2-bit numbers is shown in Fig. 7. Twelve MZIs are being used to design the proposed structure. The continuous wave (CW) optical source is applied to first input port of MZI1. The output ports of MZI1 are directly connected with the input ports of MZI2. The second output of MZI2 is connected to the first input port of MZI3 and first output port of MZI3 is connected to the first input ports of MZI4 and MZI7. The combination of output of MZI4, MZI7 and MZI10 gives the result of \( AB > 0 \) at output port 1. The second output of MZI3 is connected to the first input ports of MZI5 and MZI8. The combination of MZI5, MZI8 and MZI10 gives the result of \( AB < 0 \) at output port 3. The second input optical signal is given to the MZI6 and an output of this MZI is provided to the inputs of MZI9. The first output of MZI9 is connected to the first input port of MZI10. The outputs of MZI10 are connected to the output port 1 and 3. The combination of MZI11, MZI2, MZI6, MZI9, MZI11 and MZI12 gives the result of \( AB = 0 \) at output port 2, where

Table 1

<table>
<thead>
<tr>
<th>Control Signals</th>
<th>Signal output at different ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( B )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 5. BPM layout of 1-bit comparator using three Mach–Zehnder interferometers.

Fig. 6. Results of 1-bit comparator for different combinations of control signals obtained through beam propagation method.
outputs of MZI1, MZI2 is directly taken by one PIN photo-detector, which converts the optical signal into electrical signal. But the amount of this signal is very small so with the help of amplifier, this signal can be amplified and given as the control signal to the second electrode of MZI11. Same, the output of MZI6 and MZI9 is directly taken by other PIN photo-detector, which converts the optical signal into electrical signal and amount of this signal is very small so with the help of amplifier, this signal can also amplified and given as the control signal to the second electrode of MZI12. The MZI11 and MZI12 works as the AND gate [25], The first output port of MZI12 gives the result of $A = B$ at output port 2.

The result of 2-bit comparator logic obtained from the BPM is shown in Fig. 8. It can be verified by Table 2, which is the truth table of 2-bit comparator obtained from the MATLAB (shown in Fig. 4(a)–(d)). The different combination of control signals is applied to the proposed device as shown in Fig. 7 and its corresponding responses can be discussed as follows:

Case 1: $B_1 = 0$, $B_2 = 0$, $A_1 = 0$, $A_0 = 0$

The light from continuous wave (CW) source is incident on the first input port of the MZI1 and to second input port of MZI6. As the control signals of all MZIs are OFF except MZI11 (control signal=$A_1$ XNOR $B_0$) and MZI12 (control signal=$A_1$ XNOR $B_1$), the light emerges from the second output port of MZI1 and first output port of MZI6, goes to the second input port of MZI12 and first input port of MZI9 respectively. As it is given that, $A_0=0$ (the control signal of MZI2) and $A_1=0$ (the control signal of MZI9), the output signal from MZI2 and MZI9 commonly appears on the first input port of MZI11. Due to the variation of control signal of MZI11 and MZI12 from 0 V to 6.75 V, the incident optical signal of the first input port of MZI11 is transferred to the first output port of MZI11 and then finally from the first input port of MZI12 to its output port as shown in Fig. 7. This is showing that the magnitude of $A$ is equal to the magnitude of $B$ (Port 2 in Fig. 8a).

Case 2: $B_1 = 0$, $B_2 = 0$, $A_1 = 0$, $A_0 = 1$

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI4 (via MZI2, MZI3) and MZI12 (via MZI9, MZI10, MZI11) respectively as shown in Fig. 7. As it given that $B_2=0$ (control signal at MZI4), the light emerges from the second port of MZI4 and finally appears on output port 1, verifies the logic $A > B$ as shown in Fig. 8a.

Case 3: $B_1 = 0$, $B_2 = 0$, $A_1 = 1$, $A_0 = 0$

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI11 (via MZI2) and MZI10 (via MZI9) respectively as shown in Fig. 7. As it given that $A_1=1$ (control signal at MZI9 and MZI10), the light emerges from the first port of MZI10 and finally appears on output port 1, verifies the logic $A > B$ as shown in Fig. 8a.

Case 4: $B_1 = 0$, $B_2 = 0$, $A_1 = 1$, $A_0 = 1$

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI4, MZI7 (via MZI2 and MZI3) and MZI10 (via MZI9) respectively as shown in Fig. 7. As it given that $A_1=1$ (control signal at MZI7, MZI9 and MZI10) and $B_2=0$, the light emerges from the first port of MZI10, first port of MZI7 and the second port of MZI4 and finally appears on output port 1, verifies the logic $A > B$ as shown in Fig. 8a.

Case 5: $B_1 = 0$, $B_2 = 1$, $A_1 = 0$, $A_0 = 0$

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI5 (via MZI2 and MZI3) and MZI11 (via MZI9) respectively as shown in Fig. 7. As it given that $B_2=1$ (control signal at MZI11) and $B_0=0$, the light emerges from the second port of MZI18 and appears at the port 3 ($A < B$), verifies the logic $A < B$ as shown in Fig. 8b. No light appears at the input port of MZI12 because the control signal ($A_0$ XNOR $B_0$) of MZI11 is OFF, it switches the light to the second output port of MZI11 which is not connected.

Case 6: $B_1 = 0$, $B_2 = 1$, $A_1 = 1$, $A_0 = 1$

In the presence of the signal $A_0$ of MZI1 and $B_0$ of MZI2, the incident optical signal of the first input port of MZI1 appears on the first output port of MZI1 and then the first input port of MZI2 to its first output port as shown in Fig. 7. Finally, it appears at port 2 ($A = B$) via MZI11 (control signal is HIGH) and MZI12 (control signal is HIGH) as shown in Fig. 8b.

Case 7: $B_1 = 0$, $B_2 = 1$, $A_1 = 1$, $A_0 = 0$

The light from the constant optical source is incident on the first input port of the MZI1 and terminated at the second output port of MZI5 (control signal is OFF) as well as first output port of MZI8 (control signal is HIGH) as shown in Fig. 7 while the incoming light to the second input port of MZI6 (control signal is OFF) is reaching to MZI9 (control signal is HIGH) and MZI10 (control signal is HIGH). Finally, it appears at port 1 ($A > B$) as shown in Fig. 8b.

Case 8: $B_1 = 0$, $B_2 = 1$, $A_1 = 1$, $A_0 = 1$

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI12 (via MZI2 and MZI11) and MZI10 (via MZI9) respectively as shown in Fig. 7. As it given that $B_0=0$ and $A_1=1$, the light emerges from the second port of MZI12 (control signal $A_1$ XNOR $B_0$) is being...
Fig. 8. Results of 2-bit comparator operation obtained through BPM, when magnitude of B is (a) 0 (b) 1 (c) 2 (d) 3 and magnitude of A changes from 0 to 3.
Fig. 8. (continued)
terminated while the light emerges from the first port of MZI10 reaches to the port 1 \((A > B)\), verifies the logic \(A > B\) as shown in Fig. 8b.

Case 9: \(B_1 = 1, \ B_0 = 0, \ A_1 = 0, \ A_0 = 0\)

Incoming light to MZI1 and MZI6 is reaching to the first input port of MZI12 (via MZI2, MZI11) and MZI10 (via MZI9) respectively as shown in Fig. 7. As it given that \(A_1 = 0\) (control signal at MZI9 and MZI10), the light emerges from the second port of MZI10 and finally appears on output port 3 \((A < B)\), verifies the logic \(A < B\) as shown in Fig. 8c. No light emerges at output port of MZI12 because its control signal is OFF which terminates the incoming light to second output port which is remains open as shown in Fig. 7.

Case 10: \(B_1 = 1, \ B_0 = 0, \ A_1 = 0, \ A_0 = 1\)

The light from the constant optical source is incident on the first input port of the MZI1 and terminated at second output port of MZI7 (control signal is OFF) as well as first output port of MZI4 (control signal is HIGH) as shown in Fig. 7 while the incoming light to second input port of MZI6 (control signal is ON) is reaching to MZI9 (control signal is OFF) and MZI10 (control signal is OFF). Finally it appears at output port 3 \((A < B)\), verifies the logic \(A < B\) as shown in Fig. 8c.

Case 11: \(B_1 = 1, \ B_0 = 0, \ A_1 = 1, \ A_0 = 0\)

Incoming light to MZI1 and MZI6 is commonly reaching to first input port of MZI11 via MZI2 and MZI9, MZI10 respectively as shown in Fig. 7; the light emerges from the first port of MZI11 and enters to the first input port of MZI12. Finally it appears at output port 1 \((A = B)\), verifies the logic \(A = B\) as shown in Fig. 8c.

Case 12: \(B_1 = 1, \ B_0 = 0, \ A_1 = 1, \ A_0 = 1\)

As the control signal to MZI11 is OFF \((A_0 \text{XOR} B_0)\), the emerge light is terminated at the second output port of MZI11. But the incoming light to MZI1 is reaching to the port 1 via MZI2, MZI3, MZI7. This is showing that the magnitude of A is greater than the magnitude of B as shown in Fig. 8c.

Case 13: \(B_1 = 1, \ B_0 = 1, \ A_1 = 0, \ A_0 = 0\)

Incoming optical signal to MZI1 and MZI6 is reaching to output port 3 \((A < B)\) via MZI2, MZI3, MZI5, MZI8 and via MZI9, MZI10 respectively. Finally it verifies the logic \(A < B\) as shown in Fig. 8d.

Case 14: \(B_1 = 1, \ B_0 = 1, \ A_1 = 0, \ A_0 = 1\)
The light from the constant optical source is incident on the first input port of the MZI1 and terminated at second output port of MZI12 (control signal is OFF) as shown in Fig. 7 while the incoming light to second input port of MZI16 (control signal is ON) is reaching to second input port of MZI19 (control signal is OFF) and then first input port of MZI10 (control signal is OFF). Finally it appears at output port 3 \( A < B \), verifies the logic \( A < B \) as shown in Fig. 8d.

Case 15: \( B_1 = 1, B_0 = 1, A_1 = 1, A_0 = 0 \)

In the absence of the control signal \((A_0, \text{XOR} B_0)\) to MZI11, the incident optical signal at first input port of MZI11 is being terminated. But the optical signal to MZI1 is reaches to the output port3 \( A < B \) via MZI2, MZI3 and MZI5. Finally it verifies the logic \( A < B \) as shown in
Case 16: $B_1 = 1$, $B_0 = 1$, $A_1 = 1$, $A_0 = 1$

Incoming optical signal to MZI1 and MZI6 is commonly reaching to first input port of MZI11 via MZI2, MZI3 and MZI9, respectively as shown in Fig. 7. As it given that all control signals are ON, the light emerges from the first input port of MZI11 and then enters to the first input port of MZI12 and finally it appears at the output port 2, verifies the logic $A = B$ as shown in Fig. 8d.

Table 2 shows the optical signal of 2-bit magnitude comparator due to different combination of control signals. Where, $A$ and $B$ show the 2 bit numbers and Port 1, 2 and 3 represents ($A > B$), ($A = B$) and ($A < B$) respectively.

4. Study and analysis of some factors influencing the performances of proposed devices

The performance of a single MZI used in the construction of proposed device is examined by performing the 2D isotropic simulation using the paraxial BPM with finite difference engine scheme parameter of 0.5, propagation step of 1.3 μm and transparent boundary condition. The global data has been taken as refractive index MODAL and TM polarized test signals with wavelength 1.3 μm is considered. The chromatic dispersion of a single LiNbO$_3$ based MZI is given by [21];
Fig. 15. Representation of the cross talk level with the variation of power imbalance (Switch state: Cross) for operating wavelength 1.3 μm.

Fig. 16. Calculated cross-talk levels due to variation in Ti-thickness for operating wavelength 1.3 μm; (switch state: Cross).

Fig. 17. Calculated cross-talk levels of the switch (in bar state) as a variation of switching voltage for \( \lambda = 1.3 \, \mu m \) (with \( t_i = 0.05 \, \mu m \)).
\[
D_c = - \left( \frac{L_m \Delta \lambda_c}{c} \right) \left( \frac{d^2 n_e}{d\lambda^2} \right)
\]

(14)

Where \( L_m \) is the length of a single MZI, \( \lambda \) is the operating wavelength and \( \Delta \lambda \) is the spectral line width of optical source. \( n_e \) is the effective refractive index of the material (LiNbO\(_3\)) MZI. The modal dispersion, \( D_{\text{modal}} \) for a multimode step-index electro-optic device with length \( L_m \) is given by

\[
D_m = \frac{L_m n_e \Delta n_e}{c}
\]

(15)

\( \Delta n_e \) is the relative refractive index difference. The total dispersion coefficient \( D_{\text{total}} \) is given by

\[
D_{\text{total}} = D_m + D_c
\]

(16)

The rise time of system in terms of bit rate (BR) for non return to zero (NRZ) pulse is given by

\[
\text{BR} = \frac{0.7}{D_{\text{total}}}
\]

(17)

Fig. 9 shows the variation of chromatic dispersion in a single MZI at an operating wavelength of 1.3 \( \mu \)m, as the length of MZI is varied.
from 500 μm to 20,000 μm. It is apparent from the plot that with the increase in length of MZI, chromatic dispersion goes on increasing in negative direction. Similarly, Fig. 10 shows the variation of modal dispersion with the length of MZI.

Modal dispersion also increases with the increase of MZI. Although, at smaller length, dispersion is less and pulse broadening is less. But, there is a check in reducing the length of MZI to very small values for a specified wavelength, because as the length of MZI reduces, switching voltage required for the operation of MZI increases as given by [23]

$$V = \frac{\lambda}{n^2} \frac{\Delta \phi}{r} \frac{d}{L}$$  \hspace{1cm} (18)

Where, $\lambda$ is the wavelength of light used, $\Delta \phi$ is the phase change due to applied electric field. $r$ is electro-optic coefficient ($\approx 36.6 \times 10^{-12} \text{ m/V for LiNbO}_3$). $d$ is the separation between electrodes and $L$ is the substantial length. Also, the available length is not sufficient to couple energy from one waveguide to other. But still, by optimizing various parameters like length of MZI, switching voltage and wavelength of operation, bit rate of the order of 20–200 Gbps can be obtained as shown in Fig. 11. So, by selecting appropriate length of MZI, low power consumption structure with low latency and high speed can be obtained.

As the transmission rates goes on increasing, LiNbO3 based MZI becomes susceptible to polarization-associated deterioration. The LiNbO3 waveguides exploiting electro-optic effect can be utilized as a polarization controller [32]. This scheme is shown in Fig. 12. A polarization controller comprises of two phase modulators and a TE–TM mode converter. A phase difference of $\phi_2$ between incoming TE and TM modes is to be adjusted using first phase modulator so that the polarization controller can act with all incoming polarization states. With this condition satisfied, the central phase-matched mode converter acts as a linear polarization rotator. Even though a linear output polarization of either TE or TM is enough in some applications, for full polarization control a second shifter is needed to modify the output phase to a desired value of the elliptical output polarization.

The transition losses for straight and curved waveguides with the variation of Ti-diffusion thickness in the range of 0.04 – 0.09 μm can also be observed [24]. Fig. 13 shows that these losses can be kept at low value by taking the value of $t_i \geq 0.05$ μm for the test wavelength 1.3 μm. Now, the effect of power imbalance and their impact on the crosstalk levels introduced at the end of interferometers due to the variation of the Ti thickness ($t_i$) has been analyzed. The power level coming out from the end of the first coupler (as shown in Fig. 14) has been monitored. The power imbalance at the output splitter can be obtained using following definitions [33].

$$PI = 10 \log \left( \frac{A}{B} \right)$$  \hspace{1cm} (19)

where, $A$ and $B$ are the optical signal strength at the end of the first coupler (as shown in Fig. 14). Hence, cross talk at the end of the interferometer arms is,

$$CT = 10 \log \frac{\left( \frac{C}{D} \right)^2 - \left( \frac{A}{B} \right)^2}{\left( \frac{A}{B} \right)^2 + \left( \frac{C}{D} \right)^2}$$  \hspace{1cm} (20)

Since, $C$ and $D$ are power levels at the end of the interferometer arms and they can be expressed as follows:

$$C = \frac{1}{10^{\frac{PI}{20}}} + 1 \text{ and } D = 1 - C$$  \hspace{1cm} (21)

On the basis of the Eqs. (19)-(21) the analysis has been obtained for the basic MZI used for the proposed device. Fig. 15 projects that crosstalk levels at the end of the interferometer arms become worst as the power imbalance increases.

Fig. 16 shows the variation in crosstalk generated at the interferometers of the two arms due to the variation of the Ti-thickness in the range of 0.04–0.09 μm. On the basis of the Fig. 16, the low crosstalk can be obtained as 36.67 dB at the Ti-thickness of 0.05 μm for the operating wavelength of 1.3 μm. It is examined that switch operation is more stable for lateral diffusion length $L_{DH}=3.5$ μm and diffusion length in depth $D_{V}=4.2$ μm. However, the switch is forced to obtain its bar state due to EO effect, by applying the voltage across the electrode in the range of 5–9 V.

Fig. 17 shows the variation in the cross-talk generated at the interferometer arms due to the variation in the switching voltage in its bar state. From Fig. 17, it is clear that switching performance gives the best result for 6.75 V for the test wavelength of 1.3 μm. It can be observed that, for the switching voltage 6.75 V the crosstalk is ~24.47 dB. Basically, the ON/OFF extinction ratio should be large for optical modulator used in the digital communication system. The extinction ratio (ER) between the optical intensity at the ON-state and at the OFF-state is determined by the optical balance between the two arms. It can be calculated as follow [34]:

$$ER = \left( \frac{\text{OUT}_1 + \text{OUT}_2}{\text{OUT}_1 - \text{OUT}_2} \right)^2$$  \hspace{1cm} (22)

The ratio of $\frac{\text{OUT}_1}{\text{OUT}_2}$ represents the optical power difference between the two arms, which can be caused by different optical loss in the two arms, as well as the imbalance in the optical splitter and combiner and the Ti-diffusion thickness and switching voltages.

Fig. 18 represents the plot of the ER versus the power difference. From the Fig. 18, it is seen that in order to achieve more than 20 dB ER, the power difference has to be less than 0.8 dB. The analysis is carried out to examine the behavior of ER with respect to switching voltage and Ti-thickness. Fig. 19(a) represents the variation of ER with the variation of Ti-thickness and Fig. 19(b) shows the ER with switching voltage. The analysis is carried out for checking the suitability of Ti-thickness and switching voltage in order to achieve the satisfactory ER. The high value of ER can be obtained at specified Ti-thickness and switching voltage. The single MZI of proposed device shows the ER of 39.27 dB at Ti-thickness of 0.05 μm and switching voltage of 6.75 V.
5. Conclusion

The important aspects of electro-optic effect based Mach–Zehnder interferometer as an optical switch are presented. The layout diagrams of the proposed devices are discussed with the appropriate mathematical analysis and the results are obtained using the MATLAB simulation. The discussed method to implement the proposed devices is verified using the beam propagation method. The paper includes detailed discussion of the magnitude comparator. It presents the guideline to realize 1-bit and 2-bit magnitude comparator by using MZIs. The results furnished in this paper will be a stepping stone in the area of designing DWDM optical components for optical network systems.

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Appendix-A

For all possible minterms at the first output port of MZI12 (Output Port 2 in Fig. 3), (to perform A = B operation), the normalized power is calculated as follows;

Using the relation for single stage MZI structure in Eqs. (12) and (13) [24], we can write,

\[
\text{OUT}_{\text{MZI12}} = \left[ \begin{array}{c}
\{ \text{Je}^{-j(\phi_{\text{MZI1}})} \text{cos} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \} \\
\{ \text{Je}^{-j(\phi_{\text{MZI2}})} \text{cos} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \}
\end{array} \right] \\
+ \left[ \begin{array}{c}
\{ \text{Je}^{-j(\phi_{\text{MZI1}})} \text{sin} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \} \\
\{ \text{Je}^{-j(\phi_{\text{MZI2}})} \text{sin} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \}
\end{array} \right] \text{E}_{\text{in}}
\]

(A1)

\[
\frac{\text{OUT}_{\text{MZI12}}}{\text{E}_{\text{in}}} = \left[ \begin{array}{c}
\{ \text{Je}^{-j(\phi_{\text{MZI1}})} \text{cos} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \} \\
\{ \text{Je}^{-j(\phi_{\text{MZI2}})} \text{cos} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \}
\end{array} \right] \\
+ \left[ \begin{array}{c}
\{ \text{Je}^{-j(\phi_{\text{MZI1}})} \text{sin} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \} \\
\{ \text{Je}^{-j(\phi_{\text{MZI2}})} \text{sin} \left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \}
\end{array} \right] \text{E}_{\text{in}}
\]

(A2)
OUT1 = \[ \left( \frac{\text{OUT}_{\text{MZI12}}}{E_{\text{in}}} \right)^2 = \left\{ \cos^2\left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI9}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \right\} + \left\{ \cos^2\left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \sin^2\left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \sin^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \right\} + \left\{ \sin^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \sin^2\left( \frac{\Delta \phi_{\text{MZI12}}}{2} \right) \sin^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \right\} \]

(A3)

In the same manner, to perform \( A < B \) operation, the normalized power at second output port of MZI 8 and MZI 10 (Output Port 3 in Fig. 3) is calculated as follows;

\[
\text{OUT2}_{\text{MZI10}} = \left[ \left\{ -j e^{-j (\phi_{\text{MZI10}})} \sin \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \left\{ j e^{-j (\phi_{\text{MZI10}})} \cos \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \right] + \left[ \left\{ j e^{-j (\phi_{\text{MZI10}})} \cos \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \left\{ -j e^{-j (\phi_{\text{MZI10}})} \sin \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \right] E_{\text{in}}
\]

(A4)

\[
\text{OUT2} = \left( \frac{\text{OUT}_{\text{MZI10}}}{E_{\text{in}}} \right)^2 = \left[ \left\{ -j e^{-j (\phi_{\text{MZI10}})} \sin \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \left\{ j e^{-j (\phi_{\text{MZI10}})} \cos \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \right] + \left[ \left\{ j e^{-j (\phi_{\text{MZI10}})} \cos \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \left\{ -j e^{-j (\phi_{\text{MZI10}})} \sin \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \right] \left\{ -j e^{-j (\phi_{\text{MZI10}})} \sin \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \left\{ j e^{-j (\phi_{\text{MZI10}})} \cos \left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \right\} \right] E_{\text{in}}
\]

(A5)

\[
\text{OUT2} = \left( \frac{\text{OUT}_{\text{MZI10}}}{E_{\text{in}}} \right)^2 = \sin^2\left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI9}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) + \sin^2\left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI10}}}{2} \right) \cos^2\left( \frac{\Delta \phi_{\text{MZI2}}}{2} \right)
\]

(A6)

Similarly, to perform \( A > B \) operation, the normalized power at first output port of MZI 7 (Output Port 1 in Fig. 3) is calculated as follows;
\[
\text{OUT}_{1MZI} = \left\{ -\mathrm{e}^{-j(\varphi_{MZI})}\sin\left(\frac{\Delta \varphi_{MZI}}{2}\right), -\mathrm{e}^{-j(\varphi_{MZI})}\sin\left(\frac{\Delta \varphi_{MZI}}{2}\right) \right\}
\]

\[
\text{OUT}_{1MZI} = \left\{ -\mathrm{e}^{-j(\varphi_{MZI})}\cos\left(\frac{\Delta \varphi_{MZI}}{2}\right), -\mathrm{e}^{-j(\varphi_{MZI})}\cos\left(\frac{\Delta \varphi_{MZI}}{2}\right) \right\}
\]

\[
\frac{\text{OUT}_{1MZI}}{E_{in}} = \left\{ -\mathrm{e}^{-j(\varphi_{MZI})}\sin\left(\frac{\Delta \varphi_{MZI}}{2}\right), -\mathrm{e}^{-j(\varphi_{MZI})}\sin\left(\frac{\Delta \varphi_{MZI}}{2}\right) \right\}
\]

\[
\frac{\text{OUT}_{1MZI}}{E_{in}} = \left\{ -\mathrm{e}^{-j(\varphi_{MZI})}\cos\left(\frac{\Delta \varphi_{MZI}}{2}\right), -\mathrm{e}^{-j(\varphi_{MZI})}\cos\left(\frac{\Delta \varphi_{MZI}}{2}\right) \right\}
\]

\[
\text{OUT}_{3} = \left[ \frac{\text{OUT}_{1MZI}}{E_{in}} \right]^{T}
\]

\[
\text{OUT}_{3} = \cos^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right)\sin^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right) \sin^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right) \cos^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right)
\]

\[
\text{OUT}_{3} = \cos^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right)\sin^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right) \sin^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right) \cos^{2}\left(\frac{\Delta \varphi_{MZI}}{2}\right)
\]

For calculation of Eqs. (A1)–(A9), we have assumed,

\[
\begin{align*}
\varphi_{zMZI} = \frac{1}{2} \left[ \varphi_{1MZ1} + \varphi_{2MZ1} \right] \\
\varphi_{zMZ2} = \frac{1}{2} \left[ \varphi_{1MZ2} + \varphi_{2MZ2} \right] \\
\varphi_{zMZ3} = \frac{1}{2} \left[ \varphi_{1MZ3} + \varphi_{2MZ3} \right] \\
\varphi_{zMZ4} = \frac{1}{2} \left[ \varphi_{1MZ4} + \varphi_{2MZ4} \right] \\
\varphi_{zMZ5} = \frac{1}{2} \left[ \varphi_{1MZ5} + \varphi_{2MZ5} \right] \\
\varphi_{zMZ6} = \frac{1}{2} \left[ \varphi_{1MZ6} + \varphi_{2MZ6} \right] \\
\varphi_{zMZ7} = \frac{1}{2} \left[ \varphi_{1MZ7} + \varphi_{2MZ7} \right] \\
\varphi_{zMZ8} = \frac{1}{2} \left[ \varphi_{1MZ8} + \varphi_{2MZ8} \right] \\
\varphi_{zMZ9} = \frac{1}{2} \left[ \varphi_{1MZ9} + \varphi_{2MZ9} \right] \\
\varphi_{zMZ10} = \frac{1}{2} \left[ \varphi_{1MZ10} + \varphi_{2MZ10} \right]
\end{align*}
\]
MZI1, MZI2, MZI3, MZI4, MZI5, MZI6, MZI7, MZI8, MZI9 and MZI10 are the phase angle generated at the upper arm of MZI1, MZI2, MZI3, MZI4, MZI5, MZI6, MZI7, MZI8, MZI9 and MZI10 respectively. Also, A = \phi_0 and B = \phi_0. The MZI1 is controlled by control signal \phi_0 (the voltage applied at the second electrode, keeping other two electrodes at the ground potential). Similarly, MZI2 and MZI3 are controlled by control signal \phi_0. MZI2, MZI3, MZI4, MZI5, MZI6, MZI7, MZI8, MZI9 and MZI10 are controlled by control signal \phi_1. Basically, the control signals are 0 (0.00 V) and 1(6.75 V) at the second electrodes of each MZI. Hence the expression for 2-Bit magnitude comparator can be obtained by the following Eqsns.

For Case 1 (A > B):

\[
\text{OUT}_1 = \frac{\text{OUT}_{MZI12}}{E_{in}} = \left\{ \cos^2\left(\frac{\Delta \phi_{MZI1}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI8}}{2}\right) \right\} 
\]

(A11)

\[
+ \left\{ \cos^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI9}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI1}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \right\} 
\]

\[
+ \left\{ \cos^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \right\} 
\]

\[
+ \left\{ \sin^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI9}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI1}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \right\} 
\]

For Case 2 (A < B):

\[
\text{OUT}_2 = \frac{\text{OUT}_{MZI10}}{E_{in}} = \left\{ \sin^2\left(\frac{\Delta \phi_{MZI1}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \right\} 
\]

(A12)

\[
+ \left\{ \sin^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \right\} 
\]

\[
+ \left\{ \sin^2\left(\frac{\Delta \phi_{MZI6}}{2}\right) \cos^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \right\} 
\]

For Case 3 (B > A):

\[
\text{OUT}_3 = \frac{\text{OUT}_{MZI7}}{E_{in}} = \left\{ \cos^2\left(\frac{\Delta \phi_{MZI1}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI2}}{2}\right) \sin^2\left(\frac{\Delta \phi_{MZI10}}{2}\right) \right\} 
\]

(A13)

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References