Video of the Season
Volume 1, Number 1
www.SecondaryMathVideos.com
Fall 2016
Rob Wieman, Rowan University

Video Segment:
Australian Lesson #1 from the TIMSS Video 4:15 – 13:10
http://www.timssvideo.com/29

Mathematics Content:
Geometry (Exterior angles of a polygon, clear definitions, creating conjectures)

Mathematical Practices:
- SMP 1: Make sense of problems and persevering in solving them
- SMP 3: Construct Viable Arguments and critique the reasoning of others
- SMP 6: Attend to precision

Teacher Education Goals:
- Participants will learn how to support students in making sense of written problems, specifically in helping students identify and attend to important vocabulary
- Participants will learn how to support students in developing precise language in the context of geometric definitions, specifically by creating non-examples
- Participants will learn moves designed to support students in creating clear conjectures
- Participants will learn moves designed to support students in engaging in spoken and written mathematical communication

Description of the video clip:
- 4:15 – 5:35
  The video begins with the teacher sharing work from a previous lesson in which some students have created geometric diagrams of five pointed stars inscribed inside a circle. The teacher points out that this work has several important features. First he points out that there is a diagram, then that the angles are labeled and the angle measurements are displayed. Finally, the teacher shares some of the conclusions (conjectures) that students have made about their diagrams.
- 6:15 – 7:20
  The teacher hands out the primary task and has students read through it individually
- 7:22 – 7:45
  The teacher tells students to read through the assignment again with a partner and underline words that they "might need to know the meaning of"

- 8:59 – 10:13
  Students share words that they underlined. Teacher writes them down, and also points out cues that signal the words importance (it must be important since it occurs more than once, the actual heading says polygon)
- 10:15 – 11:15
  Students develop a definition of polygon. Teacher uses their initial incomplete definitions to create non-examples, which then prompts students to modify their definition.
- 11:15 – 13:10
  Teacher and students discuss the meaning of exterior angles, with student reading from paper, and teacher drawing exterior angles on the board.

Facilitator Notes:

Learning and Content Issues:

One challenging aspect of mathematics is the specific nature of mathematical communication, including representations, mathematical language and argumentation. This is especially relevant in the realm of geometric proof and reasoning, which involves precise, agreed upon definitions, diagrams that may be specific representations of general relationships that may not be to scale, and relational language that is central to deductive reasoning. Students often struggle to make sense of this language, which can make it difficult for them to make sense of problems. Inability to understand or use precise language, to describe specific mathematical and logical relationships can become a major barrier to reasoning, and later to proving and making sense of others' proofs.

There are three specific areas in which these issues play out in this video clip:

Definitions and vocabulary: Mathematical problems and proofs begin with, and depend on, precise and agreed upon definitions for terms and mathematical objects. Many students, in describing mathematical objects, use less formal and imprecise language. A major challenge for teachers is to access and build upon this imprecise language, but to also push for greater precision.

Conjectures: In mathematics conjectures are statements that we have reason to believe are true, but we have not established through deduction from a set of given assumptions. Conjectures are the things that mathematicians prove. Conjectures also describe relationships between mathematical objects, often taking the form of generalizations. In looking for and creating conjectures students do two things, they attend to relationships and patterns, and they work to describe those patterns and relationships in specific, precise, and hierarchical ways. If students are not able to understand the if-then structure of conjectures, then proof is an empty ritual. Without an ability to make and understand conjectures, students simply do not understand what it is that they are proving.

Reading: Often mathematical ideas are introduced in the context of written problems or situations. Students need to learn how to make sense of dense mathematical text. This requires them to identify important language, make sense of diagrams, and make connections between the two.

Video analysis:
This video shows a teacher using several moves that support students in making sense of the language demands of geometry, and in developing precise language in the context of creating definitions and conjectures.

Modeling:
In sharing examples of student conjectures at the beginning of class, the teacher highlights examples of statements that describe specific relationships using precise vocabulary. He also points out specific elements of their work (diagrams, labeled angle measurements) that will be important elements of the upcoming activity. He is making explicit aspects of communication that are specific to mathematics.

Metacognition around mathematical reading:
The teacher explicitly asks students to read through the problem twice, and the second time he has them identify important words, as well as think about their definitions. This accomplishes two things. It cues students to pay attention and look for important terms. Indeed, he makes explicit ways that readers can begin to discern important words by noticing repetition or their placement. He also pushes students to think about what those terms mean, and the relationships that they imply.

Building off a student definition to push for precision:
When a student begins to define a polygon, the teacher responds by drawing a figure that meets the students' imprecise definition, but is clearly not a polygon. This results in ever more precise definitions that exclude his non-examples.

Making explicit connections between words and pictures
The teacher asks students to identify the specific place where exterior angles are defined in the assignment. He then creates an exterior angle by mirroring the language of the definition. This not only models the connection between language and visual representation, but provides access to the content of the lesson. Students need to understand what exterior angles are before they can engage in the mathematics of the problem.

Suggested Sequence of Activities:
Doing the activity:
Begin by having participants do the activity themselves, ideally, with a partner using geometer's sketchpad, or geogebra. It is essential that participants write their own conjecture.

(The task itself can be seen by going to www.timssvideo.com/29 clicking on the "Resources" tab, and then clicking on the AU1 LessonGraph.pdf. The task is about halfway down the document in the section entitled "Private Class Work: Students Work in Groups of Two or Three on Investigation.")

After participants have finished, share their conjectures, and have them notice the different language that they use to describe the relationship that they see. Also take note of constraints (i.e. do participants explicitly limit the conjecture to convex polygons?) You may want to have participants give each other feedback on their conjectures, perhaps by having groups create posters with their conjectures, and then during a gallery walk, have other groups give feedback by putting post-its on the back of posters. Groups can then revise their conjectures to make them more precise.

Debrief the math
With participants, identify the important mathematics in the activity. Certainly, the theorem that external angles of a convex polygon sum to 360 degrees should come out, as well as angles, and shapes in general.

Debrief the practices
One suggestion is to give participants the eight mathematical practices, and ask them which ones figured most highly in their experience of doing the task. If participants do not mention it, make especial mention of attending to precision Practice 6), and the role of conjectures in argumentation and proof (Practice 3).

Anticipate student thinking
Ask participants to anticipate what difficulties students might have in doing this task. Brainstorm student difficulties, and make a list. Make sure that you mention students having difficulty with reading and making sense of the problem and with writing conjectures if participants do not anticipate these difficulties. You may want to ask participants how they imagine supporting students in making sense of the problem.

View the video:
You may want to view the video in stages. Here are some possible lenses/questions you may want to ask participants to think about while viewing:

- What specific moves is the teacher making to support students in making sense of the problem?
- What specific moves is the teacher making to support students in developing precise language?
- What specific moves is the teacher making to support students in noticing and describing mathematical relationships?

Discuss moves:

Have participants identify specific moves, and give evidence that they were effective.

Plan to use moves:
Ask participants to imagine using one or two of the moves from the video. Create a plan for using those moves in the context of an upcoming lesson.

Resources:
Readings

Driscoll, M. (2010). Geometry and proof in secondary school classrooms. In J. Lobato (Ed.), *Teaching and learning mathematics: Translating research for secondary school teachers,* Reston VA, NCTM (pp. 21-26).

- This reading describes research on proof and challenges students face in learning proof. This touches on some of the issues connected to making conjectures, precision of language and the importance of clear definitions, and the need for problem based learning to support students in meeting these challenges

Moschkovich, J. (2010). Language, culture and equity in secondary school mathematics classrooms. In J. Lobato (Ed.), *Teaching and learning mathematics: Translating research for secondary school teachers,* Reston VA, NCTM (pp. 75-80).

- This reading describes research on language and how it provides scaffolds and challenges for students in secondary classrooms. In particular, there is a section on the mathematical register, and how it is similar to, and different from, language in other disciplines. This connects to many of the moves that the teacher makes to highlight specific mathematical vocabulary, as well as mathematical modes of discourse and communication.

Transcript, Map of lesson (descriptions and times for different portions of the lesson), Researcher and teacher comments, all available for download at:
http://www.timssvideo.com/29

Bonus Feature: Creating Papers with embedded links to TIMSS videos
Your students can create a paper on the TIMSS site, and embed links to specific TIMSS videos within the paper. So, for instance, you can ask your participants to answer a question about building vocabulary, and have them respond by creating a

paper on the TIMSS site, that has a link to a specific section of this video in the paper itself.

Accessing the TIMSS Videos and Creating a Paper with Video Links

Go to the site http://www.timssvideo.com and create an account. If you already have an account log in.

To View Videos:

- Click on the link at the top, center of the page that says MATHEMATICS.
- Click on one of the countries listed under "Mathematics Lessons" on the right of the page.
- You should now see four or five different video lessons, each with a still shot, a title and a short description of the lesson. To view one of the lessons click on the picture or the title.
- This brings you to screen with the video on the left side. On the right side is an overview, a transcript of the video itself, and "resources" which include any worksheets that the students work on in the lesson, commentary by the teacher teaching the lesson, commentary by a researcher on the lesson, a Lesson Graph that gives a summary of what went on in the lesson with times for each activity.
- To watch the video hit the play button.

To Create a Paper

- Click on the link on the top right that says "MY PAPERS"
- For complete instructions, click on the words "click here" at the end of the third line of the regular text.
- Click on "Create Paper"
- To begin, type in the text box that appears on the right of your screen. To insert a video link, choose a video from the pull-down menu on the left. You can select a starting and ending time, and then insert the link into your paper.
- To save your paper, you must give it a title (in the box at the top of the page)
- Your paper is now saved, and it will appear on the "my papers" page.
- To share it, copy the url under the title and send that to someone in an e-mail