Uncertainties

or, the concept formerly known as Errors

No matter how good our experiment or measurement is, there will <u>always</u> be some uncertainty in the final answer. Why?

- Things change all the time for all sorts of reasons, *e.g.* temperature fluctuations make the size of an object change very slightly due to expansion and contraction.
- Measuring instruments have divisions. If your measurement is between divisions (*e.g.* the length of an object is between the 178 and 179 mm markings on a rule) you can't know exactly what the correct value is.
- In fact, no measurement is <u>ever</u> going to be precisely on a scale division to an infinite degree of accuracy.
- Devices with digital displays might give the impression of being perfectly accurate, but they can't be. If a digital Voltmeter reads '2.94 V', all it means is that the pd is somewhere between 2.935 V and 2.945 V.
- In fact, how can you even be *that* sure? Measuring instruments are calibrated in the factory to make sure that when it says 2.94 V it's not 3.16 V, for example. But over time, their calibration can drift due to all sorts of reasons. It is practically unheard of for a school (or many other places, for that matter) to send their meters back to the factory every so often for re-calibration, not least because it's usually expensive. This increases your uncertainty even more.

So, we can't stop results being uncertain, but we can quote the uncertainty when we give a measurement. This means that people will know how certain the result is.

> *n.b.* Uncertainties are sometimes called *errors*, especially in older textbooks. <u>Uncertainty</u> is a better term because error implies you've done something wrong, whereas uncertainties are unavoidable.

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1. Absolute uncertainty

e.g. We can measure a piece of paper with a ruler and say that:

length = 29.70 ± 0.05 cm.

This means the length of the paper is somewhere between 29.65 cm and 29.75 cm. Notice that the length is quoted up to and including the first uncertain significant figure (*i.e.* 29.7<u>0</u>, not just 29.7).

There are two ways of calculating the absolute uncertainty, depending on how you did your measurements:

a. If you are just taking **one** measurement, use this method:

$$\text{Measurement} \pm \frac{\text{smallest division on your instrument}}{2}$$

The uncertainty obviously depends on the measuring instrument used and how you were using it. If you were using a normal metre rule, you could only measure to the nearest millimetre (at best), so your uncertainty would be no less than ± <u>half</u> a millimetre*.

b. If you have taken **several** measurements you can use this method†:

$$\text{Average Measurement} \pm \frac{\text{range of measurements}}{2}$$

The range is simply ((highest value) – (lowest value)).

e.g. Measuring the length of a piece of string 5 times gives 5 different values: 9.5 mm, 9.6 mm, 9.6 mm, 8.8mm, 9.3 mm, 9.7 mm

Average = 9.54 mm‡
Range = 9.7 – 9.3 = 0.4 mm
Uncertainty = 0.4 ÷ 2 = 0.2 mm
Length of string = 9.5 ± 0.2 mm

* It could easily be ± *more* than half a millimetre, depending on what you were measuring and how you were measuring it. For example, measuring the original length of a rubber band would be quite tricky for a number of reasons, so the experimenter might increase the stated uncertainty to take account of this.

† This is a rather pessimistic method. The method used more often beyond A-level is to quote the standard deviation of the list of readings as the uncertainty (since, unless you've done something really silly in the experiment, the true value is most likely to lie ±1 standard deviations from the mean).

‡ Note that the average and range calculations do <u>not</u> include the obvious outlier/anomalous result of 8.8mm.

2. Percentage (or *Relative*) uncertainty

We can also give the uncertainty as a percentage of the quoted value. Once you know the absolute uncertainty, this is easy.

$$\text{Percentage Uncertainty} = \frac{\text{Absolute uncertainty}}{\text{Measurement}} \times 100\%$$

e.g. (with data from previous page)

$$\frac{0.05}{29.7} \times 100 = 0.17\%$$

So you can say **Length of paper = 29.70 cm ± 0.17%**

How to present your final answer

1) Quote the main value up to and including the first uncertain significant figure **but no more**.
2) Quote either the Absolute or Percentage uncertainty to no more than two significant figures – it's usually pointless to do so.
3) Only give the units once – either at the end of the whole statement if you're quoting the absolute uncertainty, or after the main reading if you're quoting the percentage uncertainty.

Good examples of presentation

- 2.340 ± 0.004 s
- 9 800 ± 200 J
- 0.6700 ± 0.0002 V
- 17.00 W ± 0.5%

Poor examples of presentation

Final answer	Correctly presented?	Why?
Length = 15 m ± 0.1 m	No	Need to quote up to and including first uncertain sf. Should be **15.0 ± 0.1 m**.
Length = 15.70 m ± 1 cm	Not ideal	Mixing units (m on main reading, cm on uncertainty) is unhelpful to the reader. **15.70 ± 0.01 m** is better.
Charge = $1.5 \times 10^{-19} \pm 2 \times 10^{-20}$ C	Not ideal	Mixing powers of ten (10^{-19} on main reading but 10^{-20} on uncertainty) is unhelpful to the reader. **$1.5 \pm 0.2 \times 10^{-19}$ C** is better.

Compound Uncertainties[§]

Very often in Physics, the final quantity is found by multiplying or dividing other quantities together:

e.g. Calculate a force using $F = ma$. Your data is:
m = 5.00 ± 0.02 kg **a = 10.0 ± 0.1 ms^{-2}**

Answer: $F = ma = 5 \times 10 = 50$ N

But how do you work out the uncertainty in the final answer of 50N? Could it be 49 or 51 N? Could it be 47 or 53 N? There are two ways you can arrive at the same result:

1. Maximum-minimum Method

This can be time-consuming, but difficult to get confused by. In our example, you need to calculate the maximum and minimum possible values of F:

Maximum: $F = 5.02 \times 10.1 =$ **50.70 N** (*i.e.* 0.70 N above 'best' answer)
Minimum: $F = 4.98 \times 9.9 =$ **49.30 N** (*i.e.* 0.70 N below 'best' answer)

You therefore quote your final result as **F = 50.0 ± 0.7 N**[**]

2. Adding Percentage Uncertainties

This is often quicker, but can be easier to lose track of what you're doing and get it wrong. Taking the same $F = ma$ example, if you work out the percentage errors in m and a and add them, you will get the same result as the maximum-minimum method gives:

% uncertainty in $m = \frac{0.02}{5} = 0.4\%$
% uncertainty in $a = \frac{0.1}{10} = 1\%$
Add the percentage uncertainties: 0.4 + 1 = 1.4%

You can then say either: **F = 50.0 N ± 1.4%**
or work out what 1.4% of 50 is and say: **F = 50.0 ± 0.7 N**[††]

Method 2 is normally preferred. Not only do percentage uncertainties allow you to make a fairer comparison between relative sources of error (*e.g.* above, the uncertainty in a (1%) has the bigger effect on the final uncertainty, so this is the one you would be well advised to try to minimize), but when you have more complex formulae than $F = ma$ to

[§] These as referred to as "propagated uncertainties" in IB
[**] Again, notice that the answer is given as 50.0 N, not 50 N. You should always quote your final answer up to and including the first uncertain significant figure.
[††] Note that this is the same answer as method 1 gave, which shows that the two methods are basically equivalent.

contend with, adding the percentage uncertainties is far easier than working out maximum and minimum values.

Dividing quantities

Treat uncertainties the same as when multiplying, *i.e.* add the percentage uncertainties:

$v = \frac{d}{t}$ with d = 6 m ± **5%** and t = 3 s ± **2%**, so **v = 2.0 m s^{-1} ± 7%**

Adding and subtracting values

If you are adding or subtracting two uncertain values, you have to add the absolute uncertainties:

C = a + b

a = 17.2 ± 0.1 J b = 64.5 ± 0.5 J

C = 81.7 ± 0.6 J

And:

Q = r - s

r = 34.0 ± 0.1 C s = 13.0 ± 0.3 C

Q = 21.0 ± 0.4 C

Powers

If you are raising something by a power, you multiply the percentage uncertainty by the power.

$A = \pi r^2$

r = 23 m ± 3%

A = 1700 m^2 ± 6 %[‡‡]

Remember that $\sqrt{x}$ is the same as $x^{\frac{1}{2}}$:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

m = 0.50 kg ± 0.005kg (*i.e.* 0.1%) and k = 15 Nm^{-1} ± 1 Nm^{-1} (*i.e.* 6.7%)

Best answer = 1.15s

Total uncertainty[§§] = ½ × (0.1 + 6.7) = 3.4%

Final answer: **T = 1.15 s ± 3.4%** or **T = 1.15 ± 0.04 s**

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‡‡ Note that a calculator gives $\pi(23)^2$ = 1661.9…. However, you cannot quote an answer to more sig figs than are uncertain, hence 1661.9… has to be rounded off to the nearest 100m (since 6% of 1662 is about 100m).

§§ **Add** the % uncertainties because the quantities are **divided** in the equation, and **halve** them because of the **square** root.

Practice questions

1) Usain Bolt runs 100m ± 2cm in 9.63 ± 0.005 s. Calculate his average speed and its absolute uncertainty.

2) Richard Feynman stands on a set of bathroom scales. They read 83 kg. His daughter, Michelle, runs into the bathroom and he picks her up. The scales now read 105 kg. If the scales have an uncertainty of 0.5 kg, calculate Michelle's mass and its absolute uncertainty.

3) Newton's constant of gravitation (abbreviated to *G*) is a fundamental constant which describes how gravity behaves. Our current (2010) best value is

$$G = 6.67384 \pm 0.0008 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}.$$

Our current best value for the charge on an electron is:

$$e = 1.602\ 176\ 565 \pm 0.000\ 000\ 035 \times 10^{-19}\ \text{Coulombs}.$$

Which of these constants would you describe as the most accurately known? Use calculations to justify your reasons.

4) A packet of printer paper contains 500 sheets of paper. Its height is 52 ± 0.5 mm, and its mass is 2.521 ± 0.001 kg. Calculate:

a) The thickness of one piece of the paper, and its absolute uncertainty.

b) The mass of one piece of paper, in grammes, and its percentage uncertainty.

c) The packet says each sheet of paper measures 210 mm by 297 mm. If both of these measurements have an accuracy of $\pm$ 0.5 mm, calculate the Volume of one sheet of paper (in mm^3) and its absolute uncertainty.

d) Using your answers to b) and c), calculate the density of the paper (in $g\ mm^{-3}$), and its percentage uncertainty.

(*slightly trickier*)

5) A vacuum bazooka fires a ball horizontally at a height of 1.115 ± 0.01 m above the floor. It strikes a board placed 9.50 ± 0.02 m away. It hits the board at a height of 0.710 ± 0.005 m above the floor. Using $g = 9.81$ ms^{-2} and ignoring air resistance, calculate:

a) The height it falls during its flight, and its absolute uncertainty.

b) The time it takes to hit the board from leaving the gun, and its percentage uncertainty.

c) Its muzzle velocity (*i.e.* the speed the ball leaves the gun at), and its absolute uncertainty.