Random Walks and $\sqrt{N}$

The Brownian motion you have seen through a microscope can be simply modelled as a 1-D 'random walk'. There is a simple rule to describe how far you typically get from the origin (your root-mean-square displacement) after N moves of such a walk, which is:

$$d_{rms} \propto \sqrt{N}$$

This is not the same as saying that the particle's mean displacement must be zero: an RMS value is not the same as a mean value. If you want more detail, there is an extensive proof here: galileo.phys.virginia.edu/classes/152.mf1i.spring02/RandomWalk.htm.

But how accurate is this rule in practice?

Experiment

1) **Equipment:** One unbiased coin, one counter, one walkway (PTO)

2) **Method**
   a) Place the counter at the origin of the walkway
   b) Toss the coin. If heads, move the counter one place to the right; if tails, move one to the left.
   c) Repeat for 20 coin tosses (*i.e.* $N = 20$)
   d) Record the final displacement (d) of the counter from the origin after the 20 moves (*e.g.* if it ends one place to the left of the origin, $d = -1$).
   e) Repeat steps a) – d) a number of times until you have a few (probably different) values of d after 20 moves
   f) Record some other people's results and calculate the rms value of d for the whole class's results. According to theory, this should equal $\sqrt{20} \approx 4.5$.
   g) If you have time, repeat for other values of N and see if $d_{rms} = \sqrt{N}$ each time.

3) **Results**
   a) My results for $N = 20$

Trial number	**Final displacement of counter after 20 moves (d)**
1	
2	
3	
4	
5	
RMS value	

   b) Other results for $N = 20$

Person	d_{rms}
Whole class average d_{rms}	

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10