


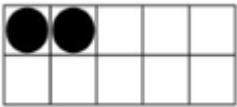
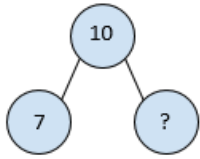


Benchmarks with Remarks/Examples

Kindergarten

Algebraic Reasoning	
<i>MA.K.AR.1 Represent and solve addition and subtraction problems within 10.</i>	
MA.K.AR.1.1	Represent addition and subtraction within 10 in multiple ways using objects, fingers, drawings, verbal explanations and equations.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed conceptual understanding by first using manipulatives or drawings then transitioning to equations. Students should understand addition as adding to and putting together and subtraction as taking apart and taking from.</p> <p><i>Remark 2:</i> Students should be able to represent one problem in multiple ways and understand how the different representations are related to each other. For instance, $3 + 4 = 7$ and $7 - 4 = 3$ can be represented by:</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>objects</p>  </div> <div style="text-align: center;"> <p>fingers</p>  </div> <div style="text-align: center;"> <p>drawing</p>  </div> <div style="text-align: center;"> <p>verbal explanation</p> <p>"I put together 3 and 4 to make 7."</p> </div> </div> <p><i>Remark 3:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations. Students are not required to independently read word problems</p>	
MA.K.AR.1.2	Solve addition and subtraction word problems within 10 using objects, drawings or equations to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should focus on understand the problem presented in order to determine if the context requires addition or subtraction. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose a specific operation or a specific strategy.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations. Students are not required to independently read the word problems.</p>	
MA.K.AR.1.3	Add and subtract within 10 using a variety of strategies.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should use a variety of strategies to add and subtract. These strategies include but are not limited to counting all, counting up or on, counting back and number bonds.</p> <p><i>Example 1:</i> $4 + 5$ <i>Example 2:</i> $7 - 3$</p>	
MA.K.AR.1.4	For any number from 1 to 9, find the number that makes 10 when added to the given number.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations. Students are not required to independently read word problems.</p> <p><i>Example 1:</i> I have 2. How many do I need to make 10?</p> <div style="display: flex; align-items: center;">  </div> <p><i>Example 2:</i> What number can I use to complete the number bond to make 10?</p>	



Example 3: $10 = 4 + ?$

MA.K.AR.1.5	Find all possible pairs of addends for sums within 10.
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Remarks/Examples:

Remark 1: Teachers should ensure students are exposed to multiple ways to add numbers to make a given number. For instance, $1 + 3$, $2 + 2$ and $3 + 1$ are all ways to make 4.

Remark 2: Problems may include both addends unknown. For examples of word problems, refer to the [Common Addition and Subtraction Situations](#). Students are not required to independently read word problems.

Example 1: There are 5 flowers to put in two vases. How many different ways can you put flowers in the red vase or the blue vase?

Example 2: What numbers can be added together to make 5?

MA.K.AR.2 Identify and extend shape and color patterns.

MA.K.AR.2.1	Identify and extend patterns consisting of shapes and colors.
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Remarks/Examples:

Remark 1: Students should be able to identify and extend the patterns present in the world around them.

Example 1: Look at the pattern below. What shape would come next?



Example 2: Is this a pattern? How do you know?



Example 3: Can you color the triangles to make a pattern?



Number Sense and Operations

MA.K.NSO.1 Count and compare groups of objects within 20.

MA.K.NSO.1.1	Count and represent a group of objects with a written numeral 0 to 20.
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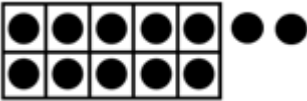










Remarks/Examples:

Remark 1: Students should be able to count objects and pictures arranged in a line, rectangular array, circle or scatter. They should also be able to answer questions about how many objects were in the group that was counted by writing the corresponding number.

Example 1: Given a group of objects, count to tell how many. Represent the count with a written numeral.

MA.K.NSO.1.2	Given a number from 0 to 20, count out that many objects. State the number of objects counted, and if the counted set is moved or rearranged, restate the number of objects in that set without having to recount.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students are expected to have an understanding of one-to-one correspondence by pairing each object with one number name and each number name with one object. <i>Remark 2:</i> Students should understand that the last number counted tells the name of the set that was counted. Students should understand that changing the arrangement of the set does not change the amount in the set. <i>Example 1:</i> Given the number 15, count out that many colored tiles. State the number counted.	
MA.K.NSO.1.3	Compare sets of objects from 0 to 20 using the terms less than, equal to or greater than.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students are not expected to use the relational symbols > or <. <i>Example 1:</i> Given 12 counters and 10 colored tiles, compare the two groups of objects. Which group has a greater amount?	

<i>MA.K.NSO.2 Count within 100 and build a foundation for place value.</i>	
MA.K.NSO.2.1	Count to 100 by ones and by tens.
<u>Remarks/Examples:</u> <i>Remark 1:</i> When counting forward by ones, students should say the number names in the standard order and understand that each successive number refers to a quantity that is one larger. When counting forward by tens, students should understand that each successive number refers to a quantity that is ten larger. <i>Remark 2:</i> Students should be verbally counting to 100 by ones and by tens. Students are not required to count to 100 by writing the numbers. <i>Example 1:</i> Count to 100 by ones. Then count to 100 by tens.	
MA.K.NSO.2.2	Starting at a given number, count forward within 100 and backwards within 20.
<u>Remarks/Examples:</u> <i>Remark 1:</i> When counting forward by ones, students should say the number names in the standard order and understand that each successive number refers to a quantity that is one larger. When counting backwards, students should understand that each successive number refers to a quantity that is one less. <i>Remark 2:</i> Students should be verbally counting to 100. Students are not required to count to 100 by writing the numbers. <i>Example 1:</i> Starting at 43, count forward to 100. <i>Example 2:</i> Starting at 15, count backward to 0.	
MA.K.NSO.2.3	Read and write numbers from 0 to 20.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to read written numerals up to 20 and write the numbers 0 to 20. <i>Example 1:</i> Given the number 12, state the number and represent the number with a written numeral.	

MA.K.NSO.2.4	Compose and decompose numbers from 11 to 19 with a group of ten ones and additional ones. Demonstrate each composition or decomposition with objects, drawings or equations.						
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should understand that the numbers 11 to 19 are composed of ten ones and one, two, three, four, five, six, seven, eight or nine additional ones. Teachers should emphasize that the “1” in a teen number means “one ten”.</p> <p><i>Example 1:</i> Show me the number 12.</p> <p><i>Possible Student Response:</i> The student used a ten frame to show the number 12.</p>  <p><i>Example 2:</i> We want to make the number 15. We have 10 circles. How many more do we need?</p> <p><i>Possible Student Response:</i> The student drew 10 circles and 5 more circles to determine 10 plus 5 equals 15.</p>  <p><i>Example 3:</i> Can you tell me what two groups to put together to make 11?</p> <p><i>Possible Student Response:</i> The student put 10 cubes in the 10 column and 1 cube in the other column to show 11 is 10 plus 1 more.</p> <table border="1" data-bbox="506 915 963 1056"> <thead> <tr> <th>10 cubes</th><th>Other cubes</th></tr> </thead> <tbody> <tr> <td></td><td></td></tr> <tr> <td></td><td></td></tr> </tbody> </table> <p>$11 = 10 + 1$</p>		10 cubes	Other cubes				
10 cubes	Other cubes						
							
							
MA.K.NSO.2.5	Compare numbers from 0 to 20 using the terms less than, equal to or greater than.						
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students are not expected to use the relational symbols > and <.</p> <p><i>Example 1:</i> Compare the numbers 11 and 5. Which one is greater?</p> <p><i>Example 2:</i> Are 5 and 15 equal? Why or why not?</p>							

Measurement	
<i>MA.K.M.1 Describe and compare the measurable attributes of objects.</i>	
MA.K.M.1.1	Describe measurable attributes of a single object such as length, capacity or weight.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be able to describe the measurable attributes, such as the length, capacity and/or weight of an object without measuring.</p> <p><i>Example 1:</i> Think about a car. What parts of a car could be measured?</p> <p><i>Example 2:</i> How can we measure a pencil?</p>	
MA.K.M.1.2	Directly compare two objects with a measurable attribute in common. Express the comparison using language to describe the difference.

Remarks/Examples:

Remark 1: Direct comparison means that one object is compared to another based on a measurable attribute that the objects share without actually measuring. To directly compare length, objects are placed next to each other with one end of each object lined up to determine which one is longer.

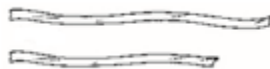
Remark 2: Examples of measurable attributes include length, capacity and weight.

Remark 3: Students should use language to compare the measurable attributes of objects. Language to compare length includes but is not limited to short, shorter, long, longer, tall, taller, high or higher. Language to compare capacity includes but is not limited to has more, has less, holds more, holds less, large, larger, small, smaller, more full, less full, full, empty, takes up more space or takes up less space. Language to compare weight includes but is not limited to heavy, heavier, light, lighter, weighs more or weighs less.

Example 1: Have one bag filled with cotton balls and another filled with rice. Do the bags weigh the same? Is one bag heavier than the other?

Example 2: Compare the lengths of the two pieces of string.

Possible Student Response: The student placed the pieces of string side-by-side and stated that one string was longer than the other.



MA.K.M.1.3

Express the length of an object, up to 20 units long, as a whole number of lengths by laying non-standard objects end to end with no gaps or overlaps.

Remarks/Examples:

Remark 1: Non-standard units of measurement are units that are not typically used, such as paper clips or colored tiles. To measure with non-standard units, students should lay multiple copies of the same object end to end with no gaps or overlaps. The length is shown by the number of objects needed.

Example 1: Can you use paper clips to measure the length of the pencil?

Possible Student Response: The student placed multiple paper clips end to end below the pencil to determine the pencil was five paper clips long.



Example 2: Will you measure the length of the rectangle using the colored tiles?



Possible Student Response: The student placed colored tiles on top of the rectangle to measure the length. The student stated that the rectangle was 11 colored tiles long.



Geometric Reasoning

MA.K.GR.1 Build a foundation for identifying and analyzing two-dimensional shapes.

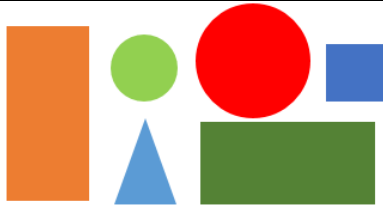
MA.K.GR.1.1

Identify two-dimensional shapes regardless of their size or orientation. Shapes are limited to circles, triangles, rectangles and squares.

Remarks/Examples:

Remark 1: Students should be shown shapes in a variety of sizes and orientations. The shapes should not be limited to those present in pattern blocks or attribute blocks.

Example 1: Which shape is a triangle? How do you know?



MA.K.GR.1.2

Compare and sort two-dimensional shapes based on their similarities and differences. Shapes are limited to circles, triangles, rectangles and squares.

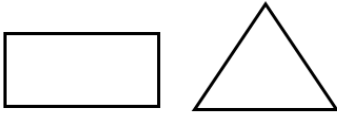
Remarks/Examples:

Remark 1: Shapes can be sorted in a variety of ways, including but not limited to the type of shape, color, size or orientation.

Remark 2: Students should be shown shapes in a variety of sizes and orientations. The shapes should not be limited to those present in pattern blocks or attribute blocks.

Remark 3: Students should use informal language to describe the similarities or differences between shapes when comparing and sorting. For instance, students may say a group of shapes are triangles because they have 3 sides and another group are rectangles because they have four sides.

Example 1: How are these shapes alike? How are they different?



Example 2: Provide students with various examples of circles, triangles, rectangles and squares. Have the students sort them in different ways.

MA.K.GR.1.3

Combine two-dimensional shapes to form a composite figure. Shapes are limited to circles, triangles, rectangles and squares.

Remarks/Examples:

Remark 1: Composite figures are figures that can be divided into more than one figure. Composite figures are not limited to the shapes listed within the benchmark.

Remark 2: Students should have an opportunity to investigate combining shapes in a variety of sizes and orientations. The shapes should not be limited to those present in pattern blocks or attribute blocks.

Example 1: Can you combine these two shapes to make something new?



Possible Student Response: Students may chose to create a representation of a house using a square and a triangle.



Example 2: Can you make a rectangle using these two squares?



Possible Student Response: The student placed the two squares next to each other to create a rectangle.




MA.K.GR.2 Build a foundation for identifying and analyzing three-dimensional figures.

MA.K.GR.2.1	Identify three-dimensional figures regardless of their size or orientation. Figures are limited to spheres, cubes, cones and cylinders.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be shown three-dimensional shapes in a variety of sizes and orientations, including those that are commercially made and real-world examples, such as cereal boxes as rectangular prisms. <i>Example 1:</i> Given a cylinder, identify the shape.	
MA.K.GR.2.2	Compare and sort three-dimensional figures based on their similarities and differences. Figures are limited to spheres, cubes, cones and cylinders.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should use informal language to describe the similarities or differences between shapes when comparing and sorting. For instance, students may say a group of shapes are cubes because they are made up of squares and another group are spheres because they have no corners and are round. <i>Remark 2:</i> Students should be shown three-dimensional shapes in a variety of sizes and orientations, including those that are commercially made and real-world examples, such as cereal boxes as rectangular prisms. <i>Example 1:</i> Show the students a cylinder and a cone. How are these shapes alike? How are they different? <i>Example 2:</i> Provide students with various examples of spheres, cubes, cones and cylinders. Have the students sort them in different ways.	
MA.K.GR.2.3	Combine three-dimensional figures to form a composite figure. Figures are limited to spheres, cubes, cones and cylinders.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Composite figures are figures that can be divided into more than one figure. Composite figures are not limited to the shapes listed within the benchmark. <i>Remark 2:</i> Students should have an opportunity to investigate combining three-dimensional shapes in a variety of sizes and orientations, including those that are commercially made and real-world examples, such as cereal boxes as rectangular prisms. <i>Example 1:</i> Provide students with a cube and a cone. Can you combine these two shapes to make something new?	

Grade 1

Algebraic Reasoning	
<i>MA.1.AR.1 Solve addition and subtraction problems within 20.</i>	
MA.1.AR.1.1	Solve addition and subtraction word problems within 20 using objects, drawings or equations to represent the problem.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should focus on understanding the problem presented in order to determine if the context requires addition or subtraction. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose a specific operation or a specific strategy. <i>Remark 2:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations .	
MA.1.AR.1.2	Add and subtract within 20 using a variety of strategies.
<u>Remarks/Examples:</u> <i>Remark 1:</i> When adding or subtracting, students should use a variety of strategies that are efficient and generalizable. Students should be able to choose a strategy that is suited to the problem. These strategies include but are not limited to counting on, counting back, doubles, doubles plus 1, number lines and making 10. <i>Example 1:</i> What is the difference of 16 and 7? What strategy did you use to subtract?	
<i>MA.1.AR.2 Apply properties of operations and the relationship between addition and subtraction.</i>	
MA.1.AR.2.1	Apply the Commutative Property of Addition and the Associative Property of Addition as strategies to add within 20.
<u>Remarks/Examples:</u> <i>Remark 1:</i> The Commutative Property of Addition states that numbers can be added in any order and the result will remain the same. $2 + 3 = 5$ and $3 + 2 = 5$ <i>Remark 2:</i> The Associative Property of Addition states that numbers can be grouped in any way prior to adding and the result will remain the same. $(2 + 3) + 1 = 6$ and $2 + (3 + 1) = 6$ <i>Remark 3:</i> Students are not required to know the names of the properties but should be able to apply them. <i>Remark 4:</i> Students are not required to utilize parentheses when working with the Commutative or Associative Properties of Addition. <i>Example 1:</i> Add $4 + 8 + 6$. <i>Possible Student Response:</i> The student added 4 and 6 to make 10, then added 8 to the 10. <i>Example 2:</i> $7 + 4$ <i>Possible Student Response:</i> The student knew 4 plus 7 equaled 11, so the student determined 7 plus 4 also equaled 11.	
MA.1.AR.2.2	Use the inverse relationship between addition and subtraction to restate a subtraction problem within 20 as a missing addend problem.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Inverse operations, such as addition and subtraction, are the opposite of one another meaning they will undo each other to result in the initial number, $3 + 2 = 5$ and $5 - 2 = 3$. <i>Example 1:</i> What is an addition equation that relates to this subtraction equation? $17 - 5 = \underline{\hspace{1cm}}$	

<i>Example 2:</i> How can $14 - 9$ be restated as an addition problem?	
MA.1.AR.2.3	Determine and explain if equations involving addition and subtraction within 20 are true or false.
<u>Remarks/Examples:</u> <i>Remark 1:</i> When students are determining if equations are true or false, they should understand the meaning of the equal sign. The equal sign is a mathematical symbol to indicate equality showing that what is on the left of the sign is equal in value or amount to what is on the right. The equal sign is placed between two quantities or expressions to state they have the same value or represent the same value. $3 + 4 = 7$ 60 seconds = 1 minute 5 tens and 4 ones = 54 <i>Example 1:</i> Is this equation true or not true? How do you know? $2 + 5 = 3 + 4$ <i>Possible Student Response:</i> The student determined the equation was true because both sides equal 7. <i>Example 2:</i> Determine if the following are true or false. Can you make the false equations true? $8 - 5 = 3$ $8 = 8$ $6 + 5 = 10$ $9 = 5 - 4$	
MA.1.AR.2.4	Determine the unknown whole number in an addition or subtraction equation within 20 with the unknown in any position.
<u>Remarks/Examples:</u> <i>Remark 1:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations . <i>Example 1:</i> Find the unknown number that would make the equation true. What strategy did you use? $9 + \underline{\quad} = 12$ <i>Example 2:</i> Find the missing number in each of the equations below. $5 + 9 = ?$ $3 + ? = 14$ $? + 7 = 12$ $? = 16 - 5$ $18 - ? = 16$ $? - 12 = 4$	

<i>MA.1.AR.3 Identify and extend shape, color and number patterns.</i>	
MA.1.AR.3.1	Identify and extend patterns consisting of shapes, colors and numbers. Limit numeric patterns to adding 1 or skip counting by 2s, 5s or 10s.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to identify and extend the patterns present in the world around them. <i>Example 1:</i> What is the pattern? What number comes next in this pattern? 2, 4, 6, 8, 10, ____ <i>Example 2:</i> What shape is missing in the pattern? 	

Number Sense and Operations	
<i>MA.1.NSO.1 Extend counting sequences and understand the place value of two-digit numbers.</i>	
MA.1.NSO.1.1	Starting at a given number, count forward and backward within 120.
<u>Remarks/Examples:</u>	

Remark 1: When counting forward by ones, students should say the number names in the standard order and understand that each successive number refers to a quantity that is one larger. When counting backwards, students should understand that each successive number refers to a quantity that is one less.

Example 1: Starting at 45, count forward to 120.

Example 2: Starting at 37, count backward to 0.

MA.1.NSO.1.2 Skip count forward by 2s to 20 and by 5s to 100.

Remarks/Examples:

Remark 1: When counting forward by 2s, students should understand that each successive number refers to a quantity that is two larger. When counting forward by 5s, students should understand that each successive number refers to a quantity that is five larger.

Remark 2: Skip counting by 2's, 5's, and 10's will help students find multiples of 2, 5, and 10, which will build a foundation for multiplication in later grades.

Example 1: Skip count by 2s to 20.

Example 2: Skip count by 5s to 100.

MA.1.NSO.1.3 Read and write numbers within 120 using standard form and word form.

Remarks/Examples:

Remark 1: There are many ways to write a number. Standard form is a way to write numbers using numerals. Word form is a way to write numbers using words.

Standard form: 34 Word form: thirty-four

Example 1: What is seventy-five in standard form?

Example 2: Write 43 in word form.

MA.1.NSO.1.4 Compose and decompose two-digit numbers in multiple ways using tens and ones.
Demonstrate each composition or decomposition with objects, drawings or equations.

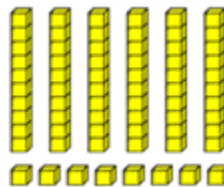
Remarks/Examples:

Remark 1: Students should be exposed to and encouraged to use multiple compositions and decompositions of two-digit numbers. The exact number of tens and ones in a two-digit number should be emphasized because this decomposition tells the place value by stating exactly how many tens and ones make up the number. This knowledge is fundamental for understanding the place value system.

Remark 2: In order to compose and decompose two-digit numbers in multiple ways, students should have an understanding of the place value system through the tens place. For instance, 47 can be thought of as 4 tens 7 ones or as 0 tens 47 ones.

Example 1: Show me the tens and ones in the number 68.

Possible Student Response: The student used base ten blocks to show 68.



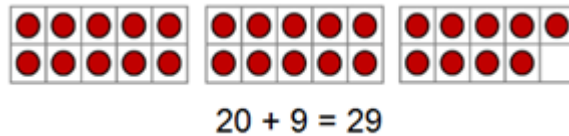
Example 2: Decompose 34 in three different ways.

Possible Student Response: The student showed three different ways to make 34.

$$\begin{array}{r}
 3 \text{ tens } 4 \text{ ones} \\
 \underline{\quad} \text{ tens } \underline{34} \text{ ones} \\
 2 \text{ tens } \underline{14} \text{ ones}
 \end{array}$$

Example 3: Represent $20 + 9$ and state the number.

Possible Student Response: The student used ten frames and an equation to represent $20 + 9$.



MA.1.NSO.1.5	Compare two two-digit numbers based on the values of the tens and ones digits using the terms less than, equal to or greater than and the symbols $<$, $=$, or $>$.
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Remarks/Examples:

Remark 1: Students should connect the terms less than, equal to or greater than to the corresponding relational symbols.

Example 1: Use $<$, $=$ or $>$ to compare the following numbers.

78 ____ 75 23 ____ 13 56 ____ 65

Example 2: Is 86 greater than 68?

MA.1.NSO.1.6	Identify the number that is one more, one less, ten more and ten less than a given two-digit number.
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Remarks/Examples:

Remark 1: Students should be able to use mental math to identify one more, one less, ten more and ten less than a number.

Example 1: What number is one more than 67?

Example 2: What number is ten less than 89?

<i>MA.1.NSO.2 Build a foundation for adding and subtracting two-digit numbers.</i>

MA.1.NSO.2.1	Add a two-digit number and a one-digit number within 100 using a variety of strategies.
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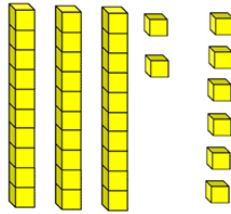
Remarks/Examples:

Remark 1: Students should combine ones with ones and tens with tens when adding two-digit numbers. They should also understand that sometimes it is necessary to compose ten ones into one ten. Students should have opportunities to act out the process of composing a ten when necessary.

Remark 2: Emphasis should be placed on conceptual understanding through the use of manipulatives, drawings and strategies based on place value.

Example 1: Add 6 to 32.

Possible Student Response: The student used base ten blocks to add 6 to 32. The student stated the sum was 38.



Example 2: $43 + 7$

Possible Student Response: The student modeled 43 and 7 by drawing lines to represent tens and circles to represent ones. Then the student found the total number by composing a new ten and counting the sum.



Example 3: What is the sum of $27 + 5$?

Possible Student Response: The student decomposed 27 as $20 + 7$. The student added 7 and 5 to get 12. Then added 20 to 12 to determine 32 was the sum.

$$20 + 7 + 5$$

$$20 + 12$$

$$32$$

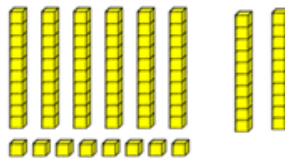
MA.1.NSO.2.2	Add a two-digit number and a multiple of 10 within 100 using a variety of strategies.
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Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives or drawings based on place value. Counting charts may also be a useful tool when students are adding multiples of ten.

Example 1: $68 + 20$

Possible Student Response: The student showed 68 and 20 with base ten blocks. Then the student counted the total to state the sum was 88.



Example 2: $23 + 20$

Possible Student Response: The student decomposed 23. Then added the tens to tens and then the ones.

$$20 + 3 + 20$$

$$20 + 20 + 3$$

$$40 + 3$$

Fractions

MA.1.FR.1 Build a foundation for fractions by partitioning shapes into halves and fourths.

MA.1.FR.1.1

Partition circles and rectangles into two and four equal-sized parts. Name the parts using appropriate language including halves or fourths. Describe the inverse relationship between the size of the parts and the number of parts.

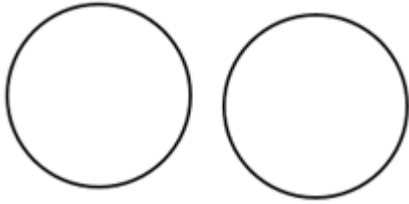
Remarks/Examples:

Remark 1: Sectioning a shape into equal-sized parts is called partitioning. Students should understand that when partitioning into two equal-sized parts halves are created and when partitioning into four equal-sized parts fourths are created.

Remark 2: There is an inverse relationship between the size of the parts and the number of parts in a fractional representation. In order to describe this relationship, students should understand the more number of parts, the smaller the parts and the less number of parts, the larger the parts.

Example 1: Divide the paper into two parts so that each part has an equal amount. How could you describe each part?

Example 2: Pretend each of these circles is a cake. The first one is for two people. Can you show how you would cut the cake for 2? The second cake is for 4 people. Can you show how you would cut the cake for 4?



Will you get a smaller piece when you cut the cake in half or fourths?

Measurement

MA.1.M.1 Compare and measure the length of objects.

MA.1.M.1.1	Compare and order the length of up to three objects using direct comparison.
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Remarks/Examples:

Remark 1: Direct comparison is a way to compare objects based on a measurable attribute that the objects share without actually measuring. To directly compare length, objects are placed next to each other with one end of each object lined up to determine which one is longer.

Example 1: Which of the pieces of yarn is the longest? How do you know?

Example 2: Can you place the three books in order from shortest to tallest?

MA.1.M.1.2	Estimate and measure the length of an object to the nearest inch or centimeter.
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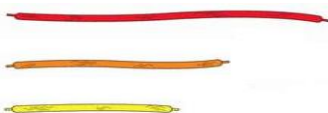
Remarks/Examples:

Remark 1: Students should recognize that a ruler is a tool that can be used to measure the attribute of length. They should understand that the length is the distance between the zero point and endpoint. Students should also recognize that rulers can be used to measure in two units of length, inches or centimeters. The markings on the ruler indicate the unit by marking equal distances with no gaps or overlaps.

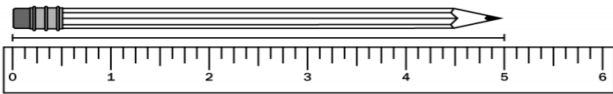
Remark 2: When estimating length, students should be able to give a reasonable amount of inches or centimeters for the length of a given object.

Remark 3: Teachers should provide students with an opportunity to investigate measuring the attribute of length by providing students with objects to measure and rulers.

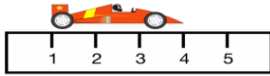
Example 1: Estimate in centimeters the length of the shoe laces. Then measure to find the actual length in centimeters.



Example 2: What is the length of the pencil measured to the nearest inch?



Example 3: Kyle was measuring the length of his toy car. Did Kyle measure his toy car correctly? If not, what error did he make?



MA.1.M.2 Tell time and identify the value of coins and combinations of coins and dollar bills.

MA.1.M.2.1 Tell and write time in hours and half-hours using analog and digital clocks.

Remarks/Examples:

Remark 1: Students are not required to understand Military time.

Example 1: What time is shown on the clock?



Example 2: Ben will get home from school at the time shown on the clock. What time will Ben be home from school?



MA.1.M.2.2 Identify pennies, nickels, dimes and quarters, and express their values using the ¢ symbol. State how many of each coin equal a dollar.

Remarks/Examples:

Remark 1: Students should be able to identify pennies, nickels, dimes and quarters regardless of their orientation and express the value of each coin. Students should also be able to state how many of each coin are needed to equal one dollar. For instance, when shown a nickel the student should be able to state it is a nickel, it is worth five cents, and it takes 20 nickels to make a dollar.

Remark 2: Students should understand the value of coins in cents. They are not required to use decimal values.

Example 1: What can you tell me about this coin?



Example 2: What do you know about a penny? How many pennies does it take to equal one dollar?

MA.1.M.2.3 Find the value of combinations of pennies and dimes up to one dollar, and the value of one and ten dollar bills up to \$100. Use the ¢ and \$ symbols appropriately.

Remarks/Examples:

Remark 1: The intent of the benchmark is to relate place value to the concept of money. Teachers should connect the combinations of pennies and dimes or one and ten dollar bills to place value.

Remark 2: Students should be able to compute the value of combinations of pennies and dimes or one and ten dollar bills by skip counting by tens to count the dimes or ten dollar bills and counting forward by ones to count the pennies or one dollar bills.

Remark 3: Students should understand the value of coins in cents and dollar bills in dollars. They are not required to use decimal values.

Example 1: There are three dimes and seven pennies on the table. What is the total value of the coins?

Example 2: What is the value of the coins shown?



Example 3: What is the value of the dollar bills shown?



Geometric Reasoning

MA.1.GR.1 Identify and analyze two-dimensional shapes based on their defining attributes.

MA.1.GR.1.1 Identify two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.

Remarks/Examples:

Remark 1: Students should use the defining attributes of number of sides, number of vertices and side lengths to identify two-dimensional shapes.

Remark 2: Students should be shown shapes in a variety of sizes and orientations. The shapes should not be limited to those present in pattern blocks or attribute blocks.

Example 1: Which of the shapes is a hexagon? How do you know?

Example 2: This is a rectangle. What makes this a rectangle?

MA.1.GR.1.2 Draw two-dimensional shapes when given defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.

Remarks/Examples:

Remark 1: Students should use the defining attributes of number of sides, number of vertices and side lengths to draw two-dimensional shapes.

Remark 2: Teachers may need to provide graph paper, grid paper or dot paper to assist students with drawing shapes.

Example 1: Can you draw a triangle? How do you know it is a triangle?

Example 2: Draw a shape with six sides and six vertices. What shape did you draw?

MA.1.GR.1.3 Compare and sort two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.

Remarks/Examples:

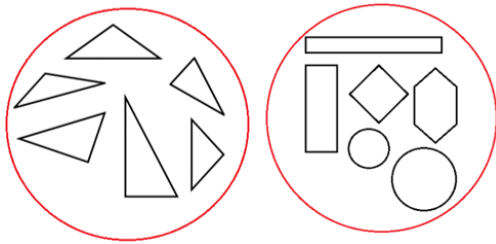
Remark 1: Students should use the defining attributes of number of sides, number of vertices and side lengths to compare and sort two-dimensional shapes.

Example 1: Compare the two shapes. Describe how they are similar and how they are different.



Example 2: Provide students with a mixture of two-dimensional shapes. Have them sort the shapes based on a defining attribute and explain their methods for sorting.

Example 3: Josey sorted her shapes into two groups, as shown below. How did she sort her shapes?

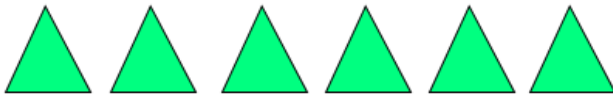


MA.1.GR.1.4	Combine two-dimensional shapes to form a composite shape. Decompose a composite shape into two-dimensional shapes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.
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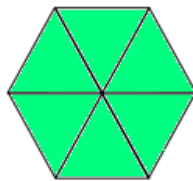
Remarks/Examples:

Remark 1: Composite figures are figures that can be divided into more than one figure. Composite figures are not limited to the shapes listed within the benchmark.

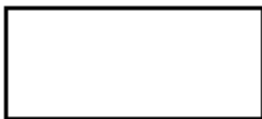
Example 1: Can you create a new shape from these triangles?



Possible Student Response: The student rearranges the triangles to create a hexagon.



Example 2: What shapes can you make from this rectangle?





Possible Student Response: The student notices that a rectangle can be decomposed into two triangles. The student cuts the rectangle into two triangles.



MA.1.GR.2 Identify and analyze three-dimensional figures based on their defining attributes.

MA.1.GR.2.1	Identify three-dimensional figures based on their defining attributes. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
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Remarks/Examples:

<p><i>Remark 1:</i> Students should use the defining attributes of number of faces, shape of faces, number of edges and number of vertices to identify three-dimensional shapes.</p> <p><i>Remark 2:</i> Students should be shown three-dimensional shapes in a variety of sizes and orientations, including those that are commercially made and real world, such as cereal boxes as rectangular prisms.</p> <p><i>Example 1:</i> Can you identify this shape? What is it? What makes this shape a rectangular prism?</p> <p><i>Example 2:</i> Show the student a cylinder. What shapes is this? How do you know?</p>	
MA.1.GR.2.2	Compare and sort three-dimensional figures based on their defining attributes. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should use the defining attributes of number of faces, shape of faces, number of edges and number of vertices to compare and sort three-dimensional shapes.</p> <p><i>Remark 2:</i> Students should be shown three-dimensional shapes in a variety of sizes and orientations, including those that are commercially made and real-world examples, such as cereal boxes as rectangular prisms.</p> <p><i>Example 1:</i> Compare a cylinder and a cone. Describe how they are similar and how they are different.</p> <p><i>Example 2:</i> Provide students with a mixture of three-dimensional shapes. Have them sort the shapes based on a defining attribute and explain their methods for sorting.</p>	
MA.1.GR.2.3	Combine three-dimensional figures to form a composite figure. Decompose a composite figure into three-dimensional figures. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Composite figures are figures that can be divided into more than one figure. Composite figures are not limited to the shapes listed within the benchmark.</p> <p><i>Example 1:</i> Can you combine two figures to create a new shape?</p> <p><i>Possible Student Response:</i> The student combined a cube and a cylinder to make a composite figure.</p> <div style="text-align: center;">  </div> <p><i>Example 2:</i> What shapes have been combined to make this new shape?</p> <div style="text-align: center;">  </div> <p><i>Possible Student Response:</i> The student identified a cylinder and a cone were combined to make the new shape.</p>	

Statistics and Probability	
<i>MA.1.SP.1 Build a foundation for collecting, representing and interpreting data.</i>	
MA.1.SP.1.1	Collect and represent data with up to three categories using tally marks, pictographs or bar graphs.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be able to collect data with up to three categories, organize the data and represent it visually using tally marks, a pictograph or a bar graph.</p>	

Example 1: Julie wants to know the favorite flavor of milk for first grade students. Is it chocolate, vanilla or strawberry milk? Collect data from your classmates on their favorite flavor of milk. Then, represent the results using a bar graph.

Example 2: The students in Mrs. Frank’s class collected data on the color of shirts they would wear on the school field trip. Students could choose red, blue or green. Organize the data using a pictograph.

Tameka- red	Cindy- blue	Mark- blue
Randy- blue	Lin- blue	Greg- red
Raphael- green	Lisa- green	Shawna- blue

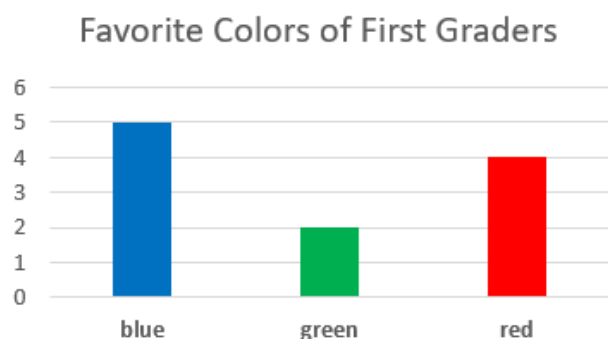
MA.1.SP.1.2

Interpret data represented with tally marks, pictographs or bar graphs by calculating the total number of data points, counting the total in each category and comparing the totals of different categories.

Remarks/Examples:

Remark 1: Students should be able to use data represented with tally marks, pictographs or bar graphs to solve word problems. For examples of word problems, refer to the [Common Addition and Subtraction Situations](#).

Example 1: Look at the graph below.



How many students chose red as their favorite color? How many more students chose blue as their favorite color over green?


Example 2: The lunchroom was serving hot dogs or hamburgers for lunch. The tally marks show the choices the students made. Each tally mark represents one student’s choice.

Choices for Lunch	
Hot dog	
Hamburger	

- How many students want hot dogs for lunch?
- How many students want hamburgers for lunch?
- How many fewer students want hot dogs than hamburgers?

Grade 2

Algebraic Reasoning	
<i>MA.2.AR.1 Solve addition and subtraction problems within 100 and develop mastery of facts to 20.</i>	
MA.2.AR.1.1	Solve one- and two-step addition and subtraction word problems within 100 with unknowns in all positions. Write an equation with a symbol for the unknown to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should focus on understanding the problem presented in order to determine if the context requires addition, subtraction or both. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose a specific operation or a specific strategy.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations.</p> <p><i>Remark 3:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p>	
MA.2.AR.1.2	Determine the unknown whole number in an addition or subtraction equation within 100 with the unknown in any position.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations.</p> <p><i>Example 1:</i> Find the unknown number that would make the equation true. What strategy did you use? $9 + \underline{\quad} = 11 + 5$</p> <p><i>Example 2:</i> Find the missing number in each of the equations below. $5 + 9 = ?$ $3 + ? = 14$ $? + 7 = 12$ $7 + ? = 16 - 5$ $18 - ? = 16 - 3$</p>	
MA.2.AR.1.3	Demonstrate mastery of addition and subtraction facts with addends to 10.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> When students demonstrate mastery of the addition and subtraction facts, they are able to efficiently, flexibly and accurately determine the sum or difference.</p> <p><i>Example 1:</i> $4 + 7$ <i>Example 2:</i> $18 - 9$</p>	

<i>MA.2.AR.2 Determine even and odd numbers and build a foundation for multiplication using arrays.</i>	
MA.2.AR.2.1	Find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns. Write an equation to express the total as a sum of equal addends.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> The intent of this benchmark is for students to make a connection between arrays and repeated addition, which will build a foundation for multiplication.</p> <p><i>Example 1:</i> Write an equation to express the array and find the total number of circles.</p>  <p><i>Example 2:</i> Create an array that would match the equation $3 + 3 + 3 + 3 + 3$?</p>	

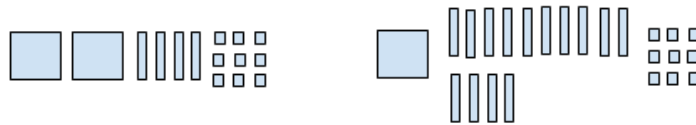
MA.2.AR.2.2	Determine whether a set of objects within 20 has an odd or even number. Write an equation to express an even number as a sum of two equal addends or an odd number as a sum of two equal addends plus 1.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to determine whether a group of objects has an even or odd number. Then, students should be able to write an equation using doubles or doubles plus 1 to represent the count of objects. <i>Example 1:</i> Here are 17 colored counters. Is there an odd or even number of counters? How do you know? <i>Example 2:</i> Find an even number of triangles. Write an equation with a doubles fact to show the number as a sum.	

MA.2.AR.3 Identify and extend numeric patterns.	
MA.2.AR.3.1	Identify and extend patterns numeric patterns. Limit patterns to addition or subtraction with sums to 100 or skip counting by 2s, 5s or 10s.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to identify and extend the patterns present in the world around them. <i>Example 1:</i> What is the pattern? What number comes next in this pattern? 21, 18, 15, 12, 9, ____. <i>Example 2:</i> What number is missing in the pattern? 10, 15, 20, ____, 30, 35	

Number Sense and Operations	
MA.2.NSO.1 Understand the place value of three-digit numbers.	
MA.2.NSO.1.1	Read and write numbers to 1,000 using standard form, word form and expanded form.
<u>Remarks/Examples:</u> <i>Remark 1:</i> There are many ways to write a number. Standard form is a way to write numbers using numerals. Word form is a way to write numbers using words. Expanded form is a way to write numbers to show the value of each digit. Standard form: 234 Word form: two hundred thirty-four Expanded form: $200 + 30 + 4$ <i>Remark 2:</i> Students should make connections between expanded form and place value. <i>Example 1:</i> Three numbers are shown in expanded form. Write the numbers in standard form and word form. $400 + 50 + 8$ $600 + 20 + 4$ $800 + 9$ <i>Example 2:</i> Write the expanded form of the number 406.	
MA.2.NSO.1.2	Compose and decompose three-digit numbers in multiple ways using hundreds, tens, and ones. Demonstrate each composition or decomposition with objects, drawings or equations.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be exposed to and encouraged to use multiple compositions and decompositions of three-digit numbers. The exact number of hundreds, tens and ones in a three-digit number, should be emphasized because this decomposition tells the place value by stating exactly how many hundreds, tens and ones make up the number. This knowledge is fundamental for understanding the place value system. <i>Remark 2:</i> In order to compose and decompose three-digit numbers in multiple ways, students should have an understanding of the place value system through the hundreds place. For instance, 947 can be thought of as 9 hundreds 4 tens 7 ones, as 94 tens 7 ones, as 8 hundreds 14 tens 7 ones or many other ways.	

Example 1: Use base ten blocks to decompose the number 249 in two different ways.

Possible Student Response: The student used base ten blocks to decompose the number.



Example 2: The numbers below have been decomposed to show the number of hundreds, tens and ones. Write the number for each decomposition.

3 hundreds, 6 tens and 7 ones

7 hundreds and 9 ones

41 tens and 2 ones

MA.2.NSO.1.3

Compare two three-digit numbers based on the values of the hundreds, tens and ones digits, using the terms less than, equal to or greater than and the symbols $<$, $=$, or $>$.

Remarks/Examples:

Remark 1: Students should connect the terms less than, equal to or greater than to the corresponding relational symbols.

Example 1: Use the symbols $>$, $<$ or $=$ to compare the following numbers.

727 ____ 294

320 ____ 309

456 ____ 465

MA.2.NSO.1.4

Round whole numbers within 1,000 to the nearest 10 or 100.

Remarks/Examples:

Remark 1: Students should understand that rounding is a process that produces a number with a similar value that is less precise but easier to use. For instance, when adding $234 + 689$ students could round the numbers to the nearest hundred to result in 200 and 700. Then students could determine the sum of 234 and 689 would be close to 900.

Example 1: Round the number 349 to the nearest ten.

MA.2.NSO.1.5

Identify the number that is ten more, ten less, one hundred more and one hundred less than a given three-digit number.

Remarks/Examples:

Remark 1: Students should be able to use mental math to identify ten more, ten less, one hundred more and one hundred less than a number.

Example 1: What number is one hundred less than 753?

Example 2: Find ten more, ten less, one hundred more and one hundred less than 657.

MA.2.NSO.2 Add and subtract within 1,000.

MA.2.NSO.2.1

Add and subtract within 1,000 using a variety of strategies.

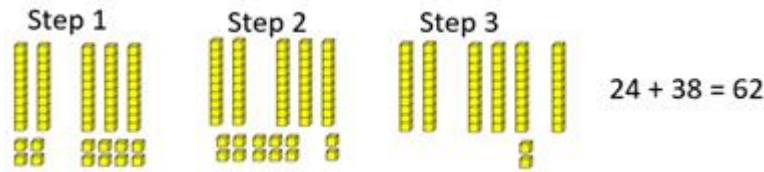
Remarks/Examples:

Remark 1: Students should combine ones with ones, tens with tens and hundreds with hundreds when adding three-digit numbers. They should also understand that sometimes it is necessary to compose ten ones into one ten or ten tens into one hundred or decompose one ten into ten ones or one hundred into ten tens.

Remark 2: When adding or subtracting, students should use a variety of strategies that are efficient and generalizable. Students should be able to choose a strategy that is suited to the problem.

Example 1: $24 + 38$

Possible Student Response: Step 1 shows how the student used base ten blocks to model 24 and 38. Step 2 shows how the student grouped ten ones together to compose a new ten. Step three shows how the student found the sum.



Example 2: $254 + 327$

Possible Student Response: The student used place value to decompose 254 as $200 + 50 + 4$ and 327 as $300 + 20 + 7$. Then, the student added the hundreds with hundreds, tens with tens and ones with ones to express the sum of 254 and 327 as 581.

$$\begin{array}{r} 200 + 300 = 500 \\ 50 + 20 = 70 \\ 4 + 7 = 11 \\ \hline 254 + 327 = 581 \end{array}$$

Example 3: What is $141 - 99$?

Possible Student Response: The student subtracted 100 from 141 and then added 1 to find a difference of 132, $141 - 100 + 1 = 132$.

Example 4: Your friend says that $247 + 65 = 897$. Without solving, explain why you think the answer is incorrect.

Fractions

MA.2.FR.1 Build a foundation for fractions by partitioning shapes into halves, thirds and fourths.

MA.2.FR.1.1	Partition circles and rectangles into two, three or four equal-sized parts. Name the parts using appropriate language, and describe the whole as two halves, three thirds or four fourths. Demonstrate that equal-sized parts may have different shapes.
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Remarks/Examples:

Remark 1: Sectioning a shape into equal-sized parts is called partitioning. Students should understand that when partitioning into two equal-sized parts halves are created, when partitioning into three equal-sized parts thirds are created and when partitioning into four equal-sized parts fourths are created.

Remark 2: Students are not expected to write the equal sized parts as a fraction with a numerator and denominator.

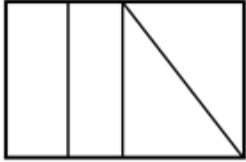
Remark 3: Students should be able to explain that equal-sized parts can have different shapes.

Example 1: Draw a rectangle and partition it into three equal-sized parts. How would you describe each piece? How would you describe the whole?

Example 2: Mykel divides this rectangle into equal parts. What would you call each part? What would you call the whole?



Example 3: Jared says he partitioned the rectangle below into fourths. Is he correct?



Measurement

MA.2.M.1 Measure the length of objects and solve problems involving length.

MA.2.M.1.1	Estimate and measure the length of an object to the nearest inch, foot, yard, centimeter or meter by selecting and using an appropriate tool. Describe the inverse relationship between the size of a unit and number of units needed to measure a given object.
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Remarks/Examples:

Remark 1: Students should recognize that there are many tools to measure length in addition to a ruler, including a yard stick, meter stick and tape measure. Students should understand that the process of measuring with any tool, finding the distance between the zero point and endpoint, is the same as using a ruler.

Remark 2: Students should have opportunities to measure objects that are larger than a 12 inch ruler. Students should also have opportunities to measure curves, such as the girth of a watermelon, using a tape measure or by using a string.

Remark 3: There is an inverse relationship between the size of a unit of measure and the number of units needed to measure a given object. When describing this relationship, students should understand that if they measure an object in one unit, that same object will have a greater or smaller value when measured with a different unit. Students are not expected to convert from one unit to another.

Example 1: Look at the length of the rug in our classroom. Answer the questions about measuring the length of the rug.

What unit would use to measure the length of the rug? _____

What tool would you use? _____

What would be a good estimate of the measurement of the length? _____

What is the length of the rug? _____

How did your estimate differ from the actual length? _____

Example 2: Suppose we measured the length of one wall in our classroom using inches. Then we measured it with yards. Would we need more inches or more yards to measure the length? Why?


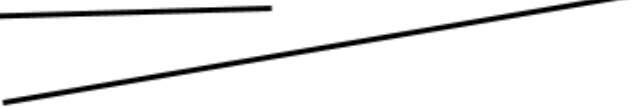
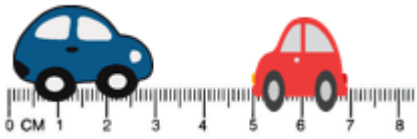
MA.2.M.1.2	Measure the length of two objects in inches, feet, yards, centimeters or meters, and determine the difference between these measurements.
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Remarks/Examples:

Remark 1: The intent of this benchmark is for students to measure two objects and find the difference in their lengths. Objects should be measured to the nearest inch, foot, yard, centimeter or meter.

Example 1: Measure the length of the two pencils. What is the difference in the measurements of their lengths?

Example 2: How much longer is line B than line A? How do you know?

<p>Line A </p> <p>Line B </p>	
MA.2.M.1.3	Solve one- and two-step word problems within 100 involving addition and subtraction of lengths that are given in the same measurement units. Write an equation with a symbol for the unknown to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations.</p> <p><i>Remark 2:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p> <p><i>Example 1:</i> Write an equation that would represent the total length of the two toy cars if they were bumper to bumper. What is the length of the two toy cars if they were bumper to bumper?</p>  <p><i>Example 2:</i> Craig was making a football field in his backyard. He needed 100 yards total for the field. He already measured 23 yards. Then, he measured 45 yards. How many more yards does Craig need to make the football field?</p>	

MA.2.M.2 Tell time and solve problems involving money.	
MA.2.M.2.1	Tell and write time on analog and digital clocks to the nearest five minutes using a.m. and p.m. appropriately. Express portions of an hour using the fractional terms half an hour, half past, quarter of an hour, quarter past and quarter til.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students are not required to understand Military time.</p> <p><i>Remark 2:</i> Students may be expected to estimate a time by using a clock with only an hour hand.</p> <p><i>Example 1:</i> What time is represented on the clock? Explain your thinking?</p>  <p><i>Possible Student Response:</i> The student explained that the time shown on the clock was about 12:45 because the hour hand was almost to 1:00.</p>	
MA.2.M.2.2	Solve one- and two-step addition and subtraction word problems involving either dollar bills within \$100 or coins within 100¢ using \$ and ¢ symbols appropriately.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should understand the value of coins in cents and dollar bills in dollars. They are not required to use decimal values.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations.</p>	

Example 1: Jalissa bought a pencil at the school store for 35¢. She gave the cashier the amount shown below. How much change would Jalissa get from the cashier? What combination of coins could Jalissa receive?



MA.2.M.3 Describe perimeter and find the perimeter of polygons.

MA.2.M.3.1	Describe perimeter as an attribute of plane figures by determining the distance around the edge of the figure.
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Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding of perimeter by walking the boundary of objects, verbally describing the perimeter of objects or relating perimeter to common items like a fence or picture frame. Then, teachers should transition students to use drawings to describe perimeter.

Example 1: Describe the perimeter of the shape below.



MA.2.M.3.2	Find the perimeter of a polygon with whole number side lengths. Polygons are limited to triangles, rectangles, squares, pentagons, hexagons and octagons.
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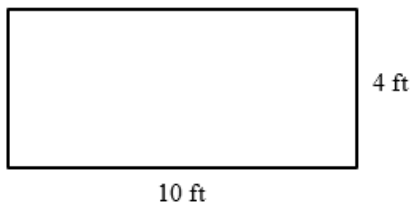
Remarks/Examples:

Remark 1: Students are not required to use a formula for finding perimeter.

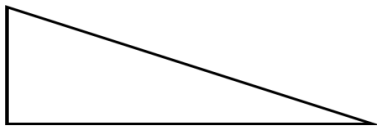
Remark 2: Side lengths can be given or students can measure the side lengths of the given shape.

Remark 3: Student responses should include the appropriate units.

Example 1: What is the perimeter of the rectangle shown below?

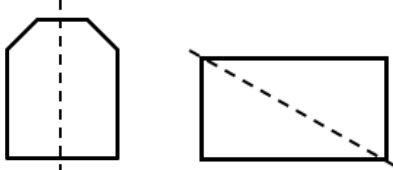
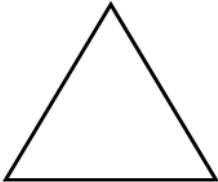


Example 2: Measure the sides of the triangle using centimeters. Then find the perimeter of the triangle.



Geometric Reasoning

MA.2.GR.1 Identify and analyze two-dimensional shapes and identify lines of symmetry.

MA.2.GR.1.1	Identify and draw two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares, pentagons, hexagons and octagons.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should use the defining attributes of number of sides, number of vertices and side lengths to identify and draw two-dimensional shapes. <i>Remark 2:</i> Teachers may need to provide graph paper, grid paper or dot paper to assist students with drawing shapes. <i>Remark 3:</i> Students should be shown shapes in a variety of sizes and orientations. The shapes should not be limited to those present in pattern blocks or attribute blocks. <i>Example 1:</i> Can you draw a pentagon? How do you know it is a pentagon? <i>Example 2:</i> Draw a shape with three sides and three vertices. What shape did you draw? <i>Example 3:</i> Given an assortment of two-dimensional shapes, identify a semi-circle. What makes the shape you chose a semi-circle?	
MA.2.GR.1.2	Identify and draw line(s) of symmetry for a two-dimensional figure. Identify line-symmetric figures.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to identify if a line is a line of symmetry for a given two-dimensional figure. Students should also be able to identify if figures are line-symmetric and draw lines of symmetry on line-symmetric two-dimensional figures. <i>Example 1:</i> State whether the line on the figures below is a line of symmetry. Then explain why or why not.	
	
<i>Example 2:</i> All of the sides of the triangle below are the same length. How many lines of symmetry does the triangle have? Draw all of the lines of symmetry.	
	

Statistics and Probability	
<i>MA.2.SP.1 Collect, represent and interpret numerical and categorical data.</i>	
MA.2.SP.1.1	Collect and represent data using tally marks, tables, pictographs or bar graphs. Use appropriate titles, labels and units.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to collect data, organize the data and represent it visually using tally marks, a pictograph or a bar graph. <i>Remark 2:</i> Data displays can be represented both horizontally and vertically.	

Example 1: Jorge wanted to buy socks for everyone in his class. He needs to make sure the socks will fit. Collect data from your classmates on the length of their feet. Then, organize the data on a table to share with Jorge.

MA.2.SP.1.2

Interpret data represented with tally marks, tables, pictographs or bar graphs by solving addition and subtraction problems.

Remarks/Examples:

Remark 1: Students should be able to use data represented with tally marks, pictographs or bar graphs to solve word problems. For examples of word problems, refer to the [Common Addition and Subtraction Situations](#).

Remark 2: Data displays can be represented both horizontally and vertically.

Example 1: Data was collected on students' favorite pets. The pictograph below represents the data.



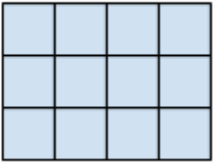
Favorite Pets	
Pet	Number of Students
Bird	♥♥♥
Cat	♥♥♥♥♥
Dog	♥♥♥♥♥♥♥
Fish	♥♥

Each ♥ represents 1 student.

How many students chose birds or dogs as their favorite pet?

How many fewer students chose fish than chose cats as their favorite pet?

Grade 3

Algebraic Reasoning	
MA.3.AR.1 Represent and solve multiplication and division problems within 100.	
MA.3.AR.1.1	Represent multiplication and division within 100 in multiple ways using arrays, equal groups, area models and equations.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed on conceptual understanding by first using manipulatives or drawings then transitioning to equations. Students should understand how the different representations are related to each other.</p> <p><i>Remark 2:</i> Students should be able to use all of the representations listed in the benchmark. In addition, students should be able to represent one problem in multiple ways. For instance, $34 = 12$ and $12 \div 4 = 3$ can be represented by:</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;"> <p>array</p>  </div> <div style="text-align: center;"> <p>equal groups</p>  </div> <div style="text-align: center;"> <p>area model</p>  </div> </div> <p><i>Remark 3:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p>	
MA.3.AR.1.2	Solve one- and two-step multiplication and division word problems with whole numbers within 100. Write an equation with a symbol for the unknown to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should focus on understanding the problem presented in order to determine if the context requires multiplication, division or both. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose a specific operation or a specific strategy.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p> <p><i>Remark 3:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p>	
MA.3.AR.1.3	Multiply and divide whole numbers within 100 using a variety of strategies.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> When multiplying or dividing, students should use a variety of strategies that are efficient and generalizable. Students should be able to choose a strategy that is suited to the problem. These strategies include, but are not limited to, area models, arrays, equal groups, number lines, repeated addition and skip counting.</p> <p><i>Example 1:</i> 7×6</p> <p><i>Example 2:</i> $48 \div 6$</p>	
MA.3.AR.1.4	Demonstrate mastery of multiplication and division facts with factors to 10.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> When students demonstrate mastery of the multiplication and division facts, they are able to efficiently, flexibly, and accurately determine the product or quotient.</p> <p><i>Example 1:</i> $42 \div 7$</p> <p><i>Example 2:</i> 4×9</p>	

MA.3.AR.2 Apply properties of operations and the relationship between multiplication and division.

MA.3.AR.2.1

Apply the Commutative Property of Multiplication, the Associative Property of Multiplication and the Distributive Property as strategies to multiply whole numbers with products to 100.

Remarks/Examples:

Remark 1: The Commutative Property of Multiplication states that numbers can be multiplied in any order and the result will remain the same.

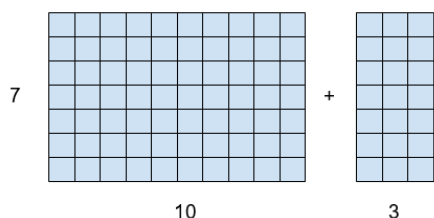
$$8 \times 4 = 4 \times 8$$

Remark 2: The Associative Property of Multiplication states that numbers can be grouped in any way prior to multiplying and the result will remain the same.

$$2 \times (3 \times 4) = (2 \times 3) \times 4$$

Remark 3: The Distributive Property states that when multiplying by a sum, multiplying each addend separately and adding the sums will give the same result as adding the addends then multiplying the sum.

$$7 \times 13 = 7 \times (10 + 3) = (7 \times 10) + (7 \times 3)$$



Remark 4: Students are not required to know the names of the properties but should be able to apply them.

Remark 5: Students are not required to utilize parentheses when working with the Commutative, Associative or Distributive Properties.

Example 1: $2 \times 10 \times 4$

Possible Student Response: The student multiplied 2 and 4 and then multiplied the result by 10.

Example 2: 12×6

Possible Student Response: The student found the product by multiplying 6×10 and 6×2 , and then adding the products together.

MA.3.AR.2.2

Use the inverse relationship between multiplication and division to restate a division problem within 100 as a missing factor problem.

Remarks/Examples:

Remark 1: Inverse operations, such as multiplication and division, are the opposite of one another meaning they will undo each other to result in the initial number, $3 \times 2 = 6$ and $6 \div 2 = 3$.

Example 1: What is a multiplication equation that relates to this division equation? $72 \div 9 = \underline{\hspace{1cm}}$

Example 2: How can $42 \div 6$ be restated as a multiplication problem?

MA.3.AR.2.3

Determine and explain whether an equation involving multiplication or division within 100 is true or false.

Remarks/Examples:

Remark 1: When students are determining if equations are true or false, they should understand the meaning of the equal sign. The equal sign is a mathematical symbol to indicate equality showing that what is on the left of the sign is equal in value or amount to what is on the right. The equal sign is placed between two quantities or expressions to state they have the same value or represent the same value.

$$3 + 4 = 7 \quad 60 \text{ seconds} = 1 \text{ minute} \quad 5 \text{ tens and 4 ones} = 54$$

<p><i>Remark 2:</i> Students should be shown equations in a variety of formats, including but not limited to product/quotient = expression ($16 = 8 \times 2$) and expression = expression ($4 \times 4 = 8 \times 2$).</p> <p><i>Example 1:</i> Determine if the following equations are true or false and explain why. $16 \div 4 = 2 \times 2$ $6 \times 4 = 3 \times 7$ $56 = 7 \times 8$</p>	
MA.3.AR.2.4	Determine the unknown whole number in a multiplication or division equation within 100 with the unknown in any position.
<p><u>Remarks/Examples:</u> <i>Remark 1:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p>	

MA.3.AR.3 Identify and analyze arithmetic patterns.	
MA.3.AR.3.1	Identify, describe and extend numeric patterns. Limit patterns to multiplication and division with products to 100.
<p><u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be able to identify and extend the patterns present in the world around them.</p> <p><i>Example 1:</i> What is the pattern? What numbers are missing in the pattern? 48, 42, ____, 30, 24, ____, 12, 6</p> <p><i>Example 2:</i> Examine a multiplication table. What patterns can you identify in the table?</p>	

Number Sense and Operations	
MA.3.NSO.1 Understand the place value of four-digit numbers.	
MA.3.NSO.1.1	Read and write numbers to 10,000 using standard form, word form and expanded form.
<p><u>Remarks/Examples:</u> <i>Remark 1:</i> There are many ways to write a number. Standard form is a way to write numbers using numerals. Word form is a way to write numbers using words. Expanded form is a way to write numbers to show the value of each digit. Standard form: 1,234 Word form: one thousand two hundred thirty-four Expanded form: $1,000 + 200 + 30 + 4$</p> <p><i>Remark 2:</i> Students should understand that expanded form can be written in multiple ways. The number 4,937 can be written in expanded form including but not limited to the following examples:</p> <ol style="list-style-type: none"> $4,000 + 900 + 30 + 7$ $(4 \times 1000) + (9 \times 100) + (3 \times 10) + (7 \times 1)$ 4 thousands, 9 hundreds, 3 tens, 7 ones $(3 \times 1000) + (19 \times 100) + (3 \times 10) + (7 \times 1)$ <p><i>Example 1:</i> A number is shown in word form. Write the number in standard form and expanded form. four thousand twenty-three</p> <p><i>Example 2:</i> Write the expanded form of 5,629 in two different ways.</p>	
MA.3.NSO.1.2	Compose and decompose four-digit numbers in multiple ways using thousands, hundreds, tens and ones. Demonstrate each composition or decomposition using objects, drawings or equations.
<p><u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be exposed to and encouraged to use multiple compositions and decompositions of four-digit numbers. In order to compose and decompose four-digit numbers in multiple ways, students should have</p>	

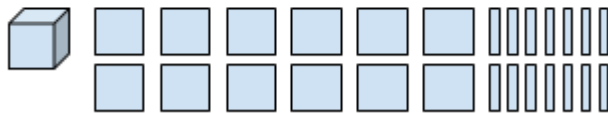
an understanding of the place value system through the thousands place. For instance, 1,947 can be thought of as 1 thousand 9 hundreds 4 tens 7 ones, as 1 thousand 94 tens 7 ones, as 18 hundreds 14 tens 7 ones or many other ways.

Example 1: Decompose the number 4,937 in three different ways.

Possible Student Response: The student used a table to show three different decompositions for 4,937.

Thousands	Hundreds	Tens	Ones
4	9	3	7
3	19	3	7
4	9	1	27

Example 2: Explain how 2,340 can be decomposed to the following.



MA.3.NSO.1.3	Compare two four-digit numbers based on the values of the thousands, hundreds, tens and ones digits using the symbols $<$, $=$, or $>$.
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Remarks/Examples:

Remark 1: The terms greater than, less than or equal to may also be used in comparisons.

Example 1: Compare the two numbers using the symbols $<$, $=$, or $>$.

7,595 ____ 7,620

Example 2: Which number is greater, 3,428 or 2,999?

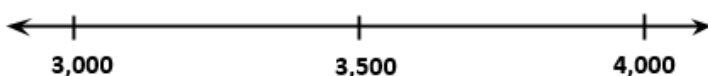
MA.3.NSO.1.4	Round whole numbers within 10,000 to the nearest 10, 100 or 1,000.
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Remarks/Examples:

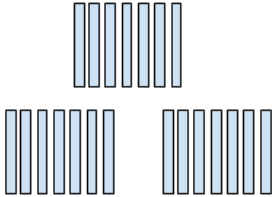
Remark 1: Students should understand that rounding is a process that produces a number with a similar value that is less precise but easier to use. For instance, when adding $234 + 689$ students could round the numbers to the nearest hundred to result in 200 and 700. Then students could determine the sum of 234 and 689 would be close to 900.

Remark 2: Students may have to find a range of numbers that round to a specific place value.

Example 1: Use the number line below to round 3,408 to the nearest 1,000.



Example 2: What is the greatest number that will round to 3,000 when rounded to the nearest hundred?

MA.3.NSO.2 Add and subtract multi-digit numbers and build a foundation for multiplication and division of two-digit numbers.	
MA.3.NSO.2.1	Add and subtract whole numbers within 10,000 using a variety of strategies, including the standard algorithm.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should build on their prior understanding to add and subtract efficiently, flexibly, and accurately. <i>Example 1:</i> What is the difference? $3,856 - 1,798$ <i>Possible Student Response:</i> The student used the standard algorithm to find the difference. <div style="text-align: center;"> $\begin{array}{r} 7^{\text{th}} 4^{\text{th}} 1^{\text{st}} \\ 3,856 \\ - 1,798 \\ \hline 2,058 \end{array}$ </div> <i>Example 2:</i> $7,908 + 1,245$ <i>Possible Student Response:</i> The student added the thousands together, the hundreds together, the tens together, and the ones together. Then the student added those sums to determine $7,908 + 1,245 = 9,153$. <div style="text-align: center;"> $\begin{array}{r} 7,908 \\ + 1,245 \\ \hline 8,000 \\ 1,100 \\ 40 \\ + 13 \\ \hline 9,153 \end{array}$ </div>	
MA.3.NSO.2.2	Multiply a one-digit whole number by a multiple of 10 using a variety of strategies.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives, drawings and strategies based on place value. <i>Example 1:</i> 3×70 <i>Possible Student Response:</i> The student used base ten blocks to determine 210 is the product of 3 and 70. <div style="text-align: center;">  </div>	

Fractions	
MA.3.FR.1 Represent and compare fractions and identify equivalent fractions.	
MA.3.FR.1.1	Represent fractions, including fractions greater than one, in multiple ways using parts of a whole, parts of a set, points on a number line, distances on a number line and area models.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives or drawings to represent fractions. Students should understand how the different representations are related to each other.

Remark 2: Students should be able to use all of the representations listed in the benchmark. In addition, students should be able to represent one fraction in multiple ways. For instance, $\frac{3}{4}$ can be represented as:

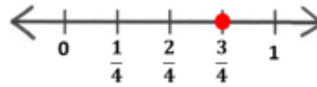
part of a whole



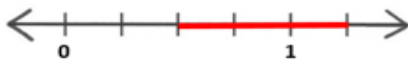
part of a set



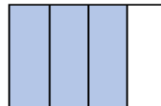
point on a number line



distance on a number line

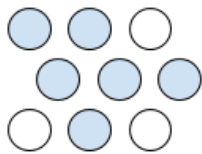


area model



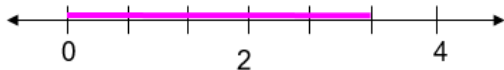
Example 1: Represent the fraction $\frac{5}{6}$ in two different ways.

Example 2: What fraction of the circles are shaded?



Example 3: Represent $\frac{11}{8}$ on a number line.

Example 4: What fraction is represented by the total length marked on the number line shown?



MA.3.FR.1.2

Explain that two fractions are equivalent if they represent equal-sized portions of the same whole or if they both lie at the same point on a number line. Identify equivalent fractions using visual fraction models.

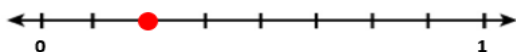
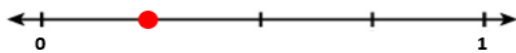
Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

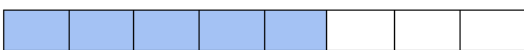
Remark 2: Students should be able to explain their reasoning for why the fractions are equivalent.

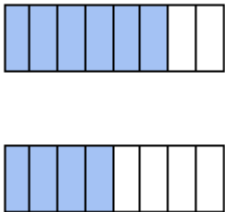
Remark 3: Students are not required to create equivalent fractions.

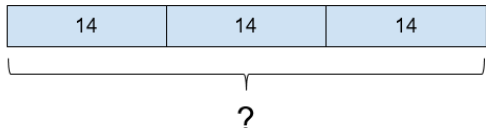
Example 1: What fractions are represented by the number lines? Are the fractions equivalent? Why or why not?



Example 2: Bella says that these two fractions are equivalent. Is Bella correct? Why or Why not?



MA.3.FR.1.3	Compare two fractions with the same numerator or the same denominator by reasoning about their size using the symbols $<$, $=$, or $>$. Determine whether the comparisons are valid based on the fractions referring to the same whole.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.</p> <p><i>Remark 2:</i> Students should be able to explain their reasoning for the comparisons.</p> <p><i>Example 1:</i> Represent each fraction and determine which fraction is greater. How do you know?</p> <p>$\frac{4}{5}$ and $\frac{4}{6}$</p> <p><i>Example 2:</i> Models of two fractions are shown. Compare the fractions using the symbols $>$, $<$, or $=$.</p> 	

Measurement	
<i>MA.3.M.1 Measure the length of objects and solve problems involving measurement.</i>	
MA.3.M.1.1	Measure the length of an object to the nearest half or quarter inch by selecting and using an appropriate tool.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> The intent of this benchmark is to show students how fractions are used in measurement.</p> <p><i>Remark 2:</i> Students should have opportunities to measure objects that are larger than a 12 inch ruler. Students should also have opportunities to measure curves, such as the girth of a watermelon, using a tape measure or by using a string.</p> <p><i>Example 1:</i> Collect pencils of varying lengths. Measure the length of the pencils to the nearest quarter inch.</p>	
MA.3.M.1.2	Solve one- and two-step word problems involving multiplication and division with whole number lengths, masses, weights, temperature or liquid volumes that are given in the same units.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Student responses should include the appropriate units.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p> <p><i>Example 1:</i> Carlos connects three ribbons together to make a bow. Each piece of ribbon is 14 cm long. How long is the total piece of ribbon he can use to make his bow?</p>  <p><i>Example 2:</i> Mr. Parker has 63 kilograms of soil for his 7 gardens. If each garden receives the same amount of soil, how many kilograms of soil will each garden get?</p>	

MA.3.M.2 Tell and write time to the nearest minute.

MA.3.M.2.1

Tell and write time on analog and digital clocks to the nearest minute using a.m. and p.m. appropriately.

Remarks/Examples:

Remark 1: Students are not required to understand Military time.

Example 1: What time is shown on the clock?



MA.3.M.3 Solve problems involving the area of rectangles.

MA.3.M.3.1

Describe area as an attribute of plane figures by covering them using unit squares without gaps or overlaps. Find areas of rectangles by counting unit squares.

Remarks/Examples:

Remark 1: The intent of this benchmark is to relate arrays to the area of rectangles.

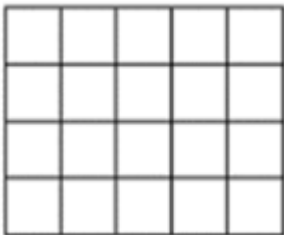
Remark 2: Emphasis should be placed on conceptual understanding through the use of manipulatives or drawings to find the area of two-dimensional figures. Students should have opportunities to create two-dimensional figures using unit squares and rearrange the unit squares to understand how the dimensions impact the area.

Remark 3: Students should understand a square with side lengths of 1 unit is called a unit square. The area of a unit square is one square unit. Unit squares can be used to find the area of two-dimensional figures by covering the figures without gaps or overlaps. When viewing student work, look for misunderstandings, such as students using non-squares as unit squares or different sized squares to find area.

Remark 4: Student responses should include the appropriate units. Students are not expected to use the exponent form of square units.

Example 1: Using unit squares, how many different rectangles can you create with an area of 12?

Example 2: The rectangle shown below was covered with unit squares without gaps or overlaps. What is the area of the rectangle pictured below?



MA.3.M.3.2

Find the area of a rectangle with whole number side lengths using visual models or a formula.

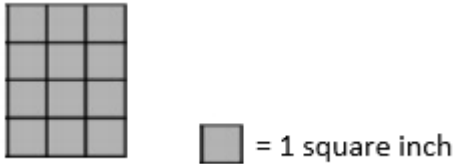
Remarks/Examples:

Remark 1: Students should be able to find the area of rectangles by covering them with unit squares and by applying the area formula. The visual provided by unit squares should lead students to the application of the area formula.

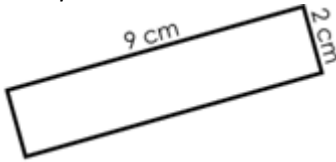
Remark 2: Students should understand there are two formulas for finding the area of rectangles and be able to apply both formulas. $A = l \times w$ or $A = b \times h$

Remark 3: Student responses should include the appropriate units. Students are not expected to use the exponent form of square units.

Example 1: What is the area of the rectangle below?



Example 2: Find the area of the rectangle.



MA.3.M.3.3	Solve problems involving area of rectangles with whole number side lengths using visual models or a formula. Write an equation with a symbol for the unknown to represent the problem.
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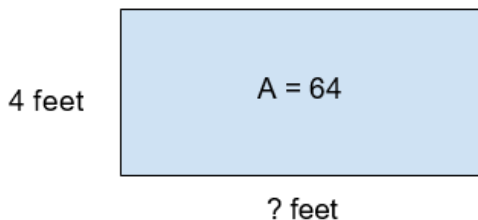
Remarks/Examples:

Remark 1: Student responses should include the appropriate units. Students are not expected to use the exponent form of square units.

Remark 2: Problems may have unknowns in any position. For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Remark 3: A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.

Example 1: Tonya is tiling her rectangular shaped room's floor with 1 square foot tiles. One wall has a length of 4 feet. She uses 64 tiles. What is the length of the other wall?



MA.3.M.3.4	Solve problems involving the area of composite figures composed of non-overlapping rectangles with whole number side lengths. Write an equation with a symbol for the unknown to represent the problem.
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Remarks/Examples:

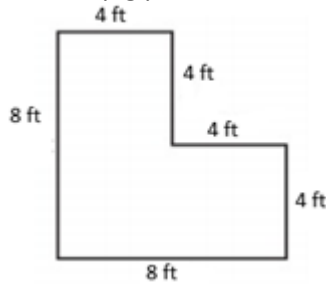
Remark 1: Composite figures are figures that can be divided into more than one figure. In order for students to be able to determine the area of composite figures, the figures must be composed of non-overlapping rectangles.

Remark 2: Student responses should include the appropriate units. Students are not expected to use the exponent form of square units.

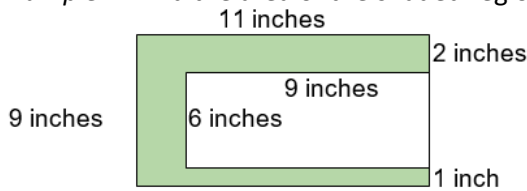
Remark 3: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Remark 4: A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.

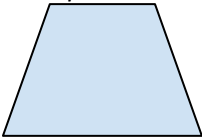
Example 1: Frederic is building a new pen for his pigs. The picture below shows the length of each side, in feet (ft) of the new pig pen. What is the area, in square feet, of Frederic's pig pen?



Example 2: Find the area of the shaded region below.



Possible Student Response: First, the student found the area by partitioning the shaded region into three rectangles, finding the area of each rectangle, and adding the three areas. Then, the student realized there could be a more efficient way. The student found the area of the entire region and subtracted the area of the cut out section to find the area of the shaded region.

Geometric Reasoning	
MA.3.GR.1 Describe and identify lines and classify quadrilaterals.	
MA.3.GR.1.1	Describe and draw points, lines, line segments, rays, perpendicular lines and parallel lines. Identify these in two-dimensional figures.
<p><u>Remarks/Examples:</u></p> <p>Remark 1: Teachers may need to provide graph paper, grid paper or dot paper to assist students with drawing lines, line segments, rays, perpendicular lines and parallel lines.</p> <p>Example 1: Draw a pair of parallel lines and a pair of perpendicular lines. What is different between the two pairs of lines?</p> <p>Example 2: The shape below is a trapezoid. Does this shape have any perpendicular lines? How do you know?</p> 	
MA.3.GR.1.2	Classify quadrilaterals into different categories based on shared defining attributes. Explain why a quadrilateral would or would not belong to a category. Quadrilaterals include parallelograms, rhombuses, rectangles, squares and trapezoids.
<p><u>Remarks/Examples:</u></p> <p>Remark 1: Trapezoids are defined as quadrilaterals having exactly one pair of parallel sides.</p> <p>Example 1: Provide students with a group of shapes. Select all of the parallelograms. What makes these parallelograms?</p>	

Example 2: Name the shapes that are shown below. List three attributes that these shapes share. What word could describe both of these shapes?



Statistics and Probability

MA.3.SP.1 Collect, represent and interpret data

MA.3.SP.1.1

Collect and represent numerical and categorical data with whole number values using tables, pictographs, bar graphs or line plots. Use appropriate titles, labels and units.

Remarks/Examples:

Remark 1: Students should be able to collect data, organize the data and represent it visually using a table, a pictograph, a bar graph or a line plot.

Remark 2: Numerical data are quantitative values, such as a person's height, the number of teeth a dog has, or how many pages were read in a book. Categorical data represent characteristics or groups, such as a person's gender, a favorite color, types of cars, or someone's hometown.

Remark 3: Data displays can be represented both horizontally and vertically.

Example 1: Manuel wants to know the favorite holidays of third grade students. Collect data from your classmates on their favorite holidays. Represent the data using a bar graph or pictograph.

MA.3.SP.1.2

Interpret data with whole number values represented with tables, pictographs, bar graphs or line plots by solving one- and two-step problems.

Remarks/Examples:

Remark 1: Students should be able to use data represented with tally marks, pictographs or bar graphs to solve one- and two-step word problems. For examples of word problems, refer to the [Common Addition and Subtraction Situations](#).

Remark 2: Data displays can be represented both horizontally and vertically.

Example 1: Spring City began collecting plastic bottles to recycle. The table below shows the bottles they collected each day for seven days. Use the table to answer the questions.

**Plastic Bottles Collected
Each Day by Spring City**

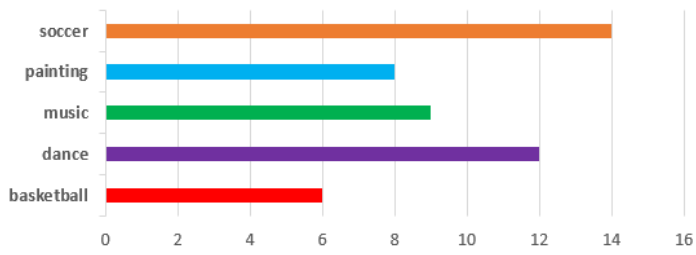
Day	Number of Bottles
1	23
2	45
3	67
4	78
5	54
6	57
7	62

How many times more bottles were collected on day 3 than on day 1?

If Sally dropped off 7 more bottles on day 5, what would be the difference between the number of bottles collected on day 7 and day 5?

Example 2: Bridget collected data on the favorite after school activities of third graders. The data is represented on a bar graph below.

Favorite After School Activities for Third Graders



How many total students told Bridget their favorite after school activity?

How many times more was dance chosen than basketball as the favorite activity?

Grade 4

Algebraic Reasoning	
<i>MA.4.AR.1 Represent and solve problems involving the four operations.</i>	
MA.4.AR.1.1	Solve multiplication and division word problems of whole numbers within 1,000, including problems in which remainders must be interpreted. Write an equation with a symbol for the unknown number to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should focus on understanding the problem presented in order to determine if the context requires multiplication or division. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose a specific operation or a specific strategy.</p> <p><i>Remark 2:</i> Students should be able to interpret remainders based on the context of the problem. Sometimes the problem will require students to drop the remainder, add 1 to the quotient or use the remainder as the answer. Example 1 shows a problem and possible student response that requires interpreting the remainder.</p> <p><i>Remark 3:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p> <p><i>Remark 4:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p> <p><i>Example 1:</i> Susan and Mateo baked 274 cupcakes. They brought the cupcakes to school for their friends in cupcake trays. Each tray holds 9 cupcakes. How many cupcakes will be in the partially full tray? After solving, determine how you used the remainder.</p> <p style="padding-left: 40px;"><i>Possible Student Response:</i> The student divided to determine there would be 30 full trays and 4 remaining, so the partially full tray would have 4 cupcakes. The student then explained that the remainder was the answer to the problem.</p>	
MA.4.AR.1.2	Solve multi-step word problems involving any combination of the four operations with whole numbers, including problems in which remainders must be interpreted. Write an equation with a symbol for the unknown number to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should focus on understanding the problem presented in order to determine what operation or operations are required by the context. Teachers should provide opportunities for students to discuss problems and ask questions about why the student chose specific operations or specific strategies.</p> <p><i>Remark 2:</i> The emphasis should not be placed on the order of operations; therefore, several equations may be needed to represent the problem. Example 1 and 2 show possible student responses that include multiple equations to represent the problem.</p> <p><i>Remark 3:</i> Students should be able to interpret remainders based on the context of the problem. Sometimes the problem will require students to drop the remainder, add 1 to the quotient or use the remainder as the answer. Example 2 shows a problem and possible student response that requires interpreting the remainder.</p> <p><i>Remark 4:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations and the Common Multiplication and Division Situations.</p> <p><i>Remark 5:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p> <p><i>Example 1:</i> Melanie worked at a dog park. There were 16 dogs at the park when she arrived. Six people showed up with three dogs each. How many dogs are there now?</p> <p style="padding-left: 40px;"><i>Possible Student Response:</i> First the student multiplied 6 times 3 to determine 18 dogs came to the dog park. Then the student added 16 to 18 to determine there were 34 dogs at the dog park.</p>	

$$6 \times 3 = ?$$

$$6 \times 3 = 18$$

$$18 + 16 = ?$$

34 dogs

Example 2: 165 students signed up to play football before tryouts. During tryouts 97 students signed up to play. If 7 people can play on each team, how many full teams can be made?

Possible Student Response: The student added 165 and 97. Then the student divided the sum by 7 to find the number of teams that can be made. The student stated that the remainder was dropped because the problem asked for the number of full teams.

$$165 + 97 = ?$$

$$262 \div 7 = ?$$

37

MA.4.AR.1.3

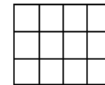
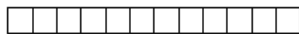
Identify factor pairs for a whole number up to 100. Determine whether the number is prime or composite.

Remarks/Examples:

Remark 1: A factor pair is a set of two numbers that when multiplied together result in a specific number. Students should understand that the number of factor pairs determine if a number is prime or composite. Numbers with only one factor pair, the given number and one, are prime. Numbers with more than one factor pair are composite.

Example 1: Find all of the factor pairs of 12. State whether 12 is prime or composite.

Possible Student Response: The student used area models to find the factor pairs of 12: 1 and 12, 2 and 6, 3 and 4. The student then stated that 12 was composite.



Example 2: Is 7 prime or composite? How do you know?

Possible Student Response: The student stated 7 is prime because the only array that can be made is 7 rows and 1 column or 7 columns and 1 row.



MA.4.AR.1.4

Determine whether a whole number up to 100 is a multiple of a given one-digit number.

Remarks/Examples:

Remark 1: A multiple is a number that can be divided by another number without a remainder. Students should be able to determine if a number is a multiple of a given number by using multiplication or division.

Example 1: Is 54 a multiple of 6? How do you know?

Example 2: If I have some packs of pencils with 8 pencils in each pack, could there be a total of 52? Why or why not?

MA.4.AR.2 Determine whether equations are true or false and find unknowns in equations.

MA.4.AR.2.1

Determine and explain whether an equation, including any combination of the four operations, is true or false.

Remarks/Examples:

Remark 1: When students are determining if equations are true or false, they should understand the meaning of the equal sign. The equal sign is a mathematical symbol to indicate equality showing that what is on the left of the sign is equal in value or amount to what is on the right. The equal sign is placed between two quantities or expressions to state they have the same value or represent the same value.

$3 + 4 = 7$ $60 \text{ seconds} = 1 \text{ minute}$ $5 \text{ tens and } 4 \text{ ones} = 54$

Remark 2: Students should be shown equations that include any of the four operations. The equations should include a variety of formats, but emphasis should not be placed on the order of operations.

Example 1: Determine whether the equations are true or false, and explain why.

$16 + 4 = 2 \times 10$ $6 - 4 = 3 + 7$ $56 = 7 \times 8$ $24 \div 4 = 2 \times 3$

Example 2: Determine whether the equation $27 \times 5 = 9 \times 23$ is true or false. How do you know?

MA.4.AR.2.2	Determine the unknown whole number in an equation including any combination of the four operations with the unknown in any position.
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Remarks/Examples:

Remark 1: For examples of word problems, refer to the [Common Addition and Subtraction Situations](#) and the [Common Multiplication and Division Situations](#).

Example 1: Find the unknown number that would make the equation $15 \times ? = 45 \times 7$ true? Explain your thinking.

Example 2: What number can replace the question mark to make the equation true?

$72 \div 9 = 72 - ?$

MA.4.AR.3 Generate and extend numerical patterns that follow a given rule.

MA.4.AR.3.1	Generate, describe and extend a numerical pattern that follows a given rule.
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Remarks/Examples:

Remark 1: Students should be able to identify and extend the patterns present in the world around them. They should also be able to create models to display the patterns present in their world.

Example 1: The pattern follows the rule Multiply by 4. If the first number is 8, what are the next four numbers in the pattern?

Example 2: The pattern below follows the rule Add 25. What number is missing in the pattern?

33, 58, 83, ____, 133

Number Sense and Operations

MA.4.NSO.1 Understand place value for multi-digit numbers.

MA.4.NSO.1.1	Express how the value of a digit in a multi-digit whole number changes if it moves one place to the left or right.
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Remarks/Examples:

Remark 1: Students should understand the place value system and begin to generalize place value for all multi-digit whole numbers.

Example 1: How does the value of the 5 in 50 compare to the value of 5 in 500?

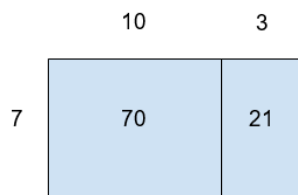
Example 2: On Saturday, 14,498 people attended a music festival. How does the value represented by the 4 in the hundreds place compare with the value of the 4 in the thousands place?

MA.4.NSO.1.2	Read and write multi-digit whole numbers within 1,000,000 using standard form, word form and expanded form.
<u>Remarks/Examples:</u> <i>Remark 1:</i> There are many ways to write a number. Standard form is a way to write numbers using numerals. Word form is a way to write numbers using words. Expanded form is a way to write numbers to show the value of each digit. Standard form: 1,234 Word form: one thousand two hundred thirty-four Expanded form: $1,000 + 200 + 30 + 4$ <i>Remark 2:</i> Students should understand that expanded form can be written in multiple ways. The number 4,937 can be written in expanded form including but not limited to the following examples: <ul style="list-style-type: none"> a. $4,000 + 900 + 30 + 7$ b. $(4 \times 1000) + (9 \times 100) + (3 \times 10) + (7 \times 1)$ c. 4 thousands, 9 hundreds, 3 tens, 7 ones d. $(3 \times 1000) + (19 \times 100) + (3 \times 10) + (7 \times 1)$ <i>Example 1:</i> Write the expanded form of 389,037,629 in two different ways. <i>Example 2:</i> A number is shown in expanded form. Write the number in standard form and word form. $(4 \times 100000) + (34 \times 10000) + (5 \times 1000) + (67 \times 10) + (8 \times 1)$	
MA.4.NSO.1.3	Compare two multi-digit whole numbers within 1,000,000 based on values of the digits in each place using the symbols $<$, $=$, or $>$.
<u>Remarks/Examples:</u> <i>Remark 1:</i> The terms greater than, less than or equal to may also be used in comparisons. <i>Example 1:</i> Compare the two numbers using the symbols $<$, $=$, or $>$. 296,569 ____ 325,432 <i>Example 2:</i> Which number is greater 434,976 or 434,679?	
MA.4.NSO.1.4	Round whole numbers within 1,000,000 to the nearest 100, 1,000, 10,000 or 100,000.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should understand that rounding is a process that produces a number with a similar value that is less precise but easier to use. For instance, when adding $234 + 689$ students could round the numbers to the nearest hundred to result in 200 and 700. Then students could determine the sum of 234 and 689 would be close to 900. <i>Remark 2:</i> Students may have to find a range of numbers that round to a specific place value. <i>Example 1:</i> Round the number 657,879 to the nearest 10,000 and 100,000. Describe the differences of the rounded values. <i>Example 2:</i> What is the greatest number that will round to 30,000 when rounded to the nearest hundred?	

<i>MA.4.NSO.2 Build a foundation for multiplying and dividing multi-digit numbers.</i>	
MA.4.NSO.2.1	Multiply a whole number up to four digits by a one-digit whole number using a variety of strategies.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives, drawings and strategies based on place value. These strategies include, but are not limited to, area models, the Distributive Property and partial products. <i>Remark 2:</i> Students are not expected to use the standard algorithm for multiplication.	

Example 1: 13×7

Possible Student Response: The student used an area model to find the product.



Example 2: $3,428 \times 4$

Possible Student Response: The student used the Distributive Property to multiply by stating 3,428 times 4 is equivalent to $4 \times 3,000 + 4 \times 400 + 4 \times 20 + 4 \times 8$.

$$3,000 + 400 + 20 + 8$$

$$4 \times 3,000 = 12,000$$

$$4 \times 400 = 1600$$

$$4 \times 20 = 80$$

$$4 \times 8 = 32$$

$$13,712$$

MA.4.NSO.2.2	Multiply a two-digit by a two-digit whole number using a variety of strategies.
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Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives, drawings and strategies based on place value. These strategies include, but are not limited to, area models, the Distributive Property and partial products.

Remark 2: Students are not expected to use the standard algorithm for multiplication.

Example 1: 48×26

Possible Student Response: The student used the partial products strategy to find the product.

48	(40 + 8)
$\times 26$	(20 + 6)
800	40×20
240	40×6
160	8×20
$+ 48$	8×6
1248	

MA.4.NSO.2.3	Divide a whole number up to four digits by a one-digit whole number using a variety of strategies. Represent remainders as fractional parts of the divisor.
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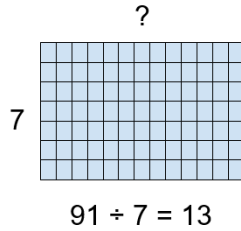
Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives, drawings and strategies based on place value. These strategies include, but are not limited to, area models, the Distributive Property and partial quotients.

Remark 2: Students are not expected to use the standard algorithm for division.

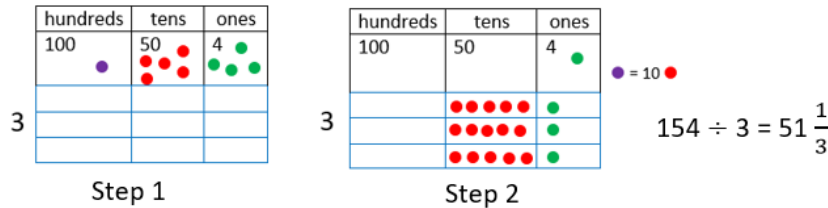
Example 1: $91 \div 7$

Possible Student Response: The student used an area model to divide.



Example 2: $154 \div 3$

Possible Student Response: The student used manipulatives to divide by starting with purple counters to represent the hundreds place, red counters to represent the tens place and green counters to represent the ones place. The student realized it was necessary to decompose the 100 into 10 tens. Then, the student divided the counters in the tens place by 3 and the counters in the ones place by 3 to find the quotient.



Fractions

MA.4.FR.1 Determine fraction equivalence and compare fractions.

MA.4.FR.1.1	Identify and generate equivalent fractions, including mixed numbers and fractions greater than 1, referring to the same whole using visual models or algorithms.
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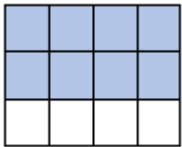
Remarks/Examples:

Remark 1: Remark 1 for benchmark MA.3.FR.1.1 shows examples of visual fraction models.

Remark 2: Students should be able to explain how the numerator and denominator are affected when an equivalent fraction is created.

Example 1: Represent $\frac{7}{8}$ and $\frac{3}{4}$ on a number line. Are the fractions $\frac{7}{8}$ and $\frac{3}{4}$ equivalent? How do you know?

Example 2: What fraction is represented by the visual model below? Generate two fractions that are equivalent.



MA.4.FR.1.2	Compare two fractions, including mixed numbers and fractions greater than 1, with different numerators and different denominators using the symbols <, =, or >.
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Remarks/Examples:

Remark 1: Students should be able to use benchmark quantities, such as 0, and 1, to compare fractions.

Example 1: Becca and Tiger share their yard. Becca uses $\frac{5}{12}$ of the yard for a garden. Tiger uses $\frac{1}{3}$ of the yard for playing football. Who uses a greater portion of the yard?

Example 2: Find the missing digit to make the inequality true.

$$\frac{3}{5} > \frac{\square}{4}$$

MA.4.FR.2 Add, subtract and multiply fractions.

MA.4.FR.2.1

Decompose a fraction, including mixed numbers and fractions greater than one, into a sum of fractions with the same denominator in multiple ways. Demonstrate each decomposition with objects, drawings or equations.

Remarks/Examples:

Remark 1: Students should be exposed to and encouraged to use multiple decompositions of fractions. For instance, $\frac{5}{6}$ can be decomposed as $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, as $\frac{1}{6} + \frac{2}{6} + \frac{2}{6}$, or as $\frac{3}{6} + \frac{2}{6}$.

Example 1: Show two ways to decompose $1\frac{3}{8}$.

Possible Student Response: The student decomposed $1\frac{3}{8}$ in two different way

$$\frac{8}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{6}{8} + \frac{5}{8}$$

Example 2: Benjamin decomposed $\frac{7}{6}$ as shown in the visual model below. Veronica says that Benjamin did not decompose $\frac{7}{6}$ correctly. Do you agree with Benjamin or Veronica? Why?



MA.4.FR.2.2

Add and subtract fractions with like denominators, including mixed numbers and fractions greater than one, using a variety of strategies.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

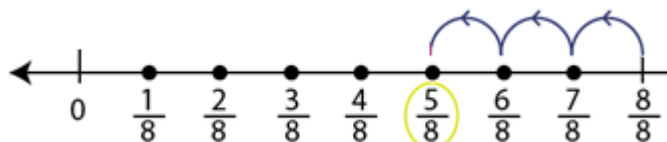
Example 1: $1\frac{2}{3} + \frac{1}{3}$

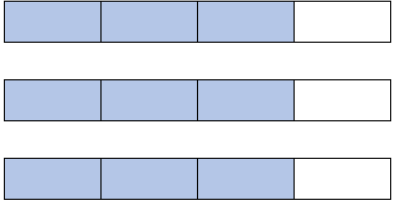
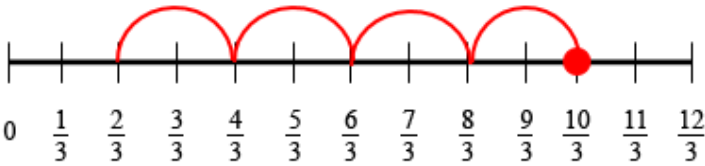
Possible Student Response: The student used a drawing to find the sum.

$$1\frac{2}{3} + \frac{1}{3} = 2$$

Example 2: $\frac{8}{8} - \frac{3}{8}$

Possible Student Response: The student used a number line to subtract.



MA.4.FR.2.3	Solve word problems involving addition and subtraction of fractions with like denominators, including mixed numbers and fractions greater than one, using visual fraction models and equations to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.</p> <p><i>Remark 2:</i> Items must reference the same whole.</p> <p><i>Remark 3:</i> Students are not required to simplify or use lowest terms.</p> <p><i>Remark 4:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations.</p> <p><i>Example 1:</i> Glenna has $\frac{7}{12}$ of a regular sized chocolate bar. Her friend eats $\frac{5}{12}$ of her chocolate bar. How much chocolate does Glenna have now?</p>	
MA.4.FR.2.4	Extend previous understanding of multiplication to multiply a fraction by a whole number or a whole number by a fraction using a variety of strategies.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.</p> <p><i>Remark 2:</i> Students are not required to simplify or use lowest terms.</p> <p><i>Example 1:</i> Write an equation that represents the model below.</p>  <p><i>Example 2:</i> $4 \times \frac{2}{3}$</p> <p><i>Possible Student Response:</i> The student used a number line to multiply.</p> 	
MA.4.FR.2.5	Solve word problems involving multiplication of a fraction by a whole number or a whole number by a fraction using visual fraction models or equations to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.</p> <p><i>Remark 2:</i> Items must reference the same whole.</p> <p><i>Remark 3:</i> Students are not required to simplify or use lowest terms.</p> <p><i>Remark 4:</i> For examples of word problems, refer to the Common Multiplication and Division Situations.</p> <p><i>Example 1:</i> Apunda is baking cookies. She needs $\frac{3}{4}$ cup of flour for each batch of cookies. How much sugar does Apunda need if she makes 6 batches of cookies?</p>	

MA.4.FR.3 Understand the relationship between fractions and decimals.

MA.4.FR.3.1

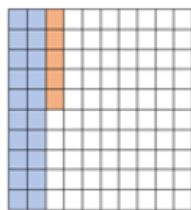
Model and express a fraction, including mixed numbers and fractions greater than one, with the denominator 10 as an equivalent fraction with the denominator 100. Use fractions with denominators of 10 and 100 to add two fractions.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models.

Example 1: Maria and Dylan are counting the number of gumballs in a gumball machine. Maria says that $\frac{2}{10}$ of all the gumballs are blue. Dylan says $\frac{5}{100}$ of all of the gumballs are red. What fraction of the gumballs are blue and red?

Possible Student Response: The student used a visual model to find the sum.



MA.4.FR.3.2

Use decimal notation to represent fractions with denominators of 10 or 100, including mixed numbers and fractions greater than one, and use fractions with denominators of 10 or 100 to represent decimals.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives, visual models and strategies based on place value.

Example 1: Write 4.5 as a fraction.

Example 2: What is $\frac{45}{100}$ written as a decimal?

MA.4.FR.3.3

Compare two decimals to hundredths by reasoning about their size using the symbols $<$, $=$, or $>$.

Remarks/Examples:

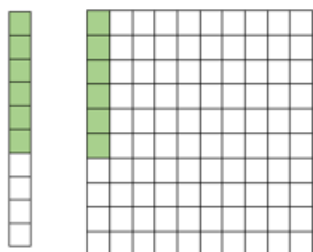
Remark 1: The terms greater than, less than or equal to may also be used in comparisons.

Remark 2: Decimals may be greater than 1, but are limited to the tenths and hundredths place.

Remarks 3: Students should be able to explain the reasoning for the comparison.

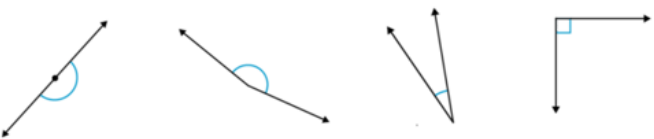
Example 1: Sadie wrote down a decimal number that is greater than 0.49 but less than 0.53. What is a possible decimal that Sadie could have written down?

Example 2: Are the decimals shown by the visual models below equal? Why or why not?



Measurement													
<i>MA.4.M.1 Measure the length of objects and solve problems involving measurement.</i>													
MA.4.M.1.1	Measure the length of an object to the nearest half, quarter or eighth inch by selecting and using an appropriate tool.												
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> The intent of this benchmark is to show students how fractions are used in measurement.</p> <p><i>Remark 2:</i> Students should have opportunities to measure objects that are larger than a 12 inch ruler. Students should also have opportunities to measure curves, such as the girth of a watermelon, using a tape measure or by using a string.</p> <p><i>Example 1:</i> Collect pencils of varying lengths. Measure the length of the pencils to the nearest eighth inch.</p>													
MA.4.M.1.2	Convert larger units to smaller units within a single system of measurement using the units yd., ft., in.; km, m, cm; lb., oz.; kg, g; gal., pt., c.; L, mL; hr., min., sec. Record the conversions on a two-column table.												
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should understand how to convert from larger units to smaller units when provided with the conversions. Students are not required to memorize the conversions.</p> <p><i>Example 1:</i> Complete the table to convert feet to inches.</p> <table border="1"> <thead> <tr> <th>Feet</th><th>Inches</th></tr> </thead> <tbody> <tr> <td>1</td><td>12</td></tr> <tr> <td>2</td><td></td></tr> <tr> <td>3</td><td></td></tr> <tr> <td>4</td><td></td></tr> <tr> <td>5</td><td></td></tr> </tbody> </table> <p><i>Example 2:</i> How many times longer is one kilometer than one meter? How do you know?</p>		Feet	Inches	1	12	2		3		4		5	
Feet	Inches												
1	12												
2													
3													
4													
5													
MA.4.M.1.3	Solve two-step word problems involving distances and intervals of time using any combination of the four operations, including problems with fractions with like denominators.												
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Problems involving fractions should only require students to add and subtract with like denominators and multiply a fraction by a whole number.</p> <p><i>Remark 2:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations and the Common Multiplication and Division Situations.</p> <p><i>Example 1:</i> Priyanka has four homework assignments. She will spend 35 minutes on each assignment. She also wants to spend 30 minutes reading before bed. If she has 200 minutes before her bed time, will she have enough time to complete her homework and read?</p> <p><i>Example 2:</i> Sue and Joaquim ran every day after school for three days. Sue ran $\frac{2}{8}$ of a mile on the first day, $\frac{5}{8}$ on the second day, and $\frac{7}{8}$ on the third day. Joaquim ran $\frac{5}{8}$ all three days. Who ran the greatest amount during the three days?</p>													

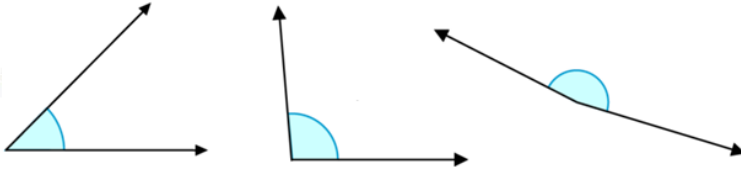
MA.4.M.2 Solve problems involving the area and perimeter of rectangles.	
MA.4.M.2.1	Solve area and perimeter word problems by applying the area and perimeter formulas for rectangles with whole number side lengths, including problems with unknown sides. Write an equation with a symbol for the unknown to represent the problem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> For examples of word problems, refer to the Common Addition and Subtraction Situations and the Common Multiplication and Division Situations.</p> <p><i>Remark 2:</i> A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.</p> <p><i>Example 1:</i> The PE teacher at Mulberry Elementary School wants a space for students to play when it is raining outside. He says he needs 900 square feet of space. The school needs to determine if an empty classroom with a perimeter of 120 feet and a width of 32 feet will be enough space for the PE teacher. Will there be enough space? Why or why not?</p> <p><i>Example 2:</i> Mrs. Bucket cleaned out her closet. She wants to put a shelf in the space that is 8 feet long. If the closet has an area of 48 square feet and a perimeter of 32 feet. Does she have enough space for her shelf? Explain your thinking.</p>	

MA.4.M.3 Draw, classify and measure angles.	
MA.4.M.3.1	Identify and classify angles as acute, right, obtuse, straight or reflex.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should recognize angles as geometric figures that are formed wherever two rays share a common endpoint, the vertex.</p> <p><i>Remark 2:</i> Students should be able to classify angles as acute, right, obtuse, straight or reflex with or without measuring them. An acute angle measures between 0° and 90°. A right angle measures exactly 90°. An obtuse angle measures between 90° and 180°. A straight angle measures exactly 180°. A reflex angle measures between 180° and 360°.</p> <p><i>Example 1:</i> Identify the angles below as acute, right, obtuse, straight or reflex. Explain why.</p> 	
MA.4.M.3.2	Estimate and measure angles in whole number degrees using a protractor, and construct angles of specified measure in whole number degrees using a protractor. Limit to within 360 degrees.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should recognize that a protractor is a tool that can be used to measure angles in degrees. They should understand that protractors can be semicircular to measure angles up to 180° or circular to measure angles up to 360°.</p> <p><i>Remark 2:</i> Students should understand that an angle is measured by using the common endpoint of the rays and the measure of the angle shows how many degrees the angle turns in reference to a circle. An angle with a measure 1° turns through $\frac{1}{360}$ of a circle.</p>	

Remark 3: When estimating angle measures, students should be able to give a reasonable amount of degrees for the measure of a given angle. Students should also be able to use benchmark angles of 45° , 90° , and 180° to estimate.

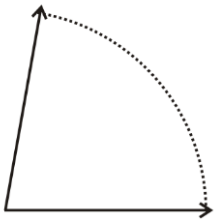
Remark 4: Teachers should provide students with an opportunity to investigate measuring and drawing angles by providing students with angles to measure and protractors.

Example 1: Estimate the measure of the angles below. Then, measure the angles using a protractor.



Example 2: Use your protractor to draw an angle that is 125° .

Example 3: Billy measured the angle below and said it was 45° . Do you think Billy is correct? Why or why not?



Geometric Reasoning

MA.4.GR.1 Classify triangles and quadrilaterals based on shared defining attributes.

MA.4.GR.1.1

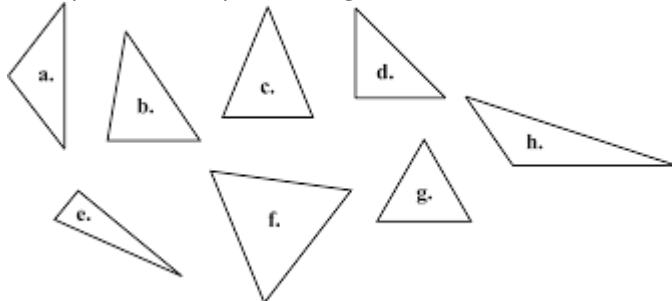
Classify triangles or quadrilaterals into different categories based on shared defining attributes. Explain why a triangle or quadrilateral would or would not belong to a category. Triangles include scalene, isosceles, equilateral, acute, obtuse, right and equiangular; quadrilaterals include parallelograms, rhombuses, rectangles, squares and trapezoids.

Remarks/Examples:

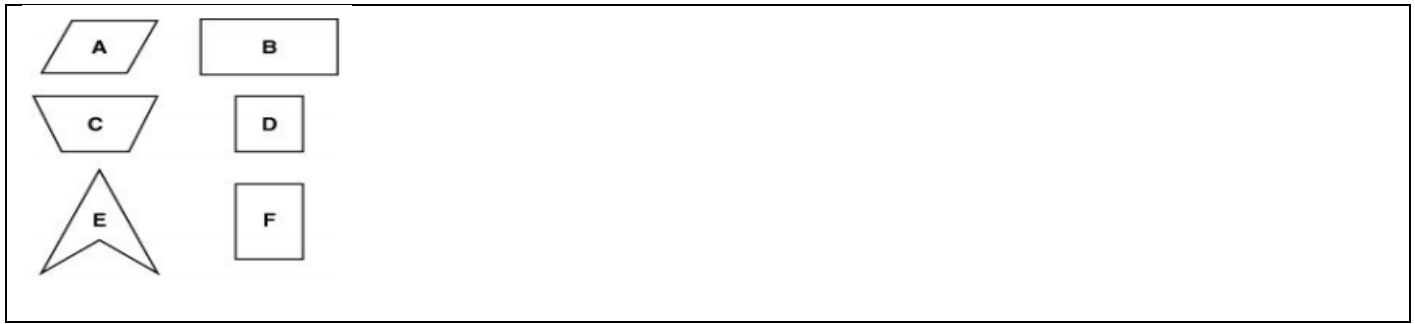
Remark 1: Students should use the defining attributes of number of sides, number of vertices, side lengths, angle measures and parallel and perpendicular sides to classify triangles and quadrilaterals.

Remark 2: Trapezoids are defined as quadrilaterals having exactly one pair of parallel sides.

Example 1: Classify the triangles below based on their angle measures. How many categories did you create? Why?



Example 2: Homer has these shapes. Classify Homer's shapes into at least two categories. Does Homer have a shape that is both a rhombus and a rectangle? Explain.



Statistics and Probability

MA.4.SP.1 Collect, represent and interpret data and find the mode, median and range of a data set.

MA.4.SP.1.1	Collect and represent numerical data with whole number values using tables or line plots. Solve multi-step problems involving any combination of the four operations using data from these representations.
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Remarks/Examples:

Remark 1: Numerical data are quantitative values, such as a person's height, the number of teeth a dog has or how many pages were read in a book.

Example 1: Mrs. Frank handed out bags of candy to her students. The weights of the bags of candy were measured in grams and are shown below. Represent the weights of the candy bags on a line plot and respond to the questions.

Weight of Candy Bags								
5	10	6	9	4	8	8	9	
8	5	6	8	8	7	7	4	
9	3	7	6	6	9	10	10	

What is the combined weight of the three lightest bags?

What is the combined weight of the three heaviest bags?

Mark collected all of the bags that weight over 7 grams and combined them in one bag. How much would this bag weigh?

MA.4.SP.1.2	Determine the mode, median or range to interpret numerical data with whole number values represented with tables or line plots.
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Remarks/Examples:

Remark 1: Numerical data are quantitative values, such as a person's height, the number of teeth a dog has or how many pages were read in a book.

Example 1: The table shows the number of homeruns hit in the 2018 season by eight players on the Braves. Use the table below to determine the range and the mode of the data. What does the range tell you? What does the mode tell you?

**Number of Homeruns Hit by Players
on the Braves in the 2018 Season**

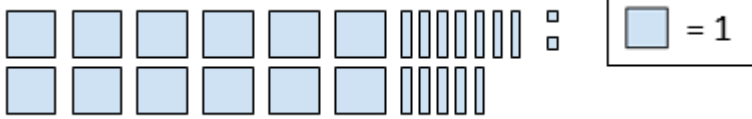
Player	Number of Homeruns
Ender Inciarte	10
Kurt Suzuki	12
Nick Markakis	14
Dansby Swanson	14
Johan Camargo	19
Freddie Freeman	23
Ozzie Albies	24
Ronald Acuna Jr.	26

Grade 5

Algebraic Reasoning	
MA.5.AR.1 Write and evaluate numerical expressions.	
MA.5.AR.1.1	Evaluate multi-step expressions including those with parentheses.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Multi-step expressions are limited to any combination of the operations of multiplication, division, addition and subtraction with parentheses.</p> <p><i>Remark 2:</i> Expressions should not include exponents or nested grouping symbols. Expressions could include whole numbers and fractions.</p> <p><i>Example 1:</i> $8 + 4 \div 2$</p> <p><i>Example 2:</i> $3 + \left(\frac{5}{12} - \frac{3}{4}\right)$</p>	
MA.5.AR.1.2	Translate written descriptions into numerical expressions and numerical expressions into written descriptions.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Expressions are limited to any combination of the operations of multiplication, division, addition and subtraction with parentheses.</p> <p><i>Remark 2:</i> Expressions should not include exponents or nested grouping symbols. Expressions could include whole numbers and fractions.</p> <p><i>Example 1:</i> Translate <u>subtract 7 from the quotient of 428 and 4</u> into a numerical expression.</p> <p><i>Example 2:</i> Write the numerical expression for: Nine less than the product of two and four.</p> <p><i>Example 3:</i> Write the written description for the numerical expression: $3 \div 17 - 2$.</p>	

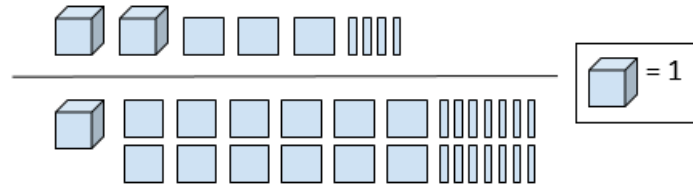
MA.5.AR.2 Analyze patterns and relationships.													
MA.5.AR.2.1	Generate, describe and extend a numerical pattern that follows a given rule. Record inputs and outputs using a two-column table. Graph the ordered pairs formed on the two-column table on the first quadrant of the coordinate grid.												
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be able to identify and extend the patterns present in the world around them. They should also be able to create models to display the patterns present in their world.</p> <p><i>Example 1:</i> Dwayne is saving money in a bank account. His account starts with \$11 on week 1 and “adds \$7” each week. Use the completed chart to create ordered pairs to graph the amount of money in Dwayne’s bank account.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Week</th><th>Amount in Dwayne’s Bank Account</th></tr> </thead> <tbody> <tr> <td>1</td><td>\$11</td></tr> <tr> <td>2</td><td></td></tr> <tr> <td>3</td><td></td></tr> <tr> <td>4</td><td></td></tr> <tr> <td>5</td><td></td></tr> </tbody> </table>		Week	Amount in Dwayne’s Bank Account	1	\$11	2		3		4		5	
Week	Amount in Dwayne’s Bank Account												
1	\$11												
2													
3													
4													
5													

Number Sense and Operations	
<i>MA.5.NSO.1 Understand the place value of multi-digit numbers with decimals to the thousandths place.</i>	
MA.5.NSO.1.1	Express how the value of a digit in a multi-digit whole number with decimals to the thousandths changes if it moves one or more places to the left or right.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should understand the place value system and generalize place value for all multi-digit whole numbers and decimals. <i>Example 1:</i> Bill looked at his odometer and it read 77.77. How does the 7 in the one's place compare to the 7 in the hundredth's place?	
MA.5.NSO.1.2	Read and write multi-digit numbers with decimals to the thousandths using standard form, word form and expanded form.
<u>Remarks/Examples:</u> <i>Remark 1:</i> There are many ways to write a number. Standard form is a way to write numbers using numerals. Word form is a way to write numbers using words. Expanded form is a way to write numbers to show the value of each place. Standard form: 194.35 Word form: one hundred ninety-four and thirty-five hundredths Expanded form: $(1 \times 100) + (9 \times 10) + (4 \times 1) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{100})$ <i>Remark 2:</i> Students should understand that expanded form can be written in multiple ways. The number 342.76 can be written in expanded form including, but not limited to, the following examples: <ul style="list-style-type: none"> a. $300 + 40 + 2 + \frac{7}{10} + \frac{6}{100}$ b. $(3 \times 100) + (4 \times 10) + (2 \times 1) + (7 \times \frac{1}{10}) + (6 \times \frac{1}{100})$ c. 3 hundreds, 4 tens, 2 ones, 7 tenths, 6 hundredths d. $(2 \times 100) + (14 \times 10) + (2 \times 1) + (76 \times \frac{1}{100})$ <i>Remark 3:</i> Students should make connections between expanded form and place value. Every number has a place value, which is determined by the value of each digit according to its place in the number. Place value is shown when writing numbers in expanded form. <i>Example 1:</i> A number is shown in word form. Write the number in standard form and expanded form. Six hundred twenty-four and thirty-seven thousandths <i>Example 2:</i> Sally wrote the expanded form for a number in two different ways. Did Sally represent the same number? Explain. <ul style="list-style-type: none"> a. $40 (\frac{1}{10}) + 5 \times 9 (\frac{1}{100}) + 8 \times (\frac{1}{1000})$ b. $4 + 5 \times (\frac{1}{10}) + 8 \times (\frac{1}{100})$ 	
MA.5.NSO.1.3	Compose and decompose multi-digit numbers with decimals to the thousandths in multiple ways using the values of the digits in each place. Demonstrate the compositions or decompositions using objects, drawings or equations.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be exposed to and encouraged to use multiple compositions and decompositions of multi-digit numbers with decimals. In order to compose and decompose multi-digit numbers in multiple ways, students should have an understanding of the place value system including decimal place values. For instance, 1,947.24 can be thought of as 1 thousand 9 hundreds 4 tens 7 ones 2 tenths 4 hundredths, as 1 thousand 94 tens 7 ones and 24 hundredths, as 18 hundreds 14 tens 7 ones 2 tenths 4 hundredths or many other ways. <i>Example 1:</i> What number is represented by the decomposition shown below?	



Example 2: Decompose 2.34 in two different ways.

Possible Student Response: The student used base ten blocks to decompose 2.34.



MA.5.NSO.1.4 Compare two multi-digit numbers with decimals to the thousandths based on the values of the digits in each place using the symbols $<$, $=$, or $>$.

Remarks/Examples:

Remark 1: The terms greater than, less than or equal to may also be used in comparisons.

Example 1: What decimal can be used for n to make the statement below true?

$28.376 > n$

MA.5.NSO.1.5 Round multi-digit numbers with decimals to the thousandths to the nearest hundredth, tenth or whole number.

Remarks/Examples:

Remark 1: Students should understand that rounding is a process that produces a number with a similar value that is less precise but easier to use. For instance, when adding $234 + 689$ students could round the numbers to the nearest hundred to result in 200 and 700. Then students could determine the sum of 234 and 689 would be close to 900.

Remark 2: Students may have to find a range of numbers that round to a specific place value.

Example 1: Carey has a number that rounds to 5.6 when rounded to the nearest tenth. What could be Carey's number?

Example 2: Round 1.03 to the nearest tenth.

MA.5.NSO.2 Perform operations with multi-digit numbers and decimals.

MA.5.NSO.2.1 Multiply multi-digit whole numbers up to five digits by two digits using a variety of strategies, including the standard algorithm.

Remarks/Examples:

Remark 1: Students should build on their prior understanding to multiply efficiently, flexibly and accurately.

Example 1: Solve $45,087 \times 65$.

Possible Student Response: The student used the standard algorithm to multiply.

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 3 & & 5 & & 4 \\
 & & 2 & & 4 & & 3 \\
 & & 4 & 5 & 0 & 8 & 7 \\
 \times & & 6 & 5 \\
 \hline
 & & 2 & 2 & 5 & 4 & 3 & 5 \\
 + & 2 & 7 & 0 & 5 & 2 & 2 & 0 \\
 \hline
 2 & 9 & 3 & 0 & 6 & 5 & 5
 \end{array}
 \end{array}$$

Example 2: What is 3,425 multiplied by 32?

Possible Student Response: The student used an area model to multiply.

	3,000	400	20	5
30	90,000	12,000	600	150
2	6,000	800	40	10

$$90,000 + 12,000 + 600 + 150 + 6,000 + 800 + 40 + 10 = 109,600$$

MA.5.NSO.2.2

Divide multi-digit whole numbers up to five digits by two digits using a variety of strategies, including the standard algorithm. Represent remainders as fractional parts of the divisor.

Remarks/Examples:

Remark 1: Students should build on their prior understanding to divide efficiently, flexibly and accurately.

Example 1: Solve $63,274 \div 28$.

Possible Student Response: The student used the standard algorithm to divide.

$$\begin{array}{r} 2259 \text{ r } 22 \\ 28 \overline{) 63,274} \\ \underline{- 56} \\ 72 \\ \underline{- 56} \\ 167 \\ \underline{- 140} \\ 274 \\ \underline{- 252} \\ 22 \end{array}$$

Example 2: $4,567 \div 7$

Possible Student Response: The student used partial quotients to divide.

$$\begin{array}{r} 652 \frac{3}{7} \\ 7 \overline{) 4,567} \\ \underline{- 3,500} 500 \\ 1,067 \\ \underline{- 700} 100 \\ 367 \\ \underline{- 350} 50 \\ 17 \\ \underline{- 14} + 2 \\ 3 652 \end{array}$$

MA.5.NSO.2.3

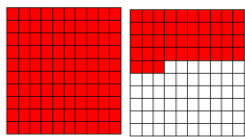
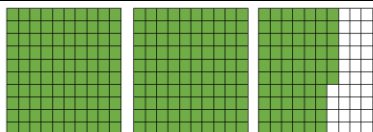
Add and subtract multi-digit numbers with decimals to the thousandths using a variety of strategies.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives, visual models and strategies based on place value, including, but not limited to, base ten blocks, hundred grids and number lines.

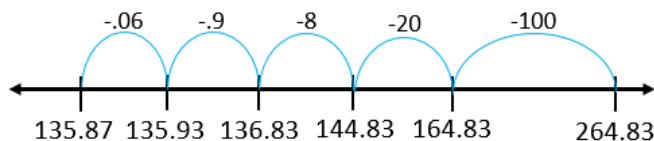
Example 1: $2.66 + 1.43$

Possible Student Response: The student used hundred grids to find the sum of 4.09.



Example 2: Find the difference. $264.83 - 128.96$

Possible Student Response: The student used a number line to subtract and find the difference.



MA.5.NSO.2.4

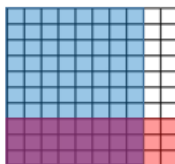
Multiply and divide multi-digit numbers with decimals to the hundredths using a variety of strategies.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives, visual models and strategies based on place value, including but not limited to base ten blocks, hundred grids and number lines.

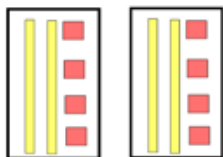
Example 1: 0.8×0.3

Possible Student Response: The student created a drawing on a hundredth charts to multiply.



$$0.8 \times 0.3 = 0.24$$

Example 2: What quotient does the model below represent?



Fractions

MA.5.FR.1 Interpret a fraction as division.

MA.5.FR.1.1

Interpret a fraction as division of the numerator by the denominator. Interpret and solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using visual fraction models or equations to represent the problem.

Remarks/Examples:

Remark 1: The intent of this benchmark is for students to relate fractions to division. Students should understand that fractions can also represent division of the numerator by the denominator.

Remark 2: Students are not required to simplify or use lowest terms.

Remark 3: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Example 1: What fraction is equivalent to $8 \div 3$?

Example 2: Ms. Clarke had 5 blocks of clay. She wanted to share them with 7 students. Write an expression to show how much clay each student would receive?

MA.5.FR.2 Perform operations with fractions.

MA.5.FR.2.1

Add and subtract fractions with unlike denominators, including mixed numbers and fractions greater than one, using a variety of strategies.

Remarks/Examples:

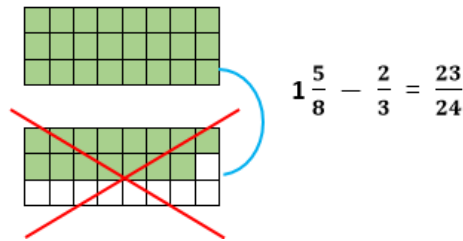
Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Remark 3: Items may require the use of equivalent fractions to find missing addends or part of a missing addend.

Example 1: $1\frac{5}{8} - \frac{2}{3}$

Possible Student Response: The student used an area model to subtract.



MA.5.FR.2.2

Solve word problems involving addition and subtraction of fractions with unlike denominators, including mixed numbers and fractions greater than one, using visual fraction models or equations to represent the problem.

Remarks/Examples:

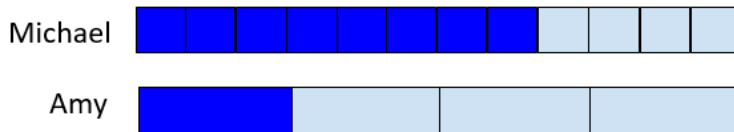
Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Remark 3: Items may require the use of equivalent fractions to find missing addends or part of a missing addend.

Remark 4: For examples of word problems, refer to the [Common Addition and Subtraction Situations](#).

Example 1: Mike and Amy each ate a fraction of a candy bar. The models are shaded to show the fraction of the candy bar each of them ate. What fraction of the candy bar did Michael and Amy eat together?



Example 2: The Bailey family started a trip with a tank $\frac{7}{9}$ full of gas. The family finished the trip with a tank $\frac{1}{3}$ full of gas. If no gas was added to the tank, how much gas was used for the trip?

MA.5.FR.2.3

Extend previous understanding of multiplication to multiply a fraction by a whole number or a fraction by a fraction using a variety of strategies.

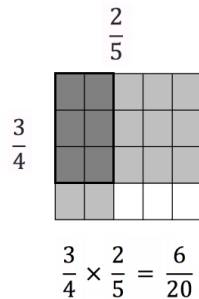
Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Example 1: Multiply $\frac{2}{5} \times \frac{3}{4}$

Possible Student Response: The student used an area model to multiply.



MA.5.FR.2.4

When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating.

Remarks/Examples:

Remark 1: Given a multiplication problem involving a fraction and whole number, students should be able to predict if the product will be greater or less than the number being multiplied. For instance, when multiplying $\frac{1}{2} \times 4$ students should be able to explain that the product will be less than 4 because $\frac{1}{2}$ is less than 1; when multiplying $\frac{3}{2} \times 4$ students should be able to explain that the product will be greater than 4 because $\frac{3}{2}$ is greater than 1.

Example 1: If I multiplied $\frac{3}{4}$ and 7, would the product be greater or less than 7? How do you know?

Example 2: What can you tell me about the product of $\frac{7}{6}$ and 2? Would the product be greater than or less than 2? Why?

MA.5.FR.2.5

Solve word problems involving multiplication of fractions, including mixed numbers and fractions greater than one, using visual fraction models or equations to represent the problem.

Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Remark 3: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Example 1: Boston is baking cookies. The recipe he is using requires $2\frac{6}{8}$ cup of sugar, but he is only making $\frac{1}{2}$ of the recipe. How much sugar does Boston need?

Possible Student Response: The student used a visual model to find that $\frac{11}{8}$ is the amount of sugar needed.



MA.5.FR.2.6

Extend previous understanding of division to divide a unit fraction by a whole number and a whole number by a unit fraction using a variety of strategies.

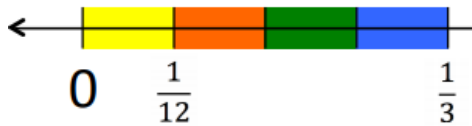
Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 for benchmark MA.3.FR.1.1 shows examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Example 1: What is the quotient $\frac{1}{3}$ divided by 4?

Possible Student Response: The student used a number line to state the quotient is $\frac{1}{12}$.



MA.5.FR.2.7

Solve word problems involving division of a unit fraction by a whole number and a whole number by a unit fraction using visual fraction models or equations to represent the problem.

Remarks/Examples:

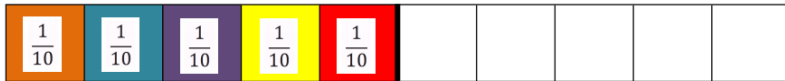
Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives and visual models. Remark 1 and 2 for benchmark MA.3.FR.1.1 show examples of visual fraction models.

Remark 2: Students are not required to simplify or use lowest terms.

Remark 3: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

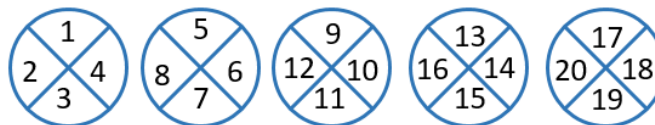
Example 1: Five people are going to run a relay race that is $\frac{1}{2}$ a mile long. If each person runs the same distance, how far will each person run?

Possible Student Response: The student used a visual model and stated each runner will run $\frac{1}{10}$ of a mile.



Example 2: How many $\frac{1}{4}$ bags of candy can you make from 5 pounds of candy?

Possible Student Response: The student used a visual model to determine 20 bags of candy can be made.



Measurement

MA.5.M.1 Convert measurement units to solve multi-step problems.

MA.5.M.1.1

Convert measurement units to equivalent measurements within a single system of measurement. Solve multi-step word problems using these conversions.

Remarks/Examples:

Remark 1: Students should understand how to convert units when provided with the conversions. Students are not required to memorize the conversions.

Remark 2: For examples of word problems, refer to the [Common Addition and Subtraction Situations](#) and the [Common Multiplication and Division Situations](#).

Example 1: Convert 12.5 centimeters to millimeters.

12.5 centimeters = _____ millimeters

Example 2: Raylynn is making 7 hair bows. She needs 9 inches of ribbon for each hair bow. How many feet of ribbon does Raylynn need to purchase for her hair bows?

Example 3: While playing a game at recess, Amar hopped three times. The distances he hopped are listed below.

- 75 centimeters
- 7.7 meters
- 83 centimeters

What is the total distance Amar hopped **in meters**?

MA.5.M.2 Find the area of rectangles with fractional side lengths.

MA.5.M.2.1	Find the area of a rectangle with fractional side lengths using visual models or a formula. Write an equation with a symbol for the unknown number to represent the problem.
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Remarks/Examples:

Remark 1: Students should understand there are two formulas for finding the area of rectangles and be able to apply both formulas. $A = l \times w$ or $A = b \times h$

Remark 2: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Remark 3: A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.

Example 1: Bodhi has a beach towel that is 3 feet long and $5\frac{4}{6}$ foot wide. What is the area of Bodhi's towel?

Example 2: A rectangle has the dimensions $\frac{2}{5}$ meter by $\frac{7}{12}$ meter. What is the area of the rectangle?

MA.5.M.3 Solve problems involving the volume of right rectangular prisms.

MA.5.M.3.1	Describe volume as an attribute of three-dimensional figures by packing them with unit cubes without gaps or overlaps. Find the volume of a right rectangular prism by counting unit cubes.
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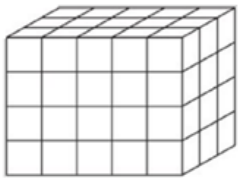
Remarks/Examples:

Remark 1: Emphasis should be placed on conceptual understanding through the use of manipulatives or drawings to find the volume of three-dimensional figures. Students should have opportunities to create three-dimensional figures using unit cubes and rearrange the unit cubes to understand how the dimensions impact the volume.

Remark 2: Students should understand a cube with side lengths of 1 unit is called a unit cube. The volume of a unit square is one cubic unit. Unit cubes can be used to find the volume of two-dimensional figures by packing the figures without gaps or overlaps. When viewing student work, look for misunderstandings, such as students using non-cubes as unit cubes or different sized cubes to find volume.

Example 1: Using centimeter cubes, how many different rectangular prisms can you create with a volume of 24?

Example 2: The rectangular prism shown below was packed with unit cubes without gaps or overlaps. What is the volume of the rectangular prism pictured below?



MA.5.M.3.2	Find the volume of a right rectangular prism with whole-number side lengths using visual models or a formula.
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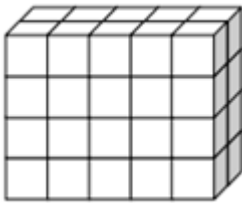
Remarks/Examples:

Remark 1: Students should be able to find the volume of right rectangular prisms by covering them with unit cubes and by applying the volume formula. The visual provided by unit cubes should lead students to the application of the volume formula.

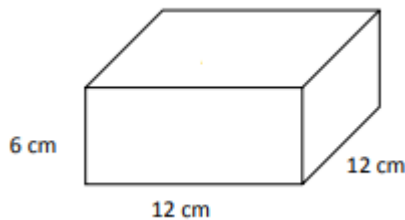
Remark 2: Students should understand there are two formulas for finding the volume of right rectangular prisms and be able to apply both formulas. $V = l \times w \times h$ or $V = B \times h$

Remark 3: Student responses should include the appropriate units. Students are not expected to use the exponent form of cubic units.

Example 1: The right rectangular prism is built from centimeter cubes. State the dimensions of the prism and find its volume.



Example 2: Find the volume for the right rectangular prism.



MA.5.M.3.3

Solve problems involving the volume of right rectangular prisms with whole number side lengths using visual models or a formula. Write an equation with a symbol for the unknown to represent the problem.

Remarks/Examples:

Remark 1: Problems may have unknowns in any position. For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Remark 2: Student responses should include the appropriate units. Students are not expected to use the exponent form of cubic units.

Remark 3: A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.

Example 1: A gardener is building a right rectangular prism shaped planter for his flowers. He wants the planter to have a volume of 324 inches. If the height of the planter is 6 inches and the width is 9 inches, what would be the length of the planter?

Example 2: Sally is filling a rectangular prism with unit cubes. The base is filled by 16 unit cubes. If the height of the prism takes 5 layers of unit cubes to fill, what is the volume of the prism?

MA.5.M.3.4

Solve problems involving the volume of composite figures composed of non-overlapping right rectangular prisms with whole number side lengths. Write an equation with a symbol for the unknown to represent the problem.

Remarks/Examples:

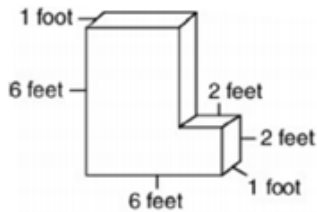
Remark 1: Composite figures are figures that can be divided into more than one figure. In order for students to be able to determine the volume of composite figures, the figures must be composed of non-overlapping right rectangular prisms.

Remark 2: Student responses should include the appropriate units. Students are not expected to use the exponent form of cubic units.

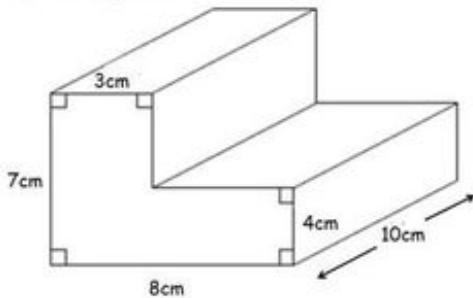
Remark 3: For examples of word problems, refer to the [Common Multiplication and Division Situations](#).

Remark 4: A variety of symbols may be used to represent the unknown such as, but not limited to, a box or question mark.

Example 1: The diagram below shows a custom swimming pool made up of rectangular prisms. What is the total volume of the swimming pool? Write an equation to represent the problem.



Example 2: Shane has a pet spider who needs 400 cubic centimeters of space. He bought a cage and wants to know if the cage will provide enough space for his spider. A model of the cage is below. Will the cage he bought have enough space for the spider?



Geometric Reasoning

MA.5.GR.1 Identify and classify three-dimensional figures based on defining attributes.

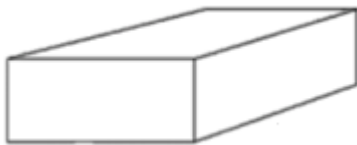
MA.5.GR.1.1	Identify and classify three-dimensional figures into categories based on their defining attributes. Three-dimensional figures are limited to pyramids, prisms, cones, cylinders, and spheres.
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Remarks/Examples:

Remark 1: Students should use the defining attributes of the number of bases, the shape of the bases, the number of faces, and the shape of the faces to identify and classify three-dimensional figures. For instance, a pyramid is a three-dimensional figure with one rectangular base.

Example 1: Margo said she is holding a cylinder because it has two bases that have a rectangular shape. Is she correct? How do you know?

Example 2: This shape is a rectangular prism. What makes it a rectangular prism?



MA.5.GR.2 Plot points and represent problems on the coordinate plane.

MA.5.GR.2.1

Identify the origin and axes in the coordinate system. Plot and label ordered pairs in the first quadrant of the coordinate plane.

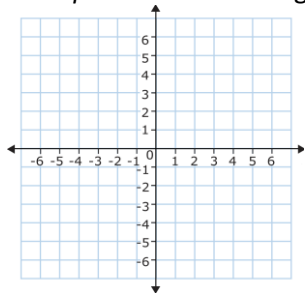
Remarks/Examples:

Remark 1: Students should understand a coordinate plane is made up of a pair of perpendicular lines called axes with the intersection of these lines occurring at the origin, the zero on each line. The horizontal line of the coordinate plane is called the x-axis and the vertical line the y-axis.

Remark 2: Students should understand that when graphing points on the coordinate plane coordinates in the form of ordered pairs are used. The first number in an ordered pair indicates how far to travel from the origin across the x-axis and the second number indicates how far to travel from the origin on the y-axis. The ordered pair (0, 0) refers to the origin.

Remark 3: Coordinate planes should have axes scaled to whole numbers. Points and ordered pairs should contain only positive values.

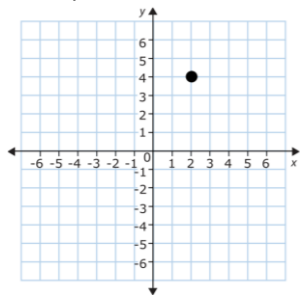
Example 1: Label the origin, the x-axis and the y-axis on the coordinate plane.



Example 2: Plot the following ordered pairs on a coordinate plane.

(2, 5), (1, 4), (4, 1), (3, 3)

Example 3: What are the coordinates of the point shown below?



MA.5.GR.2.2

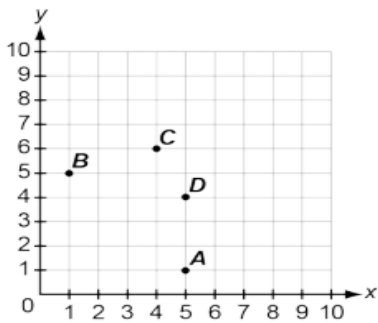
Represent real world and mathematical problems by plotting points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Remarks/Examples:

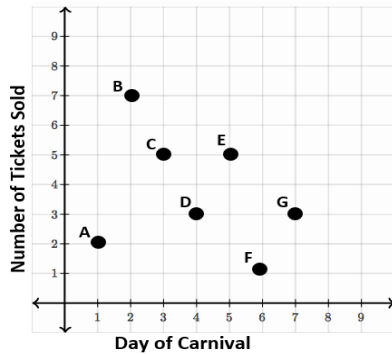
Remark 1: Coordinate planes should have axes scaled to whole numbers. Points and ordered pairs should contain only positive values.

Example 1: Point B is four units away from the origin and is on the y-axis. What could be the coordinates of point B?

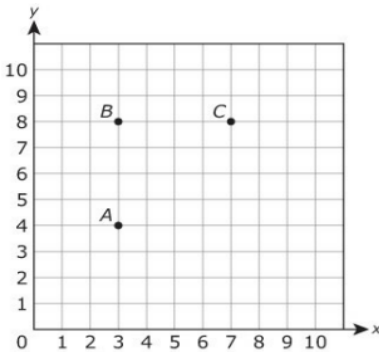
Example 2: If Sofia is at the origin, describe how she would move along the coordinate plane to get to point C.



Example 3: The coordinate plane shows how many tickets Jamal sold each day of the carnival. What does point D represent?



Example 4: Mark is drawing a square on the coordinate plane. He has placed three of the vertices for the square at points A, B, and C as shown below. At what ordered pair should Mark plot point D to complete his drawing of a square?



Statistics and Probability

MA.5.SP.1 Collect, represent and interpret data and find the mode, median and range of a data set.

MA.5.SP.1.1

Collect and represent numerical data, including fractional values, using tables, scaled pictographs, scaled bar graphs or line plots. Solve multi-step problems involving addition, subtraction and multiplication using data from these representations.

Remarks/Examples:

Remark 1: Numerical data are quantitative values, such as a person's height, the number of teeth a dog has or how many pages were read in a book.

Example 1: Claire studied the amount of water in different glasses. The data she collected is below. Use her data to create a line plot to show the amount of water in the glasses.

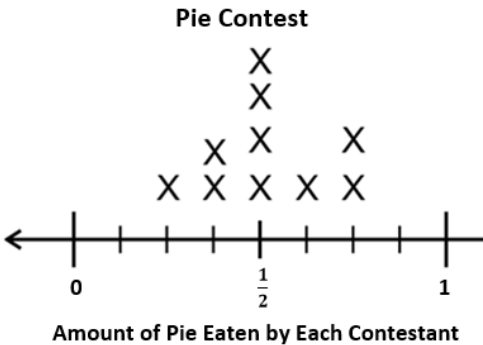
A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** $\frac{1}{8}$ **D** $\frac{1}{4}$ **E** $\frac{1}{4}$
F $\frac{5}{8}$ **G** $\frac{3}{8}$ **H** $\frac{5}{8}$ **I** $\frac{1}{8}$ **J** $\frac{1}{8}$

MA.5.SP.1.2	Determine the mode, median or range to interpret numerical data, including fractional values, represented with tables, scaled pictographs, scaled bar graphs or line plots. Limit denominators to 1 to 20.
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Remarks/Examples:

Remark 1: Numerical data are quantitative values, such as a person’s height, the number of teeth a dog has or how many pages were read in a book.

Example 1: There was a pie eating contest at the county fair. The line plot below shows the fraction of a pie each of the 10 contestants ate. Use the line plot to determine the mode, median and range of the data.



COMMON ADDITION AND SUBTRACTION SITUATIONS

The four unshaded situation types are expectations for Kindergarten students. Grade 1 and 2 students should work with

	Result Unknown	Change Unknown	Start Unknown
Add To	Three birds sat on a wire. Two more birds landed next to them. How many birds are on the wire now? $3 + 2 = ?$	Three birds sat on a wire. Some more birds landed next to them. Then there were five birds on the wire. How many birds landed on the wire next to the first three? $3 + ? = 5$	Some birds were sitting on a wire. Two more birds landed there. Then there were five birds. How many birds were on the wire to start? $? + 2 = 5$
Take From	Five snacks were on the table. Three snacks were eaten. How many snacks are on the table now? $5 - 3 = ?$	Five snacks were on the table. Some snacks were eaten. Then there were two snacks on the table. How many snacks were eaten? $5 - ? = 2$	Some snacks were on the table. Then three snacks were eaten. Now there are two snacks left on the table. How many snacks were on the table at the start? $? - 3 = 2$
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together	Three purple pens and two red pens were in the box. How many pens are in the box? $3 + 2 = ?$	Five pens are in the box. Three of them are purple, the rest are red. How many pens are red? $3 + ? = 5$ $5 - 3 = ?$	Jennifer has five pens. How many of them could be purple and how many of them could be red? $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	More: Jim has two pens. Keisha has five pens. How many more pens does Keisha have than Jim? Fewer: Jim has two pens. Keisha has five pens. How many fewer pens does Jim have than Keisha? $2 + ? = 5$ $5 - 2 = ?$	More: Keisha has three more pens than Jim. Jim has two pens. How many pens does Keisha have? Fewer: Jim has three fewer pens than Keisha. Jim has two pens. How many pens does Keisha have? $2 + 3 = ?$ $3 + 2 = ?$	More: Keisha has three more pens than Jim. Keisha has five pens. How many pens does Jim have? Fewer: Jim has three fewer pens than Keisha. Keisha has five pens. How many pens does Jim have? $5 - 3 = ?$ $? + 3 = 5$

all situation types. Darker shading indicates the four most difficult types that students should work with in Grade 1 but not need master until Grade 2.

¹The both addends unknown situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the “=” sign does not always mean *makes or results in* but always does mean *is the same value as*.

COMMON MULTIPLICATION AND DIVISION SITUATIONS

	Unknown Product	Group Size Unknown (Partitive or Fair Shares Division)	Number of Groups Unknown (Quotitive or Measurement Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ $18 \div 3 = ?$	$? \times 6 = 18$ $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example:</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example:</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example:</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays and Area	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example:</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example:</i> A rectangle has an area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example:</i> A rectangle has an area of 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Multiplicative Comparisons	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example:</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does the blue hat cost?</p> <p><i>Measurement example:</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example:</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>

The situations increase in difficulty when moving from the top of the page to the bottom and from left to right across the page. Students in grade 3 should work with all situation types but need not master the multiplicative comparisons until grade 4.

The first example in each cell include discrete items, which are easier for students. Students should be exposed to these situations before the measurement examples.

Grade 6

Number Sense and Operation

MA.6.NSO.1 Extend knowledge of numbers to negative numbers and develop an understanding of absolute value.

MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Represent quantities using positive and negative rational numbers and compare them on a number line. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: A rational number is a number that can be expressed as a quotient, or fraction, of two integers.

Remark 2: While students should be exposed to rational numbers in different forms, students should not be expected to compare a fraction to a decimal. However, the comparison of the fraction to a decimal and vice versa could be an extension of this benchmark.

Remark 3: Students should be expected to make informal verbal comparison of rational numbers using greater than and less than and it is smaller because _____ or it is larger because _____.

Example 1: New Orleans, Louisiana has an altitude of about $-6\frac{1}{2}$ feet and Miami, Florida has an altitude of about $6\frac{3}{5}$ feet. Compare the two altitudes by plotting them on a number line.

Example 2: The US Navy has two submarines named Seahorse and Flounder. The Seahorse is currently at 42.8 meters below sea level and the Flounder is currently at 64.3 feet below sea level. Write numbers that describe the position of the submarines in terms of sea level and draw a diagram to show the position of each relative to sea level.

MA.6.NSO.1.2	Plot positive and negative rational numbers on a number line. Make comparisons of the two numbers using inequality symbols.
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Remarks/Examples:

Remark 1: An inequality is a relation that holds between two quantities. Both strict inequality symbols ($<$ and $>$) and inequality symbols (\geq and \leq) should be introduced to students. In words, $<$ reads “less than”, $>$ reads “greater than”, \leq reads “less than or equal to” and \geq reads “greater than or equal to”.

Remark 2: While students should be exposed to rational numbers in different forms, students should not be expected to compare a fraction to a decimal. However, the comparison of the fraction to a decimal and vice versa could be an extension of this benchmark.

Example 1: Plot and label each of the following numbers on a number line: -3.4 , 2.7 , -7 and 1 .

Example 2: Write a comparison using an inequality of $-\frac{1}{6}$ and $-\frac{1}{3}$.

Example 3: Plot and write a comparison of -5 and 5 .

MA.6.NSO.1.3	Given a real-world situation, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.
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Remarks/Examples:

Remark 1: Absolute value is the magnitude of a real number without regard to its sign; which can be thought of as the distance away from zero. Absolute value is denoted by $|x|$ and reads “the absolute value of x ”.

Remark 2: Problem situations should include distances, temperature, differences and financials.

Example 1: What is the value of the expression $|- \frac{7}{8}|$?

Example 2: If the temperature in Chicago, IL is -7° , how many degrees below zero is the temperature?

Example 3: What is the value of the expression $|12.75|$?

Example 4: Plot 4 , -4 and 0 on the same number line. Compare 4 and -4 in relation to 0 .

MA.6.NSO.1.4	Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.				
Remarks/Examples:					
<i>Remark 1:</i> Students should be exposed to perform operations with absolute value, but should not be expected to perform more than two operations. Problems situations should include positive and negative numbers involving temperature, elevation, ions, banking, etc.					
<i>Example 1:</i> Compare $ \frac{-125}{4} $ and $ \frac{67}{2} $ using and inequality symbol.					
<i>Example 2:</i> The Philippine Trench is located 10,540 meters below sea level and the Tonga Trench is located 10,882 meters below sea level. What trench has the higher altitude?					
<i>Example 3:</i> What is the value of the expression $7 - -3 $?					
<i>Example 4:</i> The table below shows the change in rainfall for each month from the month average. Find the absolute value of each month and determine which month had the greatest change in rainfall.					
Month	March	April	May	June	July
Change in Rainfall Amount from Monthly Average (inches)	0.21	-1.64	-0.48	2.01	-2.30

MA.6.NSO.2 Add, subtract, multiply and divide positive rational numbers.	
MA.6.NSO.2.1	Solve multi-step problems involving addition, subtraction, multiplication or division of positive multi-digit decimals. Include mathematical and real-world contexts.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students have previous knowledge of operations with positive multi-digit decimals and should become fluent in grade 6. <i>Remark 2:</i> Within grade 6, students should not be expected to perform operations on decimals with more than 5 total digits. <i>Example 1:</i> Tina's SUV holds 18.5 gallons of gasoline. If she has 4.625 gallons in her car when she stops to fill it up. How much money will she spend to fill up her car if the current price for gas is \$2.57 per gallon? <i>Example 2:</i> What is the value of the expression $13.31 \div 0.125$?	
MA.6.NSO.2.2	Extend previous understanding of multiplication and division to compute quotients of positive fractions by positive fractions including mixed numbers using a variety of strategies.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Instruction should emphasize the conceptual understanding of division of fractions. Students should have practice with various strategies including manipulatives, drawings, area model, linear model, properties of operations and standard algorithm. Students can also use knowledge of common denominators to rewrite fractions and divide the numerators to find the quotient. <i>Remark 2:</i> This benchmark requires the building of conceptual understanding of how students multiply and divide fractions. This should include a progression of models to get students to understand where the standard algorithm comes from. <i>Example 1:</i> Divide $\frac{2}{3}$ by $\frac{1}{4}$. <i>Possible student response:</i> In order to divide $\frac{2}{3}$ by $\frac{1}{4}$, a student may reason that $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$. So, $(\frac{2}{3}) \div (\frac{1}{4})$ is equivalent to $(\frac{8}{12}) \div (\frac{3}{12})$, which gives the same result as $8 \div 3 = 2\frac{2}{3}$.	

<p><i>Example 2:</i> Jasmine wants to build a $2\frac{5}{6}$ meter long garden path paved with square stones that measure $\frac{1}{4}$ meter on each side. There will be no spaces between the stones. Determine how many stones are needed for a path.</p> <p><i>Example 3:</i> One-half of your yard is garden. One-fourth of your garden is a vegetable garden. What fraction of your yard is a vegetable garden? Draw an area model and write a number sentence that both describe the problem and solution.</p>	
MA.6.NSO.2.3	Solve mathematical and real-world problems involving division of positive fractions by positive fractions, including mixed numbers.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> This benchmark is intended to build a student's understanding of multiplication and division of fractions in real-world and mathematical context.</p> <p><i>Example 1:</i> How many quarter-pound hamburgers can be made from $3\frac{1}{2}$ pounds of ground beef?</p> <p><i>Example 2:</i> How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt?</p>	

MA.6.NSO.3 Use properties of operations to rewrite numbers in equivalent forms.	
MA.6.NSO.3.1	Find the greatest common factor and least common multiple of two whole numbers. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> The greatest common factor (GCF) is the largest factor shared between two or more numbers. The least common multiple (LCM) is the smallest number that is a multiple of two or more numbers.</p> <p><i>Remark 2:</i> Problem types for GCF should include splitting things into smaller groups, equally distribute two or more things into a larger grouping or to determine how many people can attend an invite. Problem types for LCM should include events that repeat, determining multiple items to have enough and determining when something may happen at some time.</p> <p><i>Example 1:</i> Mr. Davis, the band director, wants to create groups with the same number of people who play the flute, clarinet and violin for the upcoming winter concert. Mr. Davis has 15 people who play the flute, 27 people who play the clarinet and 12 people who play the violin. How many groups can Mr. Davis create with the same number of instruments in each group?</p> <p><i>Possible student response:</i> A student can use GCF to determine equally distribute the number of people evenly. The student will find the GCF of 15, 27 and 12, which is 3, so Mr. Davis can form 3 groups each with 5 people who play the flute, 9 people who play clarinet and 4 people who play violin.</p> <p><i>Example 2:</i> Parker reads a book every 12 days and Leah reads a book every 8 days. If both Parker and Leah read a book today, how many days will it be until they read a book on the same day again?</p> <p><i>Possible student response:</i> A student can use LCM to determine the soonest time this event will occur again. The student will find the LCM of 8 and 12, which is 24, to say that Leah and Parker will read again on the same day 24 days from today.</p> <p><i>Example 3:</i> What is the greatest common factor between 36 and 132?</p> <p><i>Example 4:</i> What is the least common factor between 10 and 12?</p>	
MA.6.NSO.3.2	Generate equivalent numerical expressions by rewriting the sum of two composite whole numbers as a common factor multiplied by the sum of two whole numbers.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> This benchmark develops student understanding and the foundation for the distributive property. This supports the decomposition of numbers in earlier grades and extends to future learning in algebraic reasoning in future grade levels.</p>	

Remark 2: A composite number is a positive integer that has at least a divisor other than one and itself.

Remark 3: Students should not be using the multiplication sign “ \times ” when rewriting composite numbers as a common factor multiplied by the sum of two whole numbers. In elementary school, students have seen the multiplication sign “ \times ” when using the distributive property. Students should move away from this practice in sixth grade.

Example 1: Rewrite the following numerical expression in an equivalent form using the distributive property: $24 + 36$.

Possible student response: A student can find a common factor between 24 and 36 as 12 and rewrite the expression as $12(2) + 12(3)$. The student can then use the distributive property to write the equivalent expression as $12(2 + 3)$.

MA.6.NSO.3.3	Evaluate positive rational numbers with whole number exponents.
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Remarks/Examples:

Remark 1: An exponent is a quantity representing the power in which the base, a number or quantity, is to be used as a factor.

Remark 2: Students should be exposed to taking whole number exponents of decimals, fractions and whole numbers.

Example 1: What is the value of the expression $\left(\frac{1}{3}\right)^3$?

Example 2: What is the value of the expression 2^5 ?

Example 3: What is the value of the expression 0.1^3 ?

MA.6.NSO.3.4	Express composite whole numbers as a product of prime factors with whole number exponents.
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Remarks/Examples:

Remark 1: A composite number is a positive integer that has at least a divisor other than one and itself. A prime number is a number greater than one whose only factors are one and itself.

Remark 2: Students can use a variety of use to determine the prime factors including prime factorization or using a factor tree.

Remark 3: Students should start to use the multiplication symbol of “ \cdot ”. This will allow students in sixth grade to start recognizing the difference between the variable x and representing multiplication.

Example 1: Determine all of the factors of 24. Rewrite 24 as a product of its factors using exponents.

Possible student response: A student could say that $24 = 4 \cdot 6$. The student can then say that $4 = 2 \cdot 2$ and $6 = 2 \cdot 3$. Therefore, $24 = 2 \cdot 2 \cdot 2 \cdot 3$ which can be expressed with exponents as $24 = 2^3 \cdot 3$.

Example 2: Determine all of the factors of 216. Rewrite 216 as a product of its factors using exponents.

MA.6.NSO.4 Develop an understanding of operations with integers.

MA.6.NSO.4.1	Apply and extend previous understandings of operations with whole numbers to add and subtract integers using visual or numerical representations.
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Remarks/Examples:

Remark 1: Instruction should emphasize the conceptual understanding of addition and subtraction of integers. Students should have practice with visual representations including two color counters, algebra tiles, vertical and horizontal number lines, and (+) (-) signs.

Remark 2: Students should gain an understanding that subtraction of rational numbers is adding the additive inverse, $p - q = p + (-q)$.

Example 1: Model one method for evaluating $-5 + 8 - (-2)$. Explain your work.

Possible student response: A student can begin with a group of 5 negative integer chips, and then combine it with another group (add) of 8 positive integer chips, and then remove (subtract) two negative integer chips.

The student can then remove three “zero pairs” (one positive and one negative integer chip), since $-1 + 1 = 0$. This will result in the answer of 5.

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Example 2: What is the value of the expression $9 + -12$? Use a number line to explain your reasoning.

MA.6.NSO.4.2	Apply and extend previous understandings of operations with whole numbers to multiply and divide integers using visual or numerical representations.
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Remarks/Examples:

Remark 1: Instruction should emphasize the conceptual understanding of multiplication and division of integers. Students should have practice with visual representations including two color counters, algebra tiles, vertical and horizontal number lines and (+)(-) signs.

Remark 2: Students should gain an understanding that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Students should also gain an understanding that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$.

Example 1: Model one method for evaluating -4×5 . Explain your work.

Possible student response: A student can begin with a number line and state that there are 5 groups of -4 . The student can then use the number line to illustrate the 5 groups of -4 by starting at 0 and drawing “jumps” to $-4, -8, -12, -16$ and ending at -20 .

Example 2: What is the value of the expression $-72 \div 8$? Use a model to explain your reasoning.

Algebraic Reasoning

MA.6.AR.1 Apply previous understanding of arithmetic expressions to algebraic expressions.

MA.6.AR.1.1	Translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions. Include mathematical and real-world context.
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Remark 1: An algebraic expression is built from integer constants, variables and algebraic operations.

Example 1: Rewrite the written description as an algebraic expression: 7 more than a number.

Example 2: Rewrite the algebraic expression as a written description: $10 - \frac{6}{x}$.

Example 3: The amount of money you have left after going to the mall could be described by the algebraic expression $75 - 12.75s - 9.50d$, where s is the number of shirts you purchased and d is the number of dresses purchased. Write a written description describing the situation.

MA.6.AR.1.2	Given a mathematical and real-world scenario, write an algebraic inequality from graphical representations and/or represent solutions of an inequality on a number line in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$.
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Remarks/Examples:

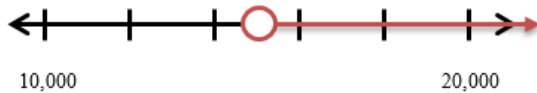
Remark 1: Instruction should emphasize the understanding of defining an algebraic inequality both numerically and graphically. Students should explore how “greater than or equal to” and strictly “greater than” are similar and different. Students should understand whether the number will be shown as a closed or an open dot on the number line and how to represent the other numbers it could represent through shading on the number line. A number line is a useful tool for modeling situations and inequalities such as “You have to be at least 40 inches tall to ride a roller coaster.”

Remark 2: Students should be able to read the inequality sign as greater than, less than, greater than or equal to, and less than or equal to. Students should be exposed to inequalities where the variable is on the left and right side of the inequality symbol.

Example 1: Graph $x > -3$ on a number line.

Example 2: Sarah is allowed to play video games no more than 4 hours over the weekend. Graph the inequality on a number line.

Example 3: The graph below describes the altitudes at which civil aircraft must provide everyone with supplemental oxygen, according to the US Federal Aviation Regulation. Write an inequality that represents this graph.



MA.6.AR.1.3	Evaluate algebraic expressions or equations, including formulas, using substitution. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Substitution is an algebraic operation in which a symbol can be replaced by a given value. An algebraic expression is built from integer constants, variables and algebraic operations whereas an equation is the statement of two equivalent expressions. In mathematics, a formula is defined by a fact or rule that uses mathematical symbols and operations.

Example 1: A car travels at an average speed of 75 miles per hour in 20 minutes. Using the formula, $d = st$ where d is distance, s is speed, and t is time, what is the distance traveled by the car?

Example 2: If $x = 3$, find $3x + 8$.

Example 3: An equation is shown. $x + 14 = 33$ What value of x can be substituted into the equation to make it true?

Example 4: The pressure exerted by a solid object on a solid surface can be calculated by using the formula, $P = \frac{F}{A}$, where the variables P , F , and A represent pressure, force, and area respectively. A newly refinished wood floor can withstand a pressure of up to 40 pounds per square inch without sustaining damage. A 120 pound woman with high heels and a 240 pound man with flat heels each enter this room. Assume that at some point all of their weight is supported equally by the heels of both of their shoes. Given that each of the woman's heels occupies an area of 0.25 in² and each of the man's heels occupies an area of 12 in², find out each person's potential for causing damage to the wood floor. Justify your answer.

MA.6.AR.1.4	Given an equation or inequality and a specified set of values, including integers, determine which values are solutions.
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Remarks/Examples:

Remark 1: Instruction should emphasize the conceptual understanding that solving an equation or inequality is a process of answering the question: which values from a specified set, if any, make the equation or inequality true? Students should also gain an understanding that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Remark 2: Students should be exposed to using set notation to list numbers, but not be expected to writing solutions in set notation. Students should also be exposed to equations where the same variable is in more than one term or on both sides of the equation or inequality.

Example 1: Which of the following solutions make the equation $3x + 8 = 14$ true? {2, 3, 4, 5, 6}

Example 2: Which of the following solutions make the inequality $4x + 5 > 25$ true? {2, 3, 4, 5, 6}

Example 3: Which of the following values from the list make the inequality $6x - 2 > -25$ false (and are not solutions)? {2, 3, -4, -5, -6}

MA.6.AR.2 Write and solve one-step equations.	
MA.6.AR.2.1	Write and solve one-step equations in one variable using addition and subtraction, where all terms and solutions are integers. Include mathematical and real-world context.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be exposed to the forms $x + p = q$ and $x - p = q$. When students write equations of the form $x + p = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems. <i>Remark 2:</i> Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand. Additionally, problem types should include patterns, model and relationships. <i>Example 1:</i> Alex has some money in his wallet. His grandmother gives him \$10 for a gift in his birthday card. He now has \$28 in his wallet. Write an equation to represent the problem. How much money did he originally have in his wallet? <i>Example 2:</i> Given $x + 15 = 3$, what is the value of x ? <i>Example 3:</i> Given $6 = x - 13$, what is the value of x ?	
MA.6.AR.2.2	Write and solve one-step equations in one variable using multiplication and division, where all terms and solutions are integers. Include mathematical and real-world context.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should see the forms $\frac{p}{x} = q$ and $px = q$. When students write equations of the form $px = q$ to solve real-world and mathematical problems, they draw on meanings of operations that they are familiar with from previous grades' work. They also begin to learn algebraic approaches to solving problems. <i>Remark 2:</i> Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand. Additionally, problem types should include patterns, model and relationships. <i>Example 1:</i> An outlet mall has 4 identical lots that can hold a total of 1,388 cars. The equation $4c = 1388$ describes the number of cars that can fit into each lot. How many cars can fit into each lot? <i>Example 2:</i> Given $\frac{x}{7} = -56$, what is the value of x ? <i>Example 3:</i> A solar panel can generate $\frac{8}{25}$ of a kilowatt of power. The average store needs to generate about 30 kilowatts of power. Write an equation to determine how many solar panels a store needs on its roof. How many solar panels does a store need?	

MA.6.AR.3 Understand ratio and unit rate concepts and use them to solve problems.	
MA.6.AR.3.1	Write and interpret ratios to show the relative sizes of two quantities using appropriate notations: a/b, a to b, or a:b where $b \neq 0$. Describe the relationship between two quantities using ratios and rates. Include mathematical and real-world context.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Instruction should emphasize the understanding of the concept of a ratio and its similarities to a fraction and division. Students should gain an understanding of a unit rate and how it is associated to a ratio. <i>Remark 2:</i> A ratio is the quantitative relationship between two numbers indicating how many times, or a comparison of, the first number contains the second. A rate is a ratio between two related quantities in different measurements. <i>Example 1:</i> Ana and Robbie both stayed after school for help on their math homework. Ana stayed for 15 minutes and Robbie stayed for 50 minutes. Write a ratio to represent the relationships between the time that Ana stayed for help and the time that Robbie stayed for help.	

Example 2: Leslie and Sabrina are both running for class president. If for every two votes Leslie receives, Sabrina receives five. Describe the relationship between the number of votes Leslie receives to the number of votes Sabrina receives as a ratio.

MA.6.AR.3.2	Calculate and interpret unit rates associated with ratios of fractions or decimals, including ratios of lengths and other quantities measured in the same measurement system. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: A unit rates describes the rate in lowest terms, or when the first quantity is compared to one unit of the second quantity.

Remark 2: Students should not be expected to convert between customary and metric systems.

Example 1: The Jones family is planning on expanding their garden so that they can plant more vegetables. The ratio of the area of the old garden to the area of the new garden is $4\frac{1}{4} : 8\frac{3}{4}$. Convert this ratio to a unit rate and explain what it means in this context.

Example 2: At the grocery store, you paid \$9.87 for 3.3 pounds of apples. What is the unit price paid per pound of apples?

MA.6.AR.3.3	Given a table of values, determine the equivalent ratio between any two pairs of values and find any missing values. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Instruction should emphasize the foundation for linear functions and slope, or the constant of proportionality.

Example 1: Given the table below, determine the common ratio between the x- and y-values.

x	y
2	8
5	20

Example 2: The table below describes the same rate in which Drew hiked along the Appalachian Trail each day. Complete the table to determine how many miles he could hike in 2 hours.

Time (hours)	5		2
Distance (miles)	20	12	

MA.6.AR.3.4	Apply ratio relationships to solve problems involving percentages using the relationship between the two quantities. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should be gain understanding that percent can be represented at percent (%) of part/whole and that this relationship can be used to find any missing aspect of percent, part, and whole.

Example 1: Find the percent of $\frac{60}{75}$.

Example 2: 15% of 80 is what?

Example 3: Benson is trying to gain muscle and keeps track of the amount of protein every day. According to the label on his protein shake, one serving contains 27 grams of protein. If 27 grams is 36% of the recommended daily amount, how many grams of protein are recommended for the whole day?

MA.6.AR.3.5	Solve mathematical and real-world problems involving ratios and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.
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Remarks/Examples:

Remark 1: Students should be exposed to a variety of problem types including comparison, mixtures, lengths and conversations. Students should not be expected to convert between customary and metric systems.

Remark 2: Students should not be expected to solve problems using proportions, however, problem types should lend to the foundation of proportions and proportional relationships.

Example 1: Marissa drove 770 miles in two days to visit her friend. On the first day, she drove 8 hours at an average speed of 55 miles per hour. She continued to drive at the same rate on the second day. How many hours did Marissa drive the second day?

Example 2: A recent study found that parking lots for offices should have a ratio of 6 spaces for every 1000 square feet of floor space. If a new office building has 19,000 square feet of floor space, how many spaces should there be in the parking lot?

Example 3: Jessica made 8 out of 24 free throws. Bob made 5 out of 20 free throws. Who has the highest free throw ratio?

Statistics and Probability

MA.6.SP.1 Develop an understanding of probability and find experimental and theoretical probabilities.

MA.6.SP.1.1 Determine the sample space for a single event.

Remarks/Examples:

Remark 1: Sample space for a single event is when you determine what is the expected outcomes are within probability. For example, consider tossing a coin, we will get single event (either head or tail) as expected result. Probability situations can include flipping a coin, rolling a fair die, picking a card from a deck, spinning a spinner, etc.

Example 1: What is the sample space of rolling a fair 6-sided die?

MA.6.SP.1.2 Given the probability of a chance event, interpret the likelihood of it occurring.

Remarks/Examples:

Remarks: Probability is the ratio of the number of ways an event can happen to the total number of outcomes. Instruction should emphasize the understanding that the probability of a chance event is a number between 0 and 1 which expresses the likelihood of the event occurring. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Remark 2: Students should have practice expressing and seeing probability as a fraction, percentage or decimal.

Example 1: Weather forecasters use a TORCON measure to determine the likelihood of a tornado for a certain area. With an incoming thunderstorm, the Tampa area has a TORCON of $\frac{2}{10}$ while the Orlando area has a TORCON of $\frac{4}{5}$. Which area is less likely to have a tornado with the incoming thunderstorm?

Example 2: Johnny has a $\frac{2}{3}$ chance of making a goal. Brian has a less likely chance of making a goal. What probability could represent Brian's chance?

MA.6.SP.1.3 Given the scenario of a simple event, find the theoretical probability.

Remarks/Examples:

Remark 1: Simple events are events in which one experiment happens at a time and will have a single outcome. When discussing events, theoretical probability is what is expected to happen while experimental probability is what actually happens.

Remark 2: Students should have practice expressing and seeing probability as a fraction, percentage or decimal.

Example 1: Each letter in the word TALLAHASSEE is written on a piece of paper and put into a hat. What is the probability that the letter “A” will be picked from the hat?

Example 2: Skip has movie collection that includes 7 Star Wars movies, 14 Marvel movies and 9 X-Men movies. What is the probability that he chooses a Marvel movie at random? Express the probability as a percentage.

MA.6.SP.2 Develop an understanding of statistics and determine measures of center and measures of variability. Summarize statistical distributions graphically and numerically.

MA.6.SP.2.1 Recognize and formulate a statistical question.

Remarks/Examples:

Remark 1: Students should gain the understanding that a statistical question is one that anticipates variability. Instruction should emphasize that a statistical question is one in which multiple answers are expected through data collection.

Example 1: Select all the questions that are statistical.

- ☐ How many hours in one day do students spend playing video games?
- ☐ How far does our 6th grade teacher live from school?
- ☐ How many students in my class have a cell phone?
- ☐ What are 6th graders favorite lunch item?
- ☐ How many pets do the students in class have?

MA.6.SP.2.2

Describe the spread of a given data set, represented numerically or graphically, and summarize it by determining a measure of center or a measure of variation. Measure of center is limited to mean and median. Measure of variation is limited to range. Include mathematical and real-world context.

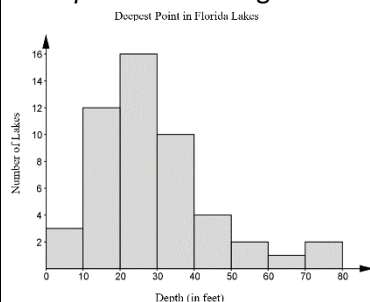
Remarks/Examples:

Remark 1: When describing distributions, skewed right, or positive skewness, refers to a graph with a tail on the right side of the distribution and skewed left, or negative skewness, refers to the graph with a tail on the left side of the distribution. If the distribution has a bell-shape, it is said to have a normal, or symmetric, distribution. If the distribution has the same frequency in all intervals, it is said to have a uniform, or rectangular distribution.

Remark 2: Instruction should emphasize that a measure of center for a data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. Students should have practice with numerical data and graphical representations including histograms, dot plots and stem-and-leaf plots.

Remark 3: Students should work with positive rational numbers in this standard.

Example 1: The histogram below the results of a study on the depth of lakes in Florida. Describe the distribution.



Example 2: The table shows the approximate number of minutes of physical exercise each student in Noah’s class gets per day. What is the median number of minutes of exercise students get per day in Noah’s class?

Time per Day (minutes)	0	15	20	30	45	60	90	120
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Number of Students	2	1	3	4	3	3	4	1
<p><i>Example 3:</i> The last math test scores for Mr. Quinn’s class are shown below. Determine the range and mean score for the last math test.</p> <p>79, 85, 90, 87, 62, 79, 97, 86, 85, 100, 85, 80, 45, 75, 90</p>								
MA.6.SP.2.3	Create a graphical representation to display a single set of numerical data within a real-world context. Graphs are limited to dot plots, histograms and stem-and-leaf plots.							
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should not be expected to know which graphical representation is best to display the data.</p> <p><i>Example 1:</i> The students in Mrs. Green’s class wrote down how many minutes they spend on math homework over a one-week period. The data is shown below. Create a dot plot to summarize the data.</p> <p>15, 20, 15, 30, 45, 60, 32, 15, 20, 32, 45, 60, 18, 22</p>								
MA.6.SP.2.4	Determine and describe how changes in data values impact measures of center and variation.							
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should emphasize understanding of distributions to recognize that adding a number into the data set that is greater than the mean will increase the mean, adding a number into the data set that is less than the mean will decrease the mean. Using the same understanding, students should recognize that removing a number from a data set that is greater than the mean it will decrease the mean and removing a number from a data set that is less than the mean it will increase the mean.</p> <p><i>Remark 2:</i> Problem types should include cases where students need to determine a measure of center or measure of variability and cases where they do not necessarily need to determine a measure of center or measure of variability.</p> <p><i>Remark 3:</i> Students should work with positive rational numbers in this standard.</p> <p><i>Example 1:</i> You went grocery shopping with your mom this past weekend. She had 13 items in her basket with a mean price of \$6.29. On the way to the checkout, you convince your mom to buy a tub of ice cream that costs \$4.25. How will adding the tub of ice cream affect the mean?</p> <p><i>Example 2:</i> Mrs. Donohue has told her students that she will remove the lowest exam score for each student at the end of the grading period. Sara received grades of 43, 78, 84, 85, 88, 78, and 90 on her exams. What will be the difference between the mean and median of her original grades and the mean and median of her six grades after Mrs. Donohue removes one grade?</p> <p><i>Example 3:</i> A group of coworkers have a salary range from \$35,000 to \$70,000. The coworker who makes \$35,000 decided to start working part-time and now has a salary of \$20,000. How will the new salary affect the variation?</p>								

Geometric Reasoning	
<i>MA.6.GR.1 Apply previous understanding of the coordinate plane to solve problems.</i>	
MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot ordered pairs in all four quadrants and on both axes. Identify the x - or y - axis as the line of reflection when two ordered pairs differ by an opposite x - or y - coordinate.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should have practice plotting more than one ordered pairs, identifying quadrants and working with positive and negative rational numbers. Students should have practice with positive and negative rational numbers and ordered pairs defined as (x, y).</p> <p><i>Remark 2:</i> Instruction with identifying the x- or y- axis as the line of reflection extends a student’s understanding of absolute value.</p>	

<p><i>Example 1:</i> Plot and label the ordered pairs listed below. $(9, 2.3)$, $(-7.4, 0)$ and $(0, -3)$</p> <p><i>Example 2:</i> Given that $x > 0$ and $y > 0$, what quadrant would you find the point $(x, -y)$?</p> <p><i>Example 3:</i> Explain the relationship between the graph of $(2, 7)$ and $(-2, 7)$. Which axis gives the reflection of the two ordered pairs?</p>	
MA.6.GR.1.2	Find distances between ordered pairs, limited to the same x -coordinate or the same y -coordinate, represented on the coordinate plane.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> When finding distances, students should be exposed to strategies such as counting from point to point or finding the absolute value between points.</p> <p><i>Remark 2:</i> Students should have practice with problems where ordered pairs are plotted on the coordinate plane and not plotted on the coordinate plane.</p> <p><i>Example 1:</i> Find the distance between the ordered pairs $(6, -2)$ and $(6, 7)$.</p> <p><i>Example 2:</i> Find the distance between the ordered pairs $(\frac{1}{2}, \frac{3}{4})$ and $(-8\frac{1}{6}, \frac{3}{4})$.</p>	
MA.6.GR.1.3	Solve problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Problem types should include finding distances between points, computing dimensions of a rectangle or determining a fourth vertex of a rectangle. Problems involving finding distances should have coordinates in which they have the same x-coordinates or y-coordinates.</p> <p><i>Remark 2:</i> Students should continue practice with positive and negative rational numbers, but instruction should focus on the use of integers to ensure fluency of using the coordinate plane to solve problems.</p> <p><i>Remark 3:</i> When finding distances, students should be exposed to strategies such as counting from point to point or finding the absolute value between points.</p> <p><i>Example 1:</i> Sandi wants to find the area of her rectangular garden. She graphed the garden on a coordinate plane so that three of the vertices are located at $(8, 2)$, $(8, -5)$ and $(1, 2)$. Find the fourth vertex and determine the area of the garden.</p> <p><i>Example 2:</i> A rectangle is defined on the coordinate plane with vertices located at $(-4, -8)$, $(-4, 5)$, $(7, 8)$ and $(7, 5)$. Plot the rectangle on a coordinate plane and find the perimeter.</p>	

MA.6.GR.2 Model and solve problems involving two- and three-dimensional figures.	
MA.6.GR.2.1	Derive and apply a formula to find the area of a triangle.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should focus on the conceptual understanding of the relationship between the area of a rectangle and the area of a triangle.</p> <p><i>Remark 2:</i> Students should be expected to know the formula for a triangle in solving problems.</p> <p><i>Remark 3:</i> Students should have practice using positive rational numbers.</p> <p><i>Example 1:</i> Explain the relationship between the area of a rectangle and the area of a triangle.</p> <p><i>Example 2:</i> Find the area of a triangle with a height of 3.75 units and a base of 7.25 units.</p>	
MA.6.GR.2.2	Solve mathematical and real-world problems involving the area of triangles and quadrilaterals by decomposing them into triangles or rectangles.

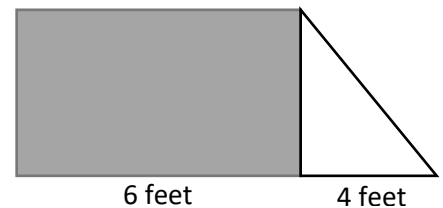
Remarks/Examples:

Remark 1: Problem types should include finding area of composite shapes and determining missing dimensions.

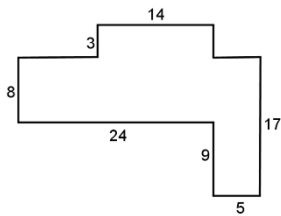
Remark 2: Students should be expected to know the formulas for triangles and rectangles.

Remark 3: Students should have practice using positive rational numbers.

Example 1: The figure to the right shows the floor of a living room. The rectangular part is covered with a carpet that covers a 22 square feet area. The house owner wants to cover the triangular part with carpet as well. Use the information provided in figure to determine the minimum additional carpet that will need to be purchased to cover the floor.



Example 2: Given the figure below, find the area and perimeter of the composite figure.



MA.6.GR.2.3

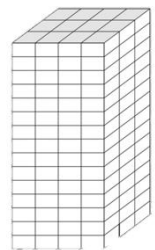
Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model or formula.

Remarks/Examples:

Remark 1: Instruction should focus on the extension of conceptual understanding of volume from grade 5. Students should move from utilizing visual models to building a formula for the volume of rectangular prisms.

Remark 2: Students should have practice with positive rational numbers in solving problems. Problem types should include students finding volume or dimensions in the rectangular prism.

Example 1: The art teacher Ms. Johnson is preparing for a lesson with clay for her 180 students. She cut a large block of clay into 180 pieces that each measured $\frac{1}{5}$ foot by $\frac{1}{4}$ foot by $\frac{1}{8}$ foot as shown in the diagram. Use these measurements to find the volume of the block of clay.



Example 2: A rectangular prism has a volume of 25 square feet with a height of 8.2 feet and a width of 2.8 feet. What is the length of the rectangular prism?

MA.6.GR.2.4

Find the surface area of right prisms and right pyramids, composed of rectangles and triangles, using the figure's net. Include mathematical and real-world context.

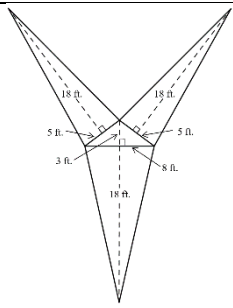
Remarks/Examples:

Remark 1: Instruction should focus on the conceptual understanding of surface area its relationship to its net.

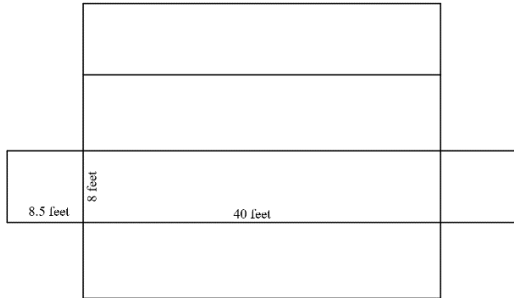
Remark 2: Students should not be expected to find dimensions using a given surface area or find lateral area. Students should have practice using positive rational numbers.

Example 1: A right rectangular prism has dimensions of 21 meters by 16 meters by 14 meters. Draw and label the net of the right rectangular prism and use it find the surface area of the prism.

Example 2: Given the net below, find the surface area of the triangular pyramid.



Example 3: A company uses metal containers and needs to paint them with anti-rust paint. Each bucket of paint will cover 800 square feet. Use the figure below to determine how many gallons of paint the company needs to buy to cover the entire container.



Grade 7

Number Sense and Operation

MA.7.NSO.1 Rewrite numbers in equivalent forms.

MA.7.NSO.1.1	Know and apply the Laws of Exponents to generate and evaluate equivalent numerical expressions, limited to whole number exponents and rational number bases.
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Remarks/Examples:

Remark 1: The Laws of Exponents students have practice with are the Product rules, Quotient rules, Power Rules, Power of Zero and Power of One.

Remark 2: Students have practice evaluating numerical expressions and recognizing equivalent expressions using the Laws of Exponents.

Remark 3: Students should continue to represent problems without the multiplication symbol of “.” that they used in sixth grade and not use the multiplication sign of “×”.

Example 1: Select all of the expressions that are equivalent to $4^3(4^2)^2$.

- 4^7
- 16^7
- 4^{12}
- 16,384
- 4^34^4

Example 2: What is the value of the expression $\left(\frac{2}{3}\right)^2 \cdot 9^2$?

MA.7.NSO.1.2	Rewrite positive and negative rational numbers in different but equivalent forms including fractions, mixed numbers, decimals and percentages. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Instruction should focus on student’s understanding of equivalent forms within a real-world context. Students should become fluent in converting rational numbers from one form to another. Students should not be expected to convert repeating decimals to fractions.

Example 1: John scored 75% on a test and Mary scored $\frac{8}{12}$ correct on the same test. Each test item is worth the same amount of points. Change Mary score to a percentage to determine who performed better on the test.

MA.7.NSO.1.3	Generate equivalent numerical expressions by rewriting the sum of two positive and negative composite rational numbers as a common factor multiplied by the sum of two positive and negative rational numbers. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Instruction should focus on the extension of the understanding of the distributive property by including positive and negative rational numbers. Students should become fluent in using the distributive property.

Example 1: Rewrite the following numerical expression in an equivalent form using the distributive property: $-\frac{3}{4} + \frac{1}{4}$.

Possible student response: A student can find a common factor between $-\frac{3}{4}$ and $\frac{1}{4}$ as $-\frac{1}{4}$ and rewrite the expression as $-\frac{1}{4}(3) + -\frac{1}{4}(-1)$. The student can then use the distributive property to write the equivalent expression as $-\frac{1}{4}(3 - 1)$.

MA.7.NSO.2 Add, subtract, multiply and divide rational numbers.

MA.7.NSO.2.1 Solve problems using multi-step order of operations with positive and negative rational numbers including parentheses, exponents and absolute value.

Remarks/Examples:

Remark 1: Students should become fluent in multi-step order of operations with positive and negative rational numbers. Students should not be expected to work with negative exponents.

Example 1: What is the expression $|-3^2| + 5(3 + 2)^2$ equivalent to?

Example 2: What is the expression $6^2 \left(\frac{16}{3} - |8| \right)$ equivalent to?

MA.7.NSO.2.2 Solve mathematical and real-world problems involving addition, subtraction, multiplication and division of positive and negative rational numbers.

Remarks/Examples:

Remark 1: Instruction should focus on real-world context and fluency of operations on positive and negative rational numbers.

Example 1: The average temperature in Jacksonville, FL in the month of January is 64.4 degrees Fahrenheit. Jerrell took the temperature every day for a week and recorded the difference between the measured temperature and the average of 64.4. The table below shows the difference in recordings. What is the average of the actual temperature taken over the 7 days?

Week	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Temperature Difference from 64.4°F	0.7	-1.1	-0.5	1.0	-3.4	0.8	-2.1

Example 2: If a woman is making \$25 an hour and gets a 10% raise, what is her new salary?

Algebraic Reasoning

MA.7.AR.1 Represent proportional relationships and use proportional reasoning to solve problems.

MA.7.AR.1.1 Write and solve an equation to represent proportional relationships between two quantities. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on students' understanding of ratios to begin work on proportional relationships. A relationship between two variables is said to be proportional if the two variables have the same ratios.

Remark 2: Students should not be expected to write and solve an equation for each problem.

Example 1: A pizza with 12 slices costs \$15.30. Write an equation that can be used to represent the total cost of n slices. What is the cost of 5 slices?

Example 2: The number of calories in a serving of oatmeal is proportional to the size of the serving. Suppose there are 160 calories in a 40 gram serving. Write an equation that models the relationship between the number of calories and the size of a serving of oatmeal.

MA.7.AR.1.2 Apply proportional reasoning to solve percent problems involving discounts, markups, simple interest, tax, tips, fees, percent increase, percent decrease and percent error. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Student should have practice with markup, simple interest, tax, percent increase, percent decrease, and percent error problems. Students should have practice using positive and negative rational numbers.

Example 1: Your mother gave you \$40 to buy new clothes for school. At the store, you buy 2 dresses that are \$12.50 each but on sale for 25% off. In addition, you buy a pair of shoes that cost \$17.99. If the sales tax for the dresses and shoes are 7.5%, do you have enough money to buy the two dresses and pair of shoes?

Example 2: In 2018 the average cost for a movie ticket was \$9.11 and in 2016 the average cost was \$8.65. Determine the percent increase in the cost of a movie ticket from 2016 to 2018.

Example 3: A student finds the density of a substance to be 1.24 g/mL. Find the percent error if the accepted value of density is 1.39 g/mL.

MA.7.AR.1.3	Apply proportional reasoning to solve problems involving the conversation between units in the same measurement system and constant speed. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should not be expected to convert between customary and metric systems. Students should have practice using positive rational numbers.

Example 1: You are riding your bike to school at a constant speed of 4 miles per hour. If your school is 1.75 miles away, how many minutes will it take you to ride your bike there?

Example 2: The table below shows the distances that two animals can run in given amounts of time. Determine the constant speed of each animal in terms of feet per minute. If the animal remains at this constant speed, how many minutes would it take for each animal to travel $425\frac{3}{4}$ feet?

Animal	Distance (miles)	Time (hours)
Roadrunner	80	4
African Elephant	50	2

MA.7.AR.2 Rewrite algebraic expressions in equivalent forms.

MA.7.AR.2.1	Generate equivalent algebraic expressions with integer coefficients.
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Remarks/Examples:

Remark 1: Instruction should focus on students using the properties of equality and operations to generate the equivalent algebraic expressions. By applying properties of operations to generate equivalent expressions, students use properties of operations that they are familiar with from previous grades' work with numbers — generalizing arithmetic in the process.

Remark 2: Seventh graders have only had experiences with linear algebraic expressions. Instruction should focus on students generating equivalent expressions using addition and subtraction situations. Students can use greatest common factor and distributive property to represent and generate equivalent expressions.

Example 1: What is the sum of the expressions $4x - 1$ and $9 - 7x$?

Example 2: What is the difference between the expressions $3x - 6$ and $x + 2$?

Example 3: Write an equivalent expression to $3(x + 3) - 2x + 4$.

MA.7.AR.2.2	Determine whether two algebraic expressions are equivalent.
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Remarks/Examples:

Remark 1: Students should recognize that by using substitution, they can determine algebraically that two expressions are equivalent. Expressions are said to be equivalent when the two expressions name the same number regardless of which value is substituted into them.

Example 1: Circle all of the expressions below that are equivalent to $6(y + 1)$.

$6y + 1$	$6y + 7$	$6(y) + 1(y)$	$6(y) + 6(1)$	$6y + 6$
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Example 2: Determine if the expressions $3x + 12$ is equivalent to $3(x + 4)$ by substituting $x = 4.5$ into each expression.

MA.7.AR.3 Write and solve equations and inequalities.

MA.7.AR.3.1	Write and solve one-step inequalities in one variable where all terms and solutions are positive and negative rational numbers and any inequality sign can be represented. Represent solutions algebraically or graphically. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should be exposed to inequalities in the forms $x + q > r$, $x + q < r$, $x + q \geq r$, and $x + q \leq r$ where p , q , and r are specific rational numbers. Problem types should include cases where students only create an inequality, only solve an inequality and ones where they create an inequality and use it to solve the task as hand. Additionally, problem types should include patterns, model and relationships.

Remark 2: Students should have practice with representing solutions to inequalities algebraically and graphically. Students should be exposed to inequalities where the variable is on the left and right side of the inequality symbol. Students should understand whether the number will be shown as a closed or an open dot on the number line and how to represent the other numbers it could represent through shading on the number line.

Example 1: Given $32x > -4$, graph the solutions.

Example 2: Solve for x and represent the solution on a number line: $\frac{5}{6} \leq x - 7$.

MA.7.AR.3.2	Write and solve two-step equations in one variable, where all terms are any positive or negative rational number. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should be exposed to equations in the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand. Additionally, problem types should include patterns, model and relationships.

Remark 2: Students should be exposed to equations where the variable is on the left and right side of the equal symbol.

Example 1: The length of a side of a square is given by the expression $2x + 1$, and its perimeter is 56. Write and solve an equation to determine the value of x .

Example 2: What is the value of x in the equation $-\frac{5}{4}\left(x - \frac{16}{3}\right) = -20$?

MA.7.AR.3.3	Write and solve two-step inequalities in one variable, where all terms are any positive or negative rational number. Represent solutions algebraically or graphically. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should be exposed to inequalities in the forms $x + q > r$, $px + q < r$, $px + q \geq r$, and $px + q \leq r$ where p , q , and r are specific rational numbers. Problem types should include cases where students only create an

inequality, only solve an inequality and ones where they create an inequality and use it to solve the task as hand.

Additionally, problem types should include patterns, model and relationships.

Remark 2: Students should have practice with representing solutions to inequalities algebraically and graphically. Students should be exposed to inequalities where the variable is on the left and right side of the inequality symbol. Students should understand whether the number will be shown as a closed or an open dot on the number line and how to represent the other numbers it could represent through shading on the number line.

Example 1: A scrap yard had 200 tons of recycled steel. They sold 15 tons per day for several days. If there are fewer than 80 tons left at the scrap yard, how many days have passed?

Example 2: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

MA.7.AR.4 Analyze and represent two-variable proportional relationships.

MA.7.AR.4.1

Determine the independent and dependent quantities described by a table, graph or written description. Include mathematical and real-world context.

Remarks/Examples:

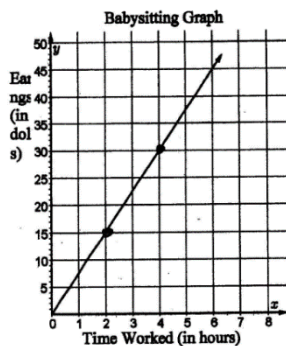
Remark 1: For any given relationship between two quantities, there is an independent variable which represents all values that is the input and dependent variable which represents all values that is the output.

Remark 2: Students should have practice using positive and negative rational numbers.

Example 1: Given the table below, determine the dependent and independent variables.

Number Correct on Test (n)	24	23	19	15
Grade on Test (g)	96	92	76	60

Example 2: Given the graph below, determine the dependent and independent variables.



MA.7.AR.4.2

Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

Remarks/Examples:

Remark 1: A relationship between two variables is said to be proportional if the two variables have the same ratios. Instruction should focus on the testing for equivalent ratios in a table or analyzing the graph on a coordinate plane to observe whether the graph is a straight line through the origin.

Remark 2: Students should have practice using a table, graph or written description to determine proportionality. Students should use the various representations to explain or justify their reasoning.

Remark 3: Students should have practice using positive and negative rational numbers.

Example 1: While on a cruise for spring break, you want to swim with dolphins. The table below gives the rate of prices from a local company. Are the two quantities, time and price, in a proportional relationship?

Time (hours)	Price (dollars)
$\frac{3}{4}$	\$90.00
$1\frac{1}{2}$	\$130.00
2	\$180.00

Example 2: Florida Elementary School has an average of six teachers per 138 second grade students. In third grade, there are 196 students for every seven teachers. The ratio of teachers to students in the fourth grade is three to 69. There are 207 fifth grade students for every nine teachers. Determine if the two quantities, number of teachers and the number of students, are proportionally related.

MA.7.AR.4.3 Determine the constant of proportionality when given a table, graph or written description of a proportional relationship. Include mathematical and real-world context.

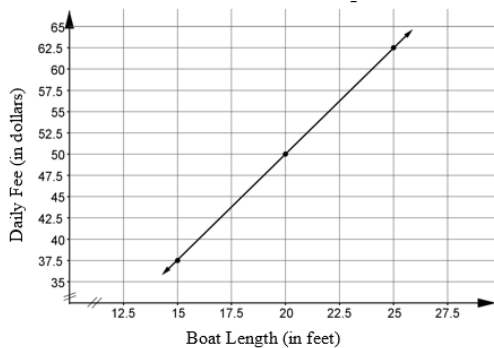
Remarks/Examples:

Remark 1: When two quantities have a proportional relationship, the constant of proportionality is the ratio between the two quantities. Instruction should focus on the foundation of linear functions, understanding the constant of proportionality is the same as the slope or rate of change for grade 8 work.

Remark 2: Students should have practice using positive and negative rational numbers.

Example 1: While driving on I-10, Geoffrey used his cruise control so that the number of miles he traveled was proportional to the time he spent driving. After four hours, Geoffrey had driven 204 miles. Determine the constant of proportionality and explain its meaning in the context of this situation.

Example 2: The docking fee at a local marina is proportional to the length of the boat. The graph below represents the relationship between the length of the boat and the fee. Determine the constant of proportionality.



MA.7.AR.4.4 Graph proportional relationships from a table, equation or a written description. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students should have practice using positive and negative rational numbers.

Example 1: The docking fee at a local marina is proportional to the length of the boat. The table below represents the relationship between the length of the boat and the fee. Graph the relationship and determine the constant of proportionality.

Boat Length (feet)	Daily Fee (dollars)
15	\$33.75
17	\$38.25
20	\$45.00
21	\$47.25

Example 2: A proportional relationship is described by the equation $y = -3x$. Graph the proportional relationship.

Statistics and Probability

MA.7.SP.1 Find and compare experimental and theoretical probabilities.

MA.7.SP.1.1

Compare the probabilities of chance events when the probabilities are given as rational numbers, including percentages.

Remarks/Examples:

Remark 1: Instruction should focus on the extension of the likeliness of an event to occur. Student practice can include using P(event) notation. Students should be fluent in comparing probabilities given as decimal, fractions or percentages.

Example 1: Ronald's probabilities for his total number of home runs for the season are below. What range of home runs is he most likely to have?

$$P(0-15 \text{ home runs}) = \frac{3}{5}$$

$$P(16-25 \text{ home runs}) = \frac{1}{4}$$

$$P(26-40 \text{ home runs}) = 30\%$$

$$P(41 \text{ or more home runs}) = 0.05$$

MA.7.SP.1.2

Given an experiment or a simulation, find experimental probabilities and compare them to theoretical probabilities within the given context.

Remarks/Examples:

Remark 1: An experiment is a repeatable procedure with a set of possible outcomes and a simulation is a way to model random events such that can simulate real-world outcomes. Problems could involve experiments that contain the "replacement" of an event.

Remark 2: Students should have practice expressing probability as a fraction, percentage or decimal.

Example 1: Gabbi rolled a die 600 times. Her results are in the table below. What is the estimated probability that she rolled an even number? Compare this with the theoretical probability of rolling an even number.

Die	Numbers Rolled
1	124
2	56
3	46
4	175
5	97
6	102

Example 2: A factory produces computer chips for video game consoles. The factory has a probability that 1% of all chips are defective. The quality control engineer notices that in a batch of 200 chips 5 are defective. Should the engineer stop production and have the machines examined?

MA.7.SP.2 Determine best measures of center and choose appropriate graphical representations. Compare data sets to draw informal comparative inferences on populations.

MA.7.SP.2.1

Determine the best measure of center, limited to mean and median, to summarize a numerical data set or graphical representation given the context and any outliers. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students should have practice with the following graphical representations: histogram, dot plot, stem-and-leaf plot, box plots. Students should have practice using positive and negative rational numbers.

Remark 2: The “best” measure of central tendency is determined by which number represents a “typical” score in the data set. When you have an outlier, the median is the best measure to describe the scores. When you don’t have an outlier, the best measure to describe your data is the mean.

Example 1: The graph shows the shoe size of each student in Charlene’s dance class. Determine and calculate which measure of center would better represent the data in that graph.

Shoe Sizes of Students in Charlene’s Class



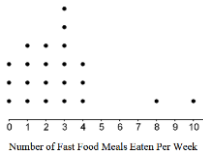
MA.7.SP.2.2	Given a numerical data set or graphical representation, identify possible outliers and describe how it may impact the measure(s) of center, limited to mean and median, or measure(s) of the variation, limited to range and interquartile range. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Students should have practice with the following graphical representations: histogram, dot plot, stem-and-leaf plot, box plots. Students should have practice using positive and negative rational numbers.

Remark 2: Students should have practice with articulating whether an outlier is increasing or decreasing the mean, median, or variability of the data set. Instruction should support student understanding of the impact an outlier has on the graphical representation.

Example 1: The graph shows how many meals per week the people in Rick’s class eat at fast food restaurants. Identify any possible outlier(s) and determine how it changes the best measure of center to summarize the data.



MA.7.SP.2.3	Given a real-world scenario, choose and create an appropriate graphical representation to display a single set of numerical data. Graphs are limited to dot plots, histograms, stem-and-leaf plots and box plots.
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Remarks/Examples:

Remark 1: When students are constructing a box plot and calculating Quartile 1 (Q1) and Quartile 3 (Q3), it should be expected that the median is excluded when a set has an odd number of data.

Remark 2: Students should have practice with the following graphical representations: histogram, dot plot, stem-and-leaf plot, box plots. Students should have practice using positive and negative rational numbers.

Remark 3: If you have a large data set, the most appropriate graphical representation is a histogram or box plot. If you have a small data set, the most appropriate graphical representation is a dot plot or stem-and-leaf plot.

Example 1: The table below shows the number of shark attacked in Florida from 2001 to 2013. Choose and construct an appropriate graphical representation of this data set.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Number of Attacks	34	29	29	12	17	21	31	28	19	14	11	27	23

MA.7.SP.2.4	Given two numerical or graphical representations of data, compare the measure(s) of center and measure(s) of variability. Measures of center are limited to mean and median. Measures of variability are limited to range and interquartile range.
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Remarks/Examples:

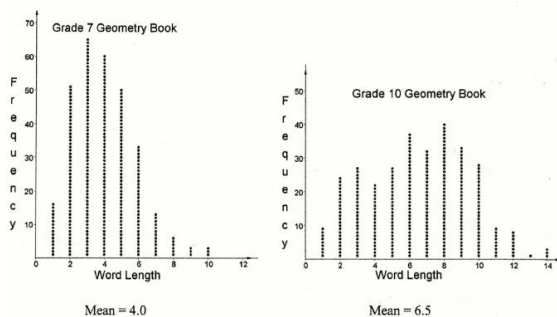
Remark 1: Students should have practice with the following graphical representations: histogram, dot plot, stem-and-leaf plot, box plots. Students should have practice using positive and negative rational numbers.

Remark 2: Instruction should focus on box plots as this is the first time students have experience constructing and analyzing them. The interquartile range is a measure of variability being equal to the difference between the 75th and 25 percentiles, or between the upper and lower quartiles of the data. Instruction should include different ways to describe the quartiles of data. For example, quartile 1, lower quartile, and 25 percentile express the same data points.

Example 1: Alfonso's bowling scores are 125, 142, 165, 138, 176, 102, 156, 130, and 142. The box and whiskers plot below represents the bowling scores of Anna. Compare the bowling scores of Alfonso and Anna. Who is a better bowler? Explain your reasoning.



Example 2: Peter is comparing the lengths of words in a seventh grade geometry book to the lengths of words in a tenth grade geometry book for a statistics project. He plotted the length of 300 randomly selected words from each book and calculated the mean for each set of data. Use the mean and the range to compare the two distributions.



MA.7.SP.2.5

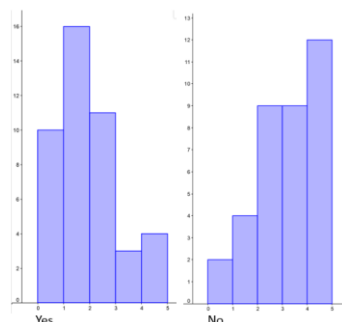
Given two numerical or graphical data sets, use measure(s) of center and measure(s) of variability to interpret results and draw valid conclusions about the two populations. Measures of center are limited to mean and median. Measures of variability are limited to range and interquartile range.

Remarks/Examples:

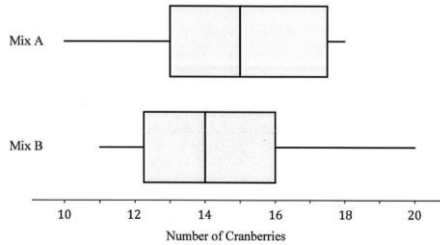
Remark 1: Instruction should focus on previous work done with measures of center and variability to draw conclusions on two populations. In drawing conclusions from box plots, students should be expected to find the difference between the medians, the percent of overlap between populations and use a comparison of ranges, maximums or minimums. In drawing conclusions from histograms, students should be expected to use the comparisons of modes, minimums or maximums.

Remark 2: Students should have practice with the following graphical representations: histogram, dot plot, stem-and-leaf plot, box plots. Students should have practice using positive and negative rational numbers.

Example 1: Data on the number of siblings Adam's classmates have and whether they have a pet are represented in the histogram below. Determine if there is a connection of having a pet or not based on how many siblings. Compare the minimum and maximum to support your conclusion.



Example 2: Reyna’s favorite part of trail mix is the dried cranberries. She wants to know whether 3 oz of snack mix A or 3 oz of snack mix B has more cranberries. She counts the number of cranberries in 10 packages and conclude that mix A has more cranberries. Her results are shown in the box plots below. Determine whether Reyna’s conclusion is valid.



Geometric Reasoning

MA.7.GR.1 Solve problems involving two-dimensional figures.

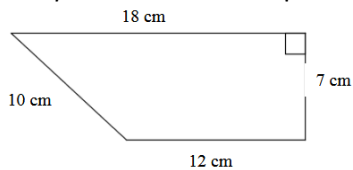
MA.7.GR.1.1 Apply a formula to find the area of trapezoids, parallelograms and rhombi.

Remarks/Examples:

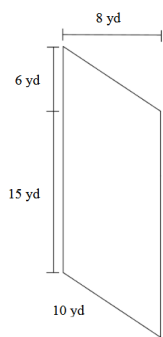
Remark 1: Instruction should focus on the conceptual understanding of the relationship between areas of trapezoids, parallelograms and rhombi and other quadrilaterals. Students should not be expected to memorize formulas for these types of quadrilaterals.

Remark 2: Problem types should include mathematical context only and students should have practice with positive rational numbers as dimensions.

Example 1: Given the trapezoid below, find the area.



Example 2: Given the parallelogram below, find the area.



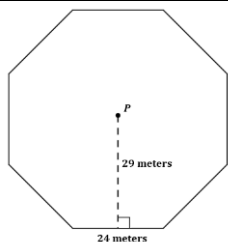
MA.7.GR.1.2 Solve mathematical or real-world problems involving the area of polygons by decomposing them into triangles or quadrilaterals.

Remarks/Examples:

Remark 1: Students should not be expected to memorize formulas for quadrilaterals except rectangles and triangles.

Remark 2: Problem types should include finding area of composite shapes and determining missing dimensions. Students should not be expected to find dimensions on the coordinate plane since that is not a grade level expectation.

Example 1: Find the area of the regular octagon with a center of P . Show all work and explain how you found your answer.



MA.7.GR.1.3

Find the approximation of Pi as the ratio of the circumference of a circle to its diameter.
Apply formulas for the area and circumference of a circle to solve mathematical and real-world problems.

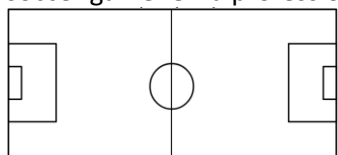
Remarks/Examples:

Remark 1: Students should not be expected to memorize formulas for circles. However, instruction should focus on that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into shapes that approximates a parallelogram.

Remark 2: Instruction on pi should focus on the conceptual understanding that students can use various circular objects to determine that the ratio of circumference to diameter approximates the value of Pi.

Remark 3: Problem types should include finding area of composite shapes and determining missing dimensions. Students should have practice in being able to give answers exactly, in terms of pi, or approximately.

Example 1: The center circle of a soccer field prohibits a defender from being near the ball at the start or restart of a soccer game. On a professional soccer field this circle is 20 yards in diameter. Find the area of this circle.



Example 2: Assume pizzas are measured in terms of its diameter, which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer.

Example 3: The circumference of the London Eye, a giant Ferris wheel on the south bank of the Thames River in London, is about 376.8 meters. Determine the diameter of the wheel.

MA.7.GR.1.4

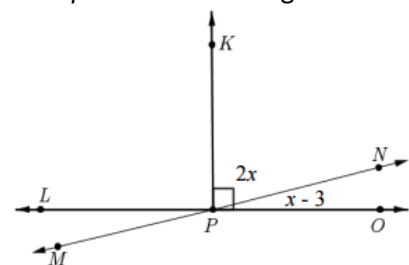
Solve mathematical and real-world problems involving the relationships between supplementary, complementary, vertical or adjacent angles.

Remarks/Examples:

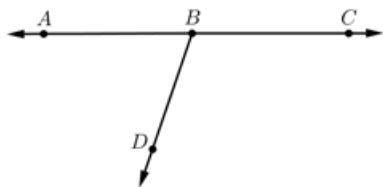
Remark 1: Instruction should support earlier work with supplementary, complementary, vertical and adjacent angles from elementary.

Remark 2: Students should not be expected to write or solve an equation that exceeds the expectation of grade 7 solving equations. Students should have practice using positive rational numbers.

Example 1: Given the figure below, write and solve an equation to determine the angle measure of OPN.



Example 2: In the diagram below, $\angle ABC$ is a straight angle. The ratio of the measure of $\angle ABD$ to the measure of $\angle CBD$ is 2:3. Write and solve an equation to find $m\angle ABD$.



MA.7.GR.1.5

Solve mathematical and real-world problems involving lengths and area of geometric figures, including scale drawings and scale factors.

Remarks/Examples:

Remark 1: The ratio of the length of something in a drawing to the length of the real thing is called a scale. A scale factor is the number in which scales, or multiplies, some quantity.

Remark 2: Problem types should include finding dimensions given scale factor or scale factor given dimensions. Students should have practice using positive rational numbers.

Example 1: On a floor plan of your school, your classroom is 9 inches long and 6 inches wide. If the scale is 1 inch = 3 ft., what is the width of your classroom in feet?

Example 2: You have a 4 in. by 5 in. photograph and you want to enlarge it to an 8 in. by 10 in. photograph. Roberto thinks that the new picture is four times as big as the old one. Dora thinks that the new picture is twice as big as the old one. Explain their thinking

MA.7.GR.2 Solve problems involving three-dimensional figures.

MA.7.GR.2.1

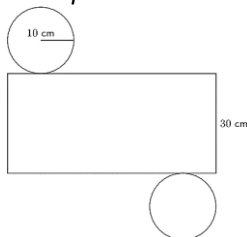
Find the surface area of right cylinders using the figure's net. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students should not be expected to memorize formulas for surface area. However, instruction should focus on the conceptual understanding of surface area its relationship to its net.

Remark 2: Students should not be expected to find dimensions using a given surface area or find lateral area. Students should have practice using positive rational numbers. Students should have practice in being able to give answers exactly, in terms of pi, or approximately. Students should have practice using positive rational numbers.

Example 1: Given the net below, determine the surface area of the cylinder.



Example 2: A right cylinder has a height of $6\frac{1}{2}$ feet and a diameter of $1\frac{1}{7}$ feet. Draw and label the net of the right rectangular prism and use it find the surface area of the prism.

MA.7.GR.2.2

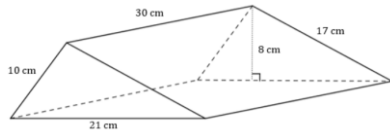
Solve mathematical and real-world problems involving surface area of right pyramids, right prisms and right cylinders.

Remarks/Examples:

Remark 1: Students should not be expected to memorize formulas for surface area, but should be expected to know the formula for the area of rectangles and triangles.

Remark 2: Students should be exposed to problems including finding a dimension of solid, finding the area of the base given the surface area, or determining the surface area. Students should have practice in being able to give answers exactly, in terms of pi, or approximately. Students should have practice using positive rational numbers.

Example 1: Given the right triangular prisms below, find the surface area.



MA.7.GR.2.3

Solve mathematical and real-world problems involving volume of right pyramids, right prisms and right cylinders.

Remark 1: Students should not be expected to memorize formulas for volume, but should be expected to know the formula for the area of a rectangles or triangles.

Remark 2: Students should be exposed to problems including finding a dimension of solid, finding the area of the base given the volume, or determining the volume. Students should have practice in being able to give answers exactly, in terms of pi, or approximately. Students should have practice using positive rational numbers.

Example 1: Until 2014, the mass of a kilogram was defined as exactly equal to the mass of a special platinum-iridium cylinder kept in Sèvres, France. The cylinder's diameter and height are both 39.2 millimeters. What is the volume of this cylinder?

Example 2: The Louvre Pyramid is a metal and glass structure that serves as the main entrance to the Louvre Museum in Paris, France. The pyramid has a volume of 8820 cubic meters. The base is a square with sides that are 35 meters long. Find the height of the pyramid.

Grade 8

Number Sense and Operation	
<i>MA.8.NSO.1 Solve problems involving rational numbers and extend the understanding of rational numbers to irrational numbers.</i>	
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Estimate the value of numerical expressions involving irrational numbers, including non-perfect squares up to 225.
<u>Remarks/Examples:</u> <i>Remark 1:</i> A rational number is a number that can be expressed as a quotient, or fraction, of two integers. An irrational number is a number that cannot be expressed as a quotient, or fraction, of two integers. <i>Example 1:</i> What is the approximation of $\sqrt{75}$? <i>Example 2:</i> Which of the following numbers are irrational? π , $\sqrt{4}$, $\frac{1}{3}$, 17	
MA.8.NSO.1.2	Compare and plot rational and irrational numbers, each represented in a different way, on a number line.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should have practice with comparing two numbers using an inequality symbol and plotting multiple numbers on a number line. <i>Example 1:</i> Plot the following numbers on a number line: $\frac{3}{4}$, $\sqrt{3}$, -1, $-\sqrt{4}$. <i>Example 2:</i> Compare the numbers $\sqrt{11}$ and $\frac{13}{4}$ using inequality symbols.	
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Evaluate and generate equivalent numerical expressions, limited to integer exponents and rational number bases.
<u>Remarks/Examples:</u> <i>Remark 1:</i> The Laws of Exponents students have practice with are the Product rules, Quotient rules, Power Rules, Power of Zero, Power of One and Negative exponents. <i>Remark 2:</i> Students have practice evaluating numerical expressions and recognizing equivalent expressions using the Laws of Exponents. <i>Example 1:</i> What is the value of the expression $\left(\frac{2}{3}\right)^{-2} \cdot 4^2$?	
MA.8.NSO.1.4	Solve multi-step problems involving the order of operations with rational numbers including exponents and radicals, limited to perfect squares up to 225 and perfect cubes up to 125. Include mathematical and real-world context.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should have practice using positive and negative rational numbers. <i>Example 1:</i> What is the value of the expression $\sqrt[3]{8} + 5(3 + 2)^2$? <i>Example 2:</i> Alex had some marbles. On his birthday, his father doubled the number of his marbles. Alex gave 5 marbles to his best friend. Then he divided the remaining marbles into three equal groups and shared them with his two brothers. Each brother got 11 marbles. What was the original number of marbles that Alex had before his birthday? Did he make a good choice of sharing his marbles? What strategy would you use if you were Alex? <i>Example 3:</i> What is the value of the expression $6^2 \left(\frac{16}{3} - \sqrt{64}\right)$?	

<i>MA.8.NSO.2 Perform operations on numbers in scientific notation.</i>	
MA.8.NSO.2.1	Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one is compared to the other. Include mathematical and real-world context.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Scientific notation is a method of writing very large or very small numbers using exponents in which a number is expressed as the product of a power of 10 and a number that is greater than or equal to one (1) and less than 10 (e.g., $7.59 \times 10^5 = 759,000$). <i>Remark 2:</i> Students should only be expected to know standard scientific notation of whole numbers and decimals to hundred billions through hundred-billionths. <i>Example 1:</i> By studying lunar samples, scientists have learned the Moon is approximately 4,600,000,000 years old. What is this number expressed in scientific notation? <i>Example 2:</i> Earth's volume is approximately $1.08 \times 10^{12} \text{ km}^3$. Sun's volume is approximately $1.41 \times 10^{18} \text{ km}^3$. How many times larger is the Sun than the Earth? <i>Example 3:</i> The average mass of a lion is approximately 2×10^2 kilograms. The average mass of an Asian elephant is approximately 4×10^3 kilograms. About how many times more mass does an Asian elephant have than a lion?	
MA.8.NSO.2.2	Solve mathematical and real-world problems involving operations with numbers expressed in scientific notation.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should only be expected to know standard scientific notation of whole numbers and decimals to hundred billions through hundred-billionths. <i>Example 1:</i> The human body produces 1.5×10^7 red blood cells every second. How many red blood cells, expressed in scientific notation, does the human body produce in a 60-second period? <i>Example 2:</i> What is the value of the expression $(2.2 \times 10^{-4}) \times (4 \times 10^{-3})$? <i>Example 3:</i> What is the value of the expression $(6 \times 10^9) / (2 \times 10^5)$?	

Algebraic Reasoning	
<i>MA.8.AR.1 Solve multi-step one-variable equations and inequalities.</i>	
MA.8.AR.1.1	Identify examples of one-variable linear equations that generate one solution, infinitely many solutions, or no solution.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students have been working informally with one-variable linear equations since as early as kindergarten. This important line of development culminates in grade 8 with the solution of general one-variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation using properties of operations. Instruction should focus on showing which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). <i>Remark 2:</i> Students should have practice with rational number coefficients. <i>Example 1:</i> What value of a would make the equation $3x - 6 = a(x - 1) - 3$ have infinitely many solutions?	
MA.8.AR.1.2	Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should be exposed to equations where the variable is on both sides of the inequality symbol. <i>Example 1:</i> What is the value of x in the equation $-4(2x + 9) + 3x = 6 - 4(x - 3)$? <i>Example 2:</i> What is the value of x in the equation $-3.5(10x - 2) = -176.75$? <i>Example 3:</i> Larry mows lawns at a base rate of \$15 with an additional rate of \$8 per lawn. Stephan mows lawns at a base rate of \$45 with an additional rate of \$6 per lawn. If they have the same income for one day, how many lawns did they mow?	
MA.8.AR.1.3	Solve multi-step linear inequalities in one variable, with rational number coefficients. Represent solutions algebraically or graphically. Include inequalities with variables on both sides.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should have practice with representing solutions to inequalities algebraically and graphically. Students should be exposed to inequalities where the variable is on the left and right side of the inequality symbol. Students should understand whether the number will be shown as a closed or an open dot on the number line and how to represent the other numbers it could represent through shading on the number line. <i>Example 1:</i> Solve for x and graph the solution on a number line: $\frac{2}{3}x - 4\frac{1}{2} \geq -8$. <i>Example 2:</i> Solve for x and graph the solution on a number line: $3(x + 1) + x < 4(x + 5) - 13$.	
MA.8.AR.1.4	Given an equation in the form of $x^2 = p$ and $x^3 = p$, where p is a positive rational number, determine the solutions.
<u>Remarks/Examples:</u> <i>Remark 1:</i> Students should not be expected to simplify radicals, but are expected to understand that solutions can be either positive or negative numbers, depending on the situation. <i>Remark 2:</i> Students should denote non-perfect squares $\pm\sqrt{p}$ and non-perfect cubes as $\sqrt[3]{p}$. <i>Example 1:</i> What values of x make the equation $x^2 = 196$ true? <i>Example 2:</i> What values of x make the equation $x^2 = 48$ true? <i>Example 3:</i> What values of x make the equation $x^3 = 81$ true?	

MA.8.AR.2 Extend understanding of proportional relationships to two-variable linear equations.	
MA.8.AR.2.1	Create an equation in slope-intercept form, $y = mx + b$, for a line with a slope of m and intersecting the vertical axis at b from a table, graph or written description.
<u>Remarks/Examples:</u> <i>Remark 1:</i> The slope of a line is the ratio of the change, or difference, in the y -coordinates to the change, or difference, in the x -coordinates. The y -intercept is a point on the graph that intersects the y -axis of the coordinate plane. <i>Example 1:</i> The height of a tree was 7 inches in the year 2000. Each year the same tree grew an additional 10 inches. Write an equation to show the height h of the tree in y years. Let y be the number of years after the year 2000. <i>Example 2:</i> Jan decided to save some money. She already had \$25. She received and saved \$5 on Friday each week for 8 weeks. Make a table and a graph of the money she would have each week. If she continues with this same savings plan, how much money will she have after 2 years?	
MA.8.AR.2.2	Graph a two-variable linear equation from a written description, an equation or a table. Include mathematical and real-world context.
<u>Remarks/Examples:</u>	

Remark 1: Instruction should not be limited to slope-intercept form. Students should be exposed to point-slope form as well. Students should recognize that they can graph a line by having any two points or a point and a slope of the line.

Remark 2: Students should have practice using positive and negative rational numbers.

Example 1: Alex works at an aquarium shop on Sunday. One Sunday, when Alex gets to work, she is asked to clean a 250-gallon reef tank. Her first job is to drain the tank. She puts a hose into the tank and starts a siphon. Alex wonders if the tank will finish draining before she leaves work. She measures the amount of water that is draining out and finds that 15.5 gallons drain out in 30 minutes. So, she figures that the rate is 31 gallons per hour. Draw a graph to represent the situation.

Example 2: Mickey has a box turtle that he is observing walking in a maze. He records how many minutes it takes the box turtle to go a specific number of feet. Graph a line to represent the situation.

Number of Feet	2	4	6	8
Time (minutes)	25	50	75	100

MA.8.AR.2.3 Determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, an equation, a table or a graph. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: The slope of a line is the ratio of the change, or difference, in the y-coordinates to the change, or difference, in the x-coordinates. The y-intercept is a point on the graph that intersects the y-axis of the coordinate plane.

Example 1: Given the table below, determine the slope and y-intercept of the linear equation.

x	y
-3	-9
-1	-5
1	-1
3	3

Example 2: The amount of water leaking out of a tank is defined by the equation $y = -\frac{4}{5}x + 138.3$, where x is the time in minutes and y is the amount of water, in liters, left in the tank. Determine the y-intercept and slope in terms of the context.

MA.8.AR.3 Develop an understanding of two-variable systems of equations.

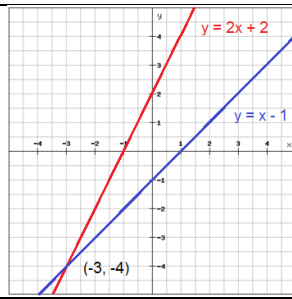
MA.8.AR.3.1 Given a system of linear equations, determine algebraically that the solution satisfies both equations simultaneously.

Remarks/Examples:

Remark 1: A system of equations is a collection of two or more equations involving the same set of variables.

Remark 2: Instructions should focus on the students' understanding that a solution to a system of linear equations in two variables corresponds to a point of intersection of their graphs. The point of intersection satisfies both equations simultaneously.

Example 1: Given the graph below, verify algebraically that the solution to the system is $(-3, -4)$.



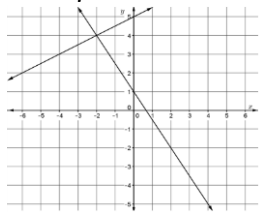
MA.8.AR.3.2

Determine whether there is one solution, no solution or infinitely many solutions when given a system of two linear equations represented on a coordinate plane.

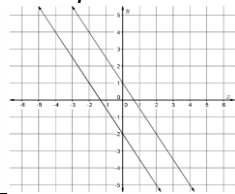
Remarks/Examples:

Remark 1: Students should recognize that intersecting lines yield a unique solution; parallel lines yield no solution; and coincidental lines yield an infinite number of solutions.

Example 1: Given the graph below, identify the solution to the system of linear equations.



Example 2: Given the graph below, identify the solution to the system of linear equations.



MA.8.AR.3.3

Solve systems of two linear equations in two variables by plotting them on a graph or using a table of values, approximating when solutions are not integers. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students should have practice with generating a table/graph to determine the solution or be given a table of values/graph the equations of a line and analyzing them to determine the solution.

Example 1: Given the system of equations below, determine the solution by graphing.

$$y = -\frac{2}{3}x + 5$$

$$y = -4x - 5$$

Example 2: Jan started with \$25 and saved \$5 each week. Bill started at the same time with no money and saved \$10 per week. Use a graph to determine if or when Bill and Jan will have the same amount of money.

Functions

MA.8.F.1 Define, evaluate and compare functions.

MA.8.F.1.1

Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.

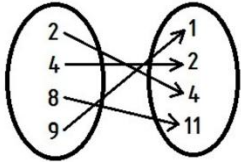
Remarks/Examples:

Remark 1: Instruction should focus on student understanding that a function is a rule that assigns to each input, the domain, exactly one output, the range.

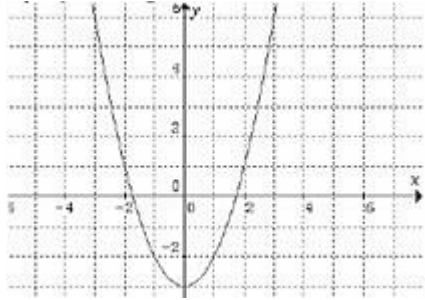
Remark 2: Students should have practice with a variety of ways to represent relations and functions and should be expected to give domain and range in set notation or using inequalities.

Example 1: Given the relation $\{(-3, -1), (2, -1), (1, 0), (2, 5)\}$, determine if the relation can be a function. If it is a function, determine the domain and range.

Example 2: Given a mapping diagram, determine if it is a function. If it is a function, determine the domain and range.



Example 3: Given the graph of the relation below, decide if this relation is a function. Explain your reasoning.



MA.8.F.1.2

Given a function defined by a table, a graph or an equation, determine whether it is a linear function.

Remarks/Examples:

Remark 1: Instruction should focus on defining linear functions as a graph of a non-vertical straight line resulting from its constant rate of change, or slope.

Example 1: Given the table below, determine whether the function is linear or nonlinear.

x	2	3	5	6
y	-7	-4.5	0.5	3

Example 2: Mark had \$100 and added \$10 to it each year. Mandy put \$100 in the bank, earned 10% interest each year on her total amount of money in the bank, and left the interest in the bank account. Determine if either function is linear and explain your reasoning.

MA.8.F.1.3

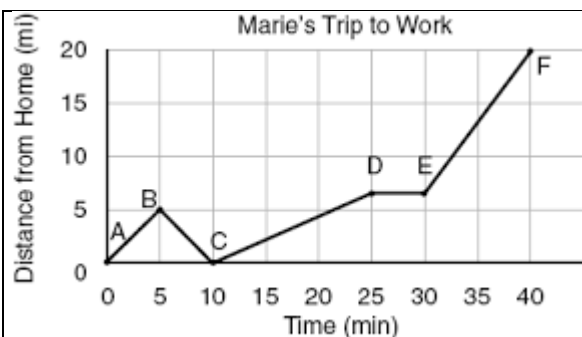
Analyze a written or graphical representation of a functional relationship of two quantities stating where the function is increasing, decreasing or constant. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on students being able to analyzing a distance-time graph. Students have some knowledge within their science courses in middle grades of these functional relationships so instruction should focus on students being able to understanding its connections to linear functions. While the focus is on functional relationships composed of linear relations, students should be exposed to nonlinear relations as well.

Remark 2: Students should have practice in analyzing a graph by stating where the function increases, decreases or is constant. Students should also have practice in writing descriptions that describe the graph, sketching a graph or determining the slope between two points on the graph.

Example 1: The graph below describes Marie's morning commute to work. Between which two point was Marie's speed at a constant rate? Describe what could have happened in between points A and C.



Example 2: Sophia gets on her bike and pedals at a constant speed of 6 miles per hour, which she maintains for 10 minutes. She then reaches a large hill where her rate slows to 2 miles per hour. She maintains this slower rate for five minutes until she gets to the top of the hill where she stops. Sketch a graph that models the relationship between Sophia's rate and the passage of time (from getting on her bike until stopping at the top of the hill). Be sure to label each axis with the appropriate variable.

Statistics and Probability

MA.8.SP.1 Represent and find compound probabilities.

MA.8.SP.1.1

Determine sample spaces and find probabilities for compound events using organized lists, tables, tree diagrams, fundamental counting principle or simulation.

Remarks/Examples:

Remark 1: Instruction should focus on the student understanding that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Remark 2: Students should have practice expressing and seeing probability as a fraction, percentage or decimal. Students should not be expected to use one model in representing sample spaces over another, however, they should have experience in representing sample spaces using organized lists, tables, tree diagrams, fundamental counting principle and simulations.

Example 1: A model of car is available in four colors (black, blue, red and white) and three body styles (coupe, sedan and SUV). Represent the sample space of the compound event using a visual model and determine the probability that the car will be a red sedan.

Example 2: Tom just got a new cell phone and wants to set up a four-digit code to keep his phone locked. He decided to use the digits 1, 3, 5, 9 and will randomly choose how to order the digits. Assuming each digit can be used more than once, represent the sample space of the compound event using a visual model.

MA.8.SP.1.2

Solve problems involving simple or compound probability.

Remarks/Examples:

Remark 1: Instruction should focus on students becoming fluent in determining the probability from one, simple, or two, compound, events. Problems could involve experiments that contain the "replacement" of an event.

Remark 2: Students should have practice expressing and seeing probability as a fraction, percentage or decimal. Students should not be expected to use one model in representing sample spaces over another, however, they should have experience in representing sample spaces using organized lists, tables, tree diagrams, fundamental counting principle and simulations.

Example 1: Provide example of simple probability.

Example 2: Provide example of compound probability.

MA.8.SP.1.3

Construct a two-way table summarizing data on two categorical variables collected from the same subjects. Express the data as frequencies and joint relative frequencies.

Remarks/Examples:

Remark 1: Instruction should focus on the connection of compound events to two-way tables as a way to determine probabilities, or relative frequencies. The joint relative frequency is the ratio of the quantity of a certain category to the total quantity within the data set.

Remark 2: Students should have practice expressing and seeing relative frequencies as a fraction, percentage or decimal. Students should be expected to construct a two-way table with only two categorical variables, with each variable having two or more responses.

Example 1: Adam asked his classmates how many siblings they had and whether they had a pet. Some of the responses are shown in the table below. Complete the frequency table and determine the joint relative frequency of students that have two siblings and a pet to the total number of students surveyed.

		How many siblings do you have?						Total
		0	1	2	3	4	5	
Do you have a pet?	Yes	10	16		3	2	2	
	No		4	9	9		4	36
Total		12	20	20		10	6	

Example 2: Suzanna took a survey of her classmates asking each whether they played a sport and whether they played a musical instrument. The results are shown in the table below. Construct a two-way table to summarize the data.

Playing Sports and Playing Musical Instruments

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Plays Team Sport	y	y	n	n	n	n	y	y	n	y	y	y	n	n	y	n	y	n	n	y
Plays Instrument	y	n	n	n	y	n	y	y	n	y	n	y	n	y	y	n	y	n	y	y

Note: y = yes; n = no

MA.8.SP.2 Investigate patterns of associations in bivariate data.

MA.8.SP.2.1

Construct scatter plots for bivariate data to investigate patterns of association between two quantities. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on student's understanding of plotting ordered pairs and linear functions to construct scatter plots and describe associations as positive, negative or no association.

Remark 2: Students should have practice constructing scatter plots with 10 to 20 data points.

Example 1: Scientists at the new company, BunG, tested their bungee cords using weights from 10 to 200 pounds. They identified a random sample of cords and measured the length that each cord stretched when different weights were applied. The table displays the average stretch length for the sample of cords for each weight. Construct a scatter plot for this set of data. Label and scale the graph appropriately.

Weight (lbs)	10	30	50	75	90	100	110	130	150	175	200
Length (ft)	12	14	16	19	17	20	18	22	24	26	25

MA.8.SP.2.2

Given a scatter plot, describe patterns of association by analyzing clustering; descriptions include outliers, positive, negative or no association, linear association and nonlinear association. Include mathematical and real-world context.

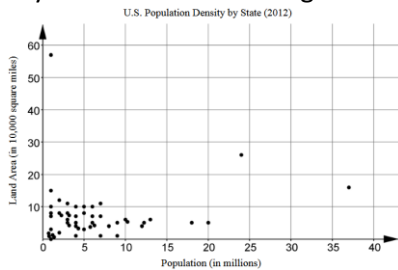
Remarks/Examples:

Remark 1: Instruction should focus on student's understanding of plotting ordered pairs and linear functions to describe associations as positive, negative or no association and describe as linear or nonlinear associations.

Instruction should also focus on student's understanding of clustering and outliers within one-variable statistics to describe possible clusters of data or outliers within the data represented on the scatterplot.

Remark 2: Students should not be expected to construct a scatter plot or make any calculations.

Example 1: Population density measures are approximations of the number of people per square unit of area. The scatter plot below represents data from each of the 50 states comparing population (in millions) to land area (in 10,000 square miles) in 2012. Describe the relationship between population and land area. Include in your description any evidence of clustering or outliers.



MA.8.SP.2.3

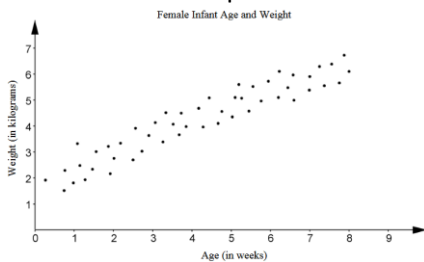
Given a scatter plot with a linear association and informally fit a straight line. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on student's understanding of linear functions to approximate a straight line through given scatter plot. Students should use a variety of tools including the use of a ruler to draw a line with approximately the same number of points above and below the line.

Remark 2: Students should not be expected to calculate the y-intercept or slope of the line, nor be expected to write the equation of the line of best fit.

Example 1: The scatterplot below shows the relationship between the ages and weights of 50 female infants. Draw a line on the scatter plot that fits the data.



MA.8.SP.2.4

Given the equation of a linear model, solve problems in the context of bivariate data by interpreting the slope and y-intercept. Include real-world context.

Remarks/Examples:

Remark 1: Problem types should include students interpreting the y-intercept and slope in terms of the context and making predictions based on the linear model.

Example 1: Yearly tuition costs at public universities were tracked since 2003. The relationship can be modeled by the equation $C = 316t + 5827$, where t is the number of years since 2003 and C is the cost of tuition. Assuming the cost keeps at the same trend, predict the average tuition cost at a public university in the year 2028.

Geometric Reasoning

MA.8.GR.1 Solve problems involving the Pythagorean Theorem and angle relationships in polygons.

MA.8.GR.1.1

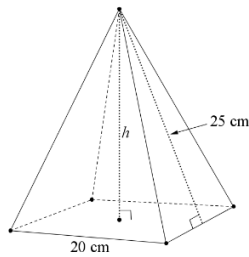
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students should be expected to find the length of a missing leg or hypotenuse within a right triangle.

Remark 2: Students should have practice in being able to give answers exactly, in terms of a radical, or approximately.

Example: Given the square pyramid below, find the height.



Example 2: You are wrapping a gift for your teacher's birthday. It is a very long and skinny pencil. You want to wrap it in a box so that your teacher can not tell what shape it is. Your friend has a shoe box that measures 10 inches by 7 inches by 5 inches. The pencil is 13 inches long. Will you be able to fit the pencil into the shoe box and close the lid?

MA.8.GR.1.2

Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on the understanding and connection between distance on the coordinate plane and right triangles. Students should not be expected to use the distance formula.

Remark 2: Students should have practice in being able to give answers exactly, in terms of a radical, or approximately. Students are not expected to reduce the radical since that is not grade appropriate.

Example 1: You are sailing your boat to Key West from Pensacola. Key West is 82°W and 25°N , and your boat is 84°W and 29°N . What is the distance from your boat to Key West? Assume 1° change in longitude or latitude is 70 miles.



Example 2: Find the distance between $(-2, 7)$ and $(1, -4)$ using the Pythagorean Theorem.

MA.8.GR.1.3

Given three sides, determine if the sides form a triangle. Given three angles, determine if the angles could be the interior angles of a triangle. Identify when given conditions create a unique triangle, more than one triangle, or no triangle using triangle sum and triangle inequality theorem.

Remarks/Examples:

Remark 1: The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees. The Triangle Inequality Theorem states that the sum of any two sides of triangle must be greater than the measure of the third side.

Remark 2: Instruction should focus on using various tools to determine and gain a conceptual understanding of the two theorems.

Example 1: Can you create a triangle from side lengths 3 ft, 6 ft, and 10 ft? If so, could they create a unique triangle or more than one triangle?

Example 2: If possible, draw and label triangle ABC so that the measure of angle A is 110 degrees, the measure of angle B is 30 degrees and the measure of angle C is 40 degrees.

Example 3: If possible, draw and label triangle XYZ so that the measure of angle X is 55 degrees, the measure of angle Y is 80 degrees and the included side, XY, measures 11 cm. Is it possible to draw a different triangle with the same measurements as described?

MA.8.GR.1.4

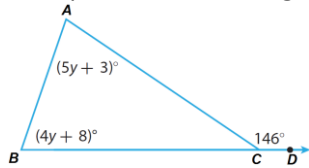
Solve problems involving the relationships of interior and exterior angles of a triangle.

Remarks/Examples:

Remark 1: During instruction, students should use various tools to determine and gain a conceptual understanding of the Exterior Angle Theorem. This theorem states that an exterior angle of a triangle is equivalent to the sum of the two non-adjacent interior angles of the triangle.

Remark 2: Students should have experience finding angles using algebraic equations.

Example 1: Given the figure below, find the measure of angle CAB.



MA.8.GR.2 Understand similarity and congruence using models and transformations.

MA.8.GR.2.1

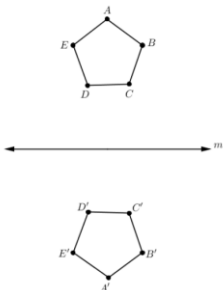
Given a preimage and image, describe a single transformation that shows the relationship between them. Determine whether the transformation preserves congruence.

Remarks/Examples:

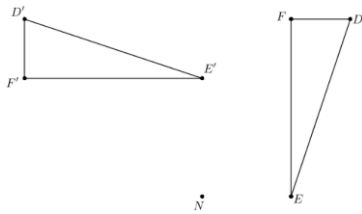
Remark 1: Instruction should focus on understanding that reflections, transformations, and rotations of figures result in congruent figures. Other transformations (such as dilations) may not preserve congruency. This begins the foundation for the understanding that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations and that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.

Remark 2: Problem types should include given the transformation and given the preimage students must create the image and given the image and preimage identify the transformation. Students should have practice with image and preimage notation.

Example 1: Describe a single transformation that shows pentagon ABCDE is congruent to pentagon $A'B'C'D'E'$.



Example 2: Describe a single transformation that shows triangle DEF is congruent to triangle $D'E'F'$.



MA.8.GR.2.2

Describe and apply the effect of a single transformation on two-dimensional figures using coordinates.

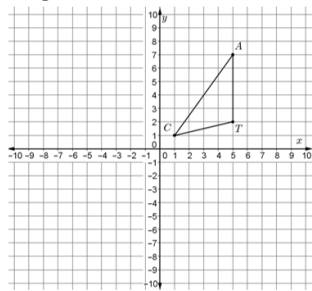
Remarks/Examples:

Remark 1: Students should have practice working all four transformations: rotation, transformation, dilation and reflection. Students should be exposed to image and preimage notation.

Remark 2: Students should only be expected to know reflections over the x- or y-axis or over lines parallel to the axes. Students should only be expected to know rotations about the origin with degrees of 90, 180, 270, or 360. Students should only be expected to dilate images from the origin and with a factor that is a unit fraction or whole number.

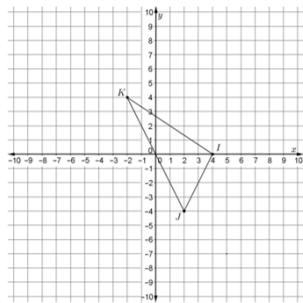
Remark 3: Problem types should include given the transformation and given the preimage students must create the image; given the image and preimage identify the transformation.

Example 1: Find the coordinates of the vertices of the image of triangle *CAT* after a 270° counterclockwise about the origin.



Example 2: Draw the triangle with vertices $(0,0)$, $(3,0)$, $(0,4)$. Translate the triangle 2 units to the right. What are the coordinates of the vertices of the new triangle?

Example 3: Find the coordinates of the vertices of the image of triangle *JK* after it is dilated by a scale factor of $\frac{1}{4}$ using the origin as the center of dilation.



MA.8.GR.2.3

Given a dilated two-dimensional figure, identify the effect on linear and area measurements.

Remarks/Examples:

Remark 1: Instruction should connect work with ratios and proportions from grades 6 and 7 and begin the development to similarity and the Proportional Perimeters and Areas Theorem.

Example 1: You have two circles one with a radius of 1 and the other that was dilated by a factor of 4. What are the ratios of the circumference and area of the circles?

Grade 9-12

9-12 Algebraic Reasoning Strand

<i>MA.912.AR.1 Interpret and rewrite algebraic expressions and equations in equivalent forms.</i>	
MA.912.AR.1.1	Identify and interpret parts of an expression or equation, such as terms, factors and coefficients that represent a quantity in terms of a mathematical or real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> For the Algebra 1 course, instruction and problem types should not go beyond linear, quadratic, or exponential functions types.</p> <p><i>Remark 2:</i> Students should not be expected to generate equivalent expressions within problem types. However, it may be necessary for students to generate equivalent forms in order to help interpret the task at hand.</p> <p><i>Example 1:</i> Last weekend, Cindy purchased two tops, a pair of pants, and a skirt at her favorite store. The equation $T = 1.075x$ can be used to calculate her total cost where x represents the pre-tax subtotal cost of her purchase. Interpret the coefficient in terms of the context of Cindy's situation.</p>	
MA.912.AR.1.2	Rearrange formulas to isolate a quantity of interest using the same reasoning as in solving equations.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> During instruction, formulas should reflect functions/equations taught within your course. In Algebra 1 it should not include exponential formulas as students have not learned how to rewrite them into logarithmic form.</p> <p><i>Example 1:</i> The formula for volume of a cone is $V = \frac{\pi hr^2}{3}$. Rewrite the equation to isolate the radius.</p>	
MA.912.AR.1.3	Apply previous understanding of integer operations to add, subtract and multiply polynomial expressions.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students are extending knowledge of operations with linear expressions to polynomial expressions. Students should be exposed to using the distributive property when performing these operations. When multiplying polynomials, students should be exposed to using an area model or manipulatives extending conceptual understanding of multiplication from elementary grades.</p> <p><i>Remark 2:</i> Students should gain an understanding of closure in that when polynomials expressions are added, subtracted and multiplied the result is a polynomial expression.</p> <p><i>Remark 3:</i> Within the Algebra 1 course, students should not be expected to perform operations on polynomials with four or more terms.</p> <p><i>Example 1:</i> What is an equivalent expression of $(-3x^4 + 7.3x^2 - 9.7) - (2.3x^4 - 2x^3 + 12.9)$?</p> <p><i>Example 2:</i> What is the product of the expressions $(x - \frac{3}{7})$ and $(x^2 - \frac{14}{3}x + 9)$?</p>	
MA.912.AR.1.4	Divide a polynomial expression by a monomial expression.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students are extending knowledge of properties of operations and Laws of Exponents with rational numbers to polynomial expressions. Students should have practice using the distributive property and Laws of Exponents and long division when dividing polynomials by monomials.</p>	

Remark 2: Students should gain understanding that unlike other operations, division of polynomials are not defined under closure because the result may not be a polynomial.

Remark 3: Within the algebra 1 course, students should not be expected to have a dividend with four or more terms.

Example 1: What is the quotient of the expressions $(8x^6 - 3x + 4)$ and $(2x^2)$?

Example 2: Simplify the polynomial expression: $\frac{9x^4 + x^3 - 3x}{3x^2}$.

MA.912.AR.1.5	Divide polynomial expressions using long division, synthetic division or algebraic manipulation.
MA.912.AR.1.6	Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials.
MA.912.AR.1.7	Apply previous understanding of rational number operations to add, subtract, multiply and divide rational expressions.
MA.912.AR.1.8	Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of rational algebraic expressions.
MA.912.AR.1.9	Apply the Binomial Theorem to expand polynomials expressions.

MA.912.AR.2 Linear equations, functions and inequalities in one- and two-variables.

MA.912.AR.2.1	Create and solve one-variable multi-step linear equations. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should focus on the fluency of solving one-variable linear equations represented in multiple ways. Multi-step equations require students to perform more than two steps in order to isolate the variable of interest. Multi-step equations can include cases where the variable occurs on both sides of the equations.</p> <p><i>Remark 2:</i> Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand.</p> <p><i>Example 1:</i> Solve the following equation for m: $\frac{1}{2}m + 2\left(\frac{3}{4}m - 1\right) = 0.25m + 6$.</p> <p><i>Example 2:</i> The Central High School soccer team is planning to raise money for new uniforms this season. They decide to sell chocolate chip cookies for \$1.25 each and oatmeal for \$0.75 each. If they spent \$73.92 to make the cookies and made \$192 on the bake sale, how many cookies they did sell? Assume that they number of chocolate chip cookies and oatmeal cookies sold are the same.</p>	
MA.912.AR.2.2	Create and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should focus on the fluency of solving one-variable linear inequalities represented in multiple ways. A compound inequality contains at least two inequalities that are separated by “and” or “or”. The solution to a compound inequality separated by “and” must satisfy both inequalities. The solution to a compound inequality separated by “or” must satisfy at least one of the inequalities.</p>	

Remark 2: Problem types should include cases where students only create an inequality, only solve an inequality and ones where they create an inequality and use it to solve the task as hand. Students should be expected to represent solutions either algebraically or on a number line. Students should be exposed to inequalities where the variable is on the left and right side of the inequality symbol.

Example 1: In a company's report for the next fiscal year it is expected that the company is going to sell between 200 units and 300 units. Each unit has a selling price of \$32.99 per unit and the cost of production is \$21.80 per unit. The number of units sold multiplied by profit per unit gives the total income the company makes. Write and solve a compound inequality to find the maximum and minimum value of the possible income that will be generated by the company in the next fiscal year.

Example 2: Solve the following inequality for x and represent solutions both algebraically and on a number line.

$$3(x - 6) + 2 \geq 5x - 4$$

MA.912.AR.2.3

Create linear functions to represent relationships between quantities from a graph, a written description or a table of values. Include mathematical and real-world context.

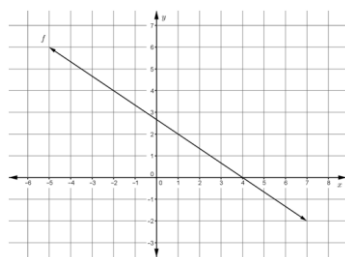
Remarks/Examples:

Remark 1: Instruction should build on the student's understanding of creating proportional relationships and linear equations from middle grades. Instruction should focus on the fluency of creating linear functions that are represented in various ways.

Remark 2: Students should be able to choose forms of linear functions to create strategically, based on the information presented to them. They should also be able to convert from one form to another. Students should have practice working with x - y notation and function notation. Students should be exposed to vertical and horizontal lines.

Example 1: Jamie bought a car in 2005 for \$28,500. By 2008, the car was worth \$23,700. Create a linear model that describes this situation.

Example 2: The graph of function f is shown below. Create a rule that corresponds to this graph.



MA.912.AR.2.4

Write a linear function for a line that is parallel or perpendicular to a given line.

Remarks/Examples:

Remark 1: Students should be able to write forms of linear functions strategically, based on the information presented to them. They should also be able to convert from one form to another. Students should have practice working with x - y notation and function notation. Students should be exposed to vertical and horizontal lines.

Example 1: Write a linear function for a line that passes through the point $(4, -3)$ and is perpendicular to $y = 0.25x + 9$.

Example 2: Write a linear function for a line that is parallel to $f(x) = \frac{1}{3}x - 8$ and passes through the point $(-7, 0)$.

Example 3: Write an equation in slope-intercept form for the line that satisfies the following condition: passes through $(3, 14)$ and is parallel to the line that passes through $(10, 2)$ and $(25, 15)$.

MA.912.AR.2.5

Solve and graph mathematical and real-world problems of linear functions. Determine and interpret key features in context.

Key features are limited to domain, range, intercepts and rate of change.

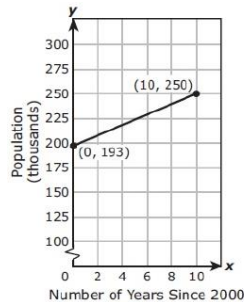
Remarks/Examples:

Remark 1: During instruction, problems should include students using multiple representations of linear functions including graphs, equations, tables or written descriptions. In addition, students should be exposed to problems where they are expected to graph the function, interpret the function, or graph and interpret the function.

Remark 2: Students should have practice working with x-y notation and function notation. Students should have practice graphing vertical and horizontal lines.

Example 1: Graph the function $g(x) = -3.6x + 7$. Identify the domain, range and x- and y-intercepts of the function.

Example 2: The population of St. John's County, Florida, from the year 2000 through 2010 is shown in the graph below. If the trend continues, what will be the population of St. John's County in 2025?



Example 3: Suppose you fill your car's tank with gasoline and drive off down the highway. Assume that, as you drive, the number of minutes since you filled the tank and the number of liters remaining in the tank are related by a linear function. After 40 minutes, you have 52 liters left. An hour after filling up, you have 50 liters left. Graph the situation as described and determine how many hours it will take to run out of gas.

MA.912.AR.2.6

Create two-variable linear inequalities to represent relationships between quantities from a graph or a written description. Include mathematical and real-world context.

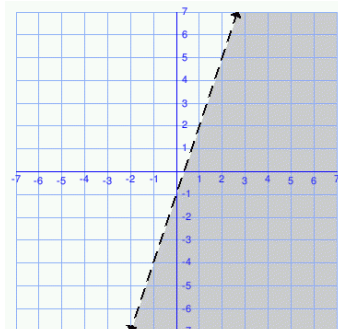
Remarks/Examples:

Remark 1: Instruction should focus on the connection of creating linear functions and one-variable linear inequalities from middle grades. Instruction should focus on the fluency of creating linear functions that are represented in various ways.

Remark 2: Students should not be expected to write the function in one form over another, but students should be able to convert from one form to another. Students should be exposed to vertical and horizontal lines.

Example 1: A carpenter makes two types of chairs: a lawn chair and a living room chair. It takes him 3 hours to make a lawn chair and 5 hours to make a living room chair. If the carpenter works a maximum of 75 hours per week, write an inequality to describe the number of possible chairs of each type he can make in a week.

Example 2: The graph of the solution set to a one-variable linear inequality is shown below. Create a linear inequality in two variables that corresponds to this graph.



MA.912.AR.2.7	Graph the solution set to a two-variable linear inequality. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should focus on the connection of graphing linear function and one-variable inequalities.</p> <p><i>Remark 2:</i> Students should have practice graphing solution sets to vertical and horizontal linear inequalities.</p> <p><i>Example 1:</i> A carpenter makes two types of chairs: a lawn chair and a living room chair. It takes him 3 hours to make a lawn chair and 5 hours to make a living room chair. If the carpenter works a maximum of 75 hours per week, graph an inequality to determine if he can make 50 lawn chairs and 20 living room chairs in a week?</p> <p><i>Example 2:</i> Graph the solution set to the inequality: $9x - 10y \geq 4$.</p>	

MA.912.AR.3 Quadratic equations, functions and inequalities in one- and two-variables.	
MA.912.AR.3.1	Create and solve one-variable quadratic equations over the real number system using a variety of strategies. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be exposed to the following ways to solve a quadratic: taking square roots, completing the square, the quadratic formula, factoring or graphing. Students should be able to determine the appropriate method of solving given the context.</p> <p><i>Remark 2:</i> Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand.</p> <p><i>Remark 3:</i> Within the Algebra 1 course, students should be introduced to the concept of non-real answers, but not expected to determine the non-real solution. Instruction with factoring and completing, instruction should reflect upon manipulatives and visual models in explaining the process of factoring and completing the square.</p> <p><i>Example 1:</i> Given the equation $x^2 + 6x = 13$, what value(s) of x satisfy the equation?</p> <p><i>Example 2:</i> A company plans to build a large multiplex theater. The financial analyst told her manager that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where x is the number of movie screens, and P(x) is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.</p>	
MA.912.AR.3.2	Create and solve one-variable quadratic equations, using a variety of strategies, over the real and complex number systems. Include mathematical and real-world context.
MA.912.AR.3.3	Create and solve one-variable quadratic inequalities, using a variety of strategies, over the real number system. Represent solutions algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.3.4	Create quadratic functions to represent relationships between quantities from a graph, a description or a table of values. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should not be expected to write the function in one form over another, but students should be able to convert from one form to another. Students should have practice working with x-y notation and function notation.</p>	

Example 1: The table of values representing the function f is shown below. Create a rule that corresponds to this quadratic function.

x	-3	-2	-1	0	1	2
H(x)	2	0	0	2	6	12

Example 2: A company is testing various mixtures of fuel for their rockets. During one of the tests, a rocket is launched from the top of a cliff reaching a maximum height of 250 yards at a horizontal distance of 4 yards from the cliff. The rocket then crashed on the ground at a horizontal distance of 9 yards from the cliff. Create a quadratic function that models the height $h(d)$ of the rocket at any given distance d yards from the cliff.

MA.912.AR.3.5	Transform expressions for quadratic functions to reveal its zeros of the associated graph using factoring techniques. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Instruction should focus on students becoming fluent in factoring for quadratic expressions. Students should not be expected to create or graph functions.

Remark 2: For the Algebra 1 course, students should not be expected to work with complex numbers.

Example 1: Given the function, $y = x^2 - 6x + 8$, rewrite the function to reveal its zeros.

Example 2: A hotel is building a 17 by 11 ft. rectangular pool with a concrete patio with the same width surrounding it. They have a total area of 315 sq. ft. to construct the pool and patio together. The function $f(x) = 4x^2 + 56x - 128$, represents the situation described. Transform the function to determine and interpret its zeros.

MA.912.AR.3.6	Transform expressions for quadratic functions to reveal the vertex of the associated graph by completing the square. Interpret the maximum or minimum value in terms of its context. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Instruction should focus on students becoming fluent in factoring for quadratic expressions. Students should not be expected to create or graph functions.

Remark 2: For the Algebra 1 course, students should not be expected to work with complex numbers.

Example 1: Given the function, $y = x^2 - 6x + 8$, rewrite the function to reveal its vertex.

Example 2: A new coffee shop wants to maximize their profit within the first year of business. They determined the function, $P(x) = -80x^2 + 480x - 540$, models the profit they can earn in thousands of dollars in terms of the price per cup of coffee, in dollars. Transform the function to determine and interpret its vertex.

MA.912.AR.3.7	Solve and graph mathematical and real-world problems of quadratic functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.
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Remarks/Examples:

Remark 1: During instruction, problems should include students using multiple representations of quadratic functions including graphs, an equation, a table or written descriptions. In addition, students should be exposed to problems where they are expected to graph the function, interpret the function, or graph and interpret the function.

Remark 2: Within the Algebra 1 course, problems should not include complex numbers since students only have worked with real number solutions of quadratic equations.

Example 1: Graph the function $f(x) = x^2 + 2x - 3$. Identify the domain, range, vertex and zeroes of the function.
Example 2: The value of a classic car produced in 1972 can be modeled by the quadratic function $v(t) = 19.25t^2 - 440t + 3,500$, where t is the number of years since 1972. After how many years does the car reach its lowest value?

MA.912.AR.4 Absolute value equations, functions and inequalities in one- and two-variables.

MA.912.AR.4.1	Create and solve one-variable absolute value equations. Include mathematical and real-world context.
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Remarks/Examples:

Remark 1: Within the Algebra 1 course, students should only be expected to solve equations involving absolute value on one side of the equal sign.

Remark 2: Problem types should include cases where students only create an equation, only solve an equation and ones where they create an equation and use it to solve the task as hand.

Example 1: The Golden Gate Bridge in California spans 4183.07 feet across San Francisco Bay. The bridge can expand or contract by as much as 2.76 feet due to changes in temperature. Write and solve an equation to find the maximum and minimum length of the Golden Gate Bridge.

Example 2: Solve the following equation for w : $\left| -\frac{7}{8} + 2w \right| = 4$.

MA.912.AR.4.2	Create and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically. Include mathematical and real-world context.
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MA.912.AR.4.3	Solve and graph mathematical and real-world problems of absolute value functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior; and symmetry.
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MA.912.AR.5 Exponential and logarithmic equations and functions in one- and two-variables.

MA.912.AR.5.1	Given a logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.
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MA.912.AR.5.2	Solve one-variable exponential equations using the properties of exponents.
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Remarks/Examples:

Remark 1: Instruction should focus on the student's knowledge of exponents and the Laws of Exponents from middle grades and expanding to algebraic situations.

Remark 2: For the Algebra 1 course, students should only be expected to solve equations in which a common base can be determined. For the Algebra 1 course, students should have practice working with rational number exponents since that is an expectation of the course.

Example 1: Solve for x : $27^{3x+3} = 81$.

MA.912.AR.5.3	Solve equations involving one-variable logarithms or exponents using a variety of strategies. Interpret solutions as viable in terms of context and identify extraneous solutions. Include mathematical and real-world context.
MA.912.AR.5.4	Classify exponential functions as representing growth or decay. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> An exponential function is classified as a decay function if the percent rate of change per unit interval is in between zero and one. Students should be exposed to $f(x) = ab^x$, where b is the percent of decay and $f(x) = a(1 - r)^x$, where $1 - r$ is the decay factor.</p> <p><i>Remark 2:</i> An exponential function is classified as a growth function if the percent rate of change per unit interval is greater than one. Students should be exposed to $f(x) = ab^x$, where b is the percent of growth and $f(x) = a(1 + r)^x$, where $1 + r$ is the growth factor.</p> <p><i>Example 1:</i> Given the exponential function $h(x) = 3(\frac{5}{6})^x$, classify whether it is an exponential growth or decay.</p> <p><i>Example 2:</i> Saturation specific humidity can be modeled as a function of temperature for the standard atmosphere. Does the model, $H(t) = 1.4(2^{0.15t})$ where H represents humidity and t represents temperature, represent exponential growth or decay?</p>	
MA.912.AR.5.5	Create exponential functions to represent relationships between quantities from a graph, a description or a table of values. Include mathematical and real-world context.
MA.912.AR.5.6	Transform expressions for exponential functions to reveal the constant percent rate of change per unit interval of the associated graph using the properties of exponents. Include mathematical and real-world context.
MA.912.AR.5.7	Solve and graph mathematical and real-world problems of exponential functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior; and asymptotes.
MA.912.AR.5.8	Solve and graph mathematical and real-world problems of logarithmic functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.

MA.912.AR.6 Polynomial equations and functions in one- and two-variables.	
MA.912.AR.6.1	When suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems. Include mathematical and real-world context.
MA.912.AR.6.2	Explain and apply the Remainder Theorem.
MA.912.AR.6.3	Solve and graph mathematical and real-world problems of polynomial functions of degree 3 or higher. Determine and interpret key features in context.

	Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetry; and end behavior.
MA.912.AR.6.4	Sketch a rough graph of a polynomial function of degree 3 or higher using zeros and knowledge of end behavior.

<i>MA.912.AR.7 Radical equations and functions in one- and two-variables.</i>	
MA.912.AR.7.1	Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. Include mathematical and real-world context.
MA.912.AR.7.2	Solve and graph mathematical and real-world problems of square or cube root functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and relative maximums and minimums.

<i>MA.912.AR.8 Rational equations and functions in one- and two-variables.</i>	
MA.912.AR.8.1	Create and solve one-variable rational equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. Include mathematical and real-world context.
MA.912.AR.8.2	Solve and graph mathematical and real-world problems of rational functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.

<i>MA.912.AR.9 Create and solve a system of two- and three-variable equations and inequalities that describe quantities and/or relationships.</i>	
MA.912.AR.9.1	Solve a system of two-variable linear equations algebraically or graphically. Include mathematical and real-world context.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should have practice solving systems by methods of graphing, substitution and elimination. Students should be able to determine the appropriate method of solving given the context.</p> <p><i>Remark 2:</i> Students should have practice with the inclusion of vertical or horizontal lines within the system.</p> <p><i>Example 1:</i> Solve the system of linear equations below using your method of choice.</p> $y - 3.75 = -2(x + 11.3)$ $13x + 8y = -9$ <p><i>Example 2:</i> A construction company received the details from the two suppliers after bidding for the machinery to be used in a project. Each supplier has a one-time charge for the cost of the machine and a transportation cost per mile.</p>	

Supplier A charges are defined by the function $A(x) = 500 + 7x$ and supplier B charges are defined by the function $B(x) = 700 + 5x$. Find the distance for which the two suppliers charge the same amount for the machinery.

MA.912.AR.9.2

Solve a system consisting of a two-variable linear equation and a nonlinear equation algebraically or graphically. Include mathematical and real-world context.

MA.912.AR.9.3

Solve a system consisting of two-variable nonlinear equations algebraically or graphically. Include mathematical and real-world context.

MA.912.AR.9.4

Graph the solution set to a system of two-variable linear inequalities. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on extending a student's understanding of graphing the solution set to one two-variable linear equality. Students should understand that the overlap of each solution set is the solution to the system.

Remark 2: Students should have practice with the inclusion of vertical or horizontal lines within the system.

Remark 3: Within the course of Algebra 1, students should not be expected to graph more than two two-variable linear inequalities.

Example 1: Graph the solution set to the system of linear inequalities shown below.

$$y \geq -\frac{8}{9}x - \frac{3}{4}$$

$$2x + 5y < -18$$

Example 2: Robin is buying plants and soil for her garden. The soil cost \$8 per bag, and the plants cost \$12 each. She wants to buy no more than 8 plants. She cannot spend more than \$120. Write and graph a system of linear inequalities to model all possible solutions to the situation.

MA.912.AR.9.5

Given a real-world context, represent constraints of systems of linear equations or inequalities. Interpret solutions as viable or non-viable options.

Remarks/Examples:

Remark 1: Instruction should focus on students creating a system of linear equations or linear inequalities to represent constraints within the context of a problem.

Example 1: A student on the weightlifting team is trying to eat healthier so that he can make it to state. The student has \$10 to spend on lunch and considers the amount of protein, at least 20 grams, and the amount of sodium, no more than 200 mg., he should eat during each meal. The student is trying to figure out how many servings he can have from Entree A and Entree B. The cost per serving of Entree A is \$1.80 and contains 8 grams of protein and 75 mg. of sodium. The cost per serving of Entree B is \$2.30 and contains 12 grams of protein and 93 mg. of sodium. Write a set of inequalities to represent the constraints on the amount of money the student can spend and the amount of protein and sodium he wants to consume per meal.

MA.912.AR.9.6

Given a real-world context, represent constraints of systems of nonlinear equations or inequalities. Interpret solutions as viable or non-viable options.

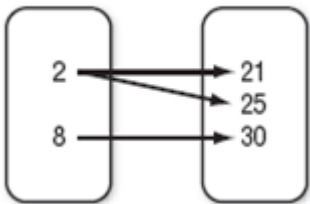
MA.912.AR.9.7

Solve problems involving linear programming. Include mathematical and real-world context.

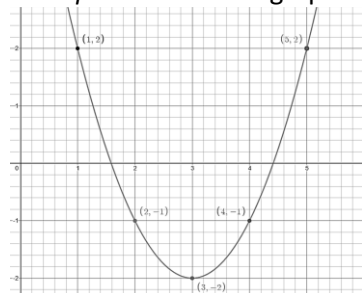
MA.912.AR.9.8	Solve a system of three-variable linear equations algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.9.9	Solve and graph mathematical and real-world problems of piecewise functions. Determine and interpret key features in context. Key features limited to intercepts, asymptotes and end behavior.

<i>MA.912.AR.10 Sequence & series equations, functions and inequalities in one- and two-variables.</i>	
MA.912.AR.10.1	Create and solve problems involving arithmetic sequences. Include mathematical and real-world context.
MA.912.AR.10.2	Create and solve problems involving geometric sequences. Include mathematical and real-world context.
MA.912.AR.10.3	Recognize and apply the formula for the sum of a finite arithmetic series to solve problems. Include mathematical and real-world context.
MA.912.AR.10.4	Recognize and apply the formula for the sum of a finite or an infinite geometric series to solve problems. Include mathematical and real-world context.
MA.912.AR.10.5	Create a sequence using function notation, defined explicitly or recursively, to represent relationships between quantities from a description. Include mathematical and real-world context.
MA.912.AR.10.6	Find the domain of a given sequence, defined recursively or explicitly. Include mathematical and real-world context.

9-12 Functions Strand

<i>MA.912.F.1 Understand, compare and analyze properties of functions.</i>	
MA.912.F.1.1	Given a set of ordered pairs, table, equation, graph or mapping diagram, identify the type of relation or function.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students have previous knowledge that a function from one set, domain, to another set, range, assigns to each element of the domain exactly one element of the range.</p> <p><i>Remark 2:</i> During instruction, functions should be represented as an equation, in a table or on a graph.</p> <p><i>Example 1:</i> Given the mapping below, identify the type of relation or function. Justify your answer.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>Domain</p> <div style="border: 1px solid black; border-radius: 10px; padding: 10px; width: 60px; margin: 0 auto;"> <p>2</p> <p>8</p> </div> </div> <div style="text-align: center;"> <p>Range</p> <div style="border: 1px solid black; border-radius: 10px; padding: 10px; width: 60px; margin: 0 auto;"> <p>21</p> <p>25</p> <p>30</p> </div> </div> </div> 	

Example 2: Given the graph below, identify the type of relation or function. Justify your answer.



MA.912.F.1.2

Evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Students are introduced to function notation for the first time. Students should extend knowledge of functions to understand that if f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x .

Example 1: Evaluate $f(24)$, when $f(x) = \frac{3}{2}x + 9$.

Example 2: Alice is helping support a charity bike race. She is in charge of putting in a water station for the bikers halfway between the start and finish lines. The function $f(x) = \left| \frac{x}{4} - 8 \right|$ models Alice's distance in miles from the water stand x minutes into the bike race. After 16 minutes, how far away from the water station would the bikers be?

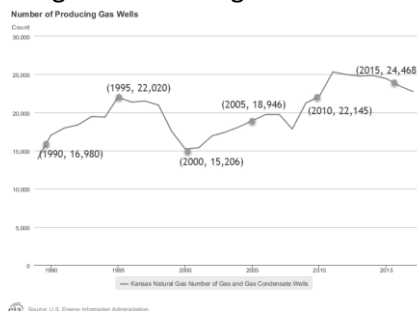
MA.912.F.1.3

Calculate and interpret the average rate of change of a real-world situation represented graphically over a specified interval.

Remarks/Examples:

Remark 1: Students should extend knowledge of rate of change from linear functions to other types of graphs and tables. Students should have practice determining and interpreting rate of change. Instruction should focus on examples outside of linear equations and functions.

Example 1: The graph below represents the change in the number of producing gas wells in Kansas. What is the average rate of change between 2000 and 2015?



MA.912.F.1.4

Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.

MA.912.F.1.5	Demonstrate understanding of the concept of limit and estimate limits from graphs and tables of values, as related to the concept of the derivative of a function.										
MA.912.F.1.6	Compare properties of a linear and a nonlinear function each represented in a different way such as algebraically, graphically, in tables or written descriptions.										
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Eighth graders have identified from tables, graphs, and equations whether a function is linear. Students should be able to distinguish if something is linear or nonlinear when represented in different form. Instruction should focus on properties for linear and nonlinear functions including intercepts, slopes, and other key features of a graph. Instruction should be able to compare the properties.</p> <p><i>Example 1:</i> Two functions, $f(x)$ and $g(x)$, are shown below. Compare the functions by stating which has a greater y-intercept, x-intercept and rate of change.</p> <div>$f(x) = 4x - 3$<table><tr><td>x</td><td>-3</td><td>-1</td><td>1</td><td>3</td></tr><tr><td>$g(x)$</td><td>13</td><td>8</td><td>3</td><td>-2</td></tr></table></div>		x	-3	-1	1	3	$g(x)$	13	8	3	-2
x	-3	-1	1	3							
$g(x)$	13	8	3	-2							
MA.912.F.1.7	Compare properties of two non-linear functions each represented in a different way such as algebraically, graphically, in tables or written descriptions.										
MA.912.F.1.8	Identify a linear, quadratic or exponential function to model a given situation.										
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should focus on strategies such as using differences, identifying rates of change, and analyzing graphs.</p> <p><i>Remark 2:</i> Problem types should describe a real-world context.</p> <p><i>Example 1:</i> A scientist is monitoring cell division and notes that a single cell divides into 4 cells within one hour. During the next hour, each of these cells divides into 4 cells. This process continues at the same rate every hour. What type of function would be used to represent this situation?</p> <p><i>Example 2:</i> Sarah is spending the summer at her grandmother’s house. The table below shows the amount of money in her bank account at the end of each week. What type of function would be used to model the total amount of money in Sarah’s bank account as a function of time?</p> <table><tr><td>Week #</td><td>Total \$</td></tr><tr><td>1</td><td>\$3400</td></tr><tr><td>2</td><td>\$3100</td></tr><tr><td>3</td><td>\$2800</td></tr><tr><td>4</td><td>\$2700</td></tr></table>		Week #	Total \$	1	\$3400	2	\$3100	3	\$2800	4	\$2700
Week #	Total \$										
1	\$3400										
2	\$3100										
3	\$2800										
4	\$2700										
MA.912.F.1.9	Compare tables and graphs of functions to verify that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically.										
<p><u>Remarks/Examples:</u></p>											

Remark 1: Students should have practice with comparing function types within a common context (finances, population, etc.).

Example 1: Mrs. Johnson gives her daughter Janae two options for payment for frosting cakes for her bakery: (1) Two dollars for each cake frosted, or (2) she will be paid for the number of cakes she frosts as follows: two cents for frosting one cake, four cents for frosting two cakes, eight cents for frosting three cakes, and so on, with the amount doubling for each additional cake frosted. If Janae frosts 6 cakes, should she opt for payment method 1 or 2? What if she frosts 12 cakes? Is there a scenario where method 2 would pay more than method 1?

MA.912.F.1.10

Determine whether a function is even, odd or neither when represented algebraically, graphically or in a table.

MA.912.F.2 Identify and describe the effects of transformations on functions. Create new functions given transformations.

MA.912.F.2.1

Identify the effect on the graph of a given function of a single transformation defined by adding or multiplying the x- or y-values by a real number.

Remarks/Examples:

Remark 1: Students should be exposed to replacing $f(x)$ with $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

Remark 2: Students should have knowledge of basic parent functions and their equations and graphs. Within the Algebra 1 course, students should be expected to work with functions including linear, quadratic, exponential and absolute value.

Example 1: Describe the effect of the transformation $f(2x)$ on the graph of $f(x) = x^2$.

MA.912.F.2.2

Identify the effect on the graph of a given function of two or more transformations defined by adding or multiplying the x- or y-values by a real number.

MA.912.F.2.3

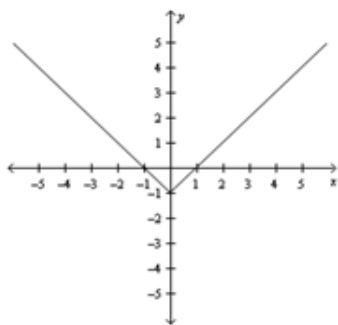
Given the graph or table of values of a single transformation of a function, find the value of the real number that defines the transformation.

Remarks/Examples:

Remark 1: Students should be exposed to replacing $f(x)$ with $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

Remark 2: Students should have knowledge of basic parent functions and their equations and graphs. Within the Algebra 1 course, students should be expected to work with functions including linear, quadratic, exponential and absolute value.

Example 1: Given the graph below, describe and determine the value of the real number that defines the transformation.



MA.912.F.2.4

Given the graph or table of values of a two or more transformations of a function, find the value of the real number that defines the transformation.

MA.912.F.2.5

Given a single transformation and a function, create the table or graph of the transformed function.

Remarks/Examples:

Remark 1: Students should be exposed to replacing $f(x)$ with $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

Remark 2: Students should have knowledge of basic parent functions and their equations and graphs. Within the Algebra 1 course, students should be expected to work with functions including linear, quadratic, exponential and absolute value.

Example 1: Graph the function $y = |x|$ after is stretched vertically by a factor of 3.

Example 2: Complete the table of values for the transformed function $f(x) = 2^x - 3$.

x	y
-1	
	-3
2	
	5
5	

MA.912.F.2.6

Given two or more transformations and a function, create the table or graph of the transformed function.

MA.912.F.2.7

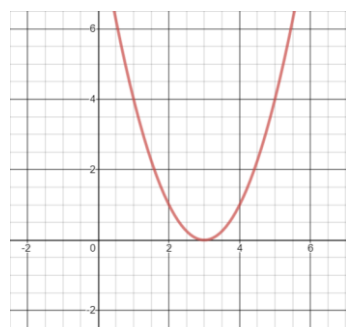
Given a graph or table of values of a single transformation of a function, write the equation of the transformed function.

Remarks/Examples:

Remark 1: Students should be exposed to replacing $f(x)$ with $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

Remark 2: Students should have knowledge of basic parent functions and their equations and graphs. Within the Algebra 1 course, students should be expected to work with functions including linear, quadratic, exponential and absolute value.

Example 1: Write the equation of the function graphed below that has been transformed from $f(x) = x^2$.



MA.912.F.2.8

Given a graph or table of values of two or more transformations of a function, write the equation of the transformed function.

MA.912.F.3 Create new functions from existing functions.

MA.912.F.3.1

Combine two functions, limited to linear and quadratic, using arithmetic operations. When appropriate, include domain restrictions for the new function. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on extending students' understanding of arithmetic operations on polynomial expressions to functions. Students should gain an understanding of when it is necessary to include domain restrictions to preserve properties of functions.

Example 1: Given $f(x) = 8x^2 + 3x - 7$ and $g(x) = 5x - 9$, find $(f - g)(x)$.

Example 2: A new cereal company decides they will make different sized boxes by changing only the width and keeping the length and height constant. The volume of the new cereal box, which can be expressed with the function $V(x) = 2x^3 + x^2 - 13x + 6$, depends on the width that is chosen, which can be expressed with the function $W(x) = x - 2$. Write a function to determine the remaining dimensions of the cereal box.

MA.912.F.3.2

Combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function. Include mathematical and real-world context.

MA.912.F.3.3

Solve problems involving functions combined using arithmetic operations. Include mathematical and real-world context.

MA.912.F.3.4

Compose functions. Determine the domain and range of the composite function. Include mathematical and real-world context.

MA.912.F.3.5

Solve problems involving composite functions. Include mathematical and real-world context.

MA.912.F.3.6

Determine if an inverse function exists by analyzing tables, graphs and equations.

MA.912.F.3.7

Represent the inverse of a function algebraically, graphically, or in a table. Use composition of functions to verify that one function is the inverse of the other.

MA.912.F.3.8

Produce an invertible function from a non-invertible function by restricting the domain.

9-12 Geometric Reasoning Strand*MA.912.GR.1 Use properties and theorems related to circles.*

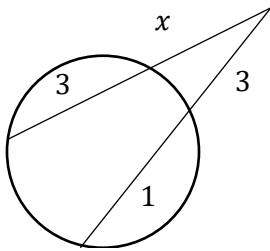
MA.912.GR.1.1

Solve mathematical and real-world problems involving the length of a secant, tangent and/or chord in a given circle.

Remarks/Examples:

Remark 1: Instruction should focus on student's understanding and application of the Chord-Chord Product Theorem, Secant-Secant Product Theorem, and the Secant-Tangent Product Theorem and length of a tangent from a point to a circle.

Example 1: Determine the value of x to the nearest tenth.



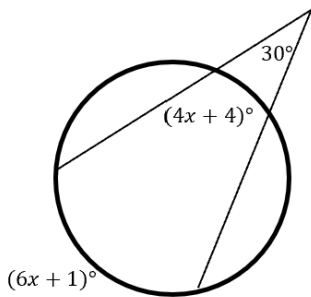
MA.912.GR.1.2

Solve mathematical and real-world problems involving the measures of arcs and related angles, limited to central, inscribed and intersections of chords, secants and/or tangents.

Remarks/Examples:

Remark 1: Instruction should focus on Inscribed Angle Theorem, central angles and angles formed by the intersection of the following: two secants, a tangent and a secant, two tangents, two chords and a perpendicular bisector and a chord. Instruction should focus on the measures of arcs and measures of angles formed based on the previously stated relationships.

Example 1: Determine the values of the intercepted arcs.



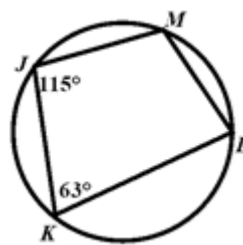
MA.912.GR.1.3

Solve problems involving quadrilaterals inscribed in a circle.

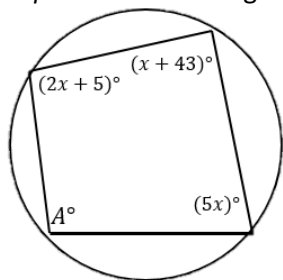
Remarks/Examples:

Remark 1: Instruction should focus on inscribed quadrilaterals.

Example 1: Given the figure to the right, find the measure of angle L.



Example 2: Given the figure below, find the value of A.



MA.912.GR.1.4

Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle.

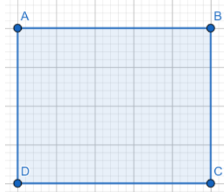
Remarks/Examples:

Remark 1: Instruction should focus on the conceptual understanding of that length of the arc intercepted by an angle is proportional to the radius.

Example 1: How far does the tip of the minute hand of a clock move in 20 minutes if the tip is 4 inches from the center of the clock?

Example 2: Paulo is planting a rectangular garden with dimensions 4 yd. by 5 yd. He can only install one automatic sprinkler that waters the lawn in a circular area.

- What area of the garden will be watered if he places a sprinkler at D that has a 4 yd. spraying radius?
- What area of the garden will be watered if he places a sprinkler at the midpoint of \overline{AB} that has a 2.5 yd. spraying radius?
- Which scenario covers more of the garden?



MA.912.GR.1.5

Apply transformations to prove that all circles are similar.

Remarks/Examples:

Remark 1: Using digital tools will be the easiest way to show this transformation. Tools such as Geogebra or Desmos will allow students to easily translate one center onto another and then dilate the original radius onto the new radius.

Example 1: Given that circle A has a radius of 3 and circle B has a radius of 5, use transformations to explain why circle A and B are similar.

MA.912.GR.2 Apply properties of transformations to describe congruence or similarity.

MA.912.GR.2.1

Describe and represent transformations as functions that take points in the plane as the domain and give other points as the range.

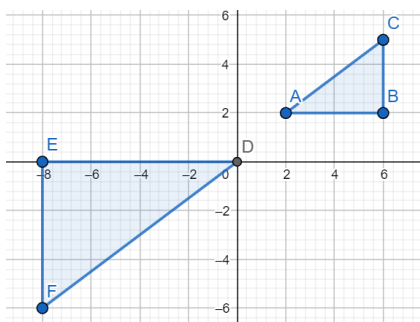
Remarks/Examples:

Remark 1: Students should be able to describe transformations algebraically and in words.

Remark 2: Students should be exposed to the algebraic descriptions of transformations.

- Translations $(x, y) \rightarrow (x + a, y + b)$
- Rotation of 90° about the origin $(x, y) \rightarrow (-y, x)$; rotation of 180° about the origin $(x, y) \rightarrow (-x, -y)$; rotation of 270° about the origin $(x, y) \rightarrow (y, -x)$
- Dilations when given the center of dilation $(x, y) \rightarrow (kx, ky)$

Example 1: Describe the transformation(s) needed to map $\triangle ABC$ onto $\triangle DEF$ as shown on the graph below.

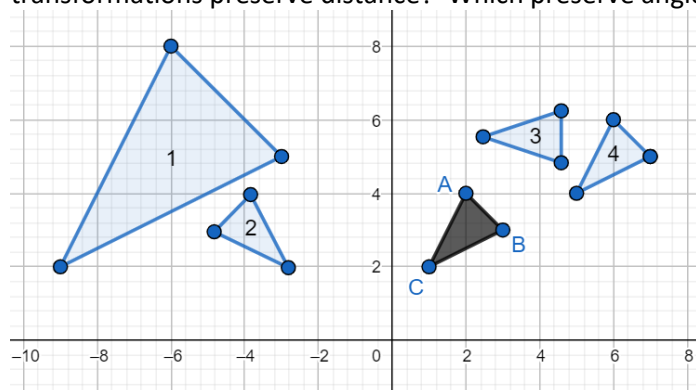


MA.912.GR.2.2 Compare transformations that do or do not preserve distance or angle measure.

Remarks/Examples:

Remark 1: Provide students with various polygons and transformations to put the polygon under to determine which preserve distance, angle measure, both or neither.

Example 1: Determine which transformation is needed to map triangle ABC onto each of the four triangles. Which transformations preserve distance? Which preserve angle measure?



MA.912.GR.2.3 Specify a sequence of two or more transformations that will map a given figure onto itself or onto another figure.

Remarks/Examples:

Remark 1: Transformation should include rotations, translations, reflection and dilations. When transforming using rotations, students should be exposed to 90° , 180° and 270° . When transforming using dilations, students should be given the center of dilation.

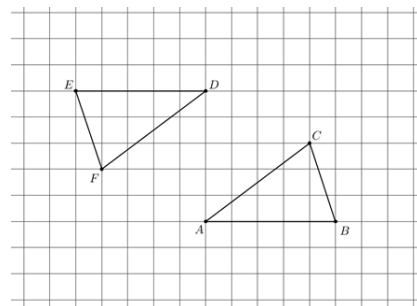
Remark 2: Students should be exposed to the algebraic descriptions of transformations.

- Translations $(x, y) \rightarrow (x + a, y + b)$
- Rotation of 90° about the origin $(x, y) \rightarrow (-y, x)$; rotation of 180° about the origin $(x, y) \rightarrow (-x, -y)$; rotation of 270° about the origin $(x, y) \rightarrow (y, -x)$
- Dilations when given the center of dilation $(x, y) \rightarrow (kx, ky)$

Remark 3: Within the geometry course, students are not expected to use matrices to describe transformations.

Example 1: Given a triangle with vertices at (1, 3), (2, 5) and (6, 1), determine the sequence of transformation to map it to a triangle with vertices (-3, 1), (-2, 1) and (2, -3). Are these triangles congruent, similar or neither?

Example 2: Describe the transformation that will map triangle ABC to triangle DEF. You may assume that all vertices are located at the intersections of grid lines.



MA.912.GR.2.4

Given a geometric figure and a sequence of two or more transformations, draw the transformed figure on a coordinate plane.

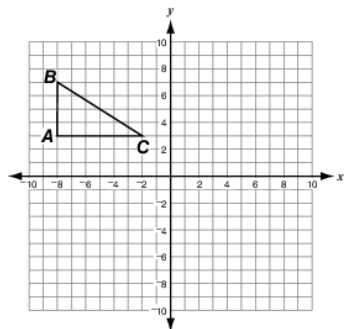
Remarks/Examples:

Remark 1: Transformation should include rotations, translations, reflection and dilations.

Remark 2: Within the geometry course, students are not expected to use matrices for transformations.

Example 1: A triangle has vertices at (1, 1), (4, 1) and (1, 3). The triangle is rotated 270 degrees about the origin and then translated 4 units to the left. Draw the described transformed figure on a coordinate plane.

Example 2: Transform $\triangle ABC$ into $\triangle A'B'C'$ by translating each segment of the triangle 4 units to the right and up 2 units. Next, Transform $\triangle A'B'C'$ into $\triangle A''B''C''$ by reflecting it over the line $y = 1$.



MA.912.GR.2.5

Apply rigid transformations to map one figure onto another and justify that corresponding sides and angles are congruent.

Remarks/Examples:

Remark 1: Rigid transformations, also called isometry, include any that will result in congruent figures.

Remark 2: Within the geometry course, students are not expected to use matrices to describe transformations.

Example 1: Two triangles are drawn in coordinate plane. The vertices of the first triangle are (2, 12), (2, 4) and (0,6). The vertices of the second triangle are (16,6), (16, 14) and (18,12). Are there a series of rigid transformations that can map the first triangle onto the second triangle? If so, list them.

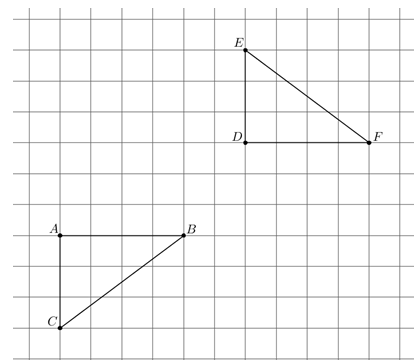
MA.912.GR.2.6

Justify the criteria for triangle congruence using the definition of congruence in terms of rigid transformations.
Criteria is limited to Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side and Hypotenuse-Leg.

Remarks/Examples:

Remark 1: Within the geometry course, students are not expected to use matrices to describe transformations.

Example 1: Justify that triangle ABC is congruent to triangle DFE using rigid transformations. Assume that all vertices are located at the intersections of grid lines.



MA.912.GR.2.7

Apply non-rigid transformations to identify similar figures and justify their corresponding proportional sides and their congruent corresponding angles.

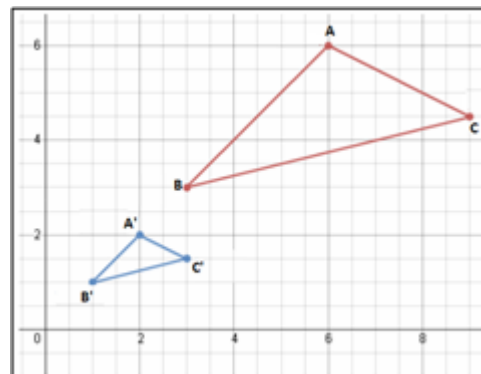
Remarks/Examples:

Remark 1: Non-rigid transformations include any that result not preserving the size or shape of the figure.

Remark 2: Within the geometry course, students are not expected to use matrices to describe transformations.

Example 1: In the figure below triangle ABC is the pre-image is triangle A'B'C' before a sequence of similarity transformations. Determine if these two figures are similar by selecting all the statements that are true.

- ☐ There was a translation 5 units right and 4 units up.
- ☐ There was a translation 5 units left and 4 units down.
- ☐ There was a dilation of scale factor $A'C'/AC$ centered at the origin.
- ☐ There was a dilation of scale factor $AC/A'C'$ centered at the origin.
- ☐ Angle A is congruent to angle A' and angle C is congruent to angle C' because dilations preserve angle measure.
- ☐ Triangle ABC is not similar to triangle A'B'C'.
- ☐ Triangle ABC is similar to triangle A'B'C'.



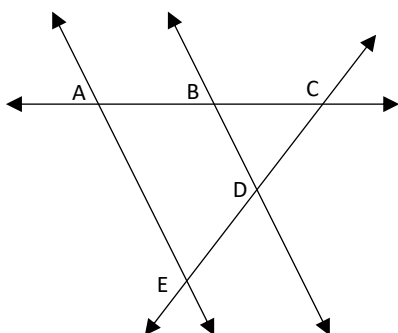
MA.912.GR.2.8

Justify the criteria for triangle similarity using the definition of similarity in terms of non-rigid transformations.
Criteria is limited to Angle-Angle, Side-Angle-Side, and Side-Side-Side.

Remarks/Examples:

Remark 1: Within the geometry course, students are not expected to use matrices to describe transformations.

Example 1: Given: $\overleftrightarrow{AE} \parallel \overleftrightarrow{BD}$ Prove: $\triangle ACE \sim \triangle BCD$



MA.912.GR.3 Prove and apply geometric theorems to solve problems.

MA.912.GR.3.1

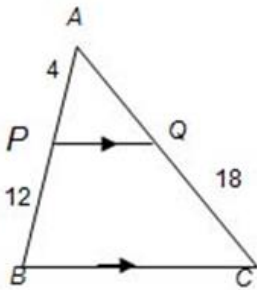
Prove triangle congruence or similarity using Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg.

Remarks/Examples:

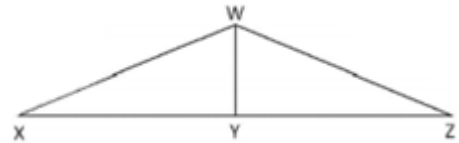
Remark 1: Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.

Remark 2: In working with proofs in the classroom, students should be exposed to two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: Given the figure below and that line PQ is parallel to line BC, prove that triangles ABC and APQ are similar.



Example 2: Given the figure to the right and that segment WY is the perpendicular bisector of segment XZ, prove that triangle WXY is congruent to WZY.



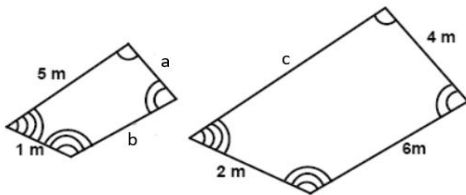
MA.912.GR.3.2

Solve problems involving congruence or similarity in two-dimensional figures. Include mathematical and real-world context.

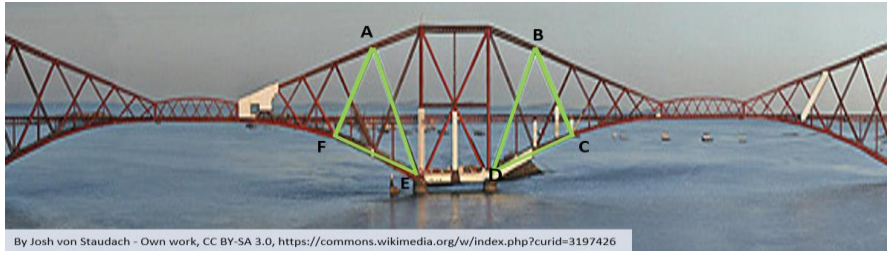
Remarks/Examples:

Remark 1: Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.

Example 1: Given the quadrilaterals below, find the lengths of sides a, b and c.



Example 2: Josh von Staudach's photo of the Forth Rail Bridge shows that the bridge used many similar triangles in its construction. Write six similarity statements about triangle FAE and triangle BCD. What measurements would you need to verify the triangles are similar?



By Josh von Staudach - Own work, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=3197426>

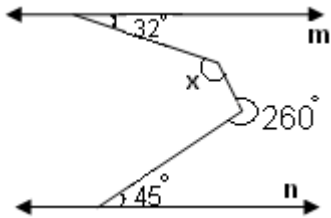
MA.912.GR.3.3

Prove theorems about lines and angles. Solve mathematical and real-world problems involving theorems of lines and angles.
Relationships and theorems are limited to vertical angles, special angles formed by parallel lines and transversals, angle bisectors, congruent supplements, congruent complements, and a perpendicular bisector of a line segment.

Remarks/Examples:

Remark 1: In working with proofs in the classroom, students should be exposed to two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: In the diagram below, the lines m and n are parallel. Find the value of x .



Example 2: Airport runway numbers refer to the number of degrees off due north. For example, Runway 15 is 150° off due north and Runway 21 is 210° off due north. The two lines marking due north are parallel to each other. Using properties of parallel lines and runway naming conventions, what is the numerical name of the runway?



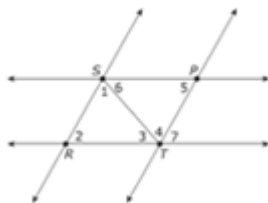
MA.912.GR.3.4

Prove theorems about triangles. Solve mathematical and real-world problems involving theorems of triangles.
Relationships, theorems and their converses are limited to interior triangle sum, base angles of isosceles triangles, mid-segment of a triangle, concurrency of medians, concurrency of angle bisectors, concurrency of perpendicular bisectors, triangle inequality, and the Hinge Theorem.

Remarks/Examples:

Remark 1: In working with proofs in the classroom, students should be exposed to two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: Given the figure below and triangle RST is congruent to triangle PTS, line SP is parallel to line RT and line SR is parallel to PT, prove that the sum of the measures of the interior angles of triangle RST is 180 degrees.



Example 2: Carla contends that the sum of the interior angles of the outlined hexagon is 1440 degrees because the hexagon is composed of two triangles and four quadrilaterals. Meisha disagrees because she can prove the sum of the interior angles of any hexagon is 720 degrees. Determine who is correct and support your choice.



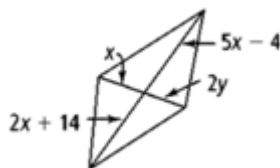
MA.912.GR.3.5

Prove theorems about parallelograms. Solve mathematical and real-world problems involving theorems of parallelograms.
Relationships, theorems and their converses are limited to opposite sides of a parallelogram, opposite angles of a parallelogram, the diagonals of a parallelogram, and consecutive angles of a parallelogram.

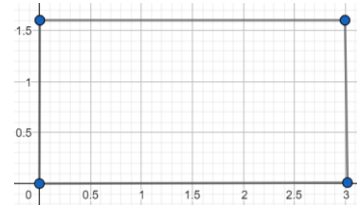
Remarks/Examples:

Remark 1: In working with proofs in the classroom, students should be exposed to two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: Given the figure below, for what values of x and y must the figure be a parallelogram?



Example 2: Number 1 on the Yellow Jacket football team is running a route where he will catch the ball at the intersection of the diagonals of the football field. Put an X on the coordinate plane where #1 will catch the ball. What are the coordinates of the location?



MA.912.GR.3.6

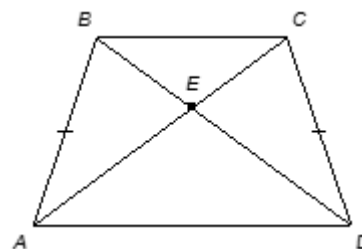
Prove theorems about trapezoids. Solve mathematical and real-world problems involving theorems of trapezoids.
Relationships and theorems are limited to the midsegment; base angles are congruent; opposite angles of an isosceles are supplementary; and diagonals of an isosceles are congruent.

Remarks/Examples:

Remark 1: In working with proofs in the classroom, students should be exposed to two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: Given the figure and that ABCD is a trapezoid with segment BC parallel to segment AD and the measure of angle BAD is congruent to the measure of angle CDA. Select all the statements that can be concluded.

- ☐ Triangle AED is congruent to triangle CEB
- ☐ Triangle AED is similar to triangle CEB
- ☐ Segment BC is congruent to segment AD
- ☐ Segment BE is congruent to segment DE
- ☐ Segment BD is congruent to segment AC
- ☐ Segment AE is congruent to segment CE



MA.912.GR.4 Use geometric measurement and dimensions to solve problems.

MA.912.GR.4.1 Identify the shapes of two-dimensional cross-sections of three-dimensional objects.

Remarks/Examples:

Remark 1: Students should have practice with cross-sections not be limited to vertical or horizontal.

Example 1: Select all of the possible cross-sections of a cone.

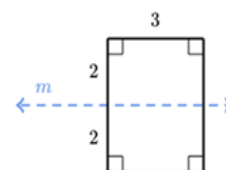
- ☐ Line
- ☐ Square
- ☐ Rectangle
- ☐ Circle
- ☐ Parabola
- ☐ Point

MA.912.GR.4.2 Identify three-dimensional objects generated by rotations of two-dimensional objects.

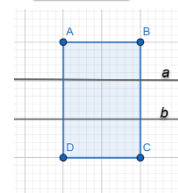
Remarks/Examples:

Remark 1: Within the course of geometry, students should only have practice with rotations about a vertical or horizontal axis.

Example 1: Given the figure to the right, what solid three-dimensional solid is produced by rotating the rectangle about line m ?



Example 2: Describe the difference between the solid formed when rotating ABCD about line a and the solid formed about line b .



MA.912.GR.4.3 Determine how changes in dimensions affect the area of two-dimensional figures and the surface area or volume of three-dimensional figures.

Remarks/Examples:

Remark 1: Instruction should focus on the effect on a specific value when it is multiplied by a scalar versus squaring or cubing.

Example 1: If the lengths of each side of a trapezoid are tripled, determine the change in its area, and justify your answer.

Example 2: Explain how changing the radius or height of a cylinder affects its surface area and volume. Justify your response algebraically.

Example 3: The volume formula for a cone is $V = \frac{1}{3}\pi hr^2$, where r is the radius of the base of the cone and h is the height of the cone. Compare what happens to the volume of the cone when the height is doubled and when the radius is doubled.

MA.912.GR.4.4

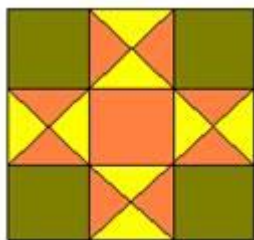
Solve mathematical and real-world problems involving the area of two-dimensional figures.

Remarks/Examples:

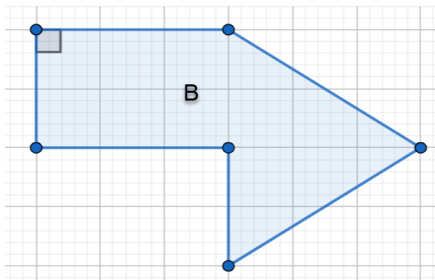
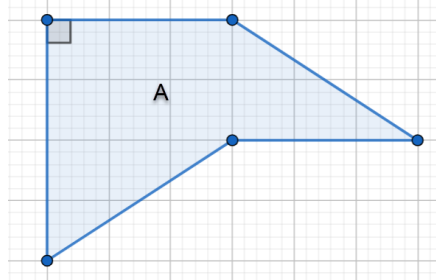
Remark 1: Students have previous knowledge of formulas for quadrilaterals. Students should be introduced to formulas for areas of polygons.

Remark 2: Problem types should include finding missing dimensions and area of composite shapes.

Example 1: The design below is called the Ohio Star. Assuming that it measures 9 inches by 9 inches, calculate the total area of all the orange patches, the total area of all the yellow patches, and the total area of all the green patches. How much fabric of each color will you need to cover an area that measures 72 inches by 90 inches?



Example 2: The grids for both polygon A and B are the same size. Describe how to determine the area of polygon A. Is the area of polygon B greater than, equal to or less than the area of polygon A?



MA.912.GR.4.5

Solve mathematical and real-world problems involving the volume of three-dimensional solids limited to cylinders, pyramids, prisms, cones and spheres.

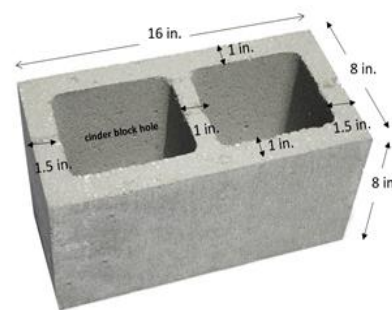
Remarks/Examples:

Remark 1: Students have previous knowledge of formulas for cylinders, prisms and pyramids. Students have only worked with right three-dimensional solids so non-right solids should be introduced and used.

Remark 2: Problem types should include finding missing dimensions and volume of composite solids.

Example 1: A gold class ring is dropped into a glass that is a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the ring?

Example 2: Joshua is going to create a garden border around three sides of his backyard deck using cinder blocks. He is going to plant a flower in each hole of the cinder block. The dimensions of the cinder blocks are 8 inches x 16 inches x 8 inches. Each hole needs to be completely filled with potting soil before the flowers can be planted. Potting soil is sold in 1 cubic foot bags. What are the dimensions of a cinder block hole? The patio is a square with a side length of 8 feet. One of the sides of the square patio is adjacent to an exterior wall of the house. If Joshua puts blocks around the other three sides of the patio, how many bags will Joshua need to purchase to fill the blocks?



Example 3: Calculate the volume of ice cream to the nearest tenth cu.in. The cone is filled with ice cream and there is a half of a scoop (hemisphere) of ice cream on the top.



MA.912.GR.4.6

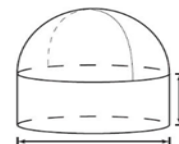
Solve mathematical and real-world problems involving the surface area of three-dimensional solids limited to cylinders, pyramids, prisms, cones and spheres.

Remarks/Examples:

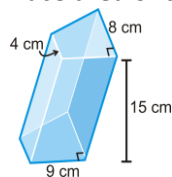
Remark 1: Students have previous knowledge of formulas for cylinders, prisms and pyramids. Students have only worked with right three-dimensional solids so non-right solids should be introduced and used.

Remark 2: Surface area is the sum of the lateral area of the solid and the area of the base(s) of the solid. Problem types should include both surface and lateral area including finding missing dimensions and composite solids.

Example 1: Joseph works at the Tasty Cupcakery and packages cupcakes in cardboard containers shaped like right circular cylinders with hemispheres on top, as shown on the diagram below. Joseph wants to wrap the containers completely in colored plastic wrap and needs to know how much wrap he will need. If the diameter of the base is 5 inches and the height of the cylinder is 3.75 inches, what is the total exterior surface area of the container?



Example 2: Find the surface area of the prism shown below.



MA.912.GR.5 Use coordinate geometry to solve problems or prove relationships.

MA.912.GR.5.1

Use coordinate geometry, definitions, properties or theorems to classify or justify properties of circles, triangles or quadrilaterals.

Remarks/Examples:

Remark 1: Students will demonstrate fluency with the use of coordinates to establish geometric results, calculate length and angle measures.

Remark 2: Students should have practice justifying using two-column proofs, paragraph/narrative proofs, pictorial proofs, flow-chart proofs or any informal proofs.

Example 1: Given a quadrilateral with vertices $(0,0)$, $(\frac{5}{2}, \frac{\sqrt{3}}{2})$, $(5,0)$, $(7, \frac{7\sqrt{3}}{3})$, prove that the diagonals of this quadrilateral are perpendicular.

Example 2: Prove or disprove that the point $(1, \sqrt{3})$ lies on a circle centered at the origin and contains the point $(0, 2)$.

Example 3: Draw the polygon defined by the following vertices $(1, 3)$, $(-1, 3)$, $(3, 1)$, $(-3, 1)$, $(1, -3)$, $(-1, -3)$, $(-3, -1)$, $(3, -1)$. Is this polygon regular? Justify your answer.

MA.912.GR.5.2

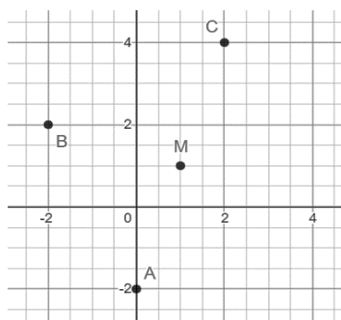
Solve geometric problems on the coordinate plane using slope criteria.

Remarks/Examples:

Remark 1: Problems should include relationships of angles formed when parallel and perpendicular lines are cut by a transversal.

Example 1: Given points $P(2, -1)$, $Q(-4, 2)$ and $M(5,3)$, find the coordinates of a point N such that line PQ and line MN are parallel. Find the coordinates of a point K such that line MK is perpendicular to line PQ .

Example 2: Point M is the midpoint of the diagonals of quadrilateral $ABCD$. Use the graph and quadrilateral properties to classify $ABCD$ and determine the coordinates of point D .



MA.912.GR.5.3

Solve mathematical and real-world problems on the coordinate plane involving finding the coordinates of a point on a line segment including the midpoint.

Remarks/Examples:

Remark 1: Problem types should include finding an endpoint when given a ratio and another endpoint on the line segment, finding the ratio when given endpoints on the line segment, and find the midpoint. The point on the segment is not limited to the midpoint.

Example 1: Given point $A(3, -4)$ and point $B(8, 6)$ on a directed line segment AB , what is the y -coordinate of point F that partitions AB in the ratio of 3:2?

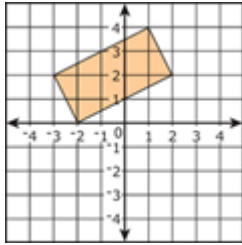
MA.912.GR.5.4

Solve mathematical and real-world problems on the coordinate plane involving the perimeter or area of polygons.

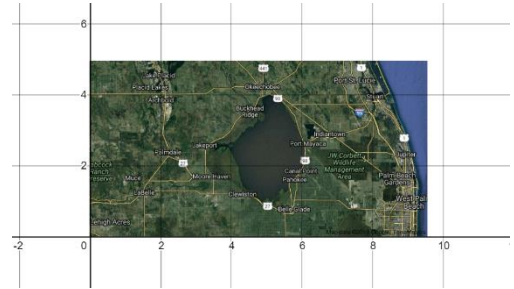
Remarks/Examples:

Remark 1: Students should be exposed to problems that involve convex, concave, regular and irregular polygons.

Example 1: Given the figure below, calculate the area and perimeter of the rectangle.



Example 2: An aerial view of Lake Okeechobee is shown on the graph. One square unit represents approximately 243 square miles. Determine the approximate area of Lake Okeechobee.



MA.912.GR.6 Apply geometric and algebraic representations of conic sections.

MA.912.GR.6.1 Identify the conic resulting from the cross-section of cones.

MA.912.GR.6.2 Derive and create the equation of a circle given key features. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Key features are limited to the center, radius, diameter and points on the circle.

Remark 2: Students have previous knowledge of completing the square from algebra 1 and should derive the equation using that technique and/or using the Pythagorean Theorem. Within the course of geometry, students should only have practice with mathematical problems.

Example 1: Write the equation of the circle with radius 10 and center (6, -3).

Example 2: The endpoints of a diameter of a circle are $(-2, 3)$ and $(2, -1)$. Write the equation of the circle. And determine the coordinates of the center and radius.

MA.912.GR.6.3 Solve and graph mathematical and real-world problems involving an equation of a circle. Determine and interpret key features in context. Key features are limited to domain, range, center and radius.

Remarks/Examples:

Remark 1: Students should not be expected to create equations of circle.

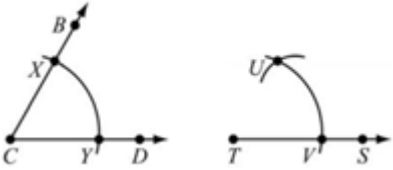
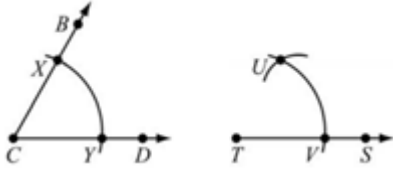
Example 1: Sketch the graph of the circle whose equation is $(x - 3)^2 + (y + 2)^2 = 16$.

MA.912.GR.6.4 Derive and create the equation of a parabola given key features. Include mathematical and real-world context.

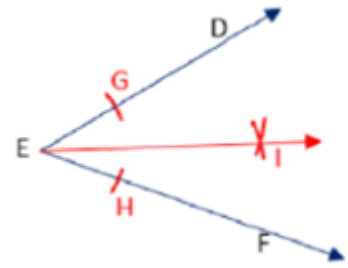
MA.912.GR.6.5 Solve and graph mathematical and real-world problems involving an equation of a parabola. Determine and interpret key features in context. Key features limited to domain, range, intercepts, focus, vertex and directrix.

MA.912.GR.6.6 Derive and create the equation of an ellipse given key features. Include mathematical and real-world context.

MA.912.GR.6.7	Solve and graph mathematical and real-world problems involving an equation of an ellipse. Determine and interpret key features in context. Key features limited to domain, range, center, foci, major axis, minor axis and vertices.
MA.912.GR.6.8	Derive and create the equation of a hyperbola given key features. Include mathematical and real-world context.
MA.912.GR.6.9	Solve and graph mathematical and real-world problems involving an equation of a hyperbola. Determine and interpret key features in context. Key features limited to domain, range, center, vertices, foci, major axis, minor axis, asymptotes and directrices.

MA.912.GR.7 Make formal geometric constructions with a variety of tools and methods.	
MA.912.GR.7.1	Construct a copy of a segment or an angle.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be able to justify steps needed to complete the construction.</p> <p><i>Remark 2:</i> Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.</p> <p><i>Example 1:</i> Louis is copying angle BCD below. What could his next step be in Louis's construction?</p> <div style="display: flex; align-items: center;">  </div> <p><i>Example 2:</i> Answer the following questions about the construction shown.</p> <ol style="list-style-type: none"> What measurement is used on the compass to make the arc that creates point U? Why does the measurement from part a guarantee that angle T is a copy of angle C? <div style="display: flex; align-items: center;">  </div>	
MA.912.GR.7.2	Construct the bisector of a segment or an angle, including the perpendicular bisector of a line segment.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be able to justify steps needed to complete the construction.</p> <p><i>Remark 2:</i> Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.</p> <p><i>Example 1:</i> Denise constructed the angle bisector of $\angle DEF$ as shown below, providing each of her steps, using her compass and straightedge. What is the missing step #4 in her construction?</p>	

1. Place your compass on vertex E and open it some width. Swing an arc that intersects both ray ED and ray EF.
2. Label these points G and H respectively.
3. Place the compass on point G and open it to swing an arc inside angle DEF.
- 4.
5. Label the point of intersection as point I.
6. Draw a ray from vertex point E to point I to create the angle bisector.



Example 2: Given a rectangle ABCD, construct square AJKD, where point J lies on line AB and K lies on line CD.

MA.912.GR.7.3

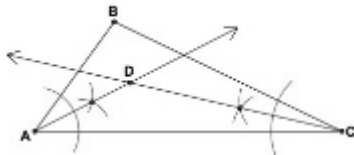
Construct the inscribed and circumscribed circles of a triangle.

Remarks/Examples:

Remark 1: Students should be able to justify steps needed to complete the construction. Problems should include obtuse, acute, right, scalene, isosceles and equilateral triangles.

Remark 2: Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Example 1: Paige has completed the first few steps for constructing the inscribed circle for triangle ABC as shown below. She started by constructing the angle bisectors for angle A and C which gives her the incenter, point D. What is the next step Paige should take?



Example 2: Classify the following statements as true or false and justify your answer.

- A. The intersection of angle bisectors of an obtuse triangle is always in the interior of the triangle.
- B. The intersection of the perpendicular bisectors of the sides of an acute triangle may be in the exterior of the triangle.
- C. The intersection of perpendicular bisectors of a right triangle is always on the triangle.
- D. The center of a circle circumscribed about a triangle is never the same point as the center of the circle inscribed about the same triangle.

MA.912.GR.7.4

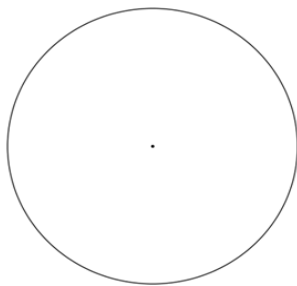
Construct a regular polygon inscribed in a circle. Regular polygons are limited to triangles, quadrilaterals and hexagons.

Remarks/Examples:

Remark 1: Students should be able to justify steps needed to complete the construction.

Remark 2: Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Example 1: Using a compass and straightedge, construct a regular hexagon inscribed in the circle provided. Leave all necessary construction marks as justification of your process.



MA.912.GR.7.5

Given a point outside a circle, construct a line tangent to the circle that passes through the given point.

Remarks/Examples:

Remark 1: Students should be able to justify steps needed to complete the construction.

Remark 2: Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Example 1: Which of the following are steps for constructing a line tangent to a given circle? Select all that apply.

- ☐ Draw a line connecting the point to the center of the circle.
- ☐ Use a straightedge to draw a ray.
- ☐ Construct the perpendicular bisector of that line.
- ☐ Place the compass on the midpoint, adjust its length to reach the endpoint, and draw an arc across the circle.
- ☐ Draw arcs above and below the line.

9-12 Number Sense & Operations Strand

MA.912.NSO.1 Perform operations on expressions involving exponents, radicals or logarithms.

MA.912.NSO.1.1

Extend previous understanding of the Laws of Exponents to include rational exponents. Evaluate and generate equivalent numerical expressions involving rational exponents.

Remarks/Examples:

Remark 1: In previous grade levels, students have been working on numerical expressions and using the Laws of Exponents. They have worked with integer exponents with rational bases. Instruction in Algebra 1 should focus on extending this knowledge by using the Laws of Exponents with rational exponents and rational bases. Students should have practice using positive and negative rational numbers.

Example 1: What is the value of the expression $\frac{5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}}}{5^{\frac{3}{4}}}$?

Example 2: What is the value of the expression $8^{\frac{2}{3}}$?

MA.912.NSO.1.2

Generate equivalent monomial algebraic expressions by using the Laws of Exponents.

Remarks/Examples:

Remark 1: In previous grade levels, students have been working on numerical expressions and using the Laws of Exponents. They have worked with integer exponents with rational bases. Instruction in Algebra 1 should focus on extending previous knowledge of the Laws of Exponents to algebraic expressions. Students should have practice using positive and negative rational numbers.

Example 1: Simplify the expression $\left(\frac{x^{-2}}{16y^{\frac{1}{2}}}\right)^{\frac{1}{4}}$.

MA.912.NSO.1.3	Generate equivalent algebraic expressions involving radicals and/or rational exponents using the properties of exponents. Radicands are limited to monomial algebraic expressions.
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> In previous grade levels, students have been working on operations with rational numbers as well as simplifying perfect square and perfect cube roots. Instruction in Algebra 1 should focus on operations with radicals, both perfect and imperfect roots, including how to simplify.</p> <p><i>Example 1:</i> Simplify the expression $2\sqrt[3]{54} - \sqrt[3]{16}$.</p>	
MA.912.NSO.1.5	Add, subtract, multiply and divide algebraic expressions involving radicals. Radicands are limited to monomial algebraic expressions.

<i>MA.912.NSO.2 Represent and perform operations on expressions within the complex number system.</i>	
MA.912.NSO.2.1	Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers.
MA.912.NSO.2.2	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane.
MA.912.NSO.2.3	Calculate the distance and midpoint between two numbers on the complex coordinate plane.
MA.912.NSO.2.4	Solve problems involving complex numbers represented algebraically or on the coordinate plane. Include mathematical and real-world context.
MA.912.NSO.2.5	Represent complex numbers on the complex plane in rectangular and polar forms. Explain why the rectangular and polar forms of a given complex number represent the same number.
MA.912.NSO.2.6	Rewrite complex numbers to trigonometric form. Multiply complex numbers in trigonometric form.

<i>MA.912.NSO.3 Represent and perform operations on vectors.</i>	
MA.912.NSO.3.1	Use appropriate notation and symbols to represent vectors in the plane as directed line segments. Determine the magnitude and direction of a vector in component form.
MA.912.NSO.3.2	Represent vectors in component form, linear form or trigonometric form. Rewrite vectors from one form to another.
MA.912.NSO.3.3	Solve problems involving velocity and other quantities that can be represented by vectors. Include mathematical and real-world context.

MA.912.NSO.3.4	Solve problems using dot product and vector projections in two-dimensions. Include mathematical and real-world context.
MA.912.NSO.3.5	Solve problems using dot product and cross product in three-dimensions. Include mathematical and real-world context.
MA.912.NSO.3.6	Add and subtract vectors algebraically or graphically.
MA.912.NSO.3.7	Given the magnitude and direction of two or more vectors, determine the magnitude and direction of their sum.
MA.912.NSO.3.8	Multiply a vector by a scalar algebraically or graphically.
MA.912.NSO.3.9	Compute the magnitude and direction of a vector scalar multiple.

<i>MA.912.NSO.5 Represent and perform operations on matrices.</i>	
MA.912.NSO.5.1	Represent and manipulate data using matrices. Include mathematical and real-world context.
MA.912.NSO.5.2	Represent and solve a system of two- or three-variable linear equations using matrices. Include mathematical and real-world context.
MA.912.NSO.5.3	Solve problems using addition, subtraction and multiplication of matrices.
MA.912.NSO.5.4	Solve problems using the inverse and determinant of matrices. Include mathematical and real-world context.
MA.912.NSO.5.5	Identify and use the additive and multiplicative identities for matrices to solve problems. Include mathematical and real-world context.

9-12 Statistics & Probability Strand

<i>MA.912.SP.1 Use and interpret independence and probability.</i>	
MA.912.SP.1.1	Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections or complements of other events.
MA.912.SP.1.2	Determine if events A and B are independent by calculating the product of their probabilities.
MA.912.SP.1.3	Calculate the conditional probability of two events and interpret the result in terms of its context.
MA.912.SP.1.4	Interpret the independence of two events using conditional probability.
MA.912.SP.1.5	Given a two-way table, interpret joint, marginal and/or conditional relative frequencies in terms of a context. Recognize possible associations in the data.
Remarks/Examples:	

Remark 1: Within the Algebra 1 course, recognizing possible associations should not include determining independence or other formal tests for association.

Example 1: Students in a film class of upperclassmen were surveyed about whether they preferred drama or horror movies. The results were recorded in the two-way frequency table below. What does the value 0.3 represent?

	Juniors	Seniors	Total
Drama	0.25	0.25	0.5
Horror	0.3	0.2	0.5
Total	0.55	0.45	1.0

Example 2: The results of a survey of 50 people about whether they like reading and video games is given in the table below. Determine if there are any associations in the given data.

	Like to read	Do not like to read	Total
Like video games	0.44	0.26	0.7
Do not like video games	0.18	0.12	0.3
Total	0.62	0.38	1.0

MA.912.SP.1.6

Decide if events are independent and approximate conditional probabilities using two-way tables as a sample space.

MA.912.SP.1.7

Given a real-world context, construct and interpret representations to find relative frequency, probabilities and conditional probabilities. Representations limited to two-way frequency tables, tree diagrams and area models.

Remarks/Examples:

Remark 1: instruction should include choosing which representation is most appropriate based on the provided context.

Example 1: Pamela asked if anyone would be interested in a co-ed soccer team. Of the 28 boys who responded, 18 said that they would play and 4 were undecided. Of the 22 girls who responded, 6 said they did not want to play and 3 were undecided. Create a two-way frequency table to represent this situation and find the relative frequency of boys that are not interested in playing co-ed to the total number of students asked.

Example 2: Jerilynn can order either a thick or thin crust pizza with either mushrooms or pepperoni. Create a tree diagram to represent this situation and find the probability Jerilynn will choose a thin crust pizza with mushrooms.

Example 3: Brandi has 3 mystery buckets of candy from which she may choose just one piece. Bucket 1 has 2 jellybeans and 3 gumballs. Bucket 2 has 4 sour patch kids and 5 jellybeans. And Bucket 3 has 1 jellybean and 3 gobstoppers. Create an area model to represent this situation and determine the probability Brandi will choose a jellybean.

MA.912.SP.1.8

Given a two-way table, construct and interpret a segmented bar graph. Determine relative frequencies, any independence and any possible associations in the data. Include mathematical and real-world context.

MA.912.SP.1.9	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.912.SP.1.10	Apply the Addition Rule for probability, taking into consideration whether the events are mutually exclusive, and interpret the result in terms of the model. Include mathematical and real-world context.
MA.912.SP.1.11	Apply the general multiplication rule for probability, taking into consideration whether the events are independent, and interpret the result in terms of the context. Include mathematical and real-world context.
MA.912.SP.1.12	Calculate the appropriate permutation or combination for a given situation.
MA.912.SP.1.13	Compute probabilities of compound events and solve problems using permutations and combinations.

<i>MA.912.SP.2 Determine methods of data collection and make inferences from collected data.</i>	
MA.912.SP.2.1	Distinguish between a population parameter and a sample statistic.
MA.912.SP.2.2	Explain how random sampling produces data that is representative of a population.
MA.912.SP.2.3	Compare and contrast sampling methods.
MA.912.SP.2.4	Generate multiple samples or simulated samples of the same size to measure the variation in estimates or predictions.
MA.912.SP.2.5	Determine if a specific model is consistent within a given process by analyzing the data distribution from a data-generating process.
MA.912.SP.2.6	Determine the appropriate design, survey, experiment or observational study, based on the purpose. Articulate the types of questions appropriate for each type of design.
MA.912.SP.2.7	Compare and contrast surveys, experiments and observational studies.
MA.912.SP.2.8	Explain how randomization relates to sample surveys, experiments and observational studies.
MA.912.SP.2.9	Estimate a population mean or proportion using data from a sample survey. Calculate and interpret the corresponding margin of error.
MA.912.SP.2.10	Draw inferences about two populations using data and statistical analysis from two random samples.
MA.912.SP.2.11	Compare two treatments from an experiment using data from a randomized experiment.
MA.912.SP.2.12	Determine if differences between parameters are significant using simulations.
MA.912.SP.2.13	Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining if a valid sampling method was used; or interpreting provided statistics.

MA.912.SP.3 Summarize, represent and interpret one- and two-variable data.

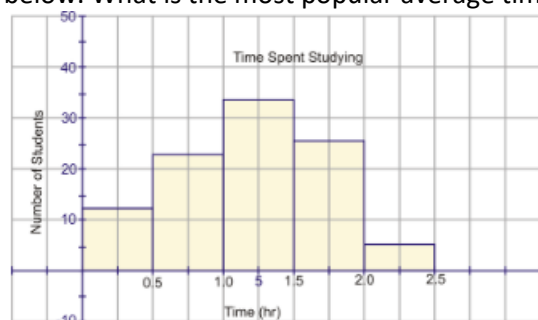
MA.912.SP.3.1

Analyze and interpret numerical data distributions represented with frequency tables, histograms, stem-and-leaf plots, dot plots and box plots.

Remarks/Examples:

Remark 1: Instruction should focus on analyzing and interpreting data distributions which builds on the fluency of creating these types of data displays in middle school.

Example 1: At the end of a school year, a principal wanted to learn more about her rising seniors and asked them to provide the average amount of time they spent on homework each night. The results are recorded in the histogram below. What is the most popular average time spent on homework each night?



MA.912.SP.3.2

Calculate and compare the appropriate measures of center and measures of variability of two or more different data sets, accounting for possible effects of outliers. Include mathematical and real-world context.

Remarks/Examples:

Remark 1: Instruction should focus on using measures of center, including the mean and median, and measures of variability, including range, interquartile range (IQR) and standard deviation.

Remark 2: Within the algebra 1 course, students should not be required to calculate the standard deviation, but should be expected to compare data sets when it is given to them.

Example 1: The city of Palm Beach is evaluating their bus systems and how many people use it. They took samples of the passenger load for five runs of three different buses. The results from the samples are shown in the table below. Which bus has the largest variability in passenger load?

Bus A	15	24	19	12	20
Bus B	18	21	16	14	16
Bus C	16	29	20	21	17

MA.912.SP.3.3

Identify if a data set is normally distributed. Include mathematical and real-world context.

MA.912.SP.3.4

Fit a normal distribution, if appropriate, and estimate population percentages using the mean and standard deviation of a data set. Include mathematical and real-world context.

MA.912.SP.3.5

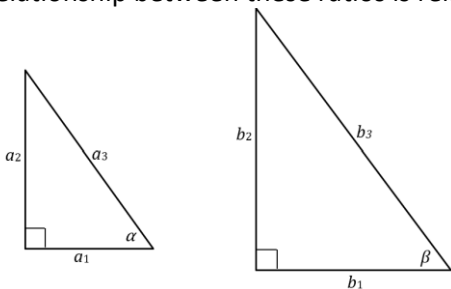
Estimate areas under the normal curve using technology, empirical rule or tables. Include mathematical and real-world context.

MA.912.SP.3.6	Fit a linear function to data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve problems in the context of the data.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Instruction should include fitting a linear function both informally and formally, with the use of technology.</p> <p><i>Remark 2:</i> Within the algebra 1 course, students should not be required to use regression, but should be expected to solve problems in the context of the data given a regression equation.</p> <p><i>Example 1:</i> When studying the growth of plants, the science club wanted to determine if the height of the plant would increase as its diameter increased. The function $H(d) = d + 1.09$ models this data, where H is the height of the plant and d is its diameter, explain what the slope and y-intercept mean in context.</p> <p><i>Example 2:</i> The Chief Operating Officer of a clothing company found that its sales followed a linear regression model based on the amount of advertising dollars spent. If the model for sales follows the equation $s = 163 + 22d$, where s represents sales in millions of dollars and d represents advertising in millions of dollars, how much should the company spend on advertising to obtain \$350 in sales next year?</p>	
MA.912.SP.3.7	Fit a quadratic function to data that suggests a quadratic association. Use the model to solve problems in the context of the data.
MA.912.SP.3.8	Fit an exponential function to data that suggests an exponential association. Use the model to solve problems in the context of the data.
MA.912.SP.3.9	Assess the fit of a function by plotting and analyzing residuals.
MA.912.SP.3.10	Use technology to compute the correlation coefficient of a linear model. Interpret the strength and direction of the correlation coefficient.
MA.912.SP.3.11	Explain the difference between correlation and causation.

MA.912.SP.4 Use probability distributions to solve problems.	
MA.912.SP.4.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.912.SP.4.2	Develop a probability distribution for a discrete random variable using theoretical probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.SP.4.3	Develop a probability distribution for a discrete random variable using empirically assigned probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.SP.4.4	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Evaluate and compare strategies on the basis of the calculated expected values.
MA.912.SP.4.5	Apply probabilities to make decisions which are equally likely, such as drawing from lots or using a random number generator.

MA.912.SP.4.6	Analyze decisions that were made and solve problems using probability concepts and strategies.
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9-12 Trigonometry Strand

<i>MA.912.T.1 Define and use trigonometric ratios, identities or functions to solve problems.</i>	
MA.912.T.1.1	Use similarity to define trigonometric ratios for acute angles in right triangles.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> During instruction, it is important to make the connection that side ratios in right triangles are properties of the angles in the triangle which leads to the definition of trigonometric ratios for acute angles.</p> <p><i>Remark 2:</i> Within the geometry course, problem types should include the coordinate plane so students can make the connection to work on the unit circle in later courses.</p> <p><i>Example 1:</i> In the given triangles below, $\alpha = \beta$. Use $>$, $<$, or $=$ to compare the ratios $\frac{a_1}{a_3}$ and $\frac{b_1}{b_3}$ and explain how the relationship between these ratios is related to the cosine of α and the cosine of β.</p> <div style="display: flex; justify-content: space-around; align-items: center;">  </div>	
MA.912.T.1.2	Apply the relationships of the side lengths of special right triangles to solve mathematical and real-world problems.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Special right triangles are defined as ones with angle measures of 30°-60°-90° and 45°-45°-90°.</p> <p><i>Example 1:</i> An isosceles right triangle has one leg 6 cm. Long. Find the lengths of the other two sides.</p> <p><i>Example 2:</i> A farmer needs to find the width of a river that flows through his pasture. He places a stake (Stake 1) on one side of the river across from a tree stump. He then places a second stake 50 yards to the right of the first (Stake 2). The angle formed by the line from Stake 1 to Stake 2 and the line from Stake 2 to the tree stump is 60°. Find the width of the river to the nearest yard.</p>	
MA.912.T.1.3	Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Students should be exposed to problems in which they use sine, cosine and tangent to determine side lengths and angles measures within right triangles.</p> <p><i>Remark 2:</i> Within the geometry course, students should be exposed to problems including angles of depression and angles of elevation.</p> <p><i>Example 1:</i> The distance of the base of a ladder from the wall it leans against should be at least $\frac{1}{3}$ of the ladder's total length. Suppose a 12-ft ladder is placed according to these guidelines. Give the minimum distance of the base of the ladder from the wall. How far up the wall will the ladder reach?</p>	

Example 2: The elevation of the Pensacola Lighthouse in Pensacola, Florida is 191 feet above sea level. From the top of the lighthouse, the angle of depression to a fishing boat in the Gulf of Mexico is determined to be 15° . How far is the fishing boat from the lighthouse?

MA.912.T.1.4	Apply the Law of Sines and the Law of Cosines to solve mathematical and real-world problems involving triangles.
MA.912.T.1.5	Solve problems involving finding the area of a triangle given two sides and the included angle.
MA.912.T.1.6	Prove Pythagorean Identities. Use Pythagorean Identities to calculate trigonometric ratios and to solve problems.
MA.912.T.1.7	Prove the Double-Angle, Half-Angle, Angle Sum and Difference formulas for sine, cosine, and tangent. Use these formulas to solve problems.
MA.912.T.1.8	Simplify expressions using trigonometric identities. Identities are limited to Double-Angle, Half-Angle, Angle Sum and Difference, Pythagorean Identities, Sum Identities, Product Identities.
MA.912.T.1.9	Solve trigonometric equations, applying inverse functions and using technology when appropriate. Include mathematical and real-world context.

MA.912.T.2 Extend trigonometric functions to the unit circle.

MA.912.T.2.1	Define the trigonometric functions for any angle by using right triangles drawn in the unit circle. Determine the values of sine, cosine and tangent of $\pi/3$, $\pi/4$ and $\pi/6$ and their multiples using special triangles.
MA.912.T.2.2	Use the unit circle to define and determine the sine, cosine, tangent, cosecant, secant and cotangent of angles.
MA.912.T.2.3	Given angles measured in radians or degrees, calculate the values of the six trigonometric functions.

MA.912.T.3 Graph and apply trigonometric relations and functions.

MA.912.T.3.1	Describe and demonstrate the connections between right triangle ratios, trigonometric functions and circular functions.
MA.912.T.3.2	On the coordinate plane, express the values of sine, cosine and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.
MA.912.T.3.3	Choose sine, cosine or tangent trigonometric functions to model periodic phenomena with specified amplitude, frequency, horizontal shift and midline. Include mathematical and real-world context.

MA.912.T.3.4	Solve and graph mathematical and real-world problems of trigonometric functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetry; end behavior; periodicity; midline; amplitude; shift(s) and asymptotes.
MA.912.T.3.5	Verify that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.912.T.3.6	Solve problems involving applications of trigonometric functions using graphing technology when appropriate. Include mathematical and real-world context.

<i>MA.912.T.4 Extend rectangular coordinates and equations to polar and parametric forms.</i>	
MA.912.T.4.1	Define polar coordinates and relate polar coordinates to Cartesian coordinates with and without the use of technology.
MA.912.T.4.2	Represent equations given in rectangular coordinates in terms of polar coordinates.
MA.912.T.4.3	Graph equations in the polar coordinate plane with and without the use of graphing technology.
MA.912.T.4.4	Identify and graph special polar equations, including circles, cardioids, limacons, rose curves and lemniscates.
MA.912.T.4.5	Sketch the graph of a curve in the plane represented parametrically, indicating the direction of motion.
MA.912.T.4.6	Convert from a parametric representation of a plane curve to a rectangular equation, and convert from a rectangular equation to a parametric representation of a plane curve.
MA.912.T.4.7	Apply parametric equations to model applications of motion in the plane.

9-12 Logic & Theory Strand

<i>MA.912.LT.1 Apply recursive methods to solve problems.</i>	
MA.912.LT.1.1	Apply recursive and iterative thinking to solve problems.
MA.912.LT.1.2	Solve problems and find explicit formulas for recurrence relations using finite differences.
MA.912.LT.1.3	Apply mathematical induction in a variety of applications.

<i>MA.912.LT.2 Apply techniques from Graph Theory to solve problems.</i>	
MA.912.LT.2.1	Solve scheduling problems using critical path analysis and Gantt charts. Create a schedule using critical path analysis.

MA.912.LT.2.2	Apply graph coloring techniques to solve problems.
MA.912.LT.2.3	Apply spanning trees, rooted trees, binary trees and decision trees to solve problems.
MA.912.LT.2.4	Create problems that can be solved using spanning trees, rooted trees, binary trees, and decision trees.
MA.912.LT.2.5	Solve problems concerning optimizing resource usage using bin-packing techniques.

<i>MA.912.LT.3 Apply techniques from Election Theory to solve problems.</i>	
MA.912.LT.3.1	Analyze election data using election theory techniques.
MA.912.LT.3.2	Decide voting power within a group using weighted voting techniques. Provide real-world examples of weighted voting and its pros and cons.
MA.912.LT.3.3	Solve problems using fair division techniques.
MA.912.LT.3.4	Solve strictly determined and non-strictly determined games by using game theory.

<i>MA.912.LT.4 Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof.</i>	
MA.912.LT.4.1	Translate propositional statements into logical arguments using propositional variables and logical connectives.
MA.912.LT.4.2	Determine truth values of simple and compound statements using truth tables.
MA.912.LT.4.3	Find the converse, inverse, and contrapositive of a statement.
<p><u>Remarks/Examples:</u></p> <p><i>Remark 1:</i> Within the geometry course, instruction should focus on the connection to proofs. Problems types should include mathematical and non-mathematical topics.</p> <p><i>Remark 2:</i> Students should have practice with the name and symbolic form of the conditionals. For the given conditional statement, it is customary to use the notation $p \rightarrow q$. For the inverse statement, it is customary to use the notation $\sim p \rightarrow \sim q$. For the converse statement, it is customary to use the notation $q \rightarrow p$. For the contrapositive statement, it is customary to use the notation $\sim q \rightarrow \sim p$.</p> <p><i>Example 1:</i> Determine the inverse, converse and contrapositive of the statement, "If it is Thursday, there will be rain."</p>	
MA.912.LT.4.4	Represent logic operations, such as AND, OR, NOT, NOR, and XOR (exclusive OR), using logical symbolism to solve problems.
MA.912.LT.4.5	Determine whether two propositions are logically equivalent.
MA.912.LT.4.6	Apply methods of direct and indirect proof, and determine whether a logical argument is valid.
<u>Remarks/Examples:</u>	

Remark 1: When proving $p \rightarrow q$ directly, one will start with the assumption that p is true and use a series of deductions to prove that q is true. When proving indirectly, one will not begin with a hypothesis but end with a conclusion using proof by contraposition or proof by contradiction.

Remark 2: Within the geometry course, instruction should focus on the connection to proofs. Problems types should include mathematical and non-mathematical topics.

Example 1: If somebody argues, “If it is Thursday, it is raining.” along with “It is raining” implies that “it is Thursday.”, is this a valid or invalid argument? Explain your answer.

MA.912.LT.4.7	Identify and give examples of undefined terms; axioms; theorems; inductive and deductive proofs; and inductive and deductive reasoning.
MA.912.LT.4.8	Construct logical arguments using laws of detachment, syllogism, tautology and contradiction.
MA.912.LT.4.9	Judge the validity of arguments, and give counterexamples to disprove statements.

Remarks/Examples:

Remark 1: An argument is said to be valid if the conclusion is logically true and all assumptions are true. Students can determine the validity of an argument by giving counterexamples, using laws of syllogism or truth tables.

Remark 2: Within the geometry course, instruction should focus on the connection to proofs. Problems types should include mathematical and non-mathematical topics.

Example 1: Jaylen makes the statements “Nymeria is a wolf.” and “Nymeria is an animal.” Does he give a valid argument? Explain your reasoning.

Possible student response: A student can state that even though it may be obvious that wolves are animals, the truth of the first statement cannot guarantee the truth of the second statement. A student can suggest that by adding the statement “All wolves are animals.” in between Jaylen’s two statement will give validity to his argument.

MA.912.LT.5 Apply properties from Set Theory to solve problems.

MA.912.LT.5.1	Given two sets, determine whether the two sets are equal, whether one set is a subset of another or if one is the power set of the other.
MA.912.LT.5.2	Given a relation on two sets, determine whether the relation is a function, determine the inverse of the relation if it exists, and identify if the relation is bijective.
MA.912.LT.5.3	Partition a set into disjoint subsets, and determine an equivalence class given the equivalence relation on a set.
MA.912.LT.5.4	Perform the set operations of union, intersection, difference, complement and cross product.
MA.912.LT.5.5	Explore relationships and patterns and make arguments about relationships between sets by using Venn Diagrams.
MA.912.LT.5.6	Prove set relations, including DeMorgan’s Laws and equivalence relations.

9-12 Financial Literacy Strand

<i>MA.912.FL.1 Determine simple and compound interest and demonstrate its relationship to functions. Calculate and use Net Present and Net Future Values.</i>	
MA.912.FL.1.1	Compare simple, compound and continuous compounded interest over time.
MA.912.FL.1.2	Solve problems involving simple, compound and continuous compounded interest, including determine the present value and future value of money.
MA.912.FL.1.3	Explain the relationship between simple interest and linear growth.
<u>Remarks/Examples:</u> <i>Remark 1:</i> If students are provided context that give the principle and the interest rate, then students can model the interest over time with a linear graph. They can also infer linear growth by considering a constant amount of interest generated each year. <i>Example 1:</i> Jonie has \$1500 in the bank that has an interest rate of 3% per year. Using a graph explain the connection between the interest she earns over time and a linear model.	
MA.912.FL.1.4	Explain the relationship between compound interest and continuous compound interest to exponential growth.
MA.912.FL.1.5	Determine the consumer price index (CPI) for goods. Interpret its value in terms of the context. Include mathematical and real-world context.
MA.912.FL.1.6	Solve problems involving maximum profit and minimal cost. Include mathematical and real-world context.

<i>MA.912.FL.2 Describe the advantages and disadvantages of short-term and long-term purchases.</i>	
MA.912.FL.2.1	Compare the advantages and disadvantages of using cash versus personal financing options or other forms of electronic payment. Include mathematical and real-world context.
MA.912.FL.2.2	Calculate the finance charges and total amount due on a bill from using various forms of credit.
MA.912.FL.2.3	Manipulate a variety of variables to compare the advantages and disadvantages of deferred payments.
MA.912.FL.2.4	Calculate the total cost of purchasing consumer durables over time given different monthly payments, down payments, financing options and fees.
MA.912.FL.2.5	Calculate the fees associated with a mortgage. Fees limited to discount prices, origination fee, maximum brokerage fee on a net or gross loan, documentary stamps prorated expenses.

MA.912.FL.2.6	Substitute values to evaluate a variety of mortgage formulas. Formulas limited to Front End Ratio, Total Debt-to-Income Ratio, Loan-to-Value Ratio (LTV), Combined Loan-to-Value Ratio (CLTV) and Amount of Interest Paid Over the Life of a Loan.
MA.912.FL.2.7	Solve problems involving student, personal and car loans, including finding the total amount to be paid, adjustable rates and refinancing options.
MA.912.FL.2.8	Calculate the final payout amount for a balloon mortgage.
MA.912.FL.2.9	Compare the cost of paying a higher interest rate and fewer mortgage points versus a lower interest rate and more mortgage points. Include mathematical and real-world context.
MA.912.FL.2.10	Calculate the total amount paid for the life of a loan including the down payment, fees and interest.
MA.912.FL.2.11	Calculate and compare, in terms of functions, the total cost for a set purchase price using a fixed rate, adjustable rate and a balloon mortgage.
MA.912.FL.2.12	Compare interest rate calculations and annual percentage rate calculations, and distinguish between the two rates.

<i>MA.912.FL.3 Develop personal financial skills and describe the advantages and disadvantages of financial and investment plans.</i>	
MA.912.FL.3.1	Develop personal budgets that fit within various income brackets.
MA.912.FL.3.2	Calculate the break-even point to determine the viability of purchasing options for housing, car and other durable goods.
MA.912.FL.3.3	Explain cash management strategies include checking and savings accounts.
MA.912.FL.3.4	Given assets and liabilities, calculate net worth.
MA.912.FL.3.5	Given a scenario, establish a plan to pay off debt.
MA.912.FL.3.6	Given a scenario, complete and calculate federal income tax, analyzing different options such as standard deductions versus itemized deductions and taxes owed based on income brackets from the tax table.
MA.912.FL.3.7	Calculate and compare various options and fees for medical, car, homeowners and life insurance.
MA.912.FL.3.8	Collect, organize and interpret data to determine an effective retirement savings plan to meet personal financial goals.
MA.912.FL.3.9	Solve problems involving different types of retirement plans, including traditional IRA, ROTH IRA, 401K, 403B and annuities.
MA.912.FL.3.10	Compare different ways that portfolios can be diversified in both investments and investment vehicles.

MA.912.FL.3.11	Purchase stock with a set amount of money, and evaluate its worth over time considering gains, losses and selling, taking into account any associated fees.
MA.912.FL.3.12	Compare income from purchase of common stock, preferred stock, and bonds.
MA.912.FL.3.13	Given current exchange rates, convert between currencies.
MA.912.FL.3.14	Apply data to compare historical rates of return on investments with investment claims to make informed decisions and identify potential fraud.