

K-12 Second Draft Mathematics Standards and Benchmarks

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K-12 Mathematical Expectations for Students

The Mathematical Thinking and Reasoning Standards provide a framework for strengthening the learning of mathematics. These standards should be infused throughout each course to promote deeper learning as indicated in the course descriptions found on CPALMS.

1. Participate actively in effortful learning both individually and with others.

Mathematicians who participate in effortful learning both individually and with others:

- Build perseverance by trying multiple strategies without becoming discouraged.
- Analyze the problem in a way that makes sense given the task.
- Ask questions that will help with solving the task.
- Stay engaged when working to solve challenging tasks.
- Help and support each other to attempt a new strategy or approach.

2. Express problems using multiple representations progressing towards efficiency.

Mathematicians who express problems using multiple representations progressing towards efficiency:

- Explore problems and build understanding through modeling and using manipulatives.
- Demonstrate solutions to problems in multiple ways using objects, drawings and equations.
- Progress from modeling problems with objects and drawings to using algorithms and equations.
- Use repeated practice to improve efficiency when performing calculations.
- Select efficient and appropriate strategies for solving problems within the given context.

3. Engage in discussions that reflect on the mathematical thinking of self and others.

Mathematicians who engage in discussions that reflect on the mathematical thinking of self and others:

- Communicate mathematical ideas, vocabulary and strategies effectively.
- Analyze the mathematical thinking of others.
- Compare the effectiveness of their strategies to those expressed by others.
- Recognize errors and make recommendations for alternative solutions.
- Justify results by explaining strategies and processes.
- Construct possible arguments based on evidence.

4. Use patterns and structure to help understand and connect mathematical concepts.

Mathematicians who use patterns and structure to help understand and connect mathematical concepts:

- Create plans and procedures to logically order events, steps or ideas to solve problems.
- Decompose a complex problem into manageable parts.
- Look for similarities among problems.
- Expand solutions of problems to more complicated large-scale situations.
- Focus on important details within a problem.

5. Assess the reasonableness of solutions.

Mathematicians who assess the reasonableness of solutions:

- Estimate to discover possible solutions.
- Use benchmark quantities to determine if a solution makes sense.
- Check calculations when solving problems.
- Verify possible solutions by explaining the strategies used.
- Evaluate results based on the given context.

6. Apply mathematics to real-world contexts.

Mathematicians who apply mathematics to real-world contexts:

- Connect mathematical concepts to every day experiences.
- Create mathematical models to represent real-world situations.
- Engineer and design methods for solving problems.
- Perform investigations to gather data or determine if a method is appropriate.
- Redesign methods to improve accuracy.

K-5 Mathematics Second Draft of Proposed Language

Kindergarten	
<i>MA.K.AR.1 Represent and solve addition and subtraction problems within 10.</i>	
MA.K.AR.1.1	Represent addition and subtraction within 10 in multiple ways using objects, fingers, drawings, verbal explanations and equations.
MA.K.AR.1.2	Solve addition and subtraction word problems within 10 using objects, drawings or equations to represent the problem.
MA.K.AR.1.3	Add and subtract within 10 using a variety of strategies.
MA.K.AR.1.4	For any number from 1 to 9, find the number that makes 10 when added to the given number.
MA.K.AR.1.5	Find all possible pairs of addends for sums within 10.
<i>MA.K.AR.2 Identify and extend shape and color patterns.</i>	
MA.K.AR.2.1	Identify and extend patterns consisting of shapes and colors.
<i>MA.K.NSO.1 Count and compare groups of objects within 20.</i>	
MA.K.NSO.1.1	Count and represent a group of objects with a written numeral 0 to 20.
MA.K.NSO.1.2	Given a number from 0 to 20, count out that many objects. State the number of objects counted, and if the counted set is moved or rearranged, restate the number of objects in that set without having to recount.
MA.K.NSO.1.3	Compare sets of objects from 0 to 20 using the terms less than, equal to or greater than.
<i>MA.K.NSO.2 Count within 100 and build a foundation for place value.</i>	
MA.K.NSO.2.1	Count to 100 by ones and by tens.
MA.K.NSO.2.2	Starting at a given number, count forward within 100 and backwards within 20.
MA.K.NSO.2.3	Read and write numbers from 0 to 20.
MA.K.NSO.2.4	Compose and decompose numbers from 11 to 19 with a group of ten ones and additional ones. Demonstrate each composition or decomposition with objects, drawings or equations.
MA.K.NSO.2.5	Compare numbers from 0 to 20 using the terms less than, equal to or greater than.
<i>MA.K.M.1 Describe and compare the measurable attributes of objects.</i>	
MA.K.M.1.1	Describe measurable attributes of a single object such as length, capacity or weight.
MA.K.M.1.2	Directly compare two objects with a measurable attribute in common. Express the comparison using language to describe the difference.
MA.K.M.1.3	Express the length of an object, up to 20 units long, as a whole number of lengths by laying non-standard objects end to end with no gaps or overlaps.
<i>MA.K.GR.1 Build a foundation for identifying and analyzing two-dimensional shapes.</i>	
MA.K.GR.1.1	Identify two-dimensional shapes regardless of their size or orientation. Shapes are limited to circles, triangles, rectangles and squares.
MA.K.GR.1.2	Compare and sort two-dimensional shapes based on their similarities and differences. Shapes are limited to circles, triangles, rectangles and squares.

MA.K.GR.1.3	Combine two-dimensional shapes to form a composite figure. Shapes are limited to circles, triangles, rectangles and squares.
<i>MA.K.GR.2 Build a foundation for identifying and analyzing three-dimensional figures.</i>	
MA.K.GR.2.1	Identify three-dimensional figures regardless of their size or orientation. Figures are limited to spheres, cubes, cones and cylinders.
MA.K.GR.2.2	Compare and sort three-dimensional figures based on their similarities and differences. Figures are limited to spheres, cubes, cones and cylinders.
MA.K.GR.2.3	Combine three-dimensional figures to form a composite figure. Figures are limited to spheres, cubes, cones and cylinders.

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Grade 1	
<i>MA.1.AR.1 Solve addition and subtraction problems within 20.</i>	
MA.1.AR.1.1	Solve addition and subtraction word problems within 20 using objects, drawings or equations to represent the problem.
MA.1.AR.1.2	Add and subtract within 20 using a variety of strategies.
<i>MA.1.AR.2 Apply properties of operations and the relationship between addition and subtraction.</i>	
MA.1.AR.2.1	Apply the Commutative Property of Addition and the Associative Property of Addition as strategies to add within 20.
MA.1.AR.2.2	Use the inverse relationship between addition and subtraction to restate a subtraction problem within 20 as a missing addend problem.
MA.1.AR.2.3	Determine and explain if equations involving addition and subtraction within 20 are true or false.
MA.1.AR.2.4	Determine the unknown whole number in an addition or subtraction equation within 20 with the unknown in any position.
<i>MA.1.AR.3 Identify and extend shape, color and number patterns.</i>	
MA.1.AR.3.1	Identify and extend patterns consisting of shapes, colors and numbers. Limit numeric patterns to adding 1 or skip counting by 2s, 5s or 10s.
<i>MA.1.NSO.1 Extend counting sequences and understand the place value of two-digit numbers.</i>	
MA.1.NSO.1.1	Starting at a given number, count forward and backward within 120.
MA.1.NSO.1.2	Skip count forward by 2s to 20 and by 5s to 100.
MA.1.NSO.1.3	Read and write numbers to 120 using standard form and word form.
MA.1.NSO.1.4	Compose and decompose two-digit numbers in multiple ways using tens and ones. Demonstrate each composition or decomposition with objects, drawings or equations.
MA.1.NSO.1.5	Compare two two-digit numbers based on the values of the tens and ones digits using the terms less than, equal to or greater than and the symbols $<$, $=$, or $>$.
MA.1.NSO.1.6	Identify the number that is one more, one less, ten more and ten less than a given two-digit number.
<i>MA.1.NSO.2 Build a foundation for adding and subtracting two-digit numbers.</i>	
MA.1.NSO.2.1	Add a two-digit number and a one-digit number within 100 using a variety of strategies.
MA.1.NSO.2.2	Add a two-digit number and a multiple of 10 within 100 using a variety of strategies.
<i>MA.1.FR.1 Build a foundation for fractions by partitioning shapes into halves and fourths.</i>	
MA.1.FR.1.1	Partition circles and rectangles into two and four equal-sized parts. Name the parts using appropriate language including halves or fourths. Describe the inverse relationship between the size of the parts and the number of parts.
<i>MA.1.M.1 Compare and measure the length of objects.</i>	
MA.1.M.1.1	Compare and order the length of up to three objects using direct comparison.
MA.1.M.1.2	Estimate and measure the length of an object to the nearest inch or centimeter.
<i>MA.1.M.2 Tell time and identify the value of coins and combinations of coins and dollar bills.</i>	

MA.1.M.2.1	Tell and write time in hours and half-hours using analog and digital clocks.
MA.1.M.2.2	Identify pennies, nickels, dimes and quarters, and express their values using the ¢ symbol. State how many of each coin equal a dollar.
MA.1.M.2.3	Find the value of combinations of pennies and dimes up to one dollar, and the value of one and ten dollar bills up to \$100. Use the ¢ and \$ symbols appropriately.
<i>MA.1.GR.1 Identify and analyze two-dimensional shapes based on their defining attributes.</i>	
MA.1.GR.1.1	Identify two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.
MA.1.GR.1.2	Draw two-dimensional shapes when given defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.
MA.1.GR.1.3	Compare and sort two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.
MA.1.GR.1.4	Combine two-dimensional shapes to form a composite shape. Decompose a composite shape into two-dimensional shapes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares and hexagons.
<i>MA.1.GR.2 Identify and analyze three-dimensional figures based on their defining attributes.</i>	
MA.1.GR.2.1	Identify three-dimensional figures based on their defining attributes. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
MA.1.GR.2.2	Compare and sort three-dimensional figures based on their defining attributes. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
MA.1.GR.2.3	Combine three-dimensional figures to form a composite figure. Decompose a composite figure into three-dimensional figures. Figures are limited to spheres, cubes, rectangular prisms, cones and cylinders.
<i>MA.1.SP.1 Build a foundation for collecting, representing and interpreting data.</i>	
MA.1.SP.1.1	Collect and represent data with up to three categories using tally marks, pictographs or bar graphs.
MA.1.SP.1.2	Interpret data represented with tally marks, pictographs or bar graphs by calculating the total number of data points, counting the total in each category and comparing the totals of different categories.

Grade 2	
<i>MA.2.AR.1 Solve addition and subtraction problems within 100 and develop mastery of facts to 20.</i>	
MA.2.AR.1.1	Solve one- and two-step addition and subtraction word problems within 100 with unknowns in all positions. Write an equation with a symbol for the unknown to represent the problem.
MA.2.AR.1.2	Determine the unknown whole number in an addition or subtraction equation within 100 with the unknown in any position.
MA.2.AR.1.3	Demonstrate mastery of addition and subtraction facts with addends to 10.
<i>MA.2.AR.2 Determine even and odd numbers and build a foundation for multiplication using arrays.</i>	
MA.2.AR.2.1	Find the total number of objects arranged in rectangular arrays with up to 5 rows and 5 columns. Write an equation to express the total as a sum of equal addends.
MA.2.AR.2.2	Determine whether a set of objects within 20 has an odd or even number. Write an equation to express an even number as a sum of two equal addends or an odd number as a sum of two equal addends plus 1.
<i>MA.2.AR.3 Identify and extend numeric patterns.</i>	
MA.2.AR.3.1	Identify and extend numeric patterns. Limit patterns to addition or subtraction with sums to 100 or skip counting by 2s, 5s or 10s.
<i>MA.2.NSO.1 Understand the place value of three-digit numbers.</i>	
MA.2.NSO.1.1	Read and write numbers to 1,000 using standard form, word form and expanded form.
MA.2.NSO.1.2	Compose and decompose three-digit numbers in multiple ways using hundreds, tens, and ones. Demonstrate each composition or decomposition with objects, drawings or equations.
MA.2.NSO.1.3	Compare two three-digit numbers based on the values of the hundreds, tens and ones digits, using the terms less than, equal to or greater than and the symbols $<$, $=$, or $>$.
MA.2.NSO.1.4	Round whole numbers within 1,000 to the nearest 10 or 100.
MA.2.NSO.1.5	Identify the number that is ten more, ten less, one hundred more and one hundred less than a given three-digit number.
<i>MA.2.NSO.2 Add and subtract within 1,000.</i>	
MA.2.NSO.2.1	Add and subtract within 1,000 using a variety of strategies.
<i>MA.2.FR.1 Build a foundation for fractions by partitioning shapes into halves, thirds and fourths.</i>	
MA.2.FR.1.1	Partition circles and rectangles into two, three or four equal-sized parts. Name the parts using appropriate language, and describe the whole as two halves, three thirds or four fourths. Demonstrate that equal-sized parts may have different shapes.
<i>MA.2.M.1 Measure and compare the length of objects and solve problems involving length.</i>	
MA.2.M.1.1	Estimate and measure the length of an object to the nearest inch, foot, yard, centimeter or meter by selecting and using an appropriate tool. Describe the inverse relationship between the size of a unit and number of units needed to measure an object.
MA.2.M.1.2	Measure the length of two objects in inches, feet, yards, centimeters or meters, and determine the difference between these measurements.

MA.2.M.1.3	Solve one- and two-step word problems within 100 involving addition and subtraction of lengths that are given in the same measurement units. Write an equation with a symbol for the unknown to represent the problem.
<i>MA.2.M.2 Tell time and solve problems involving money.</i>	
MA.2.M.2.1	Tell and write time on analog and digital clocks to the nearest five minutes using a.m. and p.m. appropriately. Express portions of an hour using the fractional terms half an hour, half past, quarter of an hour, quarter past and quarter til.
MA.2.M.2.2	Solve one- and two-step addition and subtraction word problems involving either dollar bills within \$100 or coins within 100¢ using \$ and ¢ symbols appropriately.
<i>MA.2.M.3 Describe perimeter and find the perimeter of polygons.</i>	
MA.2.M.3.1	Describe perimeter as an attribute of plane figures by determining the distance around the edge of the figure.
MA.2.M.3.2	Find the perimeter of a polygon with whole number side lengths. Polygons are limited to triangles, rectangles, squares, pentagons, hexagons and octagons.
<i>MA.2.GR.1 Identify and analyze two-dimensional shapes and identify lines of symmetry.</i>	
MA.2.GR.1.1	Identify and draw two-dimensional shapes based on their defining attributes. Shapes are limited to circles, semi-circles, triangles, rectangles, squares, pentagons, hexagons and octagons.
MA.2.GR.1.2	Identify and draw line(s) of symmetry for a two-dimensional figure. Identify line-symmetric figures.
<i>MA.2.SP.1 Collect, represent and interpret data using appropriate titles, labels and units.</i>	
MA.2.SP.1.1	Collect and represent data using tally marks, tables, pictographs or bar graphs. Use appropriate titles, labels and units.
MA.2.SP.1.2	Interpret data represented with tally marks, tables, pictographs or bar graphs by solving addition and subtraction problems.

Grade 3	
<i>MA.3.AR.1 Represent and solve multiplication and division problems within 100.</i>	
MA.3.AR.1.1	Represent multiplication and division within 100 in multiple ways using arrays, equal groups, area models and equations.
MA.3.AR.1.2	Solve one- and two-step multiplication and division word problems with whole numbers within 100. Write an equation with a symbol for the unknown to represent the problem.
MA.3.AR.1.3	Multiply and divide whole numbers within 100 using a variety of strategies.
MA.3.AR.1.4	Demonstrate mastery of multiplication and division facts with factors to 10.
<i>MA.3.AR.2 Apply properties of operations and the relationship between multiplication and division.</i>	
MA.3.AR.2.1	Apply the Commutative Property of Multiplication, the Associative Property of Multiplication and the Distributive Property as strategies to multiply whole numbers with products to 100.
MA.3.AR.2.2	Use the inverse relationship between multiplication and division to restate a division problem within 100 as a missing factor problem.
MA.3.AR.2.3	Determine and explain whether an equation involving multiplication or division within 100 is true or false.
MA.3.AR.2.4	Determine the unknown whole number in a multiplication or division equation within 100 with the unknown in any position.
<i>MA.3.AR.3 Identify and analyze arithmetic patterns.</i>	
MA.3.AR.3.1	Identify, describe and extend numeric patterns. Limit patterns to multiplication or division with products to 100.
<i>MA.3.NSO.1 Understand the place value of four-digit numbers.</i>	
MA.3.NSO.1.1	Read and write numbers to 10,000 using standard form, word form and expanded form.
MA.3.NSO.1.2	Compose and decompose four-digit numbers in multiple ways using thousands, hundreds, tens and ones. Demonstrate each composition or decomposition using objects, drawings or equations.
MA.3.NSO.1.3	Compare two four-digit numbers based on the values of the thousands, hundreds, tens and ones digits using the symbols $<$, $=$, or $>$.
MA.3.NSO.1.4	Round whole numbers within 10,000 to the nearest 10, 100 or 1,000.
<i>MA.3.NSO.2 Add and subtract multi-digit numbers and build a foundation for multiplication and division of two-digit numbers.</i>	
MA.3.NSO.2.1	Add and subtract within 10,000 using a variety of strategies, including the standard algorithm.
MA.3.NSO.2.2	Multiply a one-digit whole number by a multiple of 10 using a variety of strategies.
<i>MA.3.FR.1 Represent and compare fractions and identify equivalent fractions.</i>	
MA.3.FR.1.1	Represent fractions, including fractions greater than one, in multiple ways using parts of a whole, parts of a set, points on a number line, distances on a number line and area models.
MA.3.FR.1.2	Explain that two fractions are equivalent if they represent equal-sized portions of the same whole or if they both lie at the same point on a number line. Identify equivalent fractions using visual fraction models.

MA.3.FR.1.3	Compare two fractions with the same numerator or the same denominator by reasoning about their size using the symbols $<$, $=$, or $>$. Determine whether the comparisons are valid based on the fractions referring to the same whole.
<i>MA.3.M.1 Measure the length of objects and solve problems involving measurement.</i>	
MA.3.M.1.1	Measure the length of an object to the nearest half or quarter inch by selecting and using an appropriate tool.
MA.3.M.1.2	Solve one- and two-step word problems involving multiplication and division with whole number lengths, masses, weights, temperature or liquid volumes that are given in the same units.
<i>MA.3.M.2 Tell and write time to the nearest minute.</i>	
MA.3.M.2.1	Tell and write time on analog and digital clocks to the nearest minute using a.m. and p.m. appropriately.
<i>MA.3.M.3 Solve problems involving the area of rectangles.</i>	
MA.3.M.3.1	Describe area as an attribute of plane figures by covering them using unit squares without gaps or overlaps. Find areas of rectangles by counting unit squares.
MA.3.M.3.2	Find the area of a rectangle with whole number side lengths using visual models or a formula.
MA.3.M.3.3	Solve problems involving the area of rectangles with whole number side lengths using visual models or a formula. Write an equation with a symbol for the unknown to represent the problem.
MA.3.M.3.4	Solve problems involving the area of composite figures composed of non-overlapping rectangles with whole number side lengths. Write an equation with a symbol for the unknown to represent the problem.
<i>MA.3.GR.1 Describe and identify lines and classify quadrilaterals.</i>	
MA.3.GR.1.1	Describe and draw points, lines, line segments, rays, perpendicular lines and parallel lines. Identify these in two-dimensional figures.
MA.3.GR.1.2	Classify quadrilaterals into different categories based on shared defining attributes. Explain why a quadrilateral would or would not belong to a category. Quadrilaterals include parallelograms, rhombuses, rectangles, squares and trapezoids.
<i>MA.3.SP.1 Collect, represent and interpret numerical and categorical data.</i>	
MA.3.SP.1.1	Collect and represent numerical and categorical data with whole number values using tables, pictographs, bar graphs or line plots. Use appropriate titles, labels and units.
MA.3.SP.1.2	Interpret data with whole number values represented with tables, pictographs, bar graphs or line plots by solving one- and two-step problems.

Grade 4	
<i>MA.4.AR.1 Represent and solve problems involving the four operations.</i>	
MA.4.AR.1.1	Solve multiplication and division word problems of whole numbers within 1,000, including problems in which remainders must be interpreted. Write an equation with a symbol for the unknown number to represent the problem.
MA.4.AR.1.2	Solve multi-step word problems involving any combination of the four operations with whole numbers, including problems in which remainders must be interpreted. Write an equation with a symbol for the unknown number to represent the problem.
MA.4.AR.1.3	Identify factor pairs for a whole number up to 100. Determine whether the number is prime or composite.
MA.4.AR.1.4	Determine whether a whole number up to 100 is a multiple of a given one-digit number.
<i>MA.4.AR.2 Determine whether equations are true or false and find unknowns in equations.</i>	
MA.4.AR.2.1	Determine and explain whether an equation, including any combination of the four operations, is true or false.
MA.4.AR.2.2	Determine the unknown whole number in an equation including any combination of the four operations with the unknown in any position.
<i>MA.4.AR.3 Generate and extend numerical patterns that follow a given rule.</i>	
MA.4.AR.3.1	Generate, describe and extend a numerical pattern that follows a given rule.
<i>MA.4.NSO.1 Understand place value for multi-digit numbers.</i>	
MA.4.NSO.1.1	Express how the value of a digit in a multi-digit whole number changes if it moves one place to the left or right.
MA.4.NSO.1.2	Read and write multi-digit whole numbers within 1,000,000 using standard form, word form and expanded form.
MA.4.NSO.1.3	Compare two multi-digit whole numbers within 1,000,000 based on the values of the digits in each place using the symbols $<$, $=$, or $>$.
MA.4.NSO.1.4	Round whole numbers within 1,000,000 to the nearest 100, 1,000, 10,000 or 100,000.
<i>MA.4.NSO.2 Build a foundation for multiplying and dividing multi-digit numbers.</i>	
MA.4.NSO.2.1	Multiply a whole number up to four digits by a one-digit whole number using a variety of strategies.
MA.4.NSO.2.2	Multiply a two-digit by a two-digit whole number using a variety of strategies.
MA.4.NSO.2.3	Divide a whole number up to four digits by a one-digit whole number using a variety of strategies. Represent remainders as fractional parts of the divisor.
<i>MA.4.FR.1 Determine fraction equivalence and compare fractions.</i>	
MA.4.FR.1.1	Identify and generate equivalent fractions, including mixed numbers and fractions greater than one, referring to the same whole using visual models or algorithms.
MA.4.FR.1.2	Compare two fractions, including mixed numbers and fractions greater than one, with different numerators and different denominators using the symbols $<$, $=$, or $>$.
<i>MA.4.FR.2 Build a foundation for adding, subtracting and multiplying fractions.</i>	
MA.4.FR.2.1	Decompose a fraction, including mixed numbers and fractions greater than one, into a sum of fractions with the same denominator in multiple ways. Demonstrate each decomposition with objects, drawings or equations.
MA.4.FR.2.2	Add and subtract fractions with like denominators, including mixed numbers and fractions greater than one, using a variety of strategies.

MA.4.FR.2.3	Solve word problems involving addition and subtraction of fractions with like denominators, including mixed numbers and fractions greater than one, using visual fraction models and equations to represent the problem.
MA.4.FR.2.4	Extend previous understanding of multiplication to multiply a fraction by a whole number or a whole number by a fraction using a variety of strategies.
MA.4.FR.2.5	Solve word problems involving multiplication of a fraction by a whole number or a whole number by a fraction using visual fraction models or equations to represent the problem.
<i>MA.4.FR.3 Understand the relationship between fractions and decimals.</i>	
MA.4.FR.3.1	Model and express a fraction, including mixed numbers and fractions greater than one, with the denominator 10 as an equivalent fraction with the denominator 100. Use fractions with denominators of 10 and 100 to add two fractions.
MA.4.FR.3.2	Use decimal notation to represent fractions, including mixed numbers and fractions greater than one, with denominators of 10 or 100, and use fractions with denominators of 10 or 100 to represent decimals.
MA.4.FR.3.3	Compare two decimals to hundredths by reasoning about their size using the symbols $<$, $=$, or $>$.
<i>MA.4.M.1 Measure the length of objects and solve problems involving measurement.</i>	
MA.4.M.1.1	Measure the length of an object to the nearest half, quarter or eighth inch by selecting and using an appropriate tool.
MA.4.M.1.2	Convert larger units to smaller units within a single system of measurement using the units km, m, cm; kg, g; lb, oz.; L, mL; hr, min, sec. Record the conversions on a two-column table.
MA.4.M.1.3	Solve two-step word problems involving distances and intervals of time using any combination of the four operations, including problems with fractions with like denominators.
<i>MA.4.M.2 Solve problems involving the area and perimeter of rectangles.</i>	
MA.4.M.2.1	Solve area and perimeter word problems by applying the area and perimeter formulas for rectangles with whole number side lengths, including problems with unknown sides. Write an equation with a symbol for the unknown to represent the problem.
<i>MA.4.M.3 Draw, classify and measure angles.</i>	
MA.4.M.3.1	Identify and classify angles as acute, right, obtuse, straight or reflex.
MA.4.M.3.2	Estimate and measure angles in whole number degrees using a protractor, and construct angles of specified measure in whole number degrees using a protractor. Limit to within 360 degrees.
<i>MA.4.GR.1 Classify triangles and quadrilaterals based on shared defining attributes.</i>	
MA.4.GR.1.1	Classify triangles or quadrilaterals into different categories based on shared defining attributes. Explain why a triangle or quadrilateral would or would not belong to a category. Triangles include scalene, isosceles, equilateral, acute, obtuse, right and equiangular; quadrilaterals include parallelograms, rhombuses, rectangles, squares and trapezoids.
<i>MA.4.SP.1 Collect, represent and interpret data and find the mode, median and range of a data set.</i>	
MA.4.SP.1.1	Collect and represent numerical data with whole number values using tables or line plots. Solve multi-step problems involving any combination of the four operations using data from these representations.

MA.4.SP.1.2	Determine the mode, median or range to interpret numerical data with whole number values represented with tables or line plots.
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Grade 5	
<i>MA.5.AR.1 Write and evaluate numerical expressions.</i>	
MA.5.AR.1.1	Evaluate multi-step expressions including those with parentheses.
MA.5.AR.1.2	Translate written descriptions into numerical expressions and numerical expressions into written descriptions.
<i>MA.5.AR.2 Analyze patterns and relationships.</i>	
MA.5.AR.2.1	Generate, describe and extend a numerical pattern that follows a given rule. Record inputs and outputs using a two-column table. Graph the ordered pairs formed on the two-column table on the first quadrant of the coordinate grid.
<i>MA.5.NSO.1 Understand the place value of multi-digit numbers with decimals to the thousandths place.</i>	
MA.5.NSO.1.1	Express how the value of a digit in a multi-digit whole number with decimals to the thousandths changes if it moves one or more places to the left or right.
MA.5.NSO.1.2	Read and write multi-digit numbers with decimals to the thousandths using standard form, word form and expanded form.
MA.5.NSO.1.3	Compose and decompose multi-digit numbers with decimals to the thousandths in multiple ways using the values of the digits in each place. Demonstrate the compositions or decompositions using objects, drawings or equations.
MA.5.NSO.1.4	Compare two multi-digit numbers with decimals to the thousandths based on the values of the digits in each place using the symbols $<$, $=$, or $>$.
MA.5.NSO.1.5	Round multi-digit numbers with decimals to the thousandths to the nearest hundredth, tenth or whole number.
<i>MA.5.NSO.2 Perform operations with multi-digit numbers and decimals.</i>	
MA.5.NSO.2.1	Multiply multi-digit whole numbers up to five digits by two digits using a variety of strategies, including the standard algorithm.
MA.5.NSO.2.2	Divide multi-digit whole numbers up to five digits by two digits using a variety of strategies, including the standard algorithm. Represent remainders as fractional parts of the divisor.
MA.5.NSO.2.3	Add and subtract multi-digit numbers with decimals to the thousandths using a variety of strategies.
MA.5.NSO.2.4	Multiply and divide multi-digit numbers with decimals to the hundredths using a variety of strategies.
<i>MA.5.FR.1 Interpret a fraction as division.</i>	
MA.5.FR.1.1	Interpret a fraction as division of the numerator by the denominator. Interpret and solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers using visual fraction models or equations to represent the problem.
<i>MA.5.FR.2 Perform operations with fractions.</i>	
MA.5.FR.2.1	Add and subtract fractions with unlike denominators, including mixed numbers and fractions greater than 1, using a variety of strategies.
MA.5.FR.2.2	Solve word problems involving addition and subtraction of fractions with unlike denominators, including mixed numbers and fractions greater than one, using visual fraction models or equations to represent the problem.

MA.5.FR.2.3	Extend previous understanding of multiplication to multiply a fraction by a whole number or a fraction by a fraction using a variety of strategies.
MA.5.FR.2.4	When multiplying a given number by a fraction less than 1 or a fraction greater than 1, predict and explain the relative size of the product to the given number without calculating.
MA.5.FR.2.5	Solve word problems involving multiplication of fractions, including mixed numbers and fractions greater than one, using visual fraction models or equations to represent the problem.
MA.5.FR.2.6	Extend previous understanding of division to divide a unit fraction by a whole number and a whole number by a unit fraction using a variety of strategies.
MA.5.FR.2.7	Solve word problems involving division of a unit fraction by a whole number and a whole number by a unit fraction using visual fraction models or equations to represent the problem.
<i>MA.5.M.1 Convert measurement units to solve multi-step problems.</i>	
MA.5.M.1.1	Convert measurement units to equivalent units within a single system of measurement. Solve multi-step word problems using these conversions.
<i>MA.5.M.2 Find the area of rectangles with fractional side lengths.</i>	
MA.5.M.2.1	Find the area of a rectangle with fractional side lengths using visual models or a formula. Write an equation with a symbol for the unknown number to represent the problem.
<i>MA.5.M.3 Solve problems involving the volume of right rectangular prisms.</i>	
MA.5.M.3.1	Describe volume as an attribute of three-dimensional figures by packing them with unit cubes without gaps or overlaps. Find the volume of a right rectangular prism by counting unit cubes.
MA.5.M.3.2	Find the volume of a right rectangular prism with whole-number side lengths using visual models or a formula.
MA.5.M.3.3	Solve problems involving the volume of right rectangular prisms with whole number side lengths using visual models or a formula. Write an equation with a symbol for the unknown to represent the problem.
MA.5.M.3.4	Solve problems involving the volume of composite figures composed of non-overlapping right rectangular prisms with whole number side lengths. Write an equation with a symbol for the unknown to represent the problem.
<i>MA.5.GR.1 Identify and classify three-dimensional figures based on defining attributes.</i>	
MA.5.GR.1.1	Identify and classify three-dimensional figures into categories based on their defining attributes. Three-dimensional figures are limited to pyramids, prisms, cylinders, cones and spheres.
<i>MA.5.GR.2 Plot points and represent problems on the coordinate plane.</i>	
MA.5.GR.2.1	Identify the origin and axes in the coordinate system. Plot and label ordered pairs in the first quadrant of the coordinate plane.
MA.5.GR.2.2	Represent real world and mathematical problems by plotting points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
<i>MA.5.SP.1 Collect, represent and interpret data with fractional values and find the mode, median and range of a data set.</i>	

MA.5.SP.1.1	Collect and represent numerical data, including fractional values, using tables or line plots. Solve multi-step problems involving addition, subtraction and multiplication using data from these representations.
MA.5.SP.1.2	Determine the mode, median or range to interpret numerical data, including fractional values, represented with tables or line plots.

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6-8 Mathematics Second Draft Benchmarks with Standards

Grade 6

MA.6.NSO.1 Extend knowledge of numbers to negative numbers and develop an understanding of absolute value.

MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Represent quantities using positive and negative rational numbers and compare them on a number line. Include mathematical and real-world context.
MA.6.NSO.1.2	Plot positive and negative rational numbers on a number line. Make comparisons of the two numbers using inequality symbols.
MA.6.NSO.1.3	Given a real-world situation, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.
MA.6.NSO.1.4	Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.

MA.6.NSO.2 Add, subtract, multiply and divide positive rational numbers.

MA.6.NSO.2.1	Solve multi-step problems involving addition, subtraction, multiplication or division of positive multi-digit decimals. Include mathematical and real-world contexts.
MA.6.NSO.2.2	Extend previous understanding of multiplication and division to compute quotients of positive fractions by positive fractions including mixed numbers using a variety of strategies.
MA.6.NSO.2.3	Solve mathematical and real-world problems involving division of positive fractions by positive fractions, including mixed numbers.

MA.6.NSO.3 Use properties of operations to rewrite numbers in equivalent forms.

MA.6.NSO.3.1	Find the greatest common factor and least common multiple of two whole numbers. Include mathematical and real-world context.
MA.6.NSO.3.2	Generate equivalent numerical expressions by rewriting the sum of two composite whole numbers as a common factor multiplied by the sum of two whole numbers.
MA.6.NSO.3.3	Evaluate positive rational numbers with whole number exponents.
MA.6.NSO.3.4	Express composite whole numbers as a product of prime factors with whole number exponents.

MA.6.NSO.4 Develop an understanding of operations with integers.

MA.6.NSO.4.1	Apply and extend previous understandings of operations with whole numbers to add and subtract integers using visual or numerical representations.
MA.6.NSO.4.2	Apply and extend previous understandings of operations with whole numbers to multiply and divide integers using visual or numerical representations.

MA.6.AR.1 Apply previous understanding of arithmetic expressions to algebraic expressions.

MA.6.AR.1.1	Translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions. Include mathematical and real-world context.
MA.6.AR.1.2	Given a mathematical and real-world scenario, write an algebraic inequality from graphical representations and/or represent solutions of an inequality on a number line in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$.
MA.6.AR.1.3	Evaluate algebraic expressions or equations, including formulas, using substitution. Include mathematical and real-world context.
MA.6.AR.1.4	Given an equation or inequality and a specified set of values, including integers, determine which values are solutions.

MA.6.AR.2 Write and solve one-step equations.

MA.6.AR.2.1	Write and solve one-step equations in one variable using addition and subtraction, where all
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	terms and solutions are integers. Include mathematical and real-world context.
MA.6.AR.2.2	Write and solve one-step equations in one variable using multiplication and division, where all terms and solutions are integers. Include mathematical and real-world context.
<i>MA.6.AR.3 Understand ratio and unit rate concepts and use them to solve problems.</i>	
MA.6.AR.3.1	Write and interpret ratios to show the relative sizes of two quantities using appropriate notations: a/b , a to b , or $a:b$ where $b \neq 0$. Describe the relationship between two quantities using ratios and rates. Include mathematical and real-world context.
MA.6.AR.3.2	Calculate and interpret unit rates associated with ratios of fractions or decimals, including ratios of lengths and other quantities measured in the same measurement system. Include mathematical and real-world context.
MA.6.AR.3.3	Given a table of values, determine the equivalent ratio between any two pairs of values and find any missing values. Include mathematical and real-world context.
MA.6.AR.3.4	Apply ratio relationships to solve problems involving percentages using the relationship between the two quantities. Include mathematical and real-world context.
MA.6.AR.3.5	Solve mathematical and real-world problems involving ratios and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.
<i>MA.6.SP.1 Develop an understanding of probability and find experimental and theoretical probabilities.</i>	
MA.6.SP.1.1	Determine the sample space for a single event.
MA.6.SP.1.2	Given the probability of a chance event, interpret the likelihood of it occurring.
MA.6.SP.1.3	Given the scenario of a simple event, find the theoretical probability.
<i>MA.6.SP.2 Develop an understanding of statistics and determine measures of center and measures of variability. Summarize statistical distributions graphically and numerically.</i>	
MA.6.SP.2.1	Recognize and formulate a statistical question.
MA.6.SP.2.2	Describe the spread of a given data set, represented numerically or graphically, and summarize it by determining a measure of center or a measure of variation. Measure of center is limited to mean and median. Measure of variation is limited to range. Include mathematical and real-world context.
MA.6.SP.2.3	Create a graphical representation to display a single set of numerical data within a real-world context. Graphs are limited to dot plots, histograms and stem-and-leaf plots.
MA.6.SP.2.4	Determine and describe how changes in data values impact measures of center and variation.
<i>MA.6.GR.1 Apply previous understanding of the coordinate plane to solve problems.</i>	
MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot ordered pairs in all four quadrants and on both axes. Identify the x - or y - axis as the line of reflection when two ordered pairs differ by an opposite x - or y - coordinate.
MA.6.GR.1.2	Find distances between ordered pairs, limited to the same x -coordinate or the same y -coordinate, represented on the coordinate plane.
MA.6.GR.1.3	Solve problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle. Include mathematical and real-world context.
<i>MA.6.GR.2 Model and solve problems involving two- and three-dimensional figures.</i>	
MA.6.GR.2.1	Derive and apply a formula to find the area of a triangle.
MA.6.GR.2.2	Solve mathematical and real-world problems involving the area of triangles and quadrilaterals by decomposing them into triangles or rectangles.

MA.6.GR.2.3	Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model or formula.
MA.6.GR.2.4	Find the surface area of right prisms and right pyramids, composed of rectangles and triangles, using the figure's net. Include mathematical and real-world context.

Grade 7*MA.7.NSO.1 Rewrite numbers in equivalent forms.*

MA.7.NSO.1.1	Know and apply the Laws of Exponents to generate and evaluate equivalent numerical expressions, limited to whole number exponents and rational number bases.
MA.7.NSO.1.2	Rewrite positive and negative rational numbers in different but equivalent forms including fractions, mixed numbers, decimals and percentages. Include mathematical and real-world context.
MA.7.NSO.1.3	Generate equivalent numerical expressions by rewriting the sum of two positive and negative composite rational numbers as a common factor multiplied by the sum of two positive and negative rational numbers. Include mathematical and real-world context.

MA.7.NSO.2 Add, subtract, multiply and divide rational numbers.

MA.7.NSO.2.1	Solve problems using multi-step order of operations with positive and negative rational numbers including parentheses, exponents and absolute value.
MA.7.NSO.2.2	Solve mathematical and real-world problems involving addition, subtraction, multiplication and division of positive and negative rational numbers.

MA.7.AR.1 Represent proportional relationships and use proportional reasoning to solve problems.

MA.7.AR.1.1	Write and solve an equation to represent proportional relationships between two quantities. Include mathematical and real-world context.
MA.7.AR.1.2	Apply proportional reasoning to solve percent problems involving discounts, markups, simple interest, tax, tips, fees, percent increase, percent decrease and percent error. Include mathematical and real-world context.
MA.7.AR.1.3	Apply proportional reasoning to solve problems involving the conversation between units in the same measurement system and constant speed. Include mathematical and real-world context.

MA.7.AR.2 Rewrite algebraic expressions in equivalent forms.

MA.7.AR.2.1	Generate equivalent algebraic expressions with integer coefficients.
MA.7.AR.2.2	Determine whether two algebraic expressions are equivalent.

MA.7.AR.3 Write and solve equations and inequalities.

MA.7.AR.3.1	Write and solve one-step inequalities in one variable where all terms and solutions are positive and negative rational numbers and any inequality sign can be represented. Represent solutions algebraically or graphically. Include mathematical and real-world context.
MA.7.AR.3.2	Write and solve two-step equations in one variable, where all terms are any positive or negative rational number. Include mathematical and real-world context.
MA.7.AR.3.3	Write and solve two-step inequalities in one variable, where all terms are any positive or negative rational number. Represent solutions algebraically or graphically. Include mathematical and real-world context.

MA.7.AR.4 Analyze and represent two-variable proportional relationships.

MA.7.AR.4.1	Determine the independent and dependent quantities described by a table, graph or written description. Include mathematical and real-world context.
MA.7.AR.4.2	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.
MA.7.AR.4.3	Determine the constant of proportionality when given a table, graph or written description of a proportional relationship. Include mathematical and real-world context.

MA.7.AR.4.4	Graph proportional relationships from a table, equation or a written description. Include mathematical and real-world context.
<i>MA.7.SP.1 Find and compare experimental and theoretical probabilities.</i>	
MA.7.SP.1.1	Compare the probabilities of chance events when the probabilities are given as rational numbers, including percentages.
MA.7.SP.1.2	Given an experiment or a simulation, find experimental probabilities and compare them to theoretical probabilities within the given context.
<i>MA.7.SP.2 Determine best measures of center and choose appropriate graphical representations. Compare data sets to draw informal comparative inferences on populations.</i>	
MA.7.SP.2.1	Determine the best measure of center, limited to mean and median, to summarize a numerical data set or graphical representation given the context and any outliers. Include mathematical and real-world context.
MA.7.SP.2.2	Given a numerical data set or graphical representation, identify possible outliers and describe how it may impact the measure(s) of center, limited to mean and median, or measure(s) of the variation, limited to range and interquartile range. Include mathematical and real-world context.
MA.7.SP.2.3	Given a real-world scenario, choose and create an appropriate graphical representation to display a single set of numerical data. Graphs are limited to dot plots, histograms, stem-and-leaf plots and box plots.
MA.7.SP.2.4	Given two numerical or graphical representations of data, compare the measure(s) of center and measure(s) of variability. Measures of center are limited to mean and median. Measures of variability are limited to range and interquartile range.
MA.7.SP.2.5	Given two numerical or graphical data sets, use measure(s) of center and measure(s) of variability to interpret results and draw valid conclusions about the two populations. Measures of center are limited to mean and median. Measures of variability are limited to range and interquartile range.
<i>MA.7.GR.1 Solve problems involving two-dimensional figures.</i>	
MA.7.GR.1.1	Apply a formula to find the area of trapezoids, parallelograms and rhombi.
MA.7.GR.1.2	Solve mathematical or real-world problems involving the area of polygons by decomposing them into triangles or quadrilaterals.
MA.7.GR.1.3	Find the approximation of Pi as the ratio of the circumference of a circle to its diameter. Apply formulas for the area and circumference of a circle to solve mathematical and real-world problems.
MA.7.GR.1.4	Solve mathematical and real-world problems involving the relationships between supplementary, complementary, vertical or adjacent angles.
MA.7.GR.1.5	Solve mathematical and real-world problems involving lengths and area of geometric figures, including scale drawings and scale factors.
<i>MA.7.GR.2 Solve problems involving three-dimensional figures.</i>	
MA.7.GR.2.1	Find the surface area of right cylinders using the figure's net. Include mathematical and real-world context.
MA.7.GR.2.2	Solve mathematical and real-world problems involving surface area of right pyramids, right prisms and right cylinders.
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right pyramids, right prisms and right cylinders.

Grade 8

MA.8.NSO.1 Solve problems involving rational numbers and extend the understanding of rational numbers to irrational numbers.	
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Estimate the value of numerical expressions involving irrational numbers, including non-perfect squares up to 225.
MA.8.NSO.1.2	Compare and plot rational and irrational numbers, each represented in a different way, on a number line.
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Evaluate and generate equivalent numerical expressions, limited to integer exponents and rational number bases.
MA.8.NSO.1.4	Solve multi-step problems involving the order of operations with rational numbers including exponents and radicals, limited to perfect squares up to 225 and perfect cubes up to 125. Include mathematical and real-world context.
MA.8.NSO.2 Perform operations on numbers in scientific notation.	
MA.8.NSO.2.1	Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one is compared to the other. Include mathematical and real-world context.
MA.8.NSO.2.2	Solve mathematical and real-world problems involving operations with numbers expressed in scientific notation.
MA.8.AR.1 Solve multi-step one-variable equations and inequalities.	
MA.8.AR.1.1	Identify examples of one-variable linear equations that generate one solution, infinitely many solutions, or no solution.
MA.8.AR.1.2	Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.
MA.8.AR.1.3	Solve multi-step linear inequalities in one variable, with rational number coefficients. Represent solutions algebraically or graphically. Include inequalities with variables on both sides.
MA.8.AR.1.4	Given an equation in the form of $x^2 = p$ and $x^3 = p$, where p is a positive rational number, determine the solutions.
MA.8.AR.2 Extend understanding of proportional relationships to two-variable linear equations.	
MA.8.AR.2.1	Create an equation in slope-intercept form, $y = mx + b$, for a line with a slope of m and intersecting the vertical axis at b from a table, graph or written description.
MA.8.AR.2.2	Graph a two-variable linear equation from a written description, an equation or a table. Include mathematical and real-world context.
MA.8.AR.2.3	Determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, an equation, a table or a graph. Include mathematical and real-world context.
MA.8.AR.3 Develop an understanding of two-variable systems of equations.	
MA.8.AR.3.1	Given a system of linear equations, determine algebraically that the solution satisfies both equations simultaneously.
MA.8.AR.3.2	Determine whether there is one solution, no solution or infinitely many solutions when given a system of two linear equations represented on a coordinate plane.
MA.8.AR.3.3	Solve systems of two linear equations in two variables by plotting them on a graph or using a table of values, approximating when solutions are not integers. Include mathematical and

	real-world context.
MA.8.F.1 Define, evaluate and compare functions.	
MA.8.F.1.1	Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.
MA.8.F.1.2	Given a function defined by a table, a graph or an equation, determine whether it is a linear function.
MA.8.F.1.3	Analyze a written or graphical representation of a functional relationship of two quantities stating where the function is increasing, decreasing or constant. Include mathematical and real-world context.
MA.8.SP.1 Represent and find compound probabilities.	
MA.8.SP.1.1	Determine sample spaces and find probabilities for compound events using organized lists, tables, tree diagrams, fundamental counting principle or simulation.
MA.8.SP.1.2	Solve problems involving simple or compound probability.
MA.8.SP.1.3	Construct a two-way table summarizing data on two categorical variables collected from the same subjects. Express the data as frequencies and joint relative frequencies.
MA.8.SP.2 Investigate patterns of associations in bivariate data.	
MA.8.SP.2.1	Construct scatter plots for bivariate data to investigate patterns of association between two quantities. Include mathematical and real-world context.
MA.8.SP.2.2	Given a scatter plot, describe patterns of association by analyzing clustering; descriptions include outliers, positive, negative or no association, linear association and nonlinear association. Include mathematical and real-world context.
MA.8.SP.2.3	Given a scatter plot with a linear association and informally fit a straight line. Include mathematical and real-world context.
MA.8.SP.2.4	Given the equation of a linear model, solve problems in the context of bivariate data by interpreting the slope and y-intercept. Include real-world context.
MA.8.GR.1 Solve problems involving the Pythagorean Theorem and angle relationships in polygons.	
MA.8.GR.1.1	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles. Include mathematical and real-world context.
MA.8.GR.1.2	Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane. Include mathematical and real-world context.
MA.8.GR.1.3	Given three sides, determine if the sides form a triangle. Given three angles, determine if the angles could be the interior angles of a triangle. Identify when given conditions create a unique triangle, more than one triangle, or no triangle using triangle sum and triangle inequality theorem.
MA.8.GR.1.4	Solve problems involving the relationships of interior and exterior angles of a triangle.
MA.8.GR.2 Understand similarity and congruence using models and transformations.	
MA.8.GR.2.1	Given a preimage and image, describe a single transformation that shows the relationship between them. Determine whether the transformation preserves congruence.
MA.8.GR.2.2	Describe and apply the effect of a single transformation on two-dimensional figures using coordinates.
MA.8.GR.2.3	Given a dilated two-dimensional figure, identify the effect on linear and area measurements.

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9-12 Mathematics Second Draft Benchmarks & Standards

9-12 Algebraic Reasoning Strand	
<i>MA.912.AR.1 Rewrite algebraic expressions and equations in equivalent forms.</i>	
MA.912.AR.1.1	<p>Identify and interpret parts of an expression or equation, such as terms, factors and coefficients that represent a quantity in terms of a mathematical or real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.1.2	<p>Rearrange formulas to isolate a quantity of interest using the same reasoning as in solving equations.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.1.3	<p>Apply previous understanding of integer operations to add, subtract and multiply polynomial expressions.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.1.4	<p>Divide a polynomial expression by a monomial expression.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.1.5	Divide polynomial expressions using long division, synthetic division or algebraic manipulation.
MA.912.AR.1.6	Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of polynomials.
MA.912.AR.1.7	Apply previous understanding of rational number operations to add, subtract, multiply and divide rational expressions.
MA.912.AR.1.8	Solve mathematical and real-world problems involving addition, subtraction, multiplication or division of rational algebraic expressions.
MA.912.AR.1.9	Apply the Binomial Theorem to expand polynomials expressions.
<i>MA.912.AR.2 Linear equations, functions and inequalities in one- and two-variables.</i>	
MA.912.AR.2.1	<p>Create and solve one-variable multi-step linear equations. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.2.2	<p>Create and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>

MA.912.AR.2.3	<p>Create linear functions to represent relationships between quantities from a graph, a written description or a table of values. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.2.4	<p>Write a linear function for a line that is parallel or perpendicular to a given line.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.2.5	<p>Solve and graph mathematical and real-world problems of linear functions. Determine and interpret key features in context.</p> <p>Key features are limited to domain, range, intercepts and rate of change.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.2.6	<p>Create two-variable linear inequalities to represent relationships between quantities from a graph or a written description. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.2.7	<p>Graph the solution set to a two-variable linear inequality. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
<i>MA.912.AR.3 Quadratic equations, functions and inequalities in one- and two-variables.</i>	
MA.912.AR.3.1	<p>Create and solve one-variable quadratic equations over the real number system using a variety of strategies. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.3.2	<p>Create and solve one-variable quadratic equations, using a variety of strategies, over the real and complex number systems. Include mathematical and real-world context.</p>
MA.912.AR.3.3	<p>Create and solve one-variable quadratic inequalities, using a variety of strategies, over the real number system. Represent solutions algebraically or graphically. Include mathematical and real-world context.</p>
MA.912.AR.3.4	<p>Create quadratic functions to represent relationships between quantities from a graph, a description or a table of values. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.3.5	<p>Transform expressions for quadratic functions to reveal its zeros of the associated graph using factoring techniques. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>

MA.912.AR.3.6	<p>Transform expressions for quadratic functions to reveal the vertex of the associated graph by completing the square. Interpret the maximum or minimum value in terms of its context. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.3.7	<p>Solve and graph mathematical and real-world problems of quadratic functions. Determine and interpret key features in context.</p> <p>Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; vertex; and symmetry.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
<i>MA.912.AR.4 Absolute value equations, functions and inequalities in one- and two-variables.</i>	
MA.912.AR.4.1	<p>Create and solve one-variable absolute value equations. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.4.2	Create and solve one-variable absolute value inequalities. Represent solutions algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.4.3	<p>Solve and graph mathematical and real-world problems of absolute value functions. Determine and interpret key features in context.</p> <p>Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; vertex; end behavior; and symmetry.</p>
<i>MA.912.AR.5 Exponential and logarithmic equations and functions in one- and two-variables.</i>	
MA.912.AR.5.1	Given a logarithmic expression, generate an equivalent algebraic expression using the properties of logarithms or exponents.
MA.912.AR.5.2	<p>Solve one-variable exponential equations using the properties of exponents.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.5.3	Solve equations involving one-variable logarithms or exponents using a variety of strategies. Interpret solutions as viable in terms of context and identify extraneous solutions. Include mathematical and real-world context.
MA.912.AR.5.4	<p>Classify exponential functions as representing growth or decay. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.AR.5.5	Create exponential functions to represent relationships between quantities from a graph, a description or a table of values. Include mathematical and real-world context.

MA.912.AR.5.6	Transform expressions for exponential functions to reveal the constant percent rate of change per unit interval of the associated graph using the properties of exponents. Include mathematical and real-world context.
MA.912.AR.5.7	Solve and graph mathematical and real-world problems of exponential functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; constant percent rate of change; end behavior; and asymptotes.
MA.912.AR.5.8	Solve and graph mathematical and real-world problems of logarithmic functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
<i>MA.912.AR.6 Polynomial equations and functions in one- and two-variables.</i>	
MA.912.AR.6.1	When suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems. Include mathematical and real-world context.
MA.912.AR.6.2	Explain and apply the Remainder Theorem.
MA.912.AR.6.3	Solve and graph mathematical and real-world problems of polynomial functions of degree 3 or higher. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetry; and end behavior.
MA.912.AR.6.4	Sketch a rough graph of a polynomial function of degree 3 or higher using zeros and knowledge of end behavior.
<i>MA.912.AR.7 Radical equations and functions in one- and two-variables.</i>	
MA.912.AR.7.1	Solve one-variable radical equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. Include mathematical and real-world context.
MA.912.AR.7.2	Solve and graph mathematical and real-world problems of square or cube root functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and relative maximums and minimums.
<i>MA.912.AR.8 Rational equations and functions in one- and two-variables.</i>	
MA.912.AR.8.1	Create and solve one-variable rational equations. Interpret solutions as viable in terms of context and identify any extraneous solutions. Include mathematical and real-world context.

MA.912.AR.8.2	Solve and graph mathematical and real-world problems of rational functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive or negative; end behavior; and asymptotes.
<i>MA.912.AR.9 Create and solve a system of two- and three-variable equations and inequalities that describe quantities and/or relationships.</i>	
MA.912.AR.9.1	Solve a system of two-variable linear equations algebraically or graphically. Include mathematical and real-world context. Note: This benchmark is intended for the Algebra 1 course.
MA.912.AR.9.2	Solve a system consisting of a two-variable linear equation and a non-linear equation algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.9.3	Solve a system consisting of two-variable non-linear equations algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.9.4	Graph the solution set to a system of two-variable linear inequalities. Include mathematical and real-world context. Note: This benchmark is intended for the Algebra 1 course.
MA.912.AR.9.5	Given a real-world context, represent constraints of systems of linear equations or inequalities. Interpret solutions as viable or non-viable options. Note: This benchmark is intended for the Algebra 1 course.
MA.912.AR.9.6	Given a real-world context, represent constraints of systems of non-linear equations or inequalities. Interpret solutions as viable or non-viable options.
MA.912.AR.9.7	Solve problems involving linear programming. Include mathematical and real-world context.
MA.912.AR.9.8	Solve a system of three-variable linear equations algebraically or graphically. Include mathematical and real-world context.
MA.912.AR.9.9	Solve and graph mathematical and real-world problems of piecewise functions. Determine and interpret key features in context. Key features limited to domain, range, intercepts, asymptotes and end behavior.
<i>MA.912.AR.10 Sequence & series equations, functions and inequalities in one- and two-variables.</i>	
MA.912.AR.10.1	Create and solve problems involving arithmetic sequences. Include mathematical and real-world context.
MA.912.AR.10.2	Create and solve problems involving geometric sequences. Include mathematical and real-world context.

MA.912.AR.10.3	Recognize and apply the formula for the sum of a finite arithmetic series to solve problems. Include mathematical and real-world context.
MA.912.AR.10.4	Recognize and apply the formula for the sum of a finite or an infinite geometric series to solve problems. Include mathematical and real-world context.
MA.912.AR.10.5	Create a sequence using function notation, defined explicitly or recursively, to represent relationships between quantities from a description. Include mathematical and real-world context.
MA.912.AR.10.6	Find the domain of a given sequence, defined recursively or explicitly. Include mathematical and real-world context.

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9-12 Functions Strand	
<i>MA.912.F.1 Understand, compare and analyze properties of functions.</i>	
MA.912.F.1.1	<p>Given a set of ordered pairs, table, equation, graph or mapping diagram, identify the type of relation or function.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.2	<p>Evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.3	<p>Calculate and interpret the average rate of change of a real-world situation represented graphically over a specified interval.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.4	<p>Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.</p>
MA.912.F.1.5	<p>Demonstrate understanding of the concept of limit and estimate limits from graphs and tables of values, as related to the concept of the derivative of a function.</p>
MA.912.F.1.6	<p>Compare properties of a linear and a non-linear function each represented in a different way such as algebraically, graphically, in tables or written descriptions.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.7	<p>Compare properties of two non-linear functions each represented in a different way such as algebraically, graphically, in tables or written descriptions.</p>
MA.912.F.1.8	<p>Identify a linear, quadratic or exponential function to model a given situation.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.9	<p>Compare tables and graphs of functions to verify that a quantity increasing exponentially eventually exceeds a quantity increasing linearly and quadratically.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.F.1.10	<p>Determine whether a function is even, odd or neither when represented algebraically, graphically or in a table.</p>
<i>MA.912.F.2 Identify and describe the effects of transformations on functions. Create new functions given transformations.</i>	
MA.912.F.2.1	<p>Identify the effect on the graph of a given function of a single transformation defined by adding or multiplying the x- or y-values by a real number.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>

MA.912.F.2.2	Identify the effect on the graph of a given function of two or more transformations defined by adding or multiplying the x- or y-values by a real number.
MA.912.F.2.3	Given the graph or table of values of a single transformation of a function, find the value of the real number that defines the transformation. Note: This benchmark is intended for the Algebra 1 course.
MA.912.F.2.4	Given the graph or table of values of a two or more transformations of a function, find the value of the real number that defines the transformation.
MA.912.F.2.5	Given a single transformation and a function, create the table or graph of the transformed function. Note: This benchmark is intended for the Algebra 1 course.
MA.912.F.2.6	Given two or more transformations and a function, create the table or graph of the transformed function.
MA.912.F.2.7	Given a graph or table of values of a single transformation of a function, write the equation of the transformed function. Note: This benchmark is intended for the Algebra 1 course.
MA.912.F.2.8	Given a graph or table of values of two or more transformations of a function, write the equation of the transformed function.
<i>MA.912.F.3 Create new functions from existing functions.</i>	
MA.912.F.3.1	Combine two functions, limited to linear and quadratic, using arithmetic operations. When appropriate, include domain restrictions for the new function. Include mathematical and real-world context. Note: This benchmark is intended for the Algebra 1 course.
MA.912.F.3.2	Combine two or more functions, limited to linear, quadratic, exponential and polynomial, using arithmetic operations. When appropriate, include domain restrictions for the new function. Include mathematical and real-world context.
MA.912.F.3.3	Solve problems involving functions combined using arithmetic operations. Include mathematical and real-world context.
MA.912.F.3.4	Compose functions. Determine the domain and range of the composite function. Include mathematical and real-world context.
MA.912.F.3.5	Solve problems involving composite functions. Include mathematical and real-world context.
MA.912.F.3.6	Determine if an inverse function exists by analyzing tables, graphs and equations.

MA.912.F.3.7	Represent the inverse of a function algebraically, graphically, or in a table. Use composition of functions to verify that one function is the inverse of the other.
MA.912.F.3.8	Produce an invertible function from a non-invertible function by restricting the domain.
MA.912.F.3.9	Solve problems involving inverse functions. Include mathematical and real-world context.

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9-12 Geometric Reasoning Strand	
<i>MA.912.GR.1 Use properties and theorems related to circles.</i>	
MA.912.GR.1.1	<p>Solve mathematical and real-world problems involving the length of a secant, tangent and/or chord in a given circle.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.1.2	<p>Solve mathematical and real-world problems involving the measures of arcs and related angles, limited to central, inscribed and intersections of chords, secants and/or tangents.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.1.3	<p>Solve mathematical and real-world problems involving quadrilaterals inscribed in a circle.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.1.4	<p>Solve mathematical and real-world problems involving the arc length and area of a sector in a given circle.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.1.5	<p>Apply transformations to prove that all circles are similar.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
<i>MA.912.GR.2 Apply properties of transformations to describe congruence or similarity.</i>	
MA.912.GR.2.1	<p>Describe and represent transformations as functions that take points in the plane as the domain and give other points as the range.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.2	<p>Compare transformations that do or do not preserve distance or angle measure.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.3	<p>Specify a sequence of two or more transformations that will map a given figure onto itself or onto another figure.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.4	<p>Given a geometric figure and a sequence of two or more transformations, draw the transformed figure on a coordinate plane.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.5	<p>Apply rigid transformations to map one figure onto another and justify that corresponding sides and angles are congruent.</p> <p>Note: This benchmark is intended for the Geometry course.</p>

MA.912.GR.2.6	<p>Justify the criteria for triangle congruence using the definition of congruence in terms of rigid transformations.</p> <p>Criteria is limited to Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side and Hypotenuse-Leg.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.8	<p>Apply non-rigid transformations to identify similar figures and justify their corresponding proportional sides and their congruent corresponding angles.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.2.9	<p>Justify the criteria for triangle similarity using the definition of similarity in terms of non-rigid transformations.</p> <p>Criteria is limited to Angle-Angle, Side-Angle-Side, and Side-Side-Side.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
<i>MA.912.GR.3 Prove and apply geometric theorems to solve problems.</i>	
MA.912.GR.3.1	<p>Prove triangle congruence or similarity using Side-Side-Side, Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, Angle-Angle and Hypotenuse-Leg.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.3.2	<p>Solve problems involving congruence or similarity in two-dimensional figures. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.3.3	<p>Prove postulates, relationships and theorems about lines and angles. Solve mathematical and real-world problems involving postulates, relationships and theorems of lines and angles.</p> <p>Postulates, relationships and theorems and their converses are limited to the Vertical Angles Theorem, Same-side Interior Angles Theorem, Alternate Interior Angles Theorem, Alternate Exterior Angles Theorem, Parallel postulate, Corresponding Angles postulate, Angle Addition postulate, Congruent complements and supplements theorems, Segment Addition postulate, Linear Pair Theorem, Perpendicular Bisector Theorem, and Angle Bisector Theorem.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.3.4	<p>Prove postulates, relationships and theorems about triangles. Solve mathematical and real-world problems postulates, relationships and theorems of triangles.</p> <p>Postulates, relationships and theorems and their converses are limited to Triangle Angle Sum Theorem, Exterior angle theorem, Circumcenter Theorem, Incenter Theorem, Centroid Theorem, Triangle Inequality Theorem, Isosceles triangle theorem, Third angle theorem, Triangle Proportionality Theorem, Hinge Theorem, and Triangle Midsegment Theorem. Note: This benchmark is intended for the Geometry course.</p>

MA.912.GR.3.5	<p>Prove postulates, relationships and theorems about parallelograms. Solve mathematical and real-world problems involving postulates, relationships and theorems of parallelograms. Postulates, relationships and theorems and their converses are limited to opposite sides are parallel and congruent, opposite angles are congruent, diagonals are segment bisectors, and consecutive angles are supplementary.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.3.6	<p>Prove postulates, relationships and theorems about trapezoids. Solve mathematical and real-world problems involving postulates, relationships and theorems of trapezoids.</p> <p>Postulates, relationships and theorems and their converses are limited to the Trapezoid Midsegment Theorem, base angles are congruent (isosceles), opposite angles are supplementary (isosceles), and diagonals are congruent (isosceles).</p>
<i>MA.912.GR.4 Use geometric measurement and dimensions to solve problems.</i>	
MA.912.GR.4.1	<p>Identify the shapes of two-dimensional cross-sections of three-dimensional objects.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.4.2	<p>Identify three-dimensional objects generated by rotations of two-dimensional objects.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.4.3	<p>Determine how changes in dimensions affect the area of two-dimensional figures and the surface area or volume of three-dimensional figures.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.4.4	<p>Solve mathematical and real-world problems involving the area of two-dimensional figures.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.4.5	<p>Solve mathematical and real-world problems involving the volume of three-dimensional solids limited to cylinders, pyramids, prisms, cones and spheres.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.4.6	<p>Solve mathematical and real-world problems involving the surface area of three-dimensional solids limited to cylinders, pyramids, prisms, cones and spheres.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
<i>MA.912.GR.5 Use coordinate geometry to solve problems or prove relationships.</i>	

MA.912.GR.5.1	<p>In a mathematical or real-world context, use coordinate geometry to classify or justify definitions, properties, or theorems of circles, triangles or quadrilaterals.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.5.2	<p>Solve geometric problems on the coordinate plane using slope criteria.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.5.3	<p>Solve mathematical and real-world problems on the coordinate plane involving finding the coordinates of a point on a line segment including the midpoint.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.5.4	<p>Solve mathematical and real-world problems on the coordinate plane involving the perimeter or area of polygons.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
<i>MA.912.GR.6 Apply geometric and algebraic representations of conic sections.</i>	
MA.912.GR.6.1	Identify the conic resulting from the cross-section of cones.
MA.912.GR.6.2	<p>Derive and create the equation of a circle given key features. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.6.3	<p>Solve and graph mathematical and real-world problems involving an equation of a circle. Determine and interpret key features in context.</p> <p>Key features are limited to domain, range, center and radius.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.GR.6.4	Derive and create the equation of a parabola given key features. Include mathematical and real-world context.
MA.912.GR.6.5	<p>Solve and graph mathematical and real-world problems involving an equation of a parabola. Determine and interpret key features in context.</p> <p>Key features limited to domain, range, intercepts, focus, vertex and directrix.</p>
MA.912.GR.6.6	Derive and create the equation of an ellipse given key features. Include mathematical and real-world context.
MA.912.GR.6.7	<p>Solve and graph mathematical and real-world problems involving an equation of an ellipse. Determine and interpret key features in context.</p> <p>Key features limited to domain, range, center, foci, major axis, minor axis and vertices.</p>
MA.912.GR.6.8	Derive and create the equation of a hyperbola given key features. Include mathematical and real-world context.

MA.912.GR.6.9	Solve and graph mathematical and real-world problems involving an equation of a hyperbola. Determine and interpret key features in context. Key features limited to domain, range, center, vertices, foci, major axis, minor axis, asymptotes and directrices.
<i>MA.912.GR.7 Make formal geometric constructions with a variety of tools and methods.</i>	
MA.912.GR.7.1	Construct a copy of a segment or an angle. Note: This benchmark is intended for the Geometry course.
MA.912.GR.7.2	Construct the bisection of a segment or an angle, including the perpendicular bisector of a line segment. Note: This benchmark is intended for the Geometry course.
MA.912.GR.7.3	Construct the inscribed and circumscribed circles of a triangle. Note: This benchmark is intended for the Geometry course.
MA.912.GR.7.4	Construct a regular polygon inscribed in a circle. Regular polygons are limited to triangles, quadrilaterals and hexagons. Note: This benchmark is intended for the Geometry course.
MA.912.GR.7.5	Given a point outside a circle, construct a line tangent to the circle that passes through the given point. Note: This benchmark is intended for the Geometry course.

9-12 Number Sense & Operations Strand*MA.912.NSO.1 Perform operations on expressions involving exponents, radicals or logarithms.*

MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Evaluate and generate equivalent numerical expressions involving rational exponents. Note: This benchmark is intended for the Algebra 1 course.
MA.912.NSO.1.2	Generate equivalent monomial algebraic expressions by using the Laws of Exponents. Note: This benchmark is intended for the Algebra 1 course.
MA.912.NSO.1.3	Generate equivalent algebraic expressions involving radicals and/or rational exponents using the properties of exponents. Radicands are limited to monomial algebraic expressions.
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals. Note: This benchmark is intended for the Algebra 1 course.
MA.912.NSO.1.5	Add, subtract, multiply and divide algebraic expressions involving radicals. Radicands are limited to monomial algebraic expressions.

MA.912.NSO.2 Represent and perform operations on expressions within the complex number system.

MA.912.NSO.2.1	Extend previous understanding of the real number system to include the complex number system. Add, subtract, multiply and divide complex numbers.
MA.912.NSO.2.2	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane.
MA.912.NSO.2.3	Calculate the distance and midpoint between two numbers on the complex coordinate plane.
MA.912.NSO.2.4	Solve problems involving complex numbers represented algebraically or on the coordinate plane. Include mathematical and real-world context.
MA.912.NSO.2.5	Represent complex numbers on the complex plane in rectangular and polar forms. Explain why the rectangular and polar forms of a given complex number represent the same number.
MA.912.NSO.2.6	Rewrite complex numbers to trigonometric form. Multiply complex numbers in trigonometric form.

MA.912.NSO.3 Represent and perform operations on vectors.

MA.912.NSO.3.1	Use appropriate notation and symbols to represent vectors in the plane as directed line segments. Determine the magnitude and direction of a vector in component form.
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MA.912.NSO.3.2	Represent vectors in component form, linear form or trigonometric form. Rewrite vectors from one form to another.
MA.912.NSO.3.3	Solve problems involving velocity and other quantities that can be represented by vectors. Include mathematical and real-world context.
MA.912.NSO.3.4	Solve problems using dot product and vector projections in two-dimensions. Include mathematical and real-world context.
MA.912.NSO.3.5	Solve problems using dot product and cross product in three-dimensions. Include mathematical and real-world context.
MA.912.NSO.3.6	Add and subtract vectors algebraically or graphically.
MA.912.NSO.3.7	Given the magnitude and direction of two or more vectors, determine the magnitude and direction of their sum.
MA.912.NSO.3.8	Multiply a vector by a scalar algebraically or graphically.
MA.912.NSO.3.9	Compute the magnitude and direction of a vector scalar multiple.
<i>MA.912.NSO.4 Represent and perform operations on matrices.</i>	
MA.912.NSO.4.1	Represent and manipulate data using matrices. Include mathematical and real-world context.
MA.912.NSO.4.2	Represent and solve a system of two- or three-variable linear equations using matrices. Include mathematical and real-world context.
MA.912.NSO.4.3	Solve problems using addition, subtraction and multiplication of matrices.
MA.912.NSO.4.4	Solve problems using the inverse and determinant of matrices. Include mathematical and real-world context.
MA.912.NSO.4.5	Identify and use the additive and multiplicative identities for matrices to solve problems. Include mathematical and real-world context.

9-12 Statistics & Probability Strand	
<i>MA.912.SP.1 Use and interpret independence and probability.</i>	
MA.912.SP.1.1	Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections or complements of other events.
MA.912.SP.1.2	Determine if events A and B are independent by calculating the product of their probabilities.
MA.912.SP.1.3	Calculate the conditional probability of two events and interpret the result in terms of its context.
MA.912.SP.1.4	Interpret the independence of two events using conditional probability.
MA.912.SP.1.5	<p>Given a two-way table, interpret joint, marginal and/or conditional relative frequencies in terms of a context. Recognize possible associations in the data.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.SP.1.6	Decide if events are independent and approximate conditional probabilities using two-way tables as a sample space.
MA.912.SP.1.7	<p>Given a real-world context, construct and interpret representations to find relative frequency, probabilities and conditional probabilities. Representations limited to two-way tables, tree diagrams and area models.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.SP.1.8	Given a two-way table, construct and interpret a segmented bar graph. Determine relative frequencies, any independence and any possible associations in the data. Include mathematical and real-world context.
MA.912.SP.1.9	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.912.SP.1.10	Apply the addition rule for probability, taking into consideration whether the events are mutually exclusive, and interpret the result in terms of the model. Include mathematical and real-world context.
MA.912.SP.1.11	Apply the general multiplication rule for probability, taking into consideration whether the events are independent, and interpret the result in terms of the context. Include mathematical and real-world context.
MA.912.SP.1.12	Calculate the appropriate permutation or combination for a given situation.
MA.912.SP.1.13	Compute probabilities of compound events and solve problems using permutations and combinations.
<i>MA.912.SP.2 Determine methods of data collection and make inferences from collected data.</i>	

MA.912.SP.2.1	Distinguish between a population parameter and a sample statistic.
MA.912.SP.2.2	Explain how random sampling produces data that is representative of a population.
MA.912.SP.2.3	Compare and contrast sampling methods.
MA.912.SP.2.4	Generate multiple samples or simulated samples of the same size to measure the variation in estimates or predictions.
MA.912.SP.2.5	Determine if a specific model is consistent within a given process by analyzing the data distribution from a data-generating process.
MA.912.SP.2.6	Determine the appropriate design, survey, experiment or observational study, based on the purpose. Articulate the types of questions appropriate for each type of design.
MA.912.SP.2.7	Compare and contrast surveys, experiments and observational studies.
MA.912.SP.2.8	Explain how randomization relates to sample surveys, experiments and observational studies.
MA.912.SP.2.9	Estimate a population mean or proportion using data from a sample survey. Calculate and interpret the corresponding margin of error.
MA.912.SP.2.10	Draw inferences about two populations using data and statistical analysis from two random samples.
MA.912.SP.2.11	Compare two treatments from an experiment using data from a randomized experiment.
MA.912.SP.2.12	Determine if differences between parameters are significant using simulations.
MA.912.SP.2.13	Evaluate reports based on data from diverse media, print and digital resources by interpreting graphs and tables; evaluating data-based arguments; determining if a valid sampling method was used; or interpreting provided statistics.
<i>MA.912.SP.3 Summarize, represent and interpret one- and two-variable data.</i>	
MA.912.SP.3.1	<p>Analyze and interpret numerical data distributions represented with frequency tables, histograms, stem-and-leaf plots, dot plots and box plots.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.SP.3.2	<p>Calculate and compare the appropriate measures of center and measures of variability of two or more different data sets, accounting for possible effects of outliers. Include mathematical and real-world context.</p> <p>Note: This benchmark is intended for the Algebra 1 course.</p>
MA.912.SP.3.3	Identify if a data set is normally distributed. Include mathematical and real-world context.
MA.912.SP.3.4	Fit a normal distribution, if appropriate, and estimate population percentages using the mean and standard deviation of a data set. Include mathematical and real-world

MA.912.SP.3.5	Estimate areas under the normal curve using technology, empirical rule or tables. Include mathematical and real-world context.
MA.912.SP.3.6	Fit a linear function to data that suggests a linear association and interpret the slope and y- intercept of the model. Use the model to solve problems in the context of the data. Note: This benchmark is intended for the Algebra 1 course.
MA.912.SP.3.7	Fit a quadratic function to data that suggests a quadratic association. Use the model to solve problems in the context of the data.
MA.912.SP.3.8	Fit an exponential function to data that suggests an exponential association. Use the model to solve problems in the context of the data.
MA.912.SP.3.9	Assess the fit of a function by plotting and analyzing residuals.
MA.912.SP.3.10	Use technology to compute the correlation coefficient of a linear model. Interpret the strength and direction of the correlation coefficient.
MA.912.SP.3.11	Explain the difference between correlation and causation.
<i>MA.912.SP.4 Use probability distributions to solve problems.</i>	
MA.912.SP.4.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.912.SP.4.2	Develop a probability distribution for a discrete random variable using theoretical probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.SP.4.3	Develop a probability distribution for a discrete random variable using empirically assigned probabilities. Find the expected value and interpret it as the mean of the discrete distribution.
MA.912.SP.4.4	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Evaluate and compare strategies on the basis of the calculated expected values.
MA.912.SP.4.5	Apply probabilities to make decisions which are equally likely, such as drawing from lots or using a random number generator.
MA.912.SP.4.6	Analyze decisions that were made and solve problems using probability concepts and strategies.

9-12 Trigonometry Strand	
<i>MA.912.T.1 Define and use trigonometric ratios, identities or functions to solve problems.</i>	
MA.912.T.1.1	<p>Apply similarity to define trigonometric ratios for acute angles in right triangles.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.T.1.2	<p>Apply the relationships of the side lengths of special right triangles to solve mathematical and real-world problems.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.T.1.3	<p>Solve mathematical and real-world problems involving right triangles using trigonometric ratios and the Pythagorean Theorem.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.T.1.4	<p>Apply the Law of Sines and the Law of Cosines to solve mathematical and real-world problems involving triangles.</p>
MA.912.T.1.5	<p>Solve problems involving finding the area of a triangle given two sides and the included angle.</p>
MA.912.T.1.6	<p>Prove Pythagorean Identities. Apply Pythagorean Identities to calculate trigonometric ratios and to solve problems.</p>
MA.912.T.1.7	<p>Prove the Double-Angle, Half-Angle, Angle Sum and Difference formulas for sine, cosine, and tangent. Use these formulas to solve problems.</p>
MA.912.T.1.8	<p>Simplify expressions using trigonometric identities. Identities are limited to Double-Angle, Half-Angle, Angle Sum and Difference, Pythagorean Identities, Sum Identities, Product Identities.</p>
MA.912.T.1.9	<p>Solve trigonometric equations, applying inverse functions and using technology when appropriate. Include mathematical and real-world context.</p>
<i>MA.912.T.2 Extend trigonometric functions to the unit circle.</i>	
MA.912.T.2.1	<p>Define the trigonometric functions for any angle by using right triangles drawn in the unit circle. Determine the values of sine, cosine and tangent of $\pi/3$, $\pi/4$ and $\pi/6$ and their multiples using special triangles.</p>
MA.912.T.2.2	<p>Use the unit circle to define and determine the sine, cosine, tangent, cosecant, secant and cotangent of angles.</p>
MA.912.T.2.3	<p>Given angles measured in radians or degrees, calculate the values of the six trigonometric functions.</p>
<i>MA.912.T.3 Graph and apply trigonometric relations and functions.</i>	

MA.912.T.3.1	Describe and demonstrate the connections between right triangle ratios, trigonometric functions and circular functions.
MA.912.T.3.2	On the coordinate plane, express the values of sine, cosine and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.
MA.912.T.3.3	Choose sine, cosine or tangent trigonometric functions to model periodic phenomena with specified amplitude, frequency, horizontal shift and midline. Include mathematical and real-world context.
MA.912.T.3.4	Solve and graph mathematical and real-world problems of trigonometric functions. Determine and interpret key features in context. Key features limited to domain; range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetry; end behavior; periodicity; midline; amplitude; shift(s) and asymptotes.
MA.912.T.3.5	Verify that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.912.T.3.6	Solve problems involving applications of trigonometric functions using graphing technology when appropriate. Include mathematical and real-world context.
<i>MA.912.T.4 Extend rectangular coordinates and equations to polar and parametric forms.</i>	
MA.912.T.4.1	Define polar coordinates and relate polar coordinates to Cartesian coordinates with and without the use of technology.
MA.912.T.4.2	Represent equations given in rectangular coordinates in terms of polar coordinates.
MA.912.T.4.3	Graph equations in the polar coordinate plane with and without the use of graphing technology.
MA.912.T.4.4	Identify and graph special polar equations, including circles, cardioids, limacons, rose curves and lemniscates.
MA.912.T.4.5	Sketch the graph of a curve in the plane represented parametrically, indicating the direction of motion.
MA.912.T.4.6	Convert from a parametric representation of a plane curve to a rectangular equation, and convert from a rectangular equation to a parametric representation of a plane curve.
MA.912.T.4.7	Apply parametric equations to model applications of motion in the plane.

9-12 Logic & Theory Strand	
<i>MA.912.LT.1 Apply recursive methods to solve problems.</i>	
MA.912.LT.1.1	Apply recursive and iterative thinking to solve problems.
MA.912.LT.1.2	Solve problems and find explicit formulas for recurrence relations using finite differences.
MA.912.LT.1.3	Apply mathematical induction in a variety of applications.
<i>MA.912.LT.2 Apply techniques from Graph Theory to solve problems.</i>	
MA.912.LT.2.1	Solve scheduling problems using critical path analysis and Gantt charts. Create a schedule using critical path analysis.
MA.912.LT.2.2	Apply graph coloring techniques to solve problems.
MA.912.LT.2.3	Apply spanning trees, rooted trees, binary trees and decision trees to solve problems.
MA.912.LT.2.4	Create problems that can be solved using spanning trees, rooted trees, binary trees, and decision trees.
MA.912.LT.2.5	Solve problems concerning optimizing resource usage using bin-packing techniques.
<i>MA.912.LT.3 Apply techniques from Election Theory to solve problems.</i>	
MA.912.LT.3.1	Analyze election data using election theory techniques.
MA.912.LT.3.2	Decide voting power within a group using weighted voting techniques. Provide real-world examples of weighted voting and its pros and cons.
MA.912.LT.3.3	Solve problems using fair division techniques.
MA.912.LT.3.4	Solve strictly determined and non-strictly determined games by using game theory.
<i>MA.912.LT.4 Develop an understanding of the fundamentals of propositional logic, arguments and methods of proof.</i>	
MA.912.LT.4.1	Translate propositional statements into logical arguments using propositional variables and logical connectives.
MA.912.LT.4.2	Determine truth values of simple and compound statements using truth tables.
MA.912.LT.4.3	Find the converse, inverse, and contrapositive of a statement. Note: This benchmark is intended for the Geometry course.
MA.912.LT.4.4	Represent logic operations, such as AND, OR, NOT, NOR, and XOR (exclusive OR), using logical symbolism to solve problems.
MA.912.LT.4.5	Determine whether two propositions are logically equivalent.

MA.912.LT.4.6	<p>Apply methods of direct and indirect proof, and determine whether a logical argument is valid.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.LT.4.7	<p>Identify and give examples of undefined terms; axioms; theorems; inductive and deductive proofs; and inductive and deductive reasoning.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
MA.912.LT.4.8	<p>Construct logical arguments using laws of detachment, syllogism, tautology and contradiction.</p>
MA.912.LT.4.9	<p>Judge the validity of arguments, and give counterexamples to disprove statements.</p> <p>Note: This benchmark is intended for the Geometry course.</p>
<i>MA.912.LT.5 Apply properties from Set Theory to solve problems.</i>	
MA.912.LT.5.1	<p>Given two sets, determine whether the two sets are equal, whether one set is a subset of another or if one is the power set of the other.</p>
MA.912.LT.5.2	<p>Given a relation on two sets, determine whether the relation is a function, determine the inverse of the relation if it exists, and identify if the relation is bijective.</p>
MA.912.LT.5.3	<p>Partition a set into disjoint subsets, and determine an equivalence class given the equivalence relation on a set.</p>
MA.912.LT.5.4	<p>Perform the set operations of union, intersection, difference, complement and cross product.</p>
MA.912.LT.5.5	<p>Explore relationships and patterns and make arguments about relationships between sets by using Venn Diagrams.</p>
MA.912.LT.5.6	<p>Prove set relations, including DeMorgan's Laws and equivalence relations.</p>

9-12 Financial Literacy Strand

MA.912.FL.1 Determine simple and compound interest and demonstrate its relationship to functions. Calculate and use Net Present and Net Future Values.

MA.912.FL.1.1	Compare simple, compound and continuous compounded interest over time.
MA.912.FL.1.2	Solve problems involving simple, compound and continuous compounded interest, including determine the present value and future value of money.
MA.912.FL.1.3	Explain the relationship between simple interest and linear growth. Note: This benchmark is intended for the Algebra 1 course.
MA.912.FL.1.4	Explain the relationship between compound interest and continuous compound interest to exponential growth.
MA.912.FL.1.5	Determine the consumer price index (CPI) for goods. Interpret its value in terms of the context.
MA.912.FL.1.6	Solve problems involving maximum profit and minimal cost.
<i>MA.912.FL.2 Describe the advantages and disadvantages of short-term and long-term purchases.</i>	
MA.912.FL.2.1	Compare the advantages and disadvantages of using cash versus personal financing options or other forms of electronic payment.
MA.912.FL.2.2	Calculate the finance charges and total amount due on a bill from using various forms of credit.
MA.912.FL.2.3	Manipulate a variety of variables to compare the advantages and disadvantages of deferred payments.
MA.912.FL.2.4	Calculate the total cost of purchasing consumer durables over time given different monthly payments, down payments, financing options and fees.
MA.912.FL.2.5	Calculate the fees associated with a mortgage. Fees limited to discount prices, origination fee, maximum brokerage fee on a net or gross loan, documentary stamps prorated expenses.
MA.912.FL.2.6	Substitute values to evaluate a variety of mortgage formulas. Formulas limited to Front End Ratio, Total Debt-to-Income Ratio, Loan-to-Value Ratio (LTV), Combined Loan-to-Value Ratio (CLTV) and Amount of Interest Paid Over the Life of a Loan.
MA.912.FL.2.7	Solve problems involving student, personal and car loans, including finding the total amount to be paid, adjustable rates and refinancing options.
MA.912.FL.2.8	Calculate the final payout amount for a balloon mortgage.
MA.912.FL.2.9	Compare the cost of paying a higher interest rate and fewer mortgage points versus a lower interest rate and more mortgage points.

MA.912.FL.2.10	Calculate the total amount paid for the life of a loan including the down payment, fees and interest.
MA.912.FL.2.11	Calculate and compare, in terms of functions, the total cost for a set purchase price using a fixed rate, adjustable rate and a balloon mortgage.
MA.912.FL.2.12	Compare interest rate calculations and annual percentage rate calculations, and distinguish between the two rates.
<i>MA.912.FL.3 Develop personal financial skills and describe the advantages and disadvantages of financial and investment plans.</i>	
MA.912.FL.3.1	Develop personal budgets that fit within various income brackets.
MA.912.FL.3.2	Calculate the break-even point to determine the viability of purchasing options for housing, car and other durable goods.
MA.912.FL.3.3	Explain cash management strategies include checking and savings accounts.
MA.912.FL.3.4	Given assets and liabilities, calculate net worth.
MA.912.FL.3.5	Given a scenario, establish a plan to pay off debt.
MA.912.FL.3.6	Given a scenario, complete and calculate federal income tax, analyzing different options such as standard deductions versus itemized deductions and taxes owed based on income brackets from the tax table.
MA.912.FL.3.7	Calculate and compare various options and fees for medical, car, homeowners and life insurance.
MA.912.FL.3.8	Collect, organize and interpret data to determine an effective retirement savings plan to meet personal financial goals.
MA.912.FL.3.9	Solve problems involving different types of retirement plans, including traditional IRA, ROTH IRA, 401K, 403B and annuities.
MA.912.FL.3.10	Compare different ways that portfolios can be diversified in both investments and investment vehicles.
MA.912.FL.3.11	Purchase stock with a set amount of money, and evaluate its worth over time considering gains, losses and selling, taking into account any associated fees.
MA.912.FL.3.12	Compare income from purchase of common stock, preferred stock, and bonds.
MA.912.FL.3.13	Given current exchange rates, convert between currencies.
MA.912.FL.3.14	Apply data to compare historical rates of return on investments with investment claims to make informed decisions and identify potential fraud.