Environmentally Constrained Economic Dispatch Problem—A Unified Model

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Outline

• Introduction
• Proposed Methodology
• Case Study and Results
• Concluding Remarks
Introduction

Economic Dispatch?

Determines the generation dispatch

Minimum instantaneous operating cost

Generation = Load + Loss

used in real-time energy management

Kernel of Power System
Introduction

Emission Dispatch?

Determines the generation dispatch

Minimum amount of Emission

Pollution?

Concentration of emissions is reached to a pre-specified threshold
Introduction

Joint Dispatch?

Simultaneous Consideration of Cost and Emission

Compromise between Cost and Emission

Cost

Emission
Mathematical Formulation

\[
\min \sum_{t=1}^{T} \left[ \alpha^c \pi^0_{it} P_{d_i} + \sum_{i=1}^{n} (a_i P_{it}^2 + b_i P_{it} + c_i) \right]
\]

subject to:

\[
\min \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i P_{it}^2 + \beta_i P_{it} + \gamma_i
\]

\[
\sum_{i=1}^{n} P_{it} = P_{d_i} ; \forall t = 1...T, (\pi^0_t)
\]

\[
P_i \leq P_{it} \leq \overline{P_i} ; \forall i = 1...n, \forall t = 1...T
\]

\[
P_{it} - P_{it-1} \leq RUR_i ; \forall i = 1...n, \forall t = 1...T
\]

\[
P_{it-1} - P_{it} \leq RDR_i ; \forall i = 1...n, \forall t = 1...T
\]
Mathematical Formulation

\[ \min_{P_{d_i}^1, P_{d_i}^2} \sum_{t=1}^{T} \left[ \alpha^c \pi_t^0 P_{d_i} + \sum_{i=1}^{n} a_i P_{i t}^{l2} + b_i P_{i t}^l + c_i \right] \]

subject to:

\[ \min \sum_{i=1}^{n} \sum_{t=1}^{T} \alpha_i P_{i t}^{l2} + \beta_i P_{i t}^l + \gamma_i \]

\[ \sum_{i=1}^{n} P_{i t}^l = P_{d_i} : (\pi_t); \ \forall t = 1...T \]

\[ P_{i t}^l - \sum_{y=1}^{Y} \Delta_{i y t} = 0 : (\lambda_i); \ \forall i = 1...n, \ \forall t = 1...T \]

\[ P_{i t}^{l2} - \sum_{y=1}^{Y} m_{i y} \Delta_{i y t} = 0 : (\eta_i); \ \forall i = 1...n, \ \forall t = 1...T \]

\[ \Delta_{i y t} \leq \Delta_i : (\sigma_{i y t}); \ \forall i = 1...n, \ \forall y = 1...Y, \ \forall t = 1...T \]

\[ -\Delta_{i y t} \leq 0 : (\bar{\sigma}_{i y t}); \ \forall i = 1...n, \ \forall y = 1...Y, \ \forall t = 1...T \]
Mathematical Formulation

\[
\min \sum_{t=1}^{T} \left[ \alpha^c \pi_t P_d + \sum_{i=1}^{n} (a_i P_i^{l2} + b_i P_i^l + c_i) \right]
\]

subject to:

\[
\sum_{t=1}^{T} \left[ \pi_t P_d + \sum_{i=1}^{n} \left( \kappa_{it} \overline{P}_i - \kappa_{it} \overline{P}_i + \sum_{y=1}^{Y} \Delta_{iy} \overline{\sigma}_{iyt} \right) \right] + \sum_{i=1}^{n} \left( \overline{\tau}_{i1} (RUR_i + P_{i0}) + \tau_{i1} (RDR_i - P_{i0}) \right)
\]

\[
+ \sum_{t=2}^{T} \left[ \sum_{i=1}^{n} (\overline{\tau}_{it} RUR_i + \tau_{it} RDR_i) \right] = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i P_i^{l2} + \beta_i P_i^l + \gamma_i
\]

\[
\pi_t + \kappa_{it} - \kappa_{it} + \overline{\tau}_{it} - \overline{\tau}_{it+1} + \tau_{it+1} - \tau_{it} + \lambda_{it} = \beta_i; \quad \forall i = 1...n, \quad \forall t = 1...T - 1
\]

\[
\pi_T + \kappa_{iT} - \kappa_{iT} + \overline{\tau}_{iT} - \tau_{iT} + \lambda_{iT} = \beta_i; \quad \forall i = 1...n
\]

\[
\eta_{it} = \alpha_i; \quad \forall i = 1...n, \quad \forall t = 1...T
\]

\[-\lambda_{it} - \eta_{it} m_{iy} + \overline{\sigma}_{iyt} - \overline{\sigma}_{iyt} = 0; \quad \forall i = 1...n, \quad \forall y = 1...Y, \quad \forall t = 1...T
\]
Case Study and Results

- IEEE 30-bus system, 6-unit.
- Korean 140-unit system
- A Modeling Language for Mathematical Programming (AMPL).
- Commercial solver Cplex.
- Dynamic Dispatch
Case Study and Results

### Hourly Power Output of Units—IEEE 30-Bus System

<table>
<thead>
<tr>
<th>Unit</th>
<th>h1</th>
<th>h2</th>
<th>h3</th>
<th>h4</th>
<th>h5</th>
<th>h6</th>
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<td>39.05</td>
<td>40.625</td>
<td>50</td>
<td>50</td>
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<td>46</td>
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<td>2</td>
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<td>51.150</td>
<td>60</td>
<td>60</td>
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<tr>
<td>3</td>
<td>50.25</td>
<td>70</td>
<td>90</td>
<td>78.25</td>
<td>53.250</td>
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<tr>
<td>4</td>
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<td>61.800</td>
<td>109.8</td>
<td>83.40</td>
<td>49.200</td>
<td>58.8</td>
</tr>
<tr>
<td>5</td>
<td>50.25</td>
<td>65.200</td>
<td>95.20</td>
<td>78.35</td>
<td>53.075</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>49.20</td>
<td>51.225</td>
<td>60</td>
<td>60</td>
<td>51.750</td>
<td>57.9</td>
</tr>
</tbody>
</table>

Total cost was $4636.71; corresponding emission was 124.938 kg. Using the proposed approach, the total cost increases to $4789.43 while the corresponding emission is decreases to 114.876 kg.
Case Study and Results

- Total CPU time for the 6-unit system: 0.86 s
- For the 140-unit system, an increase in the cost about 8% yields a decrease about 21% via 2.12 s.
  - The disjoint term has been neglected.
  - The nonconvex term has been neglected as well.
Concluding Remark

- A multi-objective function with conflicting objectives has been solved.
- Using the linearization and strong duality theorem, a single objective problem has been obtained.
- Global solution has been guaranteed.
- Since it is fast enough, it can be applied to online-based problems.
Question?

Special thanks to the Session Audiences