

Environmentally Constrained Economic Dispatch Problem— A Unified Model

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Outline

- Introduction
- Proposed Methodology
- Case Study and Results
- Concluding Remarks

Introduction

Economic Dispatch ?

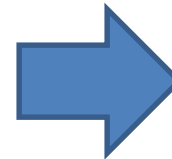
Determines the generation dispatch



Minimum instantaneous operating cost

$$\text{Generation} = \text{Load} + \text{Loss}$$

used in real-time energy management



Kernel of Power System

Introduction

Emission Dispatch ?

Determines the generation
dispatch



Minimum amount of
Emission

Pollution ?

Concentration of emissions
is reached to a pre-specified
threshold

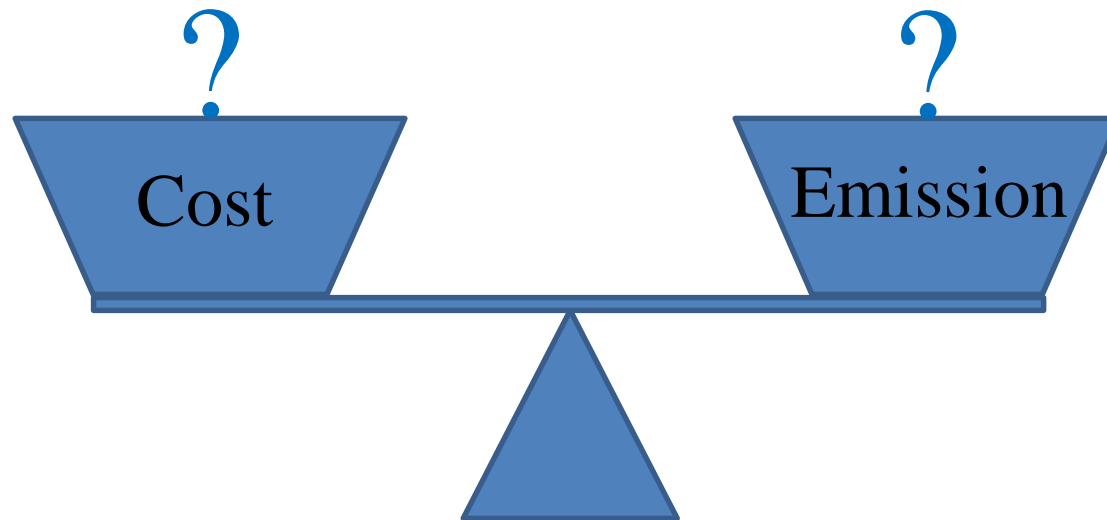
Introduction

Joint Dispatch ?

Simultaneous Consideration
of Cost and Emission



Compromise between
Cost and Emission



Mathematical Formulation

$$\min_{P_{i,t}} \sum_{t=1}^T \left[\alpha^c \pi_t^0 P_{d_t} + \sum_{i=1}^n (a_i P_{it}^2 + b_i P_{it} + c_i) \right]$$

subject to:

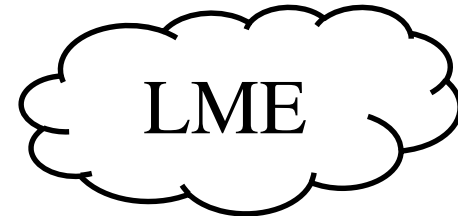
$$\min \sum_{t=1}^T \sum_{i=1}^n \alpha_i P_{it}^2 + \beta_i P_{it} + \gamma_i$$

$$\sum_{i=1}^n P_{it} = P_{d_t}; \forall t = 1 \dots T, (\pi_t^0)$$

$$\underline{P}_i \leq P_{it} \leq \bar{P}_i; \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$P_{it} - P_{it-1} \leq RUR_i; \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$P_{it-1} - P_{it} \leq RDR_i; \forall i = 1 \dots n, \forall t = 1 \dots T$$



Mathematical Formulation

$$\min_{\substack{P_{it}^l, P_{it}^{l2}, \\ \Delta_{iyt}}} \sum_{t=1}^T \left[\alpha^c \pi_t^0 P_{d_t} + \sum_{i=1}^n a_i P_{it}^{l2} + b_i P_{it}^l + c_i \right]$$

subject to:

$$\min_{P_{it}^{l2}, P_{it}^l} \sum_{t=1}^T \sum_{i=1}^n \alpha_i P_{it}^{l2} + \beta_i P_{it}^l + \gamma_i$$

$$\sum_{i=1}^n P_{it}^l = P_{d_t} : (\pi_t); \forall t = 1 \dots T$$

$$P_{it}^l - \sum_{y=1}^Y \Delta_{iyt} = 0 : (\lambda_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$P_{it}^{l2} - \sum_{y=1}^Y m_{iy} \Delta_{iyt} = 0 : (\eta_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$\Delta_{iyt} \leq \bar{\Delta}_{iy} : (\bar{\sigma}_{iyt}); \quad \forall i = 1 \dots n, \forall y = 1 \dots Y, \forall t = 1 \dots T$$

$$-\Delta_{iyt} \leq 0 : (\underline{\sigma}_{iyt}); \quad \forall i = 1 \dots n, \forall y = 1 \dots Y, \forall t = 1 \dots T$$

$$P_{it}^l \leq \bar{P}_i : (\bar{\kappa}_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$-P_{it}^l \leq -\underline{P}_i : (\underline{\kappa}_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$P_{it}^l - P_{it-1}^l \leq RUR_i : (\bar{\tau}_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$P_{it-1}^l - P_{it}^l \leq RDR_i : (\underline{\tau}_{it}); \quad \forall i = 1 \dots n, \forall t = 1 \dots T$$

Mathematical Formulation

$$\min_{\substack{P_{it}^l, P_{it}^{l2}, \pi_t, \bar{\kappa}_{it}, \\ \underline{\kappa}_{it}, \bar{\tau}_{it}, \underline{\tau}_{it}, \lambda_{it}, \eta_{it}, \\ \bar{\sigma}_{iyt}, \underline{\sigma}_{iyt}}} \sum_{t=1}^T \left[\alpha^c \pi_t P_{d_t} + \sum_{i=1}^n (a_i P_{it}^{l2} + b_i P_{it}^l + c_i) \right]$$

subject to:

$$\sum_{t=1}^T \left[\pi_t P_{d_t} + \sum_{i=1}^n \left(\bar{\kappa}_{it} \bar{P}_i - \underline{\kappa}_{it} \underline{P}_i + \sum_{y=1}^Y \bar{\Delta}_{iy} \bar{\sigma}_{iyt} \right) \right] + \sum_{i=1}^n \left(\bar{\tau}_{i1} (RUR_i + P_{i0}^l) + \underline{\tau}_{i1} (RDR_i - P_{i0}^l) \right) \\ + \sum_{t=2}^T \left[\sum_{i=1}^n (\bar{\tau}_{it} RUR_i + \underline{\tau}_{it} RDR_i) \right] = \sum_{t=1}^T \sum_{i=1}^n \alpha_i P_{it}^{l2} + \beta_i P_{it}^l + \gamma_i$$

$$\pi_t + \bar{\kappa}_{it} - \underline{\kappa}_{it} + \bar{\tau}_{it} - \bar{\tau}_{it+1} + \underline{\tau}_{it+1} - \underline{\tau}_{it} + \lambda_{it} = \beta_i; \forall i = 1 \dots n, \forall t = 1 \dots T - 1$$

$$\pi_T + \bar{\kappa}_{iT} - \underline{\kappa}_{iT} + \bar{\tau}_{iT} - \underline{\tau}_{iT} + \lambda_{iT} = \beta_i; \forall i = 1 \dots n$$

$$\eta_{it} = \alpha_i; \forall i = 1 \dots n, \forall t = 1 \dots T$$

$$-\lambda_{it} - \eta_{it} m_{iy} + \bar{\sigma}_{iyt} - \underline{\sigma}_{iyt} = 0; \forall i = 1 \dots n, \forall y = 1 \dots Y, \forall t = 1 \dots T$$

Case Study and Results

- IEEE 30-bus system, 6-unit.
- Korean 140-unit system
- A Modeling Language for Mathematical Programming (AMPL).
- Commercial solver Cplex.
- Dynamic Dispatch

Case Study and Results

➤ Hourly Power Output of Units— IEEE 30-Bus System

Unit	h1	h2	h3	h4	h5	h6
1	39.05	40.625	50	50	41.125	46
2	49.35	51.150	60	60	51.600	57.3
3	50.25	70	90	78.25	53.250	60
4	45.30	61.800	109.8	83.40	49.200	58.8
5	50.25	65.200	95.20	78.35	53.075	60
6	49.20	51.225	60	60	51.750	57.9

Total cost was \$4636.71; corresponding emission was 124.938 kg. Using the proposed approach, the total cost increases to \$4789.43 while the corresponding emission is decreases to 114.876 kg.

Case Study and Results

- Total CPU time for the 6-unit system: 0.86 s
- For the 140-unit system, an increase in the cost about 8% yeilds a decrease about 21% via 2.12 s.
 - The disjoint term has been neglected.
 - The nonconvex term has been neglected as well.

Concluding Remark

- A multi-objective function with conflicting objectives has been solved.
- Using the linearization and strong duality theorem, a single objective problem has been obtained.
- Global solution has been guaranteed.
- Since it is fast enough, it can be applied to online-based problems.

Question?

*Special thanks
to the
Session Audiences*