2017

OPTIMUM ROTOR DESIGN FOR A WIND TURBINE

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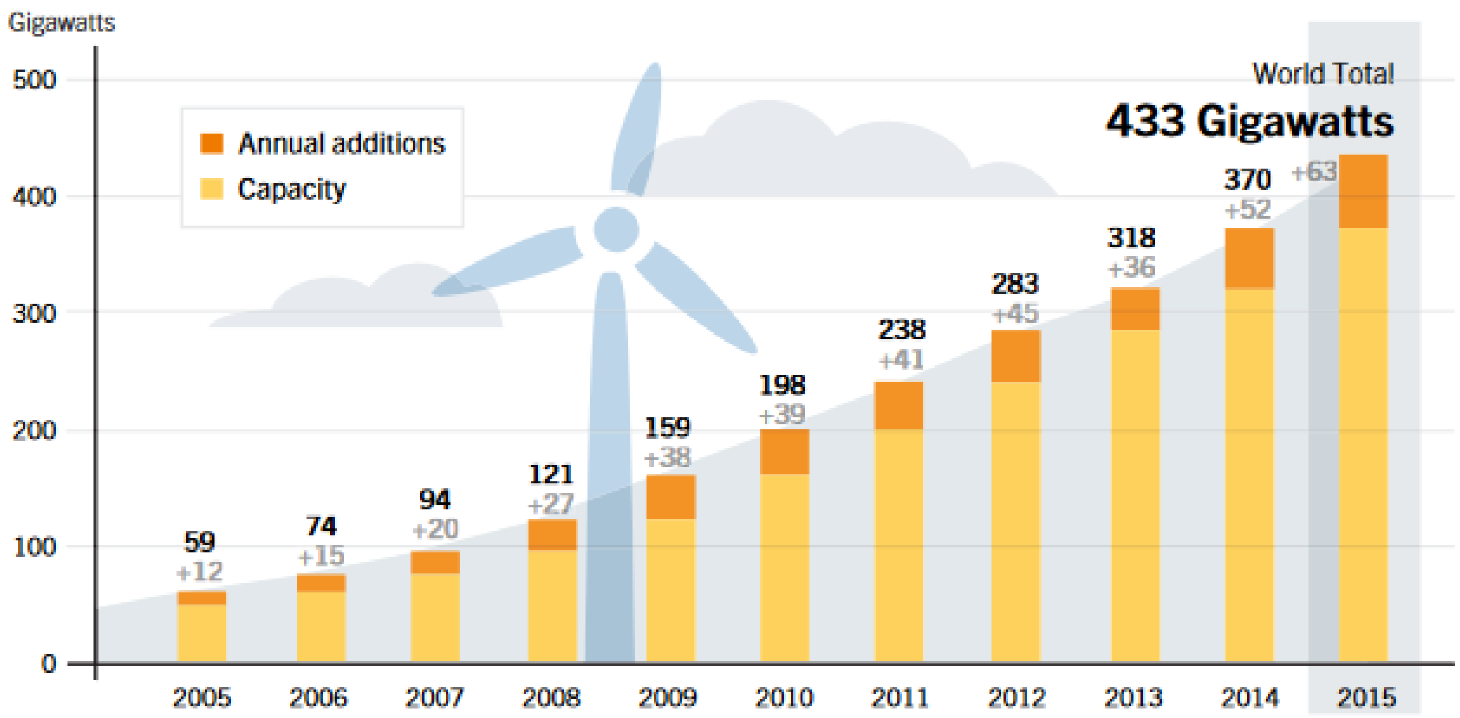
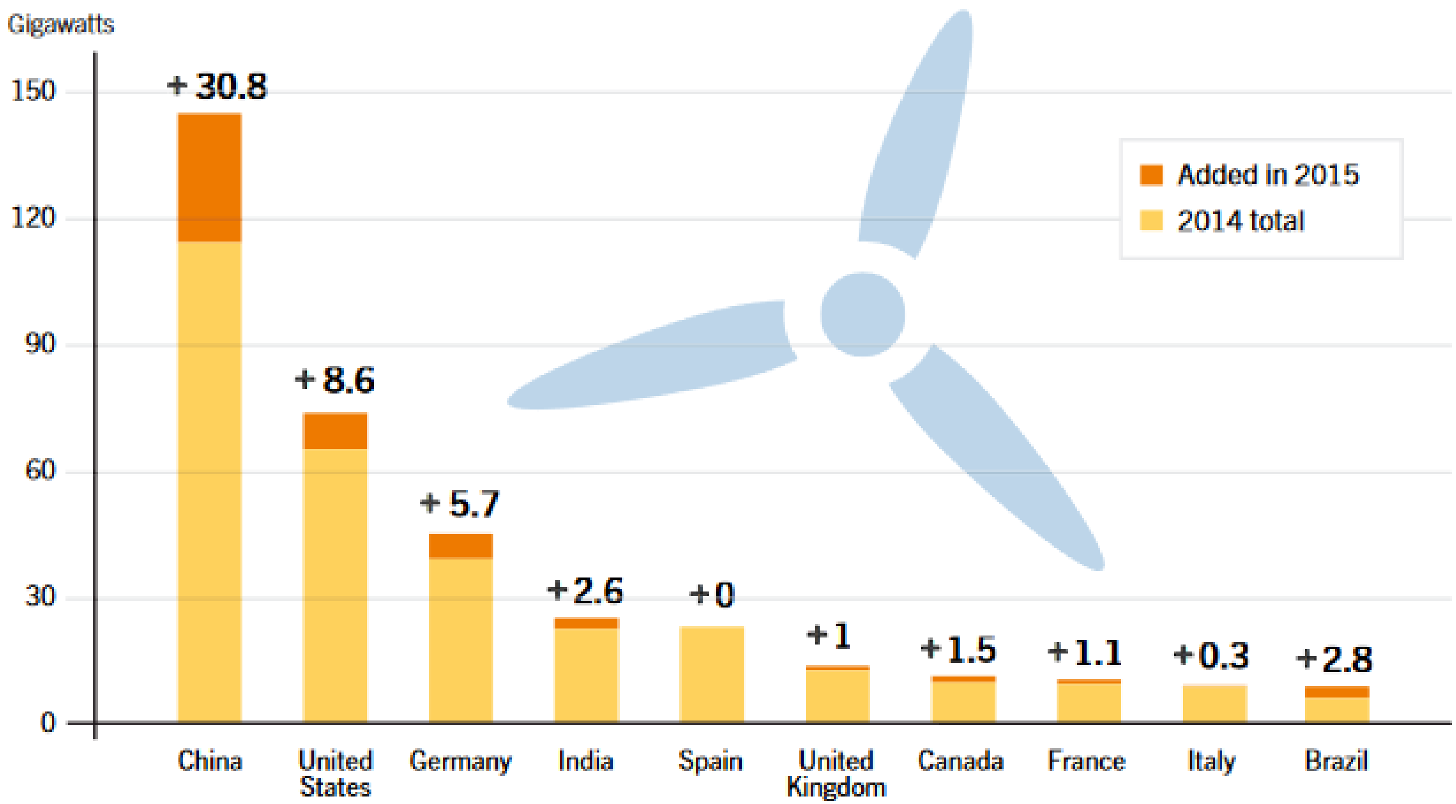
Introduction



Wind power global capacity and annual additions

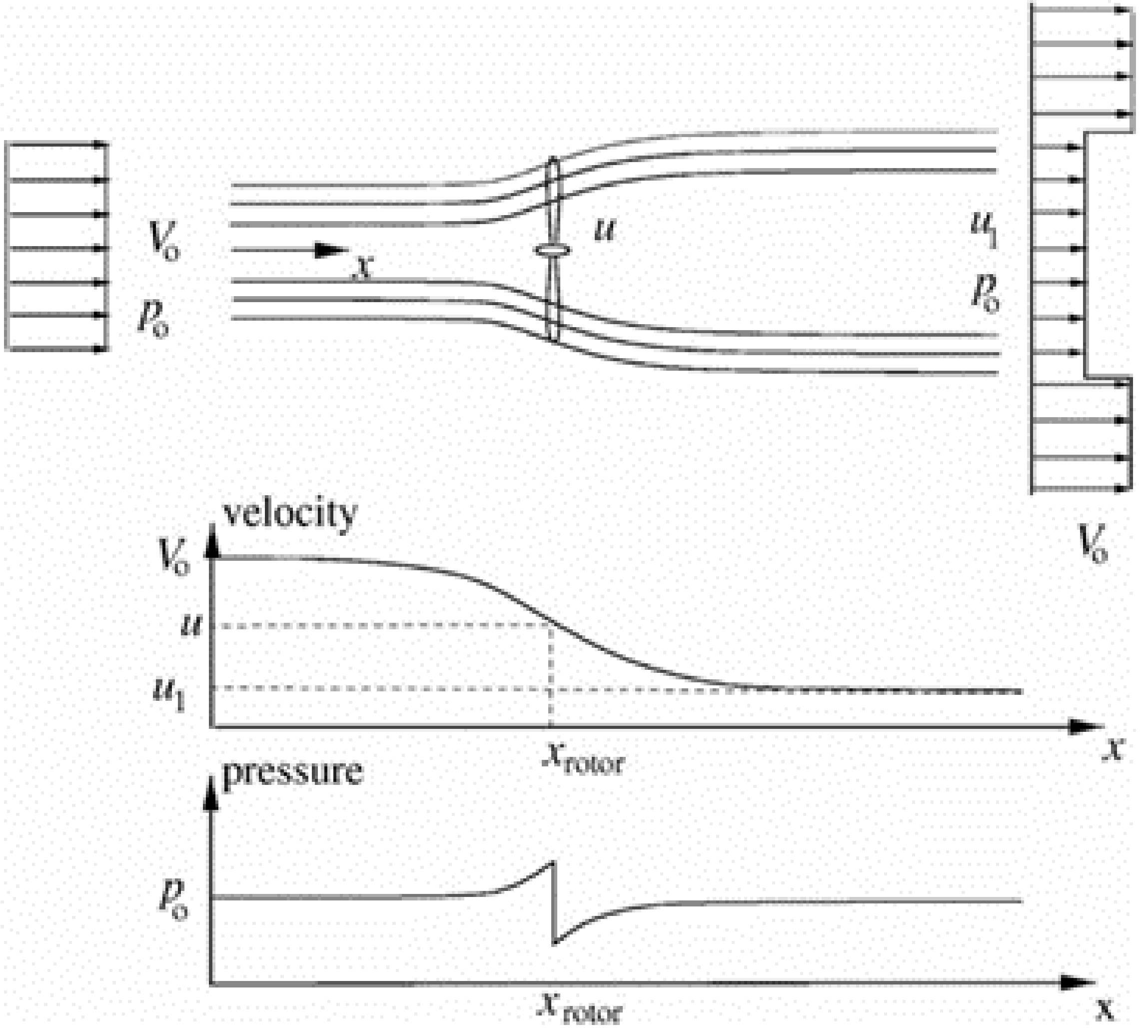


Wind power capacity and annual additions, top ten countries

Method

1D Ideal Rotor

$$P_{theoreticd} = \frac{1}{2}\rho A V_0^3$$



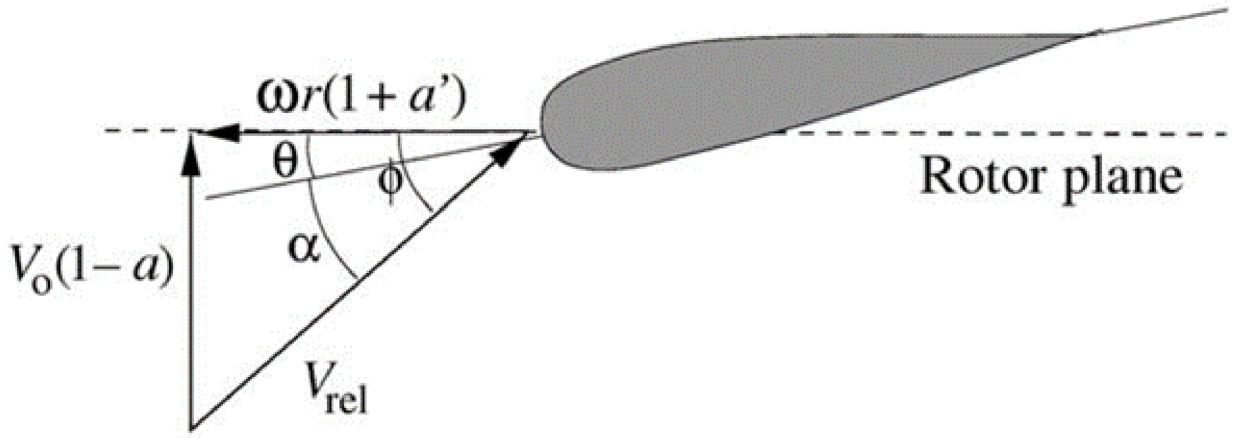
$$u = (1-a)V_0$$

$$u_1 = (1-2a)V_0$$

$$C_p = \frac{P}{\frac{1}{2}\rho A V_0^3} = 4a(1-a)^2$$

$$C_T = \frac{T}{\frac{1}{2}\rho A V_0^2} = 4a(1-a)$$

Effect of rotation



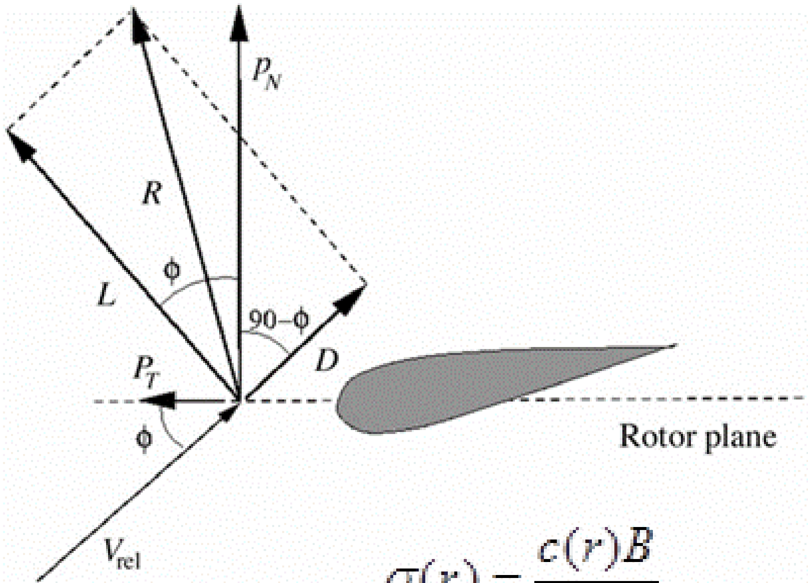
$$\tan\phi = \frac{V_a}{V_{rotor}}$$

$$C_p = \frac{8}{\lambda^2}\int_0^{\lambda} a'(1-a)x^3 dx$$

$$\lambda = \frac{\omega R}{V_0}$$

$$x = \frac{\omega r}{V_0}$$

Blade Element Momentum Theory (BEM)



$$C_n = \frac{p_N}{\frac{1}{2}\rho V_{rel}^2 c}$$

$$C_t = \frac{p_T}{\frac{1}{2}\rho V_{rel}^2 c}$$

$$\sigma(r) = \frac{c(r)B}{2\pi r}$$

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma C_n} + 1}$$

$$a' = \frac{1}{\frac{4\sin\phi\cos\phi}{\sigma C_t} - 1}$$

BEM Algorithm

1- Initialize a and a', at first a=a'=0

2- Compute flow angle, ϕ

3- Compute the local angle attack, α

4- Read off $C_l(\alpha)$ and $C_d(\alpha)$ from table

5- Compute C_n and C_t

6- Calculate new a and a'

7- If a and a' are not in a certain tolerance then go to Step 2 or else finish

8- Compute the local loads on the segment of the blades

Prandtl Tip Loss Factor

$$F = \frac{2}{\pi}\cos^{-1}(e^{-f})$$

$$f = \frac{B}{2}\frac{R-r}{r\sin\phi}$$

$$a = \frac{1}{\frac{4F\sin^2\phi}{\sigma C_n}+1}$$

$$a' = \frac{1}{\frac{4F\sin\phi\cos\phi}{\sigma C_t}-1}$$

Glauert Correction

If the axial induction factor (a) is greater than 0.4 the BEM can not be applied.

$$C_T = \begin{cases} 4a(1-a) \rightarrow a \leq a_c \\ 4a(a_c^2 + (1-2a_c)aF \rightarrow a > a_c \end{cases}$$

a_c=0.2

If a< ac

$$a = \frac{1}{\frac{4F\sin^2\phi}{\sigma C_n} + 1}$$

If a> ac then

$$a = \frac{1}{2}\left[2 + K(1-2a_c) - \sqrt{(K(1-2a_c)+2)^2 + 4(Ka_c^2 - 1)}\right]$$

$$K = \frac{4F\sin^2\phi}{\sigma C_n}$$

Optimization

$$16a^3 - 24a^2 + a(9 - 3x^2) - 1 + x^2 = 0$$

$$\theta_{opt} = \phi - \alpha_{opt}$$

$$\frac{c(x)}{R} = \frac{8\pi a x \sin^2 \phi F}{(1-a) B C_n \lambda}$$

$$C_n = C_{l,opt} \cos\phi + C_{d,opt} \sin\phi$$

Power

$$M_{i,i+1} = \int_{r_i}^{r_{i+1}} r p_T(r) dr$$

$$M_{tot} = B \sum_{1}^{N-1} M_{i,i+1}$$

$$P = M_{tot} \omega$$

Annual Energy Production

$$f_W(V_0) = \frac{k}{A} \left(\frac{V_0}{A} \right)^{k-1} \exp\left[-\left(\frac{V_0}{A} \right)^k \right]$$

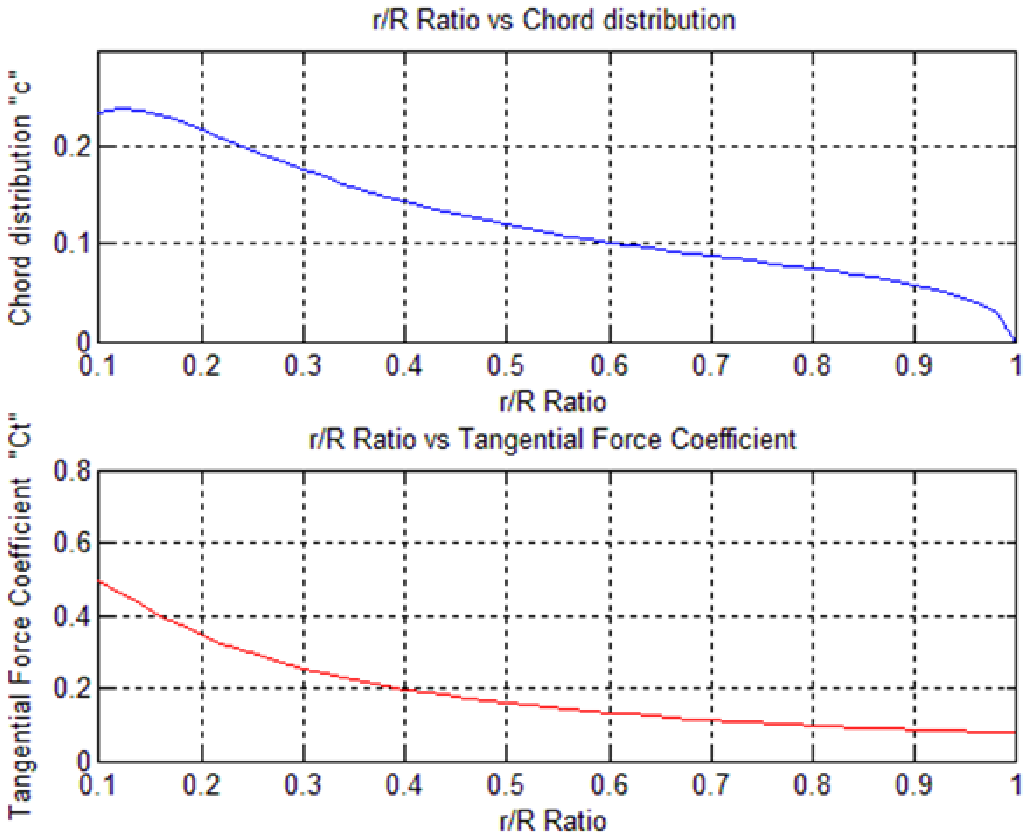
$$f(V_i < V_0 < V_{i+1}) = \exp\left[-\left(\frac{V_i}{A} \right)^k \right] - \exp\left[-\left(\frac{V_{i+1}}{A} \right)^k \right]$$

$$AEP = \sum_{1}^{N-1} \frac{1}{2} (P(V_{i+1}) + P(V_i)) f(V_i < V_0 < V_{i+1}) 8760$$

Rotor Specifications

Number of blades (B) = 3
Rotational speed (ω) = 24.9 rad/s
Wind speed (V_0) = 13.5 m/s
Radius of the rotor (R) = 31 m
Lift coefficient (C_l) = 0.8
Drag coefficient (C_d) = 0.012
Radius ratio (r/R) = 0.1-1
Airfoil type = NACA 4412

Results and Discussion



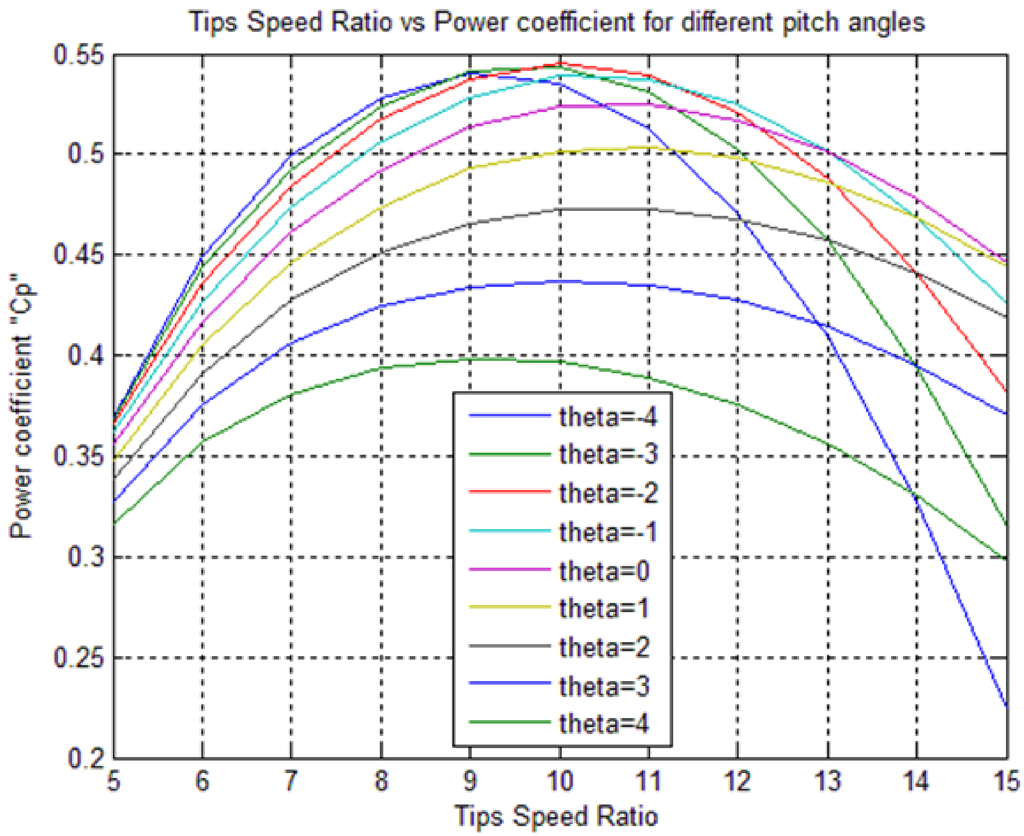
Validity of BEM code

XFOIL results

Iteration	a	a’	α	ϕ	C_n	C_t
1	0.10045	0.00615	11.20156	13.20156	0.9738	0.2274
2	0.11963	0.00598	9.8446	11.8446	0.97891	0.20428
3	0.12367	0.00595	9.60091	11.60091	0.97977	0.20011
4	0.12456	0.00595	9.54936	11.54936	0.97995	0.19923
5	0.12475	0.00594	9.5381	11.5381	0.97999	0.19904
6	0.12479	0.00594	9.53563	11.53563	0.98	0.199
7	0.1248	0.00594	9.53508	11.53508	0.98	0.19899
8	0.1248	0.00594	9.53496	11.53496	0.98	0.19899

BEM code results

Iteration	a	a’	a	f	C_n	C_t
1	0.100447	0.006154	11.20156	13.20156	0.973801	0.227404
2	0.119626	0.005982	9.844601	11.8446	0.978913	0.204279
3	0.123672	0.005952	9.600909	11.60091	0.979773	0.200114
4	0.124555	0.005946	9.54936	11.54936	0.979953	0.199232
5	0.124749	0.005945	9.538104	11.5381	0.979992	0.19904
6	0.124792	0.005944	9.535629	11.53563	0.980001	0.198997
7	0.124802	0.005944	9.535084	11.53508	0.980002	0.198988
8	0.124804	0.005944	9.534964	11.53496	0.980003	0.198986

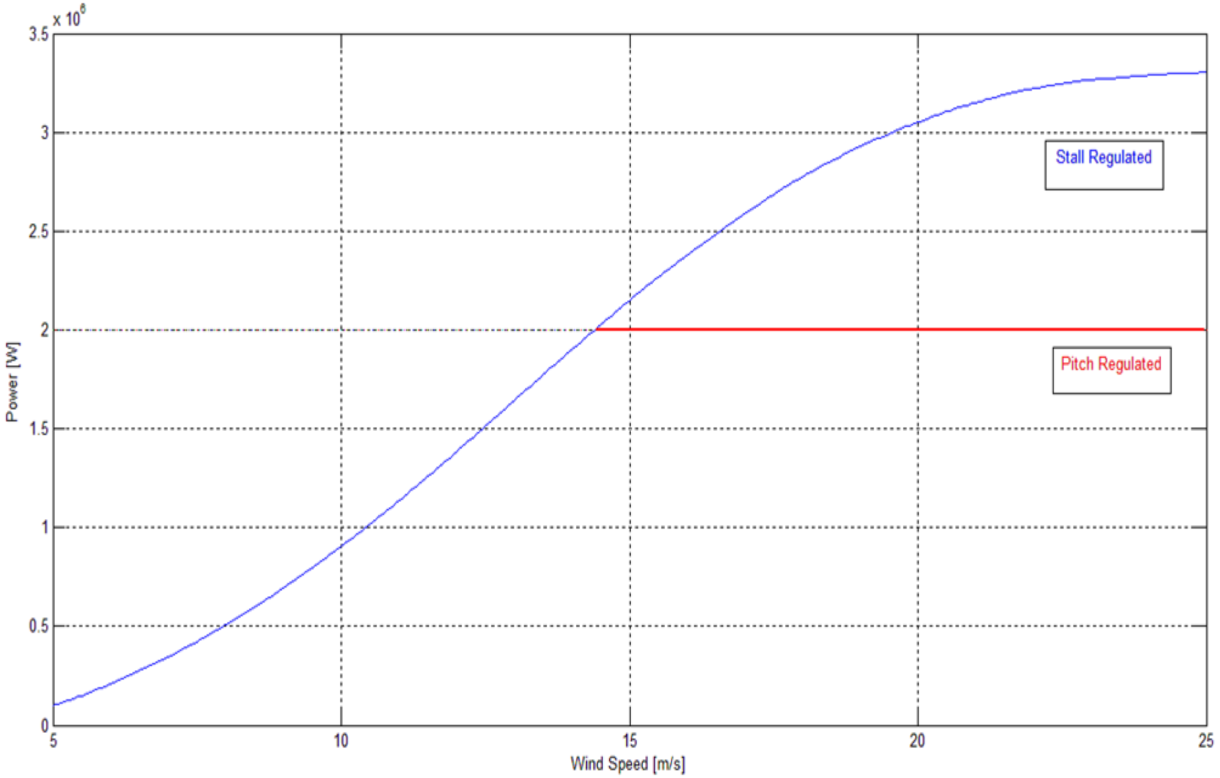

Tips Speed Ratio vs Power coefficient for different pitch angles
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
Power coefficient "Cp"
5
6
7
8
9
10
11
12
13
14
15
Tips Speed Ratio
theta=-4
theta=-3
theta=-2
theta=-1
theta=0
theta=1
theta=2
theta=3
theta=4

AEP

Weibull shape and scale parameters are taken as 1.9 and 7 m/s

AEP = 3.0161 Gwh

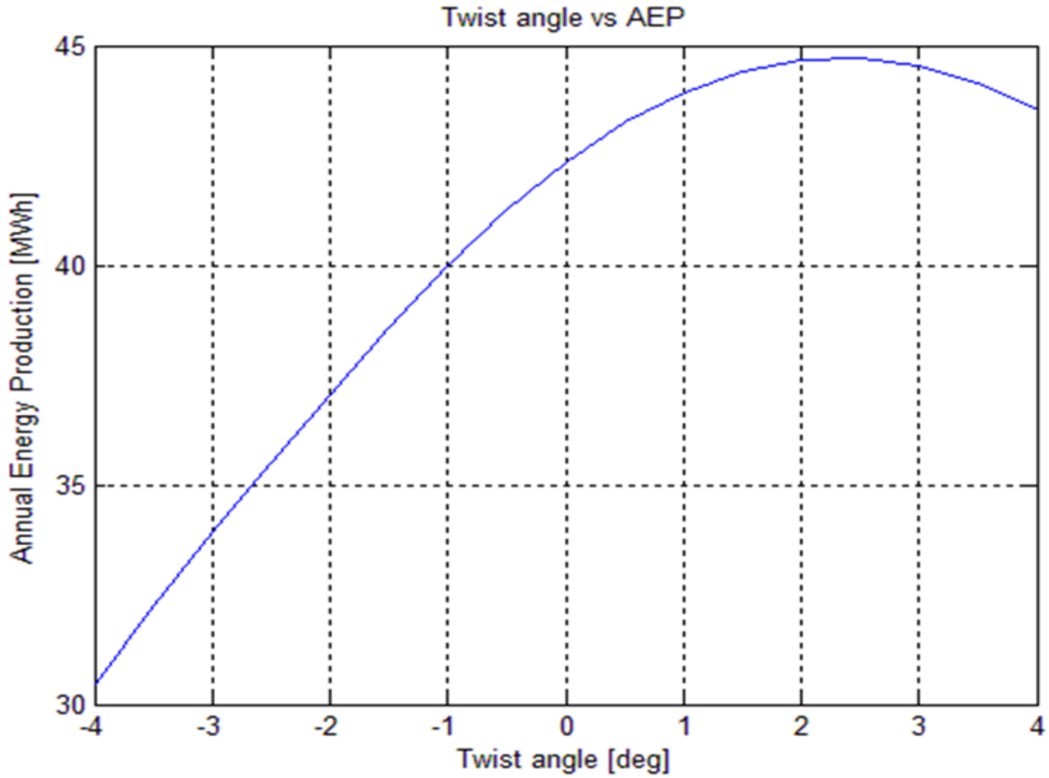
Nominal Power



Optimum Twist Angle

To find the optimum twist angle (β_{opt}) at r=24.5m for a specific wind speed range of 5-9 m/s the following equation is used.

$$\frac{dP}{dr}(\beta) = \omega r p_T$$



Conclusion

BEM theory is applied by using Prandtl Tip Loss Factor and Glauert Correction

BEM Code is validated

Optimum power coefficient $C_{p,\,opt} = 0.5455$

Optimum angle of attack $\theta_{opt} = -2°$

Optimum tip speed ratio $\lambda_{opt} = 10$

AEP = 3.0161 GWh

Nominal power for stall regulation 3.3 MW

Nominal power for pitch regulation 2 MW

Optimum twist angle at r=24.5 m for wind speed range of 5-9 m/s $\beta_{opt} = 2.5°$

Thank you for listening...

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