Answers from above examples

Q1) Prove the identity $\tan^4 x = \sec^2 x\,(\tan^2 x - 1) + 1$

$$RHS = \frac{1}{\cos^2 x}\left(\frac{\sin^2 x - \cos^2 x}{\cos^2 x}\right) + 1$$

$$= \frac{\sin^2 x - \cos^2 x}{\cos^4 x} + \frac{\cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^2 x - \cos^2 x + \cos^4 x}{\cos^4 x}$$

$$= \frac{\sin^2 x - \cos^2 x + (1 - \sin^2 x)^2}{\cos^4 x}$$

$$= \frac{\sin^2 x - \cos^2 x + 1 + \sin^4 x - 2\sin^2 x}{\cos^4 x}$$

$$= \frac{\sin^4 x - \sin^2 x - \cos^2 x + 1}{\cos^4 x}$$

$$= \frac{\sin^4 x - 1 + 1}{\cos^4 x}$$

$$= \tan^4 x = LHS\ (Shown)$$

Q2) Show that $\sec A\left(\frac{1}{\cot A} + \cot A\right) = \frac{1}{\sin A - \sin^3 A}$

$$LHS = \sec A\left(\frac{1}{\cot A} + \cot A\right)$$

$$= \frac{1}{\cos A}\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{1}{\cos A}\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

$$= \frac{1}{\cos^2 A \sin A}$$

$$= \frac{1}{(1 - \sin^2 A)\sin A}$$

$$= \frac{1}{\sin A - \sin^3 A} = RHS\ (Shown)$$

Q3) Prove the identity $1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$

$$LHS = 1 - \frac{\sin^2 x}{1 + \cos x}$$

$$= \frac{1 + \cos x}{1 + \cos x} - \frac{\sin^2 x}{1 + \cos x}$$

$$= \frac{1 + \cos x - \sin^2 x}{1 + \cos x}$$

$$= \frac{1 + \cos x - (1 - \cos^2 x)}{1 + \cos x}$$

$= \frac{\cos x + \cos^2 x}{1 + \cos x}$

$= \frac{\cos x \cancel{(1 + \cos x)}}{\cancel{1 + \cos x}}$

$= \cos x = RHS\ (Proved)$

Q9) $LHS = (1 + \sin\theta\cos\theta)(\sin\theta - \cos\theta)$

$= \sin\theta + \sin^2\theta\cos\theta - \cos\theta - \sin\theta\cos^2\theta$

$= \sin\theta + (1 - \cos^2\theta)\cos\theta - \cos\theta - \sin\theta\,(1 - \sin^2\theta)$

$= \sin\theta + \cos\theta - \cos^3\theta - \cos\theta - \sin\theta + \sin^3\theta$

$= \sin^3\theta - \cos^3\theta = RHS$