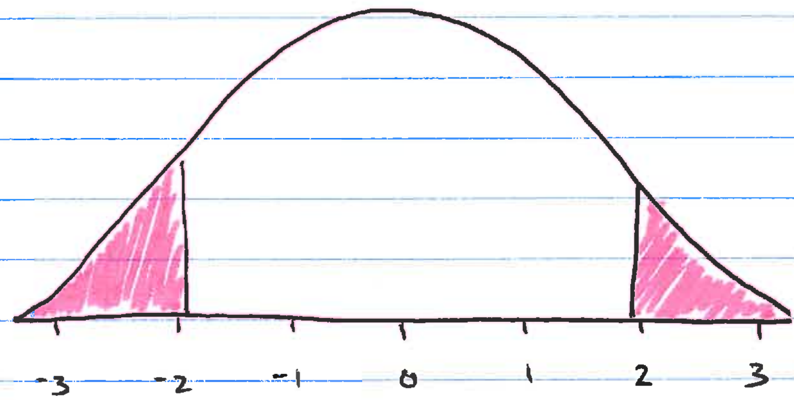
REJECT OR FAIL TO REJECT?

As previously mentioned, we have 3 methods to use to determine R or FTR, if the p-value $< \alpha$, if $Z_{cal} > Z_{\alpha}$ (or $Z_{\alpha/2}$), or if the mean $\bar{X}$ is outside a $100(1-\alpha)\%$ CI then we R. Let's review this keeping in mind that we are assuming $H_0: \mu = \mu_0$ is TRUE.

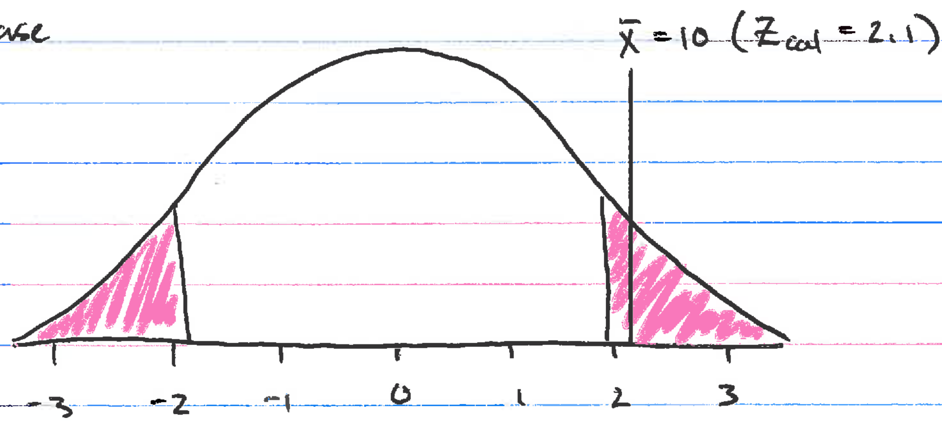
Let's assume $\alpha = 0.05$ and $H_0: \mu = \mu_0$ vs. $H_a: \mu \neq \mu_0$ (a two-tailed) therefore our RR is based on ± 1.96



(1)

so everything in pink is the RR, recall the previous statements that if $Z_{cal} < -Z_{\alpha/2}$ or $Z_{cal} > Z_{\alpha/2}$ from the text we reject (my method is if $|Z_{cal}| > |Z_{\alpha/2}|$ we reject produces the same results with less worry about, "do I need to use the $\oplus$ or $\ominus$ value?")

Thus, if we fall inside the RR we reject the null hypothesis and let's say that for some $\bar{X} = 10$ the Z-value is $Z_{cal} = 2.1$, then in this case



(2)

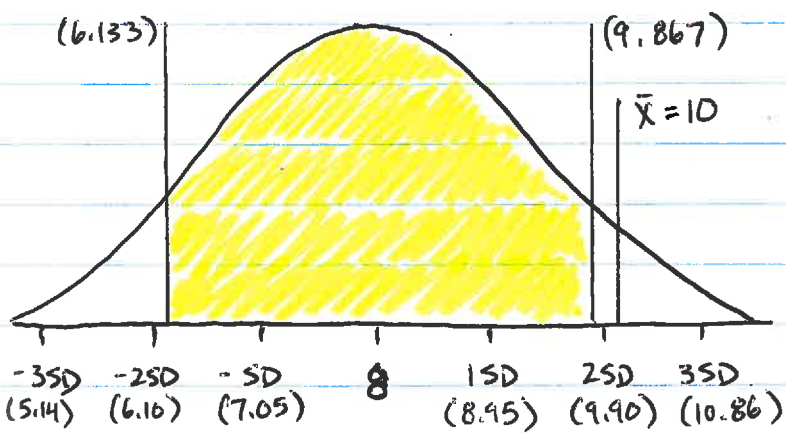
and therefore we'd (R) the null hypothesis. Now, let's say that we're testing

$$H_0: \mu = 8 \text{ vs. } H_a: \mu \neq 8, \; n = 100, \; \sigma = 9.524$$

so if we were to create a 95% confidence interval, we need to remember we still assume $H_0: \mu = 8$ is true, therefore the CI is centred on 8, so

$$\bar{X} \pm Z_{\alpha/2} \cdot \sigma_{\bar{X}}$$
$$8 \pm 1.96 \cdot 0.9524$$
$$8 \pm 1.8667$$
$$(6.133, 9.867)$$

so if we drawed that out we'd get something similar to above



(3)

but here the "ticks" are based on 0.9524 instead of 1 (remember, for a standard normal $\sigma = 1$). So everything in yellow is part of the 95% CI and the value $\bar{X} = 10$ falls outside that CI.

More importantly, this figure and the last are the same thing, just centred differently w/ a different SD, BUT the boundaries that divide pink and those that divide yellow represent the same region, THUS anything outside the 95% confidence interval is inside the RR based on $\alpha = 0.05$, which we had to split between the two tails, hence each has $\alpha/2 = 0.025$ in them (i.e. 2.5% of the data)

Finally, the relation between the p-value, which for a two-sided test is $2 \cdot P(Z > 2.1)$, and the RR. Notice that if $|Z_{cal}| > |Z_{\alpha/2}|$, i.e. $2.1 > 1.96$, we reject, but if p-value $= 2 \cdot P(Z > 2.1) < \alpha$ we also reject, so the direction of the inequality can throw people off. To better understand this let's complete the p-value calculation

$$
\begin{aligned}
\text{p-value} &= 2 \cdot P(Z > |Z_{cal}|) \\
&= 2 \cdot P(Z > 2.1) \\
&= 2(1 - P(Z \leq 2.1)) \\
&= 2(1 - 0.9821) = 2 \cdot 0.0179 \\
&= 0.0358 < 0.05 = \alpha \Rightarrow R
\end{aligned}
$$

So, as we established, the yellow portion of figure 3 is our 95% CI, which is technically: we expect the true value for μ to fall within the CI's (if we repeat this over and over again) 95 out of 100 repeat samples. However, we can also think of it as where we expect 95% of the values to lie, hence we rejected the null because $\bar{X} = 10$ wasn't inside that interval (remember, we either (1) got a very rare sample or (2) the mean isn't centred at 8). Thus, if inside yellow is 95%, then outside must split 5% evenly into 2.5%.

Recall that the p-value is the prob. of getting a more extreme value than what we calculated, so for one side of the two-tail, the prob. of getting more than 2.1 is 0.0179. Since the $\alpha/2 = 2.5\%$ is the pink region, we want to move/be to the right of it (when positive) or left of it (when negative) to be in the RR (that line is again 1.96, which we got from $Z_{\alpha/2}$). Thus, we want our p-value to move further into the RR, hence the p-value $< \alpha/2$.

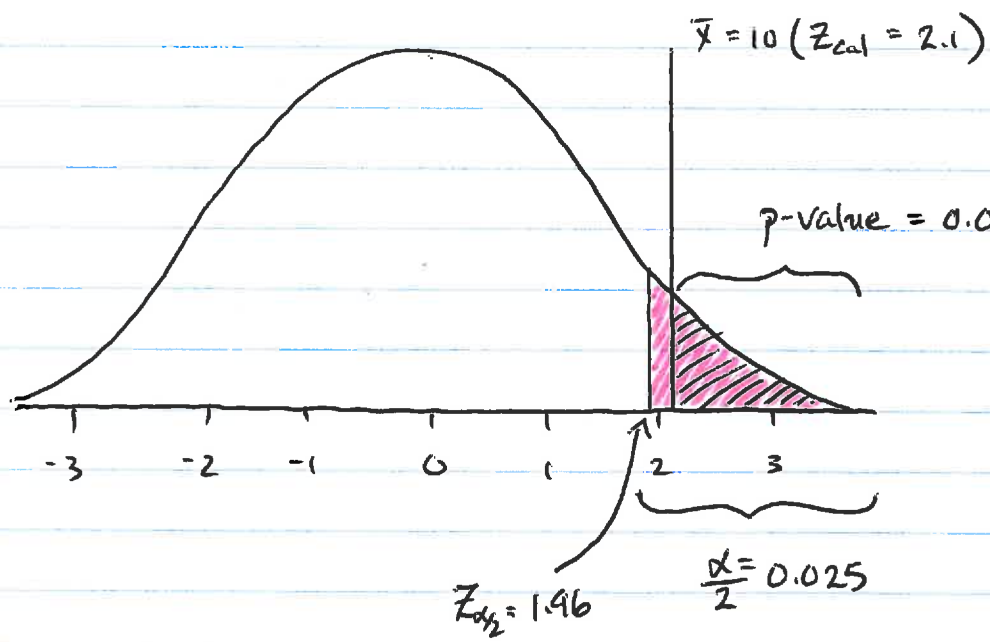
To put numbers to it, we set $\alpha = 0.05$, thus $\alpha/2 = 0.025 = 2.5\%$, the p-value

$$
\begin{aligned}
& 2 \cdot P(Z > 2.1) \text{ vs. } \alpha = 0.05 \\
= \ & 2 \cdot (0.0179) \text{ vs. } \alpha = 0.05 \\
= \ & 0.0358 < \alpha = 0.05 \Rightarrow \text{R}
\end{aligned}
$$

This is the same as

$$
\begin{aligned}
= \ & 2(0.0179) \text{ vs. } \alpha = 0.05 \\
= \ & 0.0179 < \alpha/2 = 0.05/2 = 0.025
\end{aligned}
$$

and so $\alpha/2 = 0.025$, which is the right pink region starts at $Z = 1.96$, then the p-value is the blacken region in figure 4



(4)

so this is why p-value $< \alpha/2$ is the same as $Z\text{-cal} > Z_{\alpha/2}$ because, in some sense, the Z-values move to the right and increase while p and α (which are prob's) move to the right and decrease.