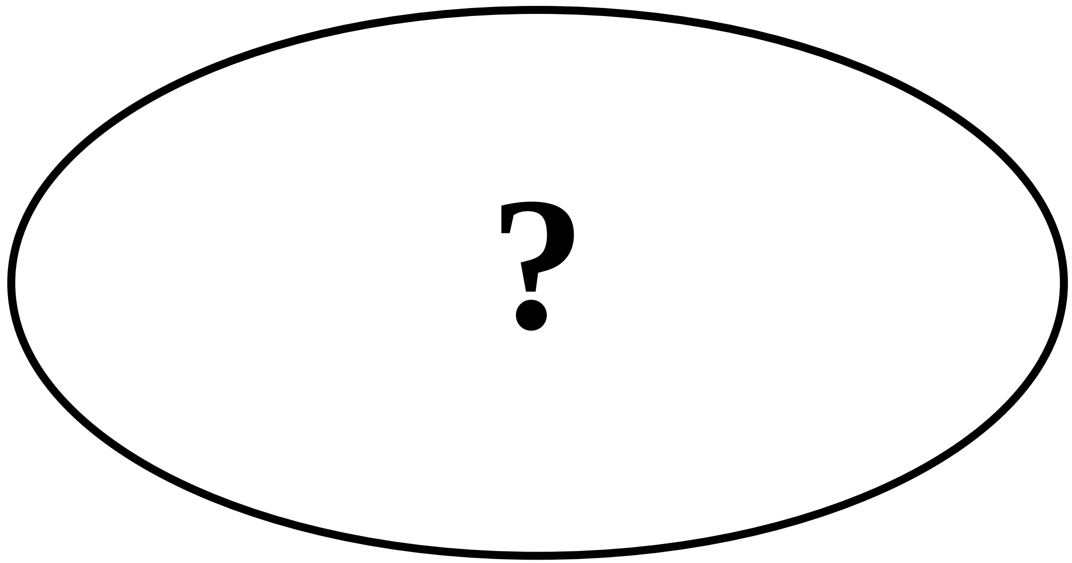
# How to calculate the circumference of an ellipse

*(by Elyes Hachicha)*

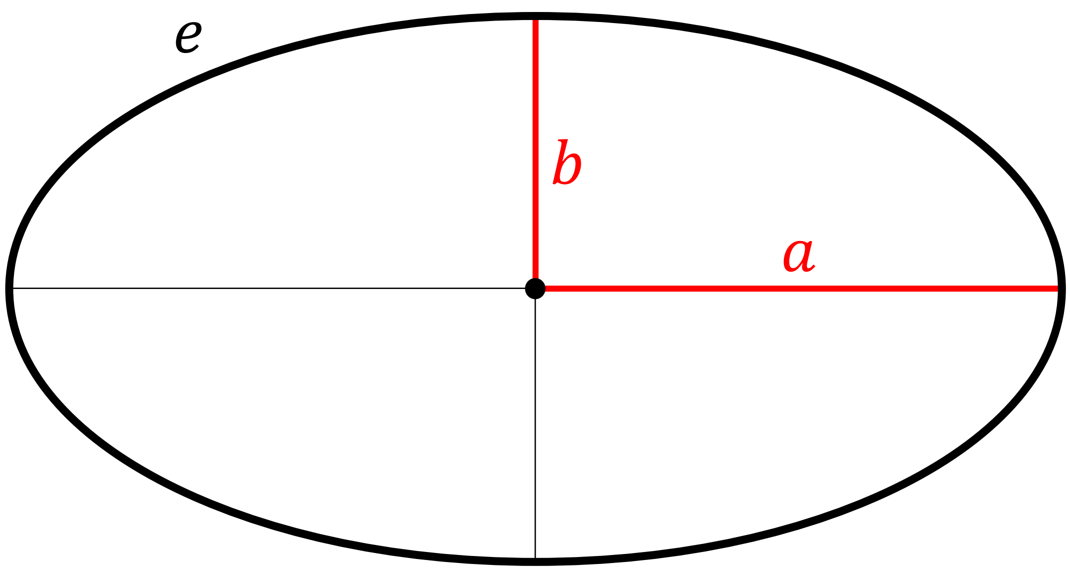



**I will propose and explain a formula to calculate the circumference of an ellipse.**

## Proposed formula:

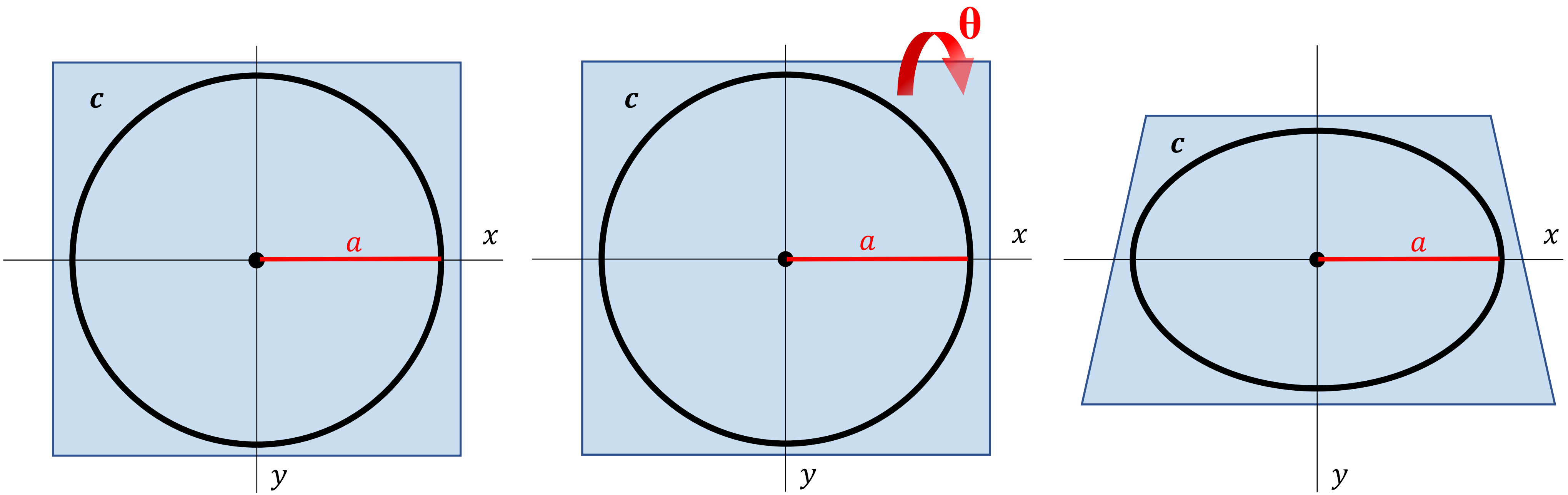
The circumference $C$ of an ellipse $e$ with a semi-major axis $a$ and a semi-minor axis $b$ is,

$$\boxed{C = 4\left(\frac{\pi}{2}\right)^{\frac{b}{a}} a}$$



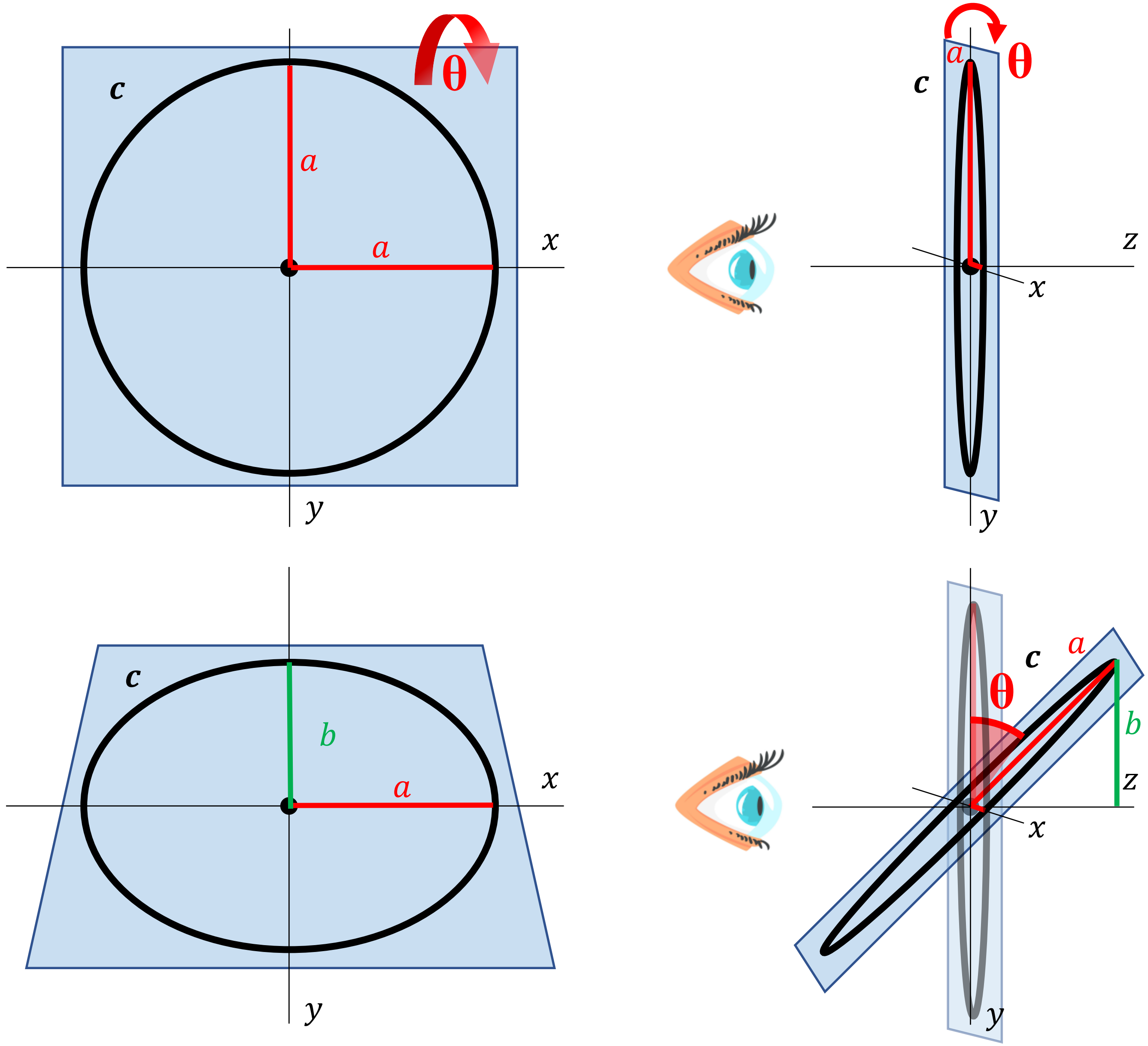
# Demonstration and explanation

If we draw a circle on a *y* plane, with the *x* axis crossing its center, and we observe from the *z* axis passing through the center of the circle, we obtain an ellipse when rotating the *y* plane around the *x* axis by an angle θ.




We can then say, that calculating the circumference of an ellipse, is calculating the observed circumference of a circle from a different angle.

After rotating the $y$ plane around the $x$ axis by the angle θ, observed from the $z$ axis passing through the center of the circle $c$, the circle $c$ with radius $a$ becomes an ellipse with a semi major-axis $a$ and a semi-minor axis $b$, where $b = a \cdot \cos\theta$

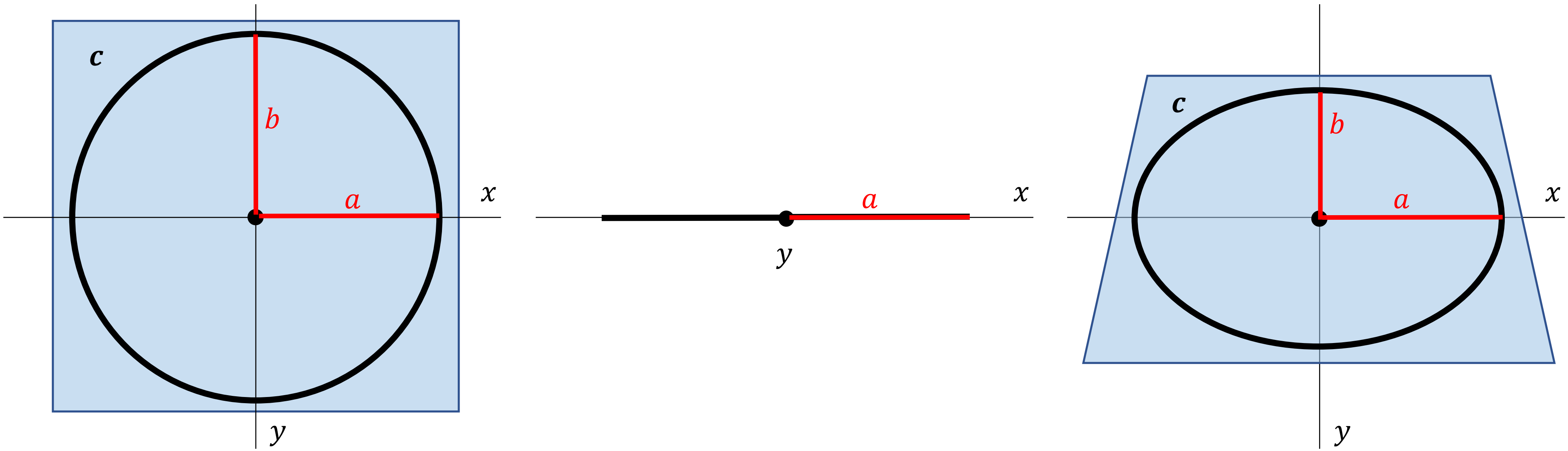


Then we can say that the circumference $C$ of the ellipse $e$ depends on the rotation of the circle $c$ by the angle θ of the $y$ plane around the $x$ axis, where the $x$ axis crosses the center of that circle.

If $\theta = 0°$,
then $b = a$,
and $C = 2\pi a$

If $\theta = 90°$,
then $b = 0$,
and $C = 4a$

If $0° < \theta < 90°$,
then $a > b > 0$,
and $2\pi a > C > 4a$




So, based on the value of θ where $\theta \in [0°; 90°]$, the circumference $C$ of the ellipse $e$ is comprised between:

$$\boldsymbol{C \in [4a; 2\pi a]}$$

where,

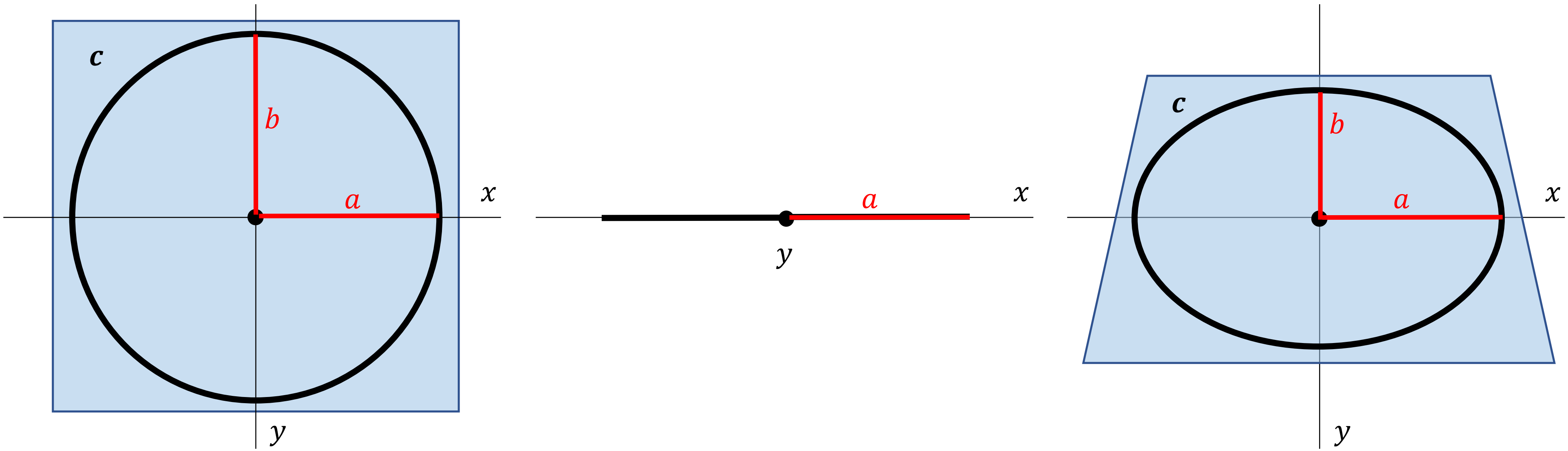
$$\boldsymbol{a > 0 \text{ and } b \in [0;\ a],}$$

If we try to propose a common equation to the previous results, we can re-write their equations as follow:

If $\theta = 0°$,
then $b = a$,
and $C = 4\left(\frac{\pi}{2}\right)a$

If $\theta = 90°$,
then $b = 0$,
and $C = 4(1)a$

If $0° < \theta < 90°$,
then $a > b > 0$,
and $C = 4(n)a$




So, based on the value of θ where θ ∈ [0°; 90°], the circumference *C* of the ellipse *e* is :

$$\boldsymbol{C = 4 \cdot a \cdot n}$$

where:

$a > 0$ and $n = \left(\frac{\pi}{2}\right)$ when $\theta = 0°$, $n = 1$ when $\theta = 90°$, and $1 < n < \left(\frac{\pi}{2}\right)$ when $0° < \theta < 90°$

We can use the exponential function to keep $n \in \left[1; \frac{\pi}{2}\right]$

We can then suggest that:

$$\boldsymbol{C = 4 \cdot a \cdot \left(\frac{\pi}{2}\right)^{p}}$$

where:

$$\boldsymbol{p \in [0; 1]}$$

Given that when,

- $\theta = 0°$, $n = \left(\frac{\pi}{2}\right)$, so $p = 1$
- $\theta = 90°$, $n = 1$, so $p = 0$
- $0° < \theta < 90°$, $1 < n < \left(\frac{\pi}{2}\right)$, so $0 < p < 1$

We can then suggest that:

$$\boldsymbol{p = \cos\theta}$$

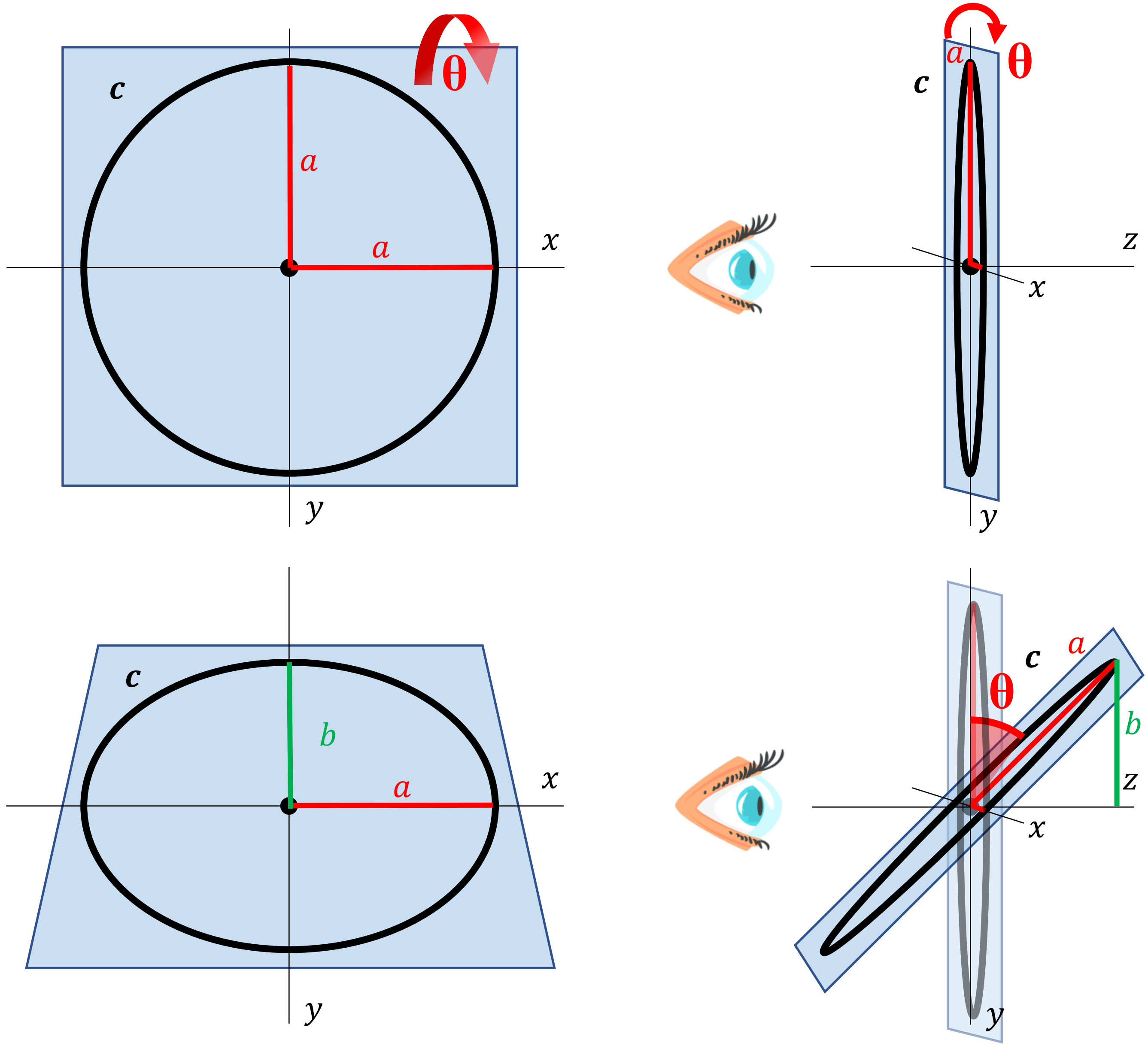
Which gives us:

$$\boldsymbol{C = 4 \cdot a \cdot \left(\frac{\pi}{2}\right)^{\cos\theta}}$$

where,

$$\boldsymbol{a > 0 \text{ and } \theta \in [0°; 90°],}$$

Remember that after rotating the *y* plane around the *x* axis by the angle θ, observed from the *z* axis passing through the center of the circle *c*, the circle *c* with radius *a* becomes an ellipse with a semi major-axis *a* and a semi-minor axis *b*, where $b = a \cdot \cos\theta$



So,

$$\cos\theta = \frac{b}{a}$$

We can then re-write our equation as:

$$C = 4 \cdot a \cdot \left(\frac{\pi}{2}\right)^{\frac{b}{a}}$$

To keep our equation resembling to the conventional formula of the circle circumference, we can re-arrange it, to obtain the following formula to calculate the circumference of an ellipse:

$$\boxed{C = 4\left(\frac{\pi}{2}\right)^{\frac{b}{a}} a}$$

where,
$C$ is the circumference of the ellipse,
$a$ is the semi-major axis of the ellipse,
$b$ is the semi-minor axis of the ellipse.

# Remarks

**Uncertainty:**
I don't really know, if the suggested formula, is an approximation or an exact result, given that I don't know if my demonstration proves anything, and I don't have the required tool to calculate practically any ellipse for any given semi-major axis and semi-minor axis to contradict my demonstration. However I know that the result is correct for an ellipse where $b=a$ or $b=0$.
Also, I don't know because I always compare my results with the already existing approximate formulas, even if I know that there are only approximations (like the Ramanujan's approximation, or the Infinite Series) and they are wrong when $b=0$ because they don't show $C=4a$.

**Alternative formulas:**
When I was trying to re-arrange my formula to get my results closer to the already existing approximate formulas, and I obtained the following formula, however, I have no explanation to this alternative formula, plus it doesn't work when $b=a$.

$$C \approx 4\left(\frac{\pi}{2}\right)^{\left(\frac{b}{a}\right)^2} a - 3\mathrm{b} \quad \text{or} \quad C \approx 4\left(\frac{\pi}{2}\right)^{\left(\frac{b}{a}\right)^2} a - \pi\mathrm{b}$$

where,
$C$ is the circumference of the ellipse,
$a$ is the semi-major axis of the ellipse,
$b$ is the semi-minor axis of the ellipse.
However those alternative formulas have no meaning to me…

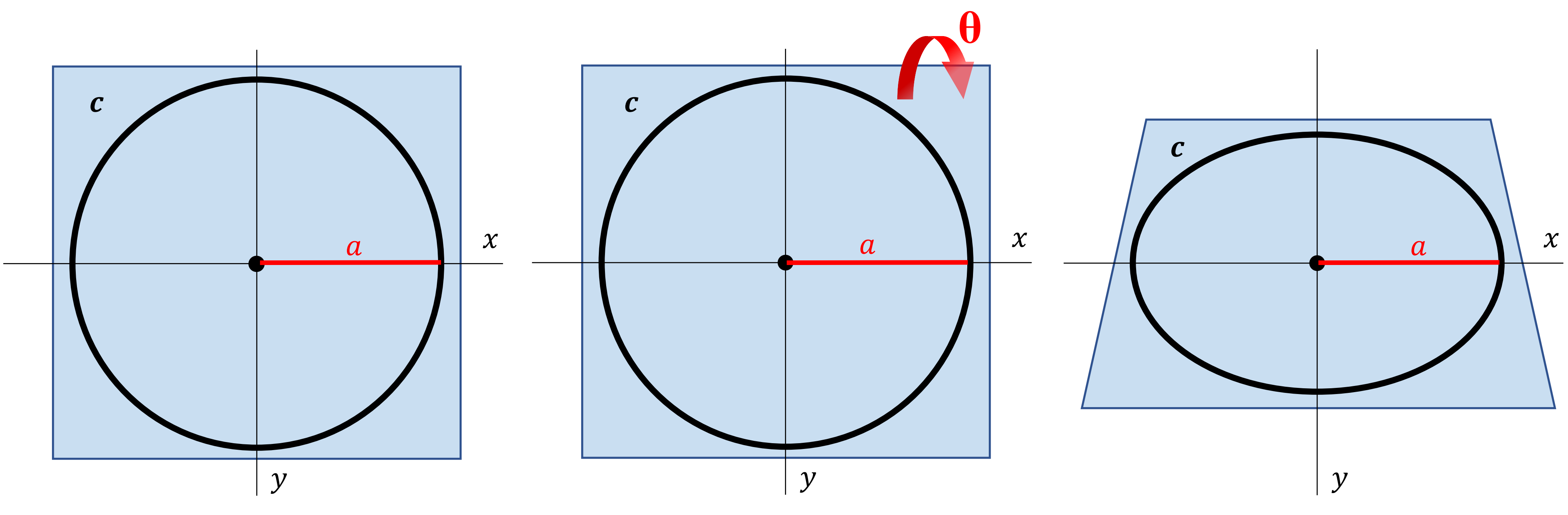
# Other application – The observable circumference of a circle

(Only if my proposed formula $\left(C = 4\left(\frac{\pi}{2}\right)^{\frac{b}{a}} a\right)$ is exact and not an approximation)

If we draw a circle *c* with a radius *r* on the *y* plane, and rotate the *y* plane by an angle θ around the *x* axis, the observable circumference *C* of the circle *c* observed from the *z* axis passing through its center is:

$$C = 4\left(\frac{\pi}{2}\right)^{|\cos\theta|} r$$

where, θ is the angle in degrees.



**Thank you for reading**

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