QUANTUM GRAVITY FROM NONNOETHERIAN SPACETIME

CHARLIE BEIL

Abstract. Aristotle proposed that time passes if and only if something changes. We investigate the consequences of incorporating Aristotle's notion of time into general relativity. We find that the resulting spacetime possesses many quantum-like properties.

Contents

1. Introduction: the indistinguishable principle and spacetime geometry

The identity of indiscernibles is the assumption, due to Leibniz, that distinct states are distinguishable; equivalently, if two states are indistinguishable, then they are the same state. We will call the identity of indiscernibles, together with its converse, the *indistinguishable principle*. In this article, we propose that this principle is the foundation for both quantum theory and the geometry of spacetime.

The indistinguishable principle is fundamental to quantum statistics. For example, consider two indistinguishable particles a and b, each contained in one of two boxes A and B. Classically, there are four possible states:

(i) $a, b \in A$;
(ii) $a, b \in B$;

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(iii) $a \in A$ and $b \in B$; or
(iv) $a \in B$ and $b \in A$.

However, states (iii) and (iv) are indistinguishable since the particles a and b are indistinguishable, and they are thus identified as the same state in quantum theory. In particular, in quantum statistics there are three distinct states, whereas in classical statistics there are four.

We claim that the indistinguishable principle is also an underlying assumption to both special and general relativity. Indeed, both Galilean and special relativity are based on the principle of relativity. The principle of relativity asserts that there is no physical frame that is at absolute rest: Galileo proposed that, given two inertial frames in relative motion, there is no experiment that can be done to determine which of the two frames is at rest. Each inertial frame is indistinguishable from being at rest, and therefore each inertial frame *is* at rest by the indistinguishable principle. Consequently, there is no single universal rest frame.

The indistinguishable principle also implies the equivalence principle, which lies at the heart of general relativity. The equivalence principle asserts that an accelerated frame is locally indistinguishable from a gravitational field. Because acceleration and gravitation are locally indistinguishable, the indistinguishable principle implies that they are locally the same. This identification led Einstein to propose that gravitation is the curvature of spacetime.

In this article, we propose that further applications of the indistinguishable principle should be incorporated into general relativity. These new applications result in a spacetime that share many of the features of quantum theory.

To begin, in Section 2 we propose that a state is a superposition of all eigenstates that are mutually indistinguishable to all the constituents of the universe. Thus, by the indistinguishable principle, superposition does not fundamentally exist.

The trade-off, however, is that spacetime is not a pseudo-Riemannian manifold. Instead, the geometry of spacetime is 'nonnoetherian'. To obtain this spacetime, we incorporate Aristotle's notion of time into general relativity. In the fourth century BCE, Aristotle proposed that time passes if and only if something changes. As is well known, Aristotle's notion of time is a direct consequence of the indistinguishable principle. We propose that it is this notion of time that gives rise to quantum nonlocality, both in regards to entanglement and wave-particle complementarity.

We present our new framework of spacetime in Section 3, which is a development of [B6]. In particular, we define a fundamental particle to be a timelike or null curve along which time does not flow. This curve is then identified as a single point of spacetime by the indistinguishable principle; we call such a 1-dimensional point a 'strand'. In an inertial frame, a strand appears as a 'strand particle'. (The worldline of a strand particle consists of a continuum of distinct points, whereas the strand itself is a single point.)

Since time does not flow along a strand, strand particles cannot undergo intrinsic change, and therefore do not admit fundamental interactions. However, strands may form bound states, and these bound state interact with each other by exchanging strands in specific ways. In Section 4, we present such a model of leptons, quarks, and gauge bosons. Our model generalizes 't Hooft's double line formalism of quarks and gluons ['tH]. A feature of our model is a new symmetry between leptons and quarks. Our model also predicts the existence of neutral and charged massive gluons, and is therefore falsifiable.

In Section 5, we present a thought experiment showing how the indistinguishable principle implies that a classical point particle propagates, at an emergent scale, as a spherical wave. In Sections 6 and 7, we introduce new spacetime formulations of the quantum-to-classical transition and state reduction, respectively, based on the indistinguishable principle. We consider, in particular, what happens when a photon encounters a beam splitter in terms of explicit Feynman-like strand diagrams.

Finally, in Section 8, we describe a modification of Einstein's equation that is sufficient for the consistency of our model. Specifically, it is not energy itself that curves spacetime, but rather the creation and annihilation of on-shell strands. This modification preserves the classicality of general relativity, and introduces new constraints in certain scattering amplitudes of quantum field theory.

To the author's knowledge, Smolin was the first to propose that the underlying principle of quantum theory should be the indistinguishable principle, in his real ensemble interpretation of quantum mechanics [S]. However, in the real ensemble interpretation, space is not fundamental, in contrast to our proposal of a fundamental nonnoetherian spacetime.

Throughout, we use the signature $(+,-,-,-)$.

2. Superposition from the indistinguishable principle

It is well known that the indistingishability of eigenstates can be a resource for quantum entanglement. We propose that *all* quantum superpositions arise from the indistinguishability of eigenstates:

A state is a superposition of all eigenstates that are mutually indistinguishable to all the constituents of the universe.

This postulate implies the following:

(a) By the indistinguishable principle, quantum superposition does not fundamentally exist.
(b) The quantum-to-classical transition is due to the collective ability of an ensemble to distinguish states that are indistinguishable to the individual constituents of the ensemble.
(c) Wavefunction collapse occurs when a set of indistinguishable eigenstates become distinguishable.

Remark 2.1. An interaction between particles or fields constitutes a measurement if an indistinguishable state becomes distinguishable in the course of the interaction. We note, however, that not all interactions transform indistinguishable states to distinguishable states, and thus not all interactions are measurements.

These consequences further imply the following:

(i) The Hilbert space of single particle considered in isolation is 1-dimensional.
(ii) The pseudo-Riemannian geometry of general relativity must be supplanted by a new sort of geometry.
(iii) The block universe interpretation of general relativity cannot hold.

Let us examine each of these claims in turn.

To justify (i), consider an electron that is in a superposition of spin up and spin down with respect to a fixed measurement basis,

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|{\uparrow}\rangle + |{\downarrow}\rangle \right). \tag{1}$$

Prior to measurement, there is no distinction between these two eigenstates: they are one and the same state,

$$|{\uparrow}\rangle = |{\downarrow}\rangle . \tag{2}$$

It is only when the electron interacts with its environment in a spin-dependent way that the states $|{\uparrow}\rangle$ and $|{\downarrow}\rangle$ separate and become distinct.

In this framework, the usual 2-dimensional Hilbert space

$$\mathcal{H} = \operatorname{span}_{\mathbb{C}} \{ |{\uparrow}\rangle , |{\downarrow}\rangle \} \cong \mathbb{C}^2$$

describes the composite system consisting of both the electron and the measuring device. In contrast, the Hilbert space of the spin of an isolated electron is 1-dimensional,

$$\mathcal{H} \cong \mathbb{C}.$$

In particular, the electron has no intrinsic degrees of freedom.

This assertion extends to all the properties of fundamental particles, such as mass and electric charge. Consequently, all properties of fundamental particles are *relational*; without reference to the exterior universe, a fundamental particle has no intrinsic properties.

Remark 2.2. Although we claim that quantum superposition does not fundamentally exist, we will continue to refer to a state such as (1) as a superposition; the eigenstate summands are simply identified as the same state until a measurement occurs, at which point they become distinct and the superposition is reduced to one of the possible eigenstates.

The second implication (ii) follows from the superposition of spacetime coordinates. For example, if a photon travels through a Mach-Zehnder interferometer, then the two paths the photon may take become identified as the same path. If an electron undergoes diffraction, then there is a corresponding continuous family of geodesic trajectories that become identified as a single trajectory. Consequently, the manifold structure of spacetime must be modified. We propose such a modification in Section 3.

Finally, to justify (iii), consider the worldline of an electron. Suppose that in a fixed inertial frame, the electron is prepared in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ at time $t = 0$, and is measured at $t = 1$. Let p_0 and p_1 be the respective spacetime points on the electrons worldline at times $t = 0$ and $t = 1$. If the two points p_0 and p_1 coexist together in the block universe, then there exists a constituent of the universe, namely the measurement of the electron at p_1, that is able to distinguish between the two eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$. Thus the electron cannot be in the initial superposition $|\psi\rangle$, contrary to assumption.

The fact that superposition has been conclusively demonstrated in experiments therefore implies that the the postulate we are proposing is incompatible with the block universe interpretation of general relativity.

3. Nonnoetherian spacetime

In formulating general relativity, Einstein replaced the gravitational field in Newton's theory of gravity with the geometry of spacetime. In a similar way, we would like to describe the particles in the standard model not as quantized fields, but as geometric properties of spacetime itself. Quantum field theory would then become an emergent description of particle physics, rather than a fundamental description.

To this end, *we define a fundamental particle to be a causal (i.e., timelike or null) curve in spacetime that is deemed to be a single point.* In particular, time does not flow along such a curve, even if it is timelike; this will be the source for entanglement-based quantum nonlocality. Furthermore, without fields at hand, the various properties of particles–such as mass, spin, electric charge, and color charge–must be defined entirely by the geometry of these curve-like points. A purely geometric description is also needed to uphold our claim in Section 2 that properties of fundamental particles are relational, and not intrinsic to the particles themselves.

Definition 3.1. Let $(\tilde{M}, \tilde{g})$ be a (3+1)-dimensional time-orientable Lorentzian manifold. Let S be a collection of smooth causal curves in $\tilde{M}$, called *strands*. We declare two points $p, q \in \tilde{M}$ to be equivalent if (and only if) there is a strand $\alpha \in S$ that contains both p and q. We define *spacetime* to be the set of equivalence classes

$$M := \{[p] : p \in \tilde{M}\}.$$

Denote by π the map

$$\pi : \tilde{M} \to M, \quad p \mapsto [p].$$

Each point p in $U := \pi(\tilde{M} \setminus \cup_{\alpha \in S} \alpha)$ has a unique preimage $\pi^{-1}(p)$. Thus, to each point $p \in U$, we may associate the unique vector space

$$T_p M := T_{\pi^{-1}(p)} \tilde{M}.$$

This allows us to make the following definitions:

- The exponential map $\exp : T_p M \to M$ at $p \in U$ is the composition
$$T_p M = T_{\pi^{-1}(p)} \tilde{M} \stackrel{\tilde{\exp}}{\longrightarrow} \tilde{M} \stackrel{\pi}{\longrightarrow} M.$$
- The metric at $p \in U$ is the metric at $\pi^{-1}(p)$,
$$g_p := \tilde{g}_{\pi^{-1}(p)} : T_p M \times T_p M \to \mathbb{R}.$$

A strand, then, is a 1-dimensional point of spacetime, and does not possess a tangent space. We refer to the (pseudo-Riemannian) manifold $\tilde{M}$ as a *depiction* of spacetime. We say two strands are *indistinguishable* if they are mapped to the same point of M under π. Such strands are thus in superposition, by our assumption of Section 2.

Note that if γ is a smooth curve with affine parameterization $\gamma : (a, b) \to \tilde{M}$, such that

$$\gamma \cap \alpha = \emptyset$$

for each $\alpha \in S$, then the length of γ in M equals its length in $\tilde{M}$:

$$\ell(\gamma) = \int_a^b \sqrt{\pm g(\dot{\gamma}(s), \dot{\gamma}(s))} ds,$$

with $+$ if γ is causal, and $-$ if γ is spacelike. In contrast, the length of each strand $\alpha \in S$ is zero in M, even if α is timelike in $\tilde{M}$.

In a given frame, a strand appears to be a classical particle moving through space, which we call the associated *strand particle*. The distinction between a worldline and a strand is that a worldline consists of a continuum of distinct points, whereas a strand is a single point. We will consider strands that are helices centered about causal geodesics in $\tilde{M}$, called 'circular strands'.

Remark 3.2. In the framework of nonnoetherian algebraic geometry introduced in [B2], algebraic varieties with nonnoetherian coordinate rings of finite Krull dimension necessarily contain positive dimensional 'smeared-out' points (see [B3, Theorem A] for a precise statement). Such a variety may contain, for example, curves that are identified as single points. The original purpose of this framework was to provide a geometric description of the vacuum moduli spaces of certain unstable quiver gauge theories in string theory [B4] (see also [B1]). It was then proposed in [B6] that this geometry could be applied to spacetime itself, with the hope that it could explain, in a suitable sense, quantum nonlocality.

3.1. **Circular strands.**

Let β be a (piece-wise) causal geodesic in $\tilde{M}$ with affine parameterization $\beta : I \subset \mathbb{R} \to \tilde{M}$. Let $\{e_a(s)\}$ be an orthonormal basis for $T_{\beta(s)}\tilde{M}$,

$$g(e_a(s), e_b(s)) = \eta_{ab},$$

parallel transported along β, such that in the corresponding local Fermi (normal) coordinates about $\beta(\tilde{s}) \in \tilde{M}$,

$$(x^a) = \exp_{\beta(\tilde{s})}(x^a e_a(\tilde{s})), \tag{3}$$

we have

$$(\beta(s)^a) = \begin{cases} (s,0,0,0) & \text{if } \beta \text{ is timelike} \\ (s,0,0,s) & \text{if } \beta \text{ is null} \end{cases}.$$

Let $r \geq 0$ and $\omega \in \mathbb{R}$. We call the strand in M associated to the curve

$$(\alpha(s)^a) = \begin{cases} (s, r\cos(\omega s), r\sin(\omega s), 0) & \text{if } \beta \text{ is timelike} \\ (s, r\cos(\omega s), r\sin(\omega s), s) & \text{if } \beta \text{ is null} \end{cases}$$

a *circular strand.*

We define the four-momentum p^a of the strand particle of α by the Planck-de Broglie relation in the direction $\dot{\beta}$ (with $\hbar = c = 1$):

$$p^a = k^a = \omega\dot{\beta}^a. \tag{4}$$

We say the strand particle is *massive* if $p^2 = p^a p_a \neq 0$, and *massless* if $p^2 = 0$.

3.2. **Mass and the mass-shell condition.**

3.2.1. *Massive circular strands.*

Suppose the geodesic β is timelike. In the inertial frame of β, $\boldsymbol{\alpha}$ is a circular trajectory of radius r, angular frequency ω, and tangential velocity

$$u := |\dot{\boldsymbol{\alpha}}| = \omega r.$$

We define the mass m of the strand α to be its spatial curvature,

$$m := |\nabla_{\dot{\boldsymbol{\alpha}}}\dot{\boldsymbol{\alpha}}| = \kappa = \frac{1}{r} = \frac{\hbar}{cr},$$

with units restored in the rightmost equality. (Note that $r = \hbar(cm)^{-1}$ is the reduced Compton wavelength of a particle of mass m.) Consequently,

$$\frac{E_0}{u} = \frac{\hbar\omega}{u} = \frac{\hbar}{r} = mc,$$

that is,

$$E_0 = mcu. \tag{5}$$

We thus derive a variant of Einstein's relation $E_0 = mc^2$; Einstein's relation holds if and only if the tangential velocity u equals the speed of light c.

Set $c = 1$. From (5), the four-momentum p^μ of the strand particle satisfies

$$p^2 = p^\mu p_\mu = E_0^2 = m^2 u^2. \tag{6}$$

This is a modification of the standard relativistic mass-shell condition $p^2 = m^2$.[1]

A particle (or field excitation) is said to be on-shell if $p^2 = m^2$, and off-shell, or virtual, if $p^2 \neq m^2$. During a scattering event, most internal particles are off-shell. Under the assumption that $E_0 = m$, such particles violate relativity (hence the name 'virtual'). However, under the assumption that $E_0 = mu$, i.e., $p^2 = m^2u^2$, off-shell massive strand particles do not violate relativity; they are simply particles whose tangential velocity is not lightlike. Consequently, a massive strand particle is

- ∗ off-shell if $u \neq 1$, and
- ∗ on-shell if $u = 1$.

The variability of u, together with $p^2 = m^2u^2$, thus enables a geometric description of off-shell massive particles for which $p^2 = E_0^2$ is always satisfied, and therefore relativity is not violated.

Lightlike tangential velocity may be viewed as a geodesic-like property: suppose a circle of radius r is rotating with tangential velocity u measured in an inertial frame. In the accelerated frame of the circle, Ehrenfest observed that the circumference is

$$C = 2\pi r(1 - u^2)^{-1/2} = 2\pi r\gamma(u).$$

Thus, if $u = 1$, then C is infinite. Let us suppose that a circle of infinite circumference may be regarded as a straight line. Then, in the strand particle's reference frame, the particle travels in a 'straight line' if and only if it travels at the speed of light $u = 1$. Off-shell strand particles are thus unstable, and as such quickly interact with neighboring strands to recover their geodesic states.

In a field-theoretic description, the condition (6) modifies the massive Klein-Gordon and Dirac equations, which are the classical equations of motion for free scalar and spinor fields, respectively. We propose the modifications:

$$(\partial^2 - m^2)\phi = 0, \qquad (i\not{\partial} - m)\psi = 0, \qquad \bar{\psi}(i\not{\partial} + m) = 0,$$

where $\bar{\psi} = \psi^\dagger \gamma^0$. These equations differ from the standard Klein-Gordon and Dirac equations in the definition of m: the standard equations are recovered by replacing m with E_0.

3.2.2. *Massless circular strands.*

Now suppose the geodesic β is null. Set $|\boldsymbol{k}| = (k^i k_i)^{\frac{1}{2}}$. The condition

$$p^2 = k^2 = 0$$

implies $\omega = |\boldsymbol{k}|$, whence the phase velocity $v := \omega|\boldsymbol{k}|^{-1}$ equals 1.

[1] Note that $p^\mu = \omega\dot{\beta}^\mu = E_0\dot{\beta}^\mu = mu\dot{\beta}^\mu$.

To account for off-shell massless strand particles, we allow the phase velocity v of α to be less than 1. If $v < 1$, then the strand particle acquires mass and is no longer centered about a null geodesic. Such a strand has parameterization

$$(\alpha(s)^a) = (s, r\cos(\omega s), r\sin(\omega s), vs).$$

Consequently, a massless strand particle is

* off-shell if $v < 1$, and
* on-shell if $v = 1$.

The variability of v thus enables a geometric description of off-shell massless particles that does not violate relativity.

3.3. **Conservation of angular momentum.**

By assumption, a massive circular strand particle $\alpha(s) \in \tilde{M}$ has mass $m = r^{-1}$. Thus, the spatial angular momentum L of $\alpha(s)$ (in the inertial frame of its central wordline β) equals its tangential velocity $u = \omega r$,

$$L = m|\boldsymbol{p}| = rmu = u.$$

Conservation of angular momentum therefore implies that its tangential velocity u is constant.

3.4. **Electric and color charge of massive strands.**

A strand is a single point of spacetime M, and appears as a worldline (or a collection of worldlines) consisting of a continuum of distinct points in the depiction $\tilde{M}$. In particular, there does not exist a tangent vector field $\dot{\alpha}$ along a strand α in M. We will use this feature of strands to give a geometric definition of electric and color charge.

Let α be a massive circular strand centered about a timelike geodesic β. Identify the tangent spaces $T_{\beta(s)}\tilde{M}$ along β via the isomorphism induced by the tetrad $\{e_a\}$,

$$T_{\beta(s)}\tilde{M} \cong T_{\beta(s')}\tilde{M} \cong \mathbb{R}^{1,3}. \tag{7}$$

Consider the spatial subspace

$$V := \operatorname{span}_{\mathbb{R}}\{e_1, e_2, e_3\} \subset T_{\beta(s)}\tilde{M}.$$

Restricted to V, α has circular trajectory

$$(\boldsymbol{\alpha}(s)^i) = (r\cos(\omega s), r\sin(\omega s), 0). \tag{8}$$

Translate the Frenet frame $\{\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}\}$ of $\boldsymbol{\alpha}$ to the origin of V. The tangent and normal vectors,

$$\boldsymbol{t}(s) = \dot{\boldsymbol{\alpha}}(s) \quad \text{and} \quad \boldsymbol{n}(s) = \ddot{\boldsymbol{\alpha}}(s),$$

vary with $s \in I$. In contrast, the binormal vector

$$\boldsymbol{b}(s) = \dot{\boldsymbol{\alpha}}(s) \times \ddot{\boldsymbol{\alpha}}(s) = (0, 0, \omega^3 r^2) = \omega^3 r^2 e_3$$

is constant, independent of s. Therefore, since the circle $\boldsymbol{\alpha}$ is a single point in M, only the binormal vector $\boldsymbol{b}$ of the Frenet frame $\{\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}\}$ may be assigned to the strand $\boldsymbol{\alpha}$, since only $\boldsymbol{b}$ is independent of s. There are two possible choices of normalized binormal vector:

$$\hat{\boldsymbol{b}} = e_3 \quad \text{or} \quad \hat{\boldsymbol{b}} = -e_3. \tag{9}$$

We may also consider wedge products of Frenet vectors. The two wedge products

$$\boldsymbol{t} \wedge \boldsymbol{b} = -\omega^4 r^3 \sin(\omega s) e_1 \wedge e_3 + \omega^4 r^3 \cos(\omega s) e_2 \wedge e_3 \quad \text{and} \quad \boldsymbol{n} \wedge \boldsymbol{b}$$

vary with $s \in I$, whereas the two wedge products

$$\boldsymbol{t} \wedge \boldsymbol{n} = \omega^3 r^2 e_1 \wedge e_2 \quad \text{and} \quad \boldsymbol{t} \wedge \boldsymbol{n} \wedge \boldsymbol{b} = \omega^6 r^4 e_1 \wedge e_2 \wedge e_3 \tag{10}$$

are independent of s. Thus, we may also assign to the strand $\boldsymbol{\alpha}$ the two wedge products $\boldsymbol{t} \wedge \boldsymbol{n}$ and $\boldsymbol{t} \wedge \boldsymbol{n} \wedge \boldsymbol{b}$ since they are independent of s. There are again two possible choices for each:

$$\hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} = \pm \operatorname{sgn}(\omega) e_1 \wedge e_2 \quad \text{and} \quad \hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} \wedge \hat{\boldsymbol{b}} = \pm e_1 \wedge e_2 \wedge e_3. \tag{11}$$

We call the set

$$\hat{\boldsymbol{b}}, \quad \hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}}, \quad \hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} \wedge \hat{\boldsymbol{b}}, \tag{12}$$

the *strand frame* of α.

However, the choices of sign $\pm$ in (9) and (11) must be consistent with each other under the symmetries of space. To determine the allowed combinations of signs, consider the action of the subgroup $O(3)$ of the Lorentz group $O(1,3)$ on V.

We first consider a single strand.

In isolation, or empty space, there is no distinguished direction of space. Thus, to obtain the strand frame (12), we may apply any orthogonal transformation $g \in O(3)$ simultaneously to e_3, $e_1 \wedge e_2$, and $e_1 \wedge e_2 \wedge e_3$, with the property that g is invariant under every orthogonal change-of-basis $h \in O(3)$:

$$h^{-1} g h = g.$$

Consequently, g is in the center of $O(3)$,

$$g \in Z(O(3)) = \{\pm I\} \cong \mathbb{Z}_2.$$

We denote the two elements of $Z(O(3))$ by

$$w^+ := I \quad \text{and} \quad w^- := -I.$$

The possible strand frames are therefore

$$\begin{aligned} \hat{\boldsymbol{b}} &= e_3, \\ \hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} &= \operatorname{sgn}(\omega) e_1 \wedge e_2, \\ \hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} \wedge \hat{\boldsymbol{b}} &= e_1 \wedge e_2 \wedge e_3. \end{aligned} \tag{13}$$

and

$$\begin{aligned}
\hat{\boldsymbol{b}} &= w^- e_3 = -e_3, \\
\hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} &= \operatorname{sgn}(\omega)(w^- e_1) \wedge (w^- e_2) = \operatorname{sgn}(\omega) e_1 \wedge e_2, \\
\hat{\boldsymbol{t}} \wedge \hat{\boldsymbol{n}} \wedge \hat{\boldsymbol{b}} &= (w^- e_1) \wedge (w^- e_2) \wedge (w^- e_3) = -e_1 \wedge e_2 \wedge e_3.
\end{aligned} \tag{14}$$

Now suppose α belongs to a bound state of circular strands, all of which are centered at β, such that the plane of $\boldsymbol{\alpha}$,

$$P = \operatorname{span}_{\mathbb{R}}\{e_1, e_2\} \subset V,$$

is fixed within the bound state. Since the plane of $\boldsymbol{\alpha}$ is fixed, its normal line

$$L = \operatorname{span}_{\mathbb{R}}\{e_3\}$$

is a distinguished direction of space. Consequently, the spatial symmetry group $O(3)$ is 'broken to' the direct product $O(2) \times O(1)$, where $O(2)$ acts on the plane P, and $O(1)$ acts on the line L.

Thus, to obtain the strand frame (12), we may apply any orthogonal transformation $g \in O(3)$ simultaneously to e_3, $e_1 \wedge e_2$, and $e_1 \wedge e_2 \wedge e_3$, with the property that g is invariant under every orthogonal change-of-basis h in the subgroup $O(2) \times O(1)$ of $O(3)$ specified by P. Consequently, g is in the center of $O(2) \times O(1)$,

$$g \in Z(O(2) \times O(1)) \cong \mathbb{Z}_2 \times \mathbb{Z}_2.$$

We denote the four elements of $Z(O(2) \times O(1))$, with respect to the basis $\{e_1, e_2, e_3\}$, by

$$w^{\pm} := \pm \operatorname{diag}(1,1,1) \quad \text{and} \quad c^{\pm} := \pm \operatorname{diag}(1,1,-1).$$

Note that $c^{\pm}$ depends on the choice of plane P.

If we apply w^+ or c^+ (resp. w^- or c^-) to obtain the strand frame, then we say α has positive (resp. negative) *strand charge*. We denote the charge of a strand α by $q(\alpha)$, with $q(\alpha) \in \{w^+, w^-, c^+, c^-\}$.

The actions of w^+ and c^- are identical on e_3, $e_1 \wedge e_2$, and $e_1 \wedge e_2 \wedge e_3$, as are the actions of w^- and c^+. The distinction between the charges $w^{\pm}$ and $c^{\mp}$ of α is only detectable by their actions on the wedge products

$$e_3', \quad e_1' \wedge e_2', \quad e_1' \wedge e_2' \wedge e_3'$$

of a strand α' in the bound state with a different fixed plane $P' = \operatorname{span}_{\mathbb{R}}\{e_1', e_2'\}$. In this way, the $O(2)$ factor of $O(2) \times O(1)$ specifies the relationship between P and the other fixed planes of the bound state.

Let $\cup\alpha$ be a bound state of strands, and let $\mathcal{P}$ be the set of fixed planes of the strands in $\cup\alpha$. The total charge of $\cup\alpha$ is the $\mathbb{Z}$-linear combination

$$q(\cup\alpha) := \sum_{\alpha} q(\alpha) = n_w w^+ + \sum_{P \in \mathcal{P}} n_P c_P^+,$$

where $n_w, n_P \in \mathbb{Z}$ are integer coefficients. A strand or bound state of strands is able to exist in isolation if and only if it has spatial group $O(3)$. Therefore, a bound state $\cup\alpha$ may exist in isolation if and only if

$$q(\cup\alpha) = n_w w^+$$

for some $n_w \in \mathbb{Z}$. This condition restricts the allowable sets of fixed planes of bound states that may exist in isolation.

The simplest bound state with color charge that may exist in isolation consists of two strands that share the same fixed plane P, but have opposite charges c_P^+ and c_P^-:

$$c_P^+ + c_P^- = 0w^+.$$

We call such a bound state a *mesonic state.*

The next simplest bound state with color charge that may exist in isolation consists of three circular strands $\alpha_1, \alpha_2, \alpha_3$, necessarily with orthogonal binormal lines, say

$$e_3(\alpha_1) = (1,0,0), \quad e_3(\alpha_2) = (0,1,0), \quad e_3(\alpha_3) = (0,0,1).$$

Their respective possible color charges are then

$$\begin{aligned} r^\pm &:= \pm \operatorname{diag}(-1,1,1) \\ g^\pm &:= \pm \operatorname{diag}(1,-1,1) \\ b^\pm &:= \pm \operatorname{diag}(1,1,-1) \end{aligned}$$

These matrices satisfy the relations

$$r^\pm + g^\pm + b^\pm = w^\pm \tag{15}$$

and

$$w^+ + w^- = r^+ + r^- = g^+ + g^- = b^+ + b^-. \tag{16}$$

Therefore the strands $\alpha_1, \alpha_2, \alpha_3$ may have color charges

$$q(\alpha_1) = r^+, \quad q(\alpha_2) = g^+, \quad q(\alpha_3) = b^+$$

or

$$q(\alpha_1) = r^-, \quad q(\alpha_2) = g^-, \quad q(\alpha_3) = b^-.$$

We call such a bound state a *baryonic state.*

There is a unique configuration of three pairwise orthogonal planes in $\mathbb{R}^3$, up to rotation. Thus there are precisely three orthogonal embeddings of $O(2) \times O(1)$ in $O(3)$, up to rotation. Consequently, *every $O(3)$ bound state must be a mesonic state, a baryonic state, a collection of strands each with spatial group $O(3)$, or a union of such states.*

The strand charge of a circular strand α is therefore an element of $\{w^\pm, r^\pm, g^\pm, b^\pm\}$. We make the following identifications between strand charges and electric and color charges:

strand charge	electric charge	color charge
w^+	$-e$ (negative)	
w^-	$+e$ (positive)	
r^+, g^+, b^+		red, green, blue
r^-, g^-, b^-		anti-red, anti-green, anti-blue

We will denote by $c^{\pm}$ an unspecified color charge $r^{\pm}, g^{\pm}, b^{\pm}$.

The *sign* of α (or $q(\alpha)$), denoted $\operatorname{sgn}(\alpha)$, is the sign $\pm$ of the superscript of $q(\alpha)$.

Remark 3.3. We emphasize that the geometric definition of electric and color charge that we are proposing is only possible under the assumption that spacetime is nonnoetherian; in a noetherian (that is, pseudo-Riemannian) spacetime, the tangent vector to a curve is uniquely determined.

3.5. **Antiparticles.**

Definition 3.4. The *antiparticle* $\bar{\alpha}$ of a strand particle α is obtained by reversing the sign of α. The antiparticle $\bar{\sigma}$ of a bound state $\sigma = \cup\alpha$ consisting of a collection of strands is obtained by reversing the sign of each strand in σ.

Remark 3.5. The Stueckelberg interpretation of antiparticles as particles that travel backwards through time [St] is obtained by replacing s with $-s$ in (8), and thus results in the respective binormal vectors

$$\hat{b} = e_3 \quad \text{and} \quad \hat{b} = -e_3,$$

in agreement with (9). However, this interpretation does not give color charge. Furthermore, time does not flow along a strand: time does not flow backwards just as it does not flow forwards.

3.6. **Apexes: creation and annihilation of strands.**

Definition 3.6. An *apex* is a point $p \in \tilde{M}$ where two strands α, $\tilde{\alpha}$ of opposite charge, $q(\alpha) = -q(\tilde{\alpha})$, are created or annihilated. Two strands that do not have opposite charge cannot create or annihilate each other.

In the model we present in Section 4, the elementary particles are replaced by bound states of strands. These bound states interact with each other by exchanging strands. However, there are no fundamental interactions between the individual strands themselves, other than pair creation/annihilation at apexes. *We propose that creation apexes only arise so that strands that are off-shell can become on-shell.* In contrast to particle-antiparticle creation in quantum field theory, apexes do not spontaneously occur without cause. In particular, there are no vacuum fluctuations of strands. A vacuum void of quantum fluctuations is conceptually similar to the smooth manifold structure of spacetime in general relativity.[2]

In an interaction of bound states of strands (that is, at a Feynman vertex), energy-momentum is always conserved. New strand particles created in an interaction of bound states thus obtain their energy from the bound states involved. Unlike particle-antiparticle creation, they do not use 'free energy' from the vacuum, allowed by the time-energy uncertainty principle, to exist. Consequently, the time-energy uncertainty principle does not constrain their lifetimes. The new strand particles may therefore exist indefinitely.

3.7. **Electric and color charge of massless strands.**

In Section 3.4, we defined the charge of a circular strand α using the inertial frame of its central worldline β. However, this is not possible if the central worldline is null.

Let $\alpha : [0, t] \to \tilde{M}$ be a massless circular strand, and let β be its null central worldline. Suppose α is created in a pair with a massive circular strand $\tilde{\alpha}$ at an apex $\alpha(0) \in \tilde{M}$, and is later annihilated at an apex $\alpha(t) \in \tilde{M}$. Since $\tilde{\alpha}$ is massive, it possesses a strand charge $q(\tilde{\alpha})$. Thus, α receives the opposite charge to $\tilde{\alpha}$ at $\alpha(0)$,

$$q(\alpha(0)) = -q(\tilde{\alpha}(0)).$$

Furthermore, the endpoints of α map to the same point in M since $\pi(\alpha)$ is a single point,

$$\pi(\alpha(0)) = \pi(\alpha(t)) \in M.$$

Therefore, the endpoints of α have the same charge,

$$q(\alpha(0)) = q(\alpha(t)).$$

However, since β is null, charge is not defined along the length of α (in $\tilde{M}$). Consequently, the charge of α resides only at its endpoints. More generally, a strand has the same fixed charge along each of its timelike segments, and no charge along each of its lightlike segments.

Remark 3.7. In the model of gauge bosons we introduce in Section 4, massless strands are always created in pairs with massive strands. Indeed, there are no 3-photon vertices, and at least two of the gluons in a 3- or 4-gluon vertex are massive.

3.8. **Spin.**

We define a circular strand to be a spin $\frac{1}{2}$ fermion, with spin up if $\omega > 0$ and spin down if $\omega < 0$. This definition will be justified in Section 4.3.1. This is encoded in the sign of the wedge product $\boldsymbol{t} \wedge \boldsymbol{n}$ by (10). Indeed, the (electric or color) charge of the strand determines the sign of $\boldsymbol{b}$ and $\boldsymbol{t} \wedge \boldsymbol{n} \wedge \boldsymbol{b}$, but does not affect the sign of $\boldsymbol{t} \wedge \boldsymbol{n}$, as shown in (13) and (14).

[2] Vacuum fluctuations account for the Casimir force and the positive cosmological constant, as well as provide a physical mechanism that corrects bare masses to renormalized masses. Further development of our framework is therefore required to address these issues.

A fundamental difference between spin and classical (spatial) angular momentum is that spin is frame-independent, in contrast to angular momentum. In our framework, this distinction is due to the indistinguishable principle: Consider a massive circular strand α with central worldline β. Identify the tangent spaces $T_{\beta(s)}\tilde{M} \cong \mathbb{R}^{1,3}$ along β as in (7). The embedding of α into $\tilde{M}$, $\alpha : I \to \tilde{M}$, is an extrinsic property of α, and is thus not detectable to α. In particular, the plane of rotation $P = \operatorname{span}_{\mathbb{R}}\{e_1, e_2\}$ of α in $\mathbb{R}^3 \subset \mathbb{R}^{1,3}$ is indistinguishable from any other choice of 2-dimensional subspace P' of $\mathbb{R}^3$. Therefore, by the indistinguishable principle, all the 2-dimensional subspaces of $\mathbb{R}^3$ are identified as the same plane. Consequently, the strand particle exists in a 2-dimensional space, not a 3-dimensional space.

3.9. **Self-interference.**

Consider two indistinguishable strands $\alpha, \tilde{\alpha}$, with respective central worldlines $\beta, \tilde{\beta}$. The strands are then in superposition in $\tilde{M}$, and equal in spacetime M, $\pi(\alpha) = \pi(\tilde{\alpha})$.

Definition 3.8. We call a point $p \in \tilde{M}$ a *branching point* (resp. *joining point*) of $\pi(\alpha) \in M$ if

$$p = \beta(s) = \tilde{\beta}(\tilde{s}),$$

and there is some $\epsilon_0 > 0$ such that for each $\epsilon \in (0, \epsilon_0]$, we have

$$\beta(s+\epsilon) \neq \tilde{\beta}(\tilde{s}+\epsilon) \quad \text{(resp. } \beta(s-\epsilon) \neq \tilde{\beta}(\tilde{s}-\epsilon)).$$

A branching point occurs, for example, when a photon encounters a beam splitter, and a joining point occurs when the two copies of the photon are brought back together, as in a Mach-Zehnder interferometer.

Suppose $p, q \in \tilde{M}$ are branching and joining points of $\pi(\alpha)$, respectively. Let

$$P_s = \operatorname{span}\{e_1(s), e_2(s)\} \subset T_{\beta(s)}\tilde{M} \quad \text{and} \quad \tilde{P}_{\tilde{s}} = \operatorname{span}\{\tilde{e}_1(\tilde{s}), \tilde{e}_2(\tilde{s})\} \subset T_{\tilde{\beta}(\tilde{s})}\tilde{M}$$

be the planes of rotation of α and $\tilde{\alpha}$, where $e_j(s)$ and $\tilde{e}_j(\tilde{s})$ are parallel transported along β and $\tilde{\beta}$. Consider the vectors

$$n_s = r\cos(\omega s)e_1(s) + r\sin(\omega s)e_2(s) \in P_s,$$
$$\tilde{n}_{\tilde{s}} = r\cos(\omega\tilde{s})\tilde{e}_1(\tilde{s}) + r\sin(\omega\tilde{s})\tilde{e}_2(\tilde{s}) \in \tilde{P}_{\tilde{s}}.$$

Since p is a branching point, the two planes coincide at p,

$$P_p = \tilde{P}_p.$$

However, in general the planes will not coincide at q,

$$P_q \neq \tilde{P}_q.$$

Despite this, we would like to compare the two vectors n_q and $\tilde{n}_q$ as points on a circle in the same plane P_q at the joining point q.

Now time does not flow along α or $\tilde{\alpha}$, since they are single points of spacetime. Consequently, all the points along α and $\tilde{\alpha}$ in $\tilde{M}$ exist ‘at the same time’, that is, simultaneously, in contrast to the points along a worldline. We may therefore parallel transport $\tilde{n}_q$ (backwards) along $\tilde{\beta}$ from q to p, and then (forwards) along β from p to q, obtaining

$$\tilde{n}'_q = r\cos(\omega\tilde{s}_q)e_1(s_q) + r\sin(\omega\tilde{s}_q)e_2(s_q) \in P_q.$$

Since $\tilde{n}'_q$ and n_q both lie on the same circle in the same plane P_q, they can be compared. In particular, the difference of their angles (from $e_1(s_q)$, say), $\theta - \tilde{\theta}$, is well-defined.

To obtain standard quantum mechanical interference, we identify the plane P_p, parallel transported along β and $\tilde{\beta}$, with the complex plane $\mathbb{C}$, via the vector space isomorphisms defined on $P_{\beta(s)}$ and $\tilde{P}_{\tilde{\beta}(\tilde{s})}$ by

$$re_1(s) \mapsto 1, \quad re_2(s) \mapsto i; \quad \text{and} \quad r\tilde{e}_1(\tilde{s}) \mapsto 1, \quad r\tilde{e}_2(\tilde{s}) \mapsto i.$$

We may thus identify the circular trajectory of α in P with the phase factor $e^{i\theta}$ of its wavefunction.

4. A strand model of leptons, quarks, and gauge bosons

We would like to describe the standard model using the framework of strands. Naively, we could associate a strand to each elementary particle. However, a primary objective of nonnoetherian spacetime is to provide a spacetime description of quantum nonlocality, and this would not be possible if each elementary particle was simply a different type of strand.

Indeed, suppose that there are 'photon strands' and 'electron strands', and consider two photons that are initially entangled in a Bell state. One photon then travels to the left, and the other to the right. Suppose the left photon passes through a polarizer, altering its polarization, but the two photons remain entangled. Since the left photon's polarization was altered, it must have interacted with an electron in the polarizer. This would constitute a scattering – an infinite superposition of Feynman diagrams (integrated over all positions and momenta) – between the photon and electron. At each electron-electron-photon vertex (in each Feynman diagram), the electron strand and photon strand would undergo a fundamental change. This change would cause time to progress, by the indistinguishable principle. But a strand is a single point in spacetime; in particular, time does not flow along a strand. Thus the photon strand and electron strand would both terminate at the vertex, and a new electron strand would be created. At a different vertex in the diagram, a new photon strand would be created that would then propagate as the left photon in the original entangled photon pair. But this new photon strand would not be connected in spacetime to the right photon strand: the left and right photon strands would no longer be 'touching'. Consequently, the framework of nonnoetherian spacetime we are proposing would not be able to account for the quantum nonlocality of the two entangled photons.

To remedy this problem, we introduce a new model of particle physics, where leptons, quarks, and gauge bosons are realized as bound states of strands. These bound states interact with each other by exchanging strands in specific ways. Our model is a generalization of 't Hooft's double line formalism ['tH] of quarks and gluons to all standard model particles, excluding the Higgs boson.

We propose two kinds of bound states of strand particles: *symmetric* and *split*. Based on their rotational symmetry, we will identify these states with bosons and fermions, respectively.

Remark 4.1. We give an informal account of the passage of time from the perspective of a strand particle: since a strand particle is non-interacting, its entire universe appears empty, void of anything from which distances of space or intervals of time can be measured. The particle lives alone in a completely empty universe, and thus never observes any change. Thus, by the indistinguishable principle, the particle never experiences the passage of time. Therefore the worldline of the particle is a single point of spacetime M.

4.1. **Symmetric states: bosons.**

Recall that two strands α and α' are indistinguishable, whence in superposition, if they belong to the same spacetime point:

$$\pi(\alpha) = \pi(\alpha') \in M.$$

Definition 4.2. We define a *symmetric state* to be a bound state of circular strands with three orbitals: an outer orbital with two strands; an inner orbital with zero or two strands; and a central orbital with zero or two indistinguishable strands. We call these strands *outer strands*, *inner strands*, and *central strands*, respectively. See Figure 1. Furthermore, the following holds:

(a) The strands all lie on a diameter, and the outer and inner strands alternate in sign along the diameter. Since the two central strands belong to the same spacetime point in M, they necessarily have the same sign.
(b) If the central orbital is nonempty, then the inner orbital is also nonempty.
(c) The two outer strands have the same spatial group $O(3)$ (electric charge) or $O(2)$ (color charge). The inner and central strands have spatial group $O(3)$.

Condition (c) is shown to be a sufficient condition for the consistency of our model in Remark 4.7.

The outer strands, inner strands, and central strands have respective parameterizations in the Fermi coordinates (3),

$$\begin{aligned}
(\alpha_{2\pm}(s)^a) &= (s, \pm r\cos(\omega s), \pm r\sin(\omega s), 0)\\
(\alpha_{1\pm}(s)^a) &= (s, \pm n_1^{-1} r\cos(n_1\omega s), \pm n_1^{-1} r\sin(n_1\omega s), 0)\\
(\alpha_{0\pm}(s)^a) &= (s, \pm n_0^{-1} r\cos(n_0\omega s), \pm n_0^{-1} r\sin(n_0\omega s), 0)
\end{aligned} \tag{17}$$

where $1 < n_1 < n_0$ are positive odd integers (n_0, n_1 must be odd in order for the split states, defined below, to be well-defined). We may equivalently say that there is only one central particle, and this particle is a superposition of the two strand particles $\alpha_{0+}(s)$ and $\alpha_{0-}(s)$.

Note that all strands in the bound state have equal tangential velocity u,

$$\omega r = (n_0\omega)(n_0^{-1}r) = (n_1\omega)(n_1^{-1}r) = u, \tag{18}$$

and thus have equal spatial angular momentum (see Section 3.3),

$$L(\alpha_0) = L(\alpha_1) = L(\alpha_2) = u. \tag{19}$$

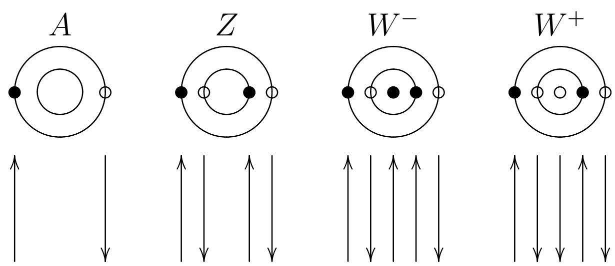


FIGURE 1. The four $O(3)$ symmetric states. The central double helix in $W^{\mp}$ is drawn as a single strand, and consists of two strands of charge $w^{\pm}$ in superposition.

4.2. **Split states: fermions.**

Definition 4.3. Each symmetric state may be 'split' into two *split states*, shown in Table 1, by a process we call a *fundamental splitting*. The rules of a fundamental splitting are as follows:

(1) The undirected graph, with the central strand omitted, is planar and symmetric with respect to reflections about the central axis of the symmetric state.
(2) Strands maintain their orbitals.
(3) The two inner strands of the symmetric state may annihilate each other.
(4) In a split state, strands in neighboring orbitals (outer-inner; inner-central; outer-central if inner is empty) have opposite sign.

Examples of condition (1) are given in Figures 2 and 3. We say a bound state is an $O(3)$ (resp. $O(2)$) bound state if the outer strand(s) have spatial group $O(3)$ (resp. $O(2) \times O(1)$).

Remark 4.4. In condition (1), the requirement that a splitting have reflective symmetry may be regarded as form of Newton's third law of motion. Moreover, the requirement that the graph of a splitting is planar–that is, the graph is 2-dimensional rather than 3-dimensional–is suggestive that there may be a holographic principle underlying the model.

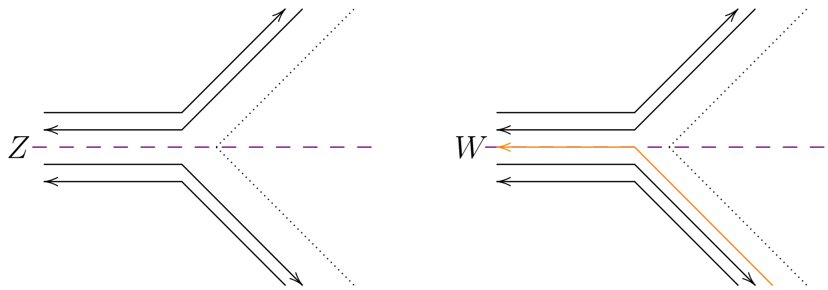


FIGURE 2. The undirected graph of a splitting, with central strand omitted, is symmetric with respect to reflections about the central axis (shown here as a dashed line) of the symmetric state. The central strand in the W splitting is drawn in orange.

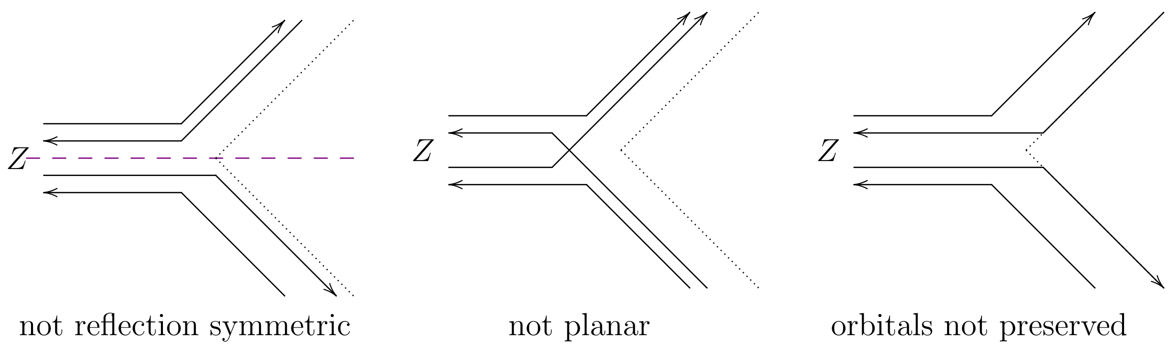


FIGURE 3. Some non-allowable interactions.

Definition 4.5. A *non-fundamental splitting* occurs when there is a strand that does not belong to the original symmetric state; see Figure 4. The rules for such a splitting are the same as for a fundamental splitting, with three additional rules:

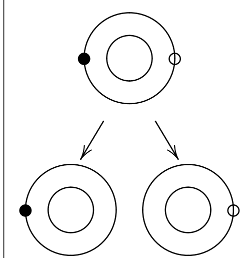
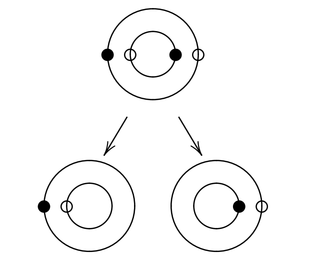
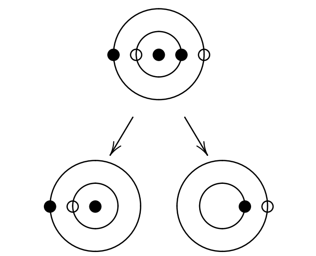
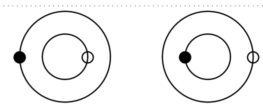
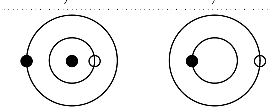
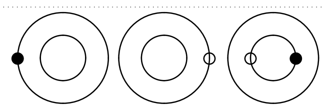
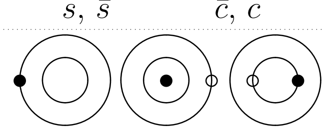
(5) For each orbital there is a bound state that contains a strand in that orbital.
(6) Each bound state with a maximum number of strands (in the interaction) and each bound state share a common strand.
(7) If all the involved bound states are symmetric, then the two inner strands of a bound state may not annihilate each other.

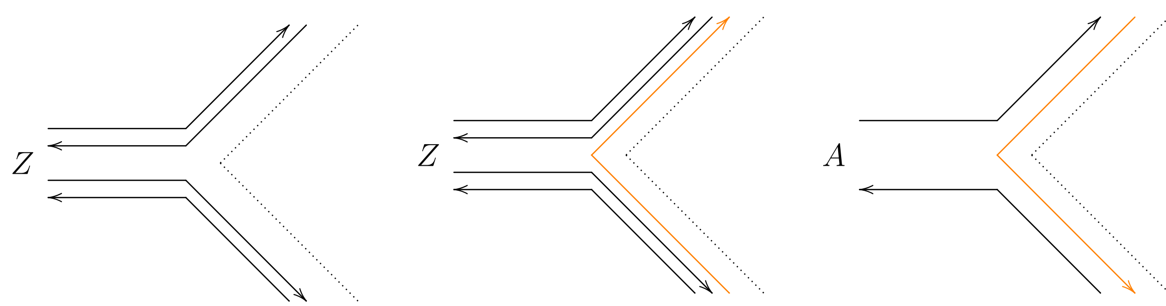
Only rule (5) is relevant for non-fundamental splittings into split states. In this case, rule (5) says that if there is a strand that does not belong to the symmetric state, then there is at least one strand in each orbital of the interaction.

4.3. **Particle identifications.**

4.3.1. *Spin and stability.* We make the following identifications:

TABLE 1. The fundamental splittings and their particle identifications.

	A	Z	W^-, W^+
symmetric state / **split state**			
$O(3)$	e $\bar{e}$	ν_τ $\bar{\nu}_\tau$	τ, $\bar{\tau}$ $\bar{\nu}_\tau$, ν_τ
$O(2)$	d $\bar{d}$	t $\bar{t}$	b, $\bar{b}$ $\bar{t}$, t
split state			
$O(3)$		ν_μ $\bar{\nu}_\mu$	μ, $\bar{\mu}$ $\bar{\nu}_\mu$, ν_μ
$O(2)$		c $\bar{c}$	s, $\bar{s}$ $\bar{c}$, c
split state			
$O(3)$		e $\bar{e}$ $\emptyset$	e, $\bar{e}$ $\bar{\nu}_e$, ν_e $\emptyset$
$O(2)$		d $\bar{d}$ $\emptyset$	d, $\bar{d}$ $\bar{u}$, u $\emptyset$



a fundamental splitting   a non-fundamental splitting   not allowed by cond. (5)

FIGURE 4. A fundamental splitting involves only strands that belong to the symmetric state. Here, central strands are drawn in orange. The last example is not allowed (which corresponds to a photon-neutrino vertex $A\nu_e\nu_e$) since the first orbital is empty in all three bound states.

- A bound state is a vector boson if its rotational symmetry (with strand signs ignored) is π, and a fermion if its rotational symmetry is 2π. Therefore:
  - A symmetric state is a vector boson.
  - A split state is a fermion.
- An $O(3)$ bound state is unstable (that is, it immediately decays) if and only if it contains at least two strands of the same sign.[3]

Based on this criteria, together with electric and color charge, we make the particle identifications given in Table 2. In particular, we identify a free strand as either an (anti-)electron or (anti-)down quark, depending on its spatial group. Since the electron and down quark have different masses, the value of r for $O(2)$ and $O(3)$ bound states must be different.

In the defining representation of $SO(3)$, a vector is returned to its initial position by a rotation of $\theta = 2\pi$, whereas in the spin-$\frac{1}{2}$ representation of $SO(3)$, a spinor is returned to its initial position by a rotation of $4\pi = 2\theta$. The ratio of rotational symmetry between vectors and spinors, namely 2, is precisely the ratio of rotational symmetry between symmetric and split bound states.

4.3.2. *Lepton interactions.* There are four $O(3)$ symmetric states: the photon A, Z-boson, and $W^{\pm}$-bosons, shown in Figure 1. Their splittings into split states are shown in Figure 5. The splittings correspond precisely to the Feynman interactions between leptons and electroweak gauge bosons.

In the propagation of a neutrino (that is, an $O(3)$ split state consisting of two strands), it may be necessary to allow the non-outer strand to slowly move between the inner and central orbitals, in order to account for neutrino mixing. By assumption (19), each strand in a bound state has the same angular momentum. Thus the total angular momentum of the bound state would remain conserved if a strand were to move between the inner and central orbitals. We leave the question of neutrino mixing and quark mixing for future work.

4.3.3. *Quark interactions.* We call the $O(2)$ symmetric states *gluons*, though they differ from gluons in quantum chromodynamics. There are three types of gluons:

- Neutral massless gluons $\tilde{A}$, called A-gluons, which only mediate color charge between down, strange, and bottom quarks.
- Neutral massive gluons $\tilde{Z}$, called Z-gluons, which mediate color charge between all quarks.
- Charged massive gluons $\tilde{W}^{\pm}$, called $W^{\pm}$-gluons, which allow flavor transformations within a generation.

The model therefore predicts the existence of both neutral and charged massive gluons. Their splittings into split states are shown in Figure 6. Note that an A-gluon (resp. Z-gluon, $W^{\pm}$-gluon) with outer strands of equal color is simply a photon (resp. Z boson, $W^{\pm}$ boson), by the relations (16); see Figure 8.

[3]Only the $O(2)$ split state of lowest rest energy, namely the up quark, is stable.

In our model, quarks do not have fractional electric charge as they do in QCD. Instead, they possess integer combinations of strand charges. Nevertheless, our model gives the correct electric charges for all baryons and mesons:

Proposition 4.6. *Upon substituting the charges*

$$w^{\pm} \mapsto \mp 1 \quad \textit{and} \quad c^{\pm} \mapsto \mp \frac{1}{3}$$

in Table 2, we obtain the fractional electric charges of the quarks in QCD. Therefore, the strand model and QCD produce the same electric charges for all baryons and mesons.

Proof. The second statement follows from the relations (15) and (16). □

Remark 4.7. We now justify condition (c) in the definition of a symmetric state.

(i) In the inertial frame of a symmetric or split state, the central strand particle, which is a superposition of the two circular strands α_{0+} and α_{0-}, has center of mass

$$\boldsymbol{\alpha}_{0+} + \boldsymbol{\alpha}_{0-} = (0, 0, 0),$$

and thus is at rest. Hence, to the central particle, there is no distinguished direction in space. Therefore its spatial group is $O(3)$, even if the other strands in the bound state have spatial group $O(2)$.

(ii) If both the inner and outer strands were allowed to have spatial group $O(2)$, then we would encounter an inconsistency in our model. Indeed, suppose both the inner and outer strands of a split state (s, t, c, or b) have spatial group $O(2)$. For concreteness, suppose we have a bound state of three split states σ_1, σ_2, σ_3, with inner and outer color charges

$$q(\sigma_1) = (r^+, g^-), \quad q(\sigma_2) = (g^+, b^-), \quad q(\sigma_3) = (b^+, r^-).$$

A gluon is then free to exchange the colors of the outer strands in σ_1 and σ_2, upon which we obtain

$$q'(\sigma_1) = (g^+, g^-), \quad q'(\sigma_2) = (r^+, b^-), \quad q'(\sigma_3) = (b^+, r^-).$$

But by the relation (16), we have

$$(g^+, g^-) = (w^+, w^-).$$

Therefore the split state σ_1 is no longer confined to the bound state of quarks, and is free to escape. We would therefore be left with an isolated bound state of two split states that does not have spatial group $O(3)$, which is forbidden since there is no distinguished direction in empty space. Consequently, a necessary condition for our model to be consistent is that the inner and outer strands do not both have spatial group $O(2)$.

Remark 4.8. Our model 'explains' two of the prominent features of QCD:

- The reason there are three color charges is because there are three dimensions of space (specifically, three pairwise orthogonal embeddings of $O(2) \times O(1)$ into $O(3)$).
- The reason that quarks cannot exist in isolation is because color charge is only possible when there is a distinguished direction of space; in isolation, there is no distinguished direction of space.

However, the different gluon types predicted by the model do not arise in QCD, and this feature may cause the model to fail.

4.3.4. *Gauge boson interactions.* By the reflective symmetry in rule (1) of Definition 4.3, interactions involving ≥ 4 bound states only involve symmetric states. Furthermore, by rule (6), there may be at most 4 bound states in a splitting. Consequently, our model exactly reproduces the standard model Feynman interactions involving the electroweak gauge bosons; these are shown in Figure 7. The standard 4-valent gluon interaction is replaced by the 4-valent electroweak boson interactions, with color charge assignments on the outer strands.

4.4. **Orderings of bound state masses.**

Consider a symmetric or split bound state $\cup\alpha$. Each strand α in the bound state has a common central worldline β (with affine parameterization). We define the total four-momentum of $\cup\alpha$ to be the sum of the signed four-momenta p^a_α of each strand, and an unknown positive binding energy δ between strands in different orbitals,

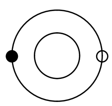
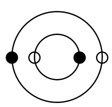
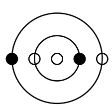
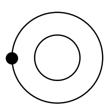
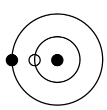
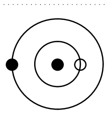
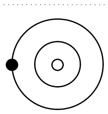
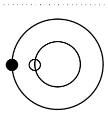
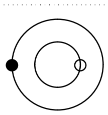
$$
\begin{aligned}
p^a &\overset{\text{(I)}}{=} \delta\dot{\beta}^a + \sum_\alpha \operatorname{sgn}(\alpha)^{\dot{\beta}^2} p^a_\alpha \\
&\overset{\text{(II)}}{=} \delta\dot{\beta}^a + \sum_\alpha \operatorname{sgn}(\alpha)^{\dot{\beta}^2} \omega_\alpha \dot{\beta}^a \\
&= \left(\delta + \sum_\alpha \operatorname{sgn}(\alpha)^{\dot{\beta}^2} \omega_\alpha \right) \dot{\beta}^a.
\end{aligned} \tag{20}
$$

Here, (I) holds since a strand α has a strand charge at a point $\alpha(s) \in \tilde{M}$ if and only if $\dot{\beta}^a(s)$ is timelike; see Section 3.7. Furthermore, (II) holds since the four-momentum of α is $p^a_\alpha = k^a_\alpha = \omega_\alpha \dot{\beta}^a$; see Section 3.1.

First suppose β is timelike, $\dot{\beta}^2 = 1$. Then in the Fermi coordinates (3), we have

$$(\dot{\beta}^a) = (1, 0, 0, 0).$$

Table 2. Particle identifications of the symmetric and split bound states

bound state	rotational symm.	$\Rightarrow$ spin	≥ 2 like charges	$\Rightarrow$ stability	strand charge	$\Rightarrow$ electric charge	particle
	π	1	no	stable -	0 $c_1^+ + c_2^-$	0 0	A A-gluon
	π	1	yes	unstable -	0 $c_1^+ + c_2^-$	0 0	Z Z-gluon
	π	1	yes	unstable -	$w^\mp$ $c_1^+ + c_2^- + w^\mp$	± 1 ± 1	$W^\pm$ $W^\pm$-gluon
	2π	$\frac{1}{2}$	no	stable -	$w^\pm$ $c^\pm$	∓ 1 $\mp \frac{1}{3}$	e, $\bar{e}$ d, $\bar{d}$
	2π	$\frac{1}{2}$	yes	unstable -	$w^\pm$ $c^\pm$	∓ 1 $\mp \frac{1}{3}$	τ, $\bar{\tau}$ b, $\bar{b}$
	2π	$\frac{1}{2}$	yes	unstable -	$w^\pm$ $c^\pm$	∓ 1 $\mp \frac{1}{3}$	μ, $\bar{\mu}$ s, $\bar{s}$
	2π	$\frac{1}{2}$	no	stable -	0 $c^\pm + w^\mp$	0 $\pm \frac{2}{3}$	ν_e, $\bar{\nu}_e$ u, $\bar{u}$
	2π	$\frac{1}{2}$	no	stable -	0 $c^\pm + w^\mp$	0 $\pm \frac{2}{3}$	ν_τ, $\bar{\nu}_\tau$ t, $\bar{t}$
	2π	$\frac{1}{2}$	no	stable -	0 $c^\pm + w^\mp$	0 $\pm \frac{2}{3}$	ν_μ, $\bar{\nu}_\mu$ c, $\bar{c}$

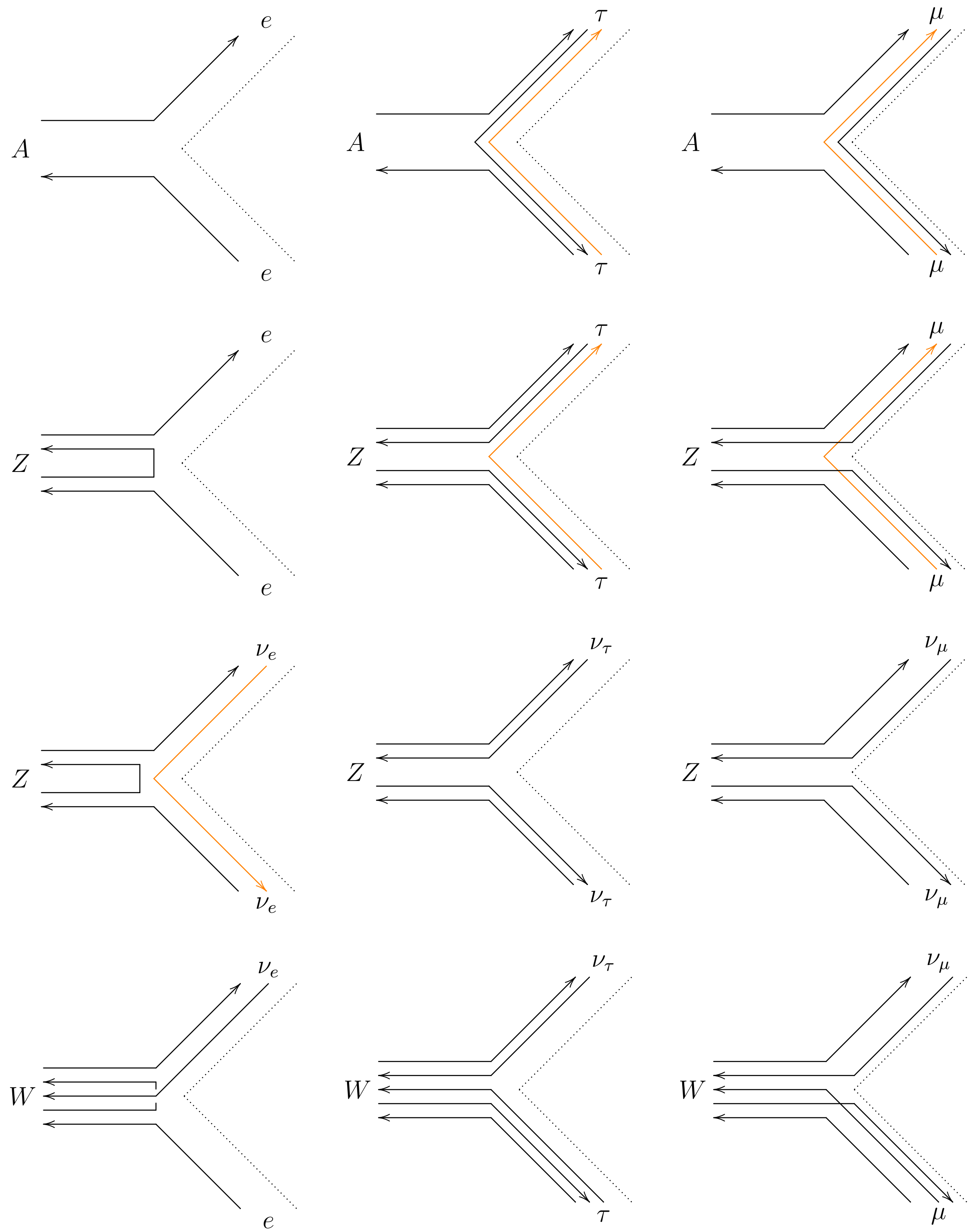


FIGURE 5. All splittings of the $O(3)$ symmetric states into split states. Each non-fundamental splitting contains a central strand that is drawn in orange.

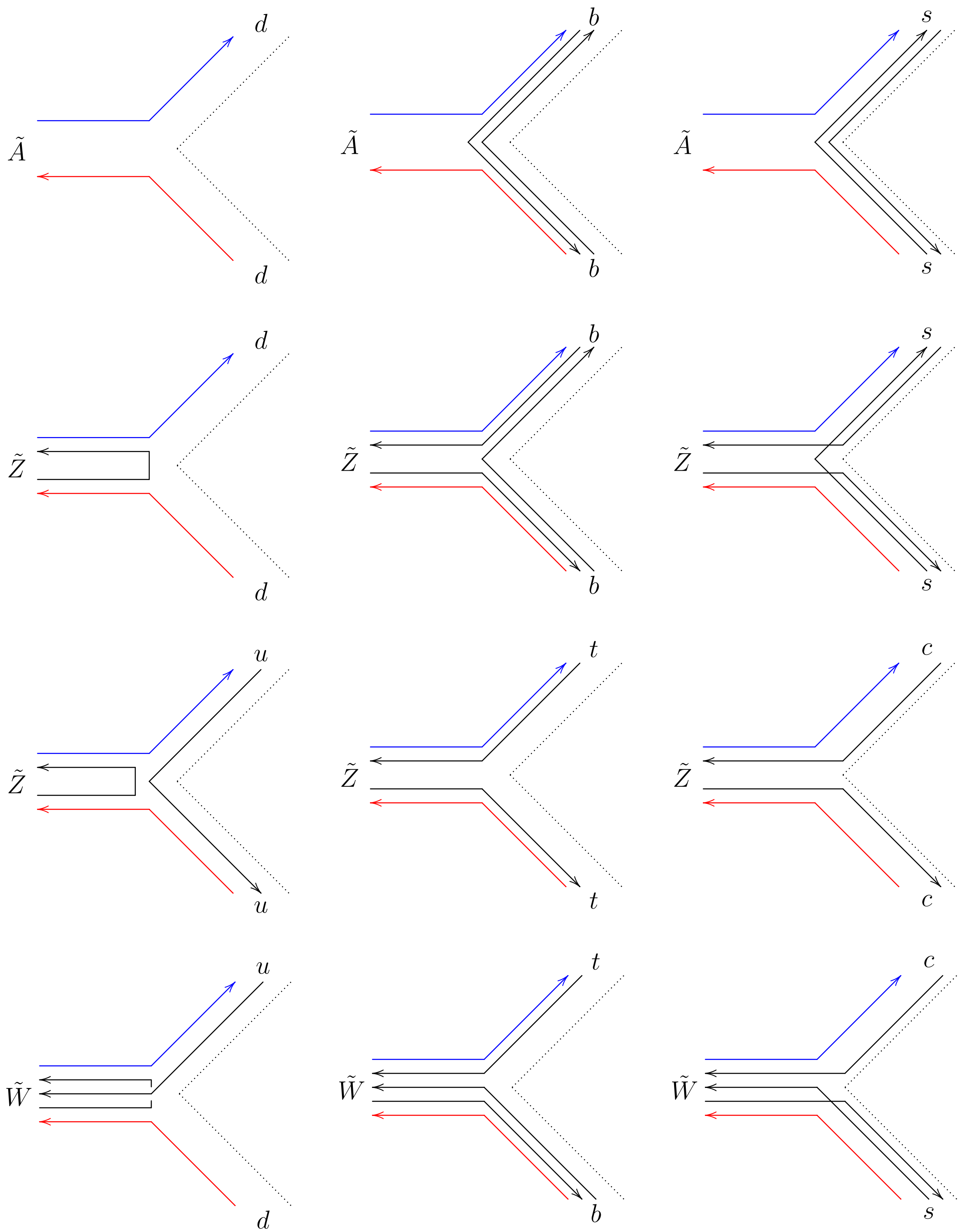


FIGURE 6. All splittings of the $O(2)$ symmetric states into split states.

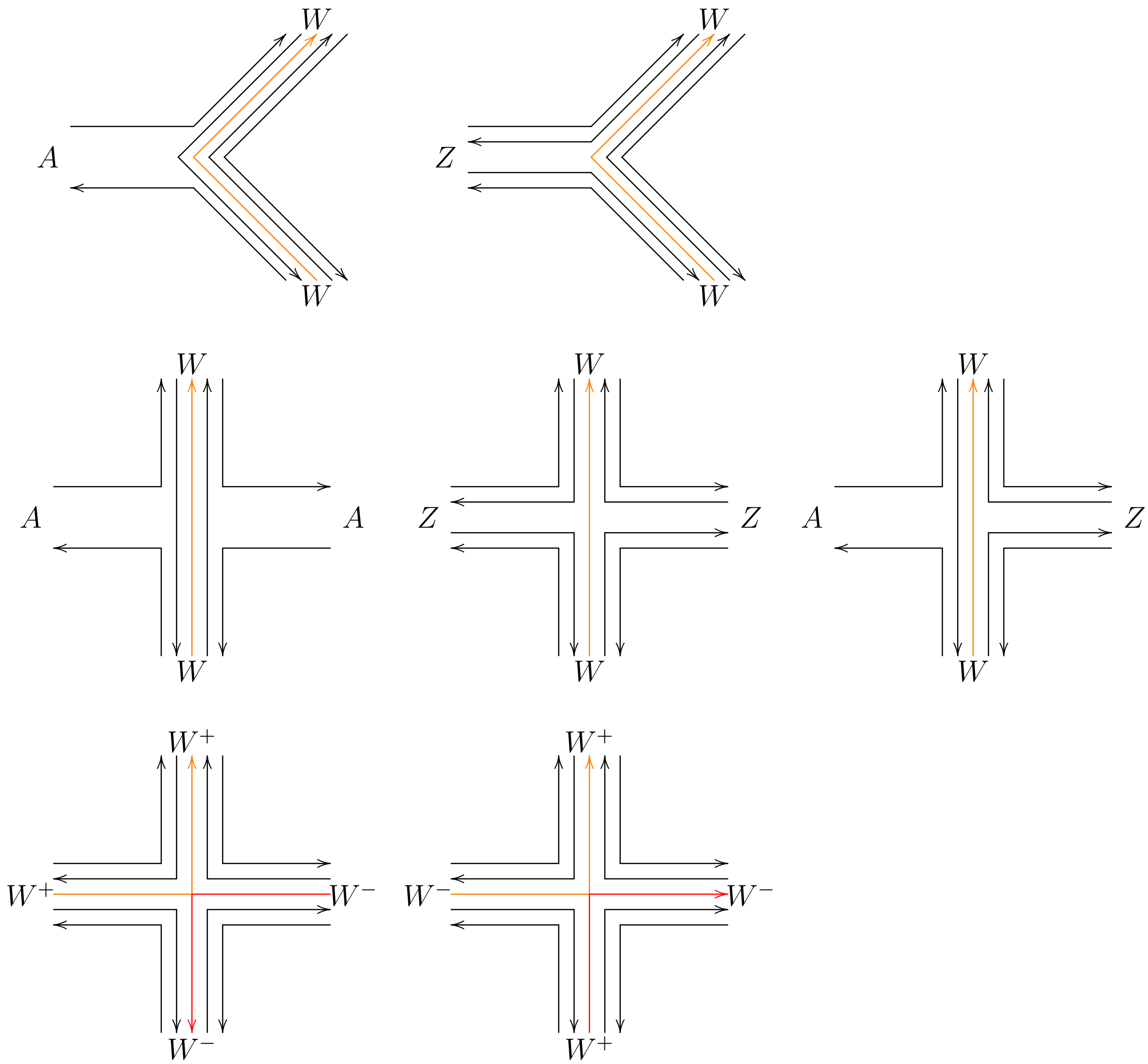


Figure 7. All splittings of the symmetric states into other symmetric states. These splittings correspond precisely to the Feynman interactions between the electroweak gauge bosons. The central strands are drawn in red and orange, and cannot cross by rule (6) of Definition 4.5.

Therefore, the rest energy of the bound state is

$$\begin{aligned} E_0 &= \sqrt{p^a p_a} \\ &= |\delta + \sum_\alpha \operatorname{sgn}(\alpha)\omega_\alpha| \\ &= |\delta + u \sum_\alpha \operatorname{sgn}(\alpha) r_\alpha^{-1}|, \end{aligned}$$

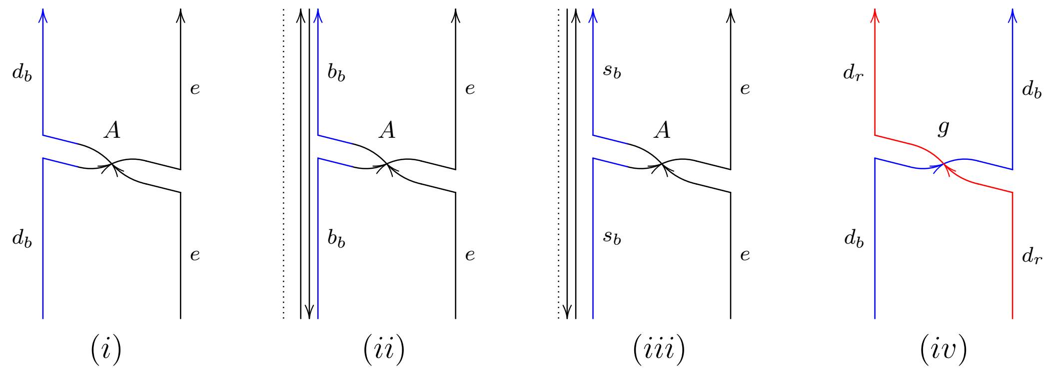


FIGURE 8. (i) - (iii): Tree-level quark-electron interactions mediated by a photon (drawn with time in the vertical direction). Here we use the relation $w^+ + w^- = b^+ + b^-$ to change the color charges of the photon strands. (iv): A tree-level quark interaction mediated by an A-gluon. The strand representation of the quark-gluon interaction (iv) coincides with 't Hooft's double line formalism.

where the last line holds since each strand particle in the bound state has the same tangential velocity u by (18). As in Section 3.2, we say the bound state is on-shell if $u = 1$, and off-shell otherwise.

If $p^a \equiv 0$ with $\dot{\beta}^2 = 1$, then the relation (20) is not valid, and thus β must be null, $\dot{\beta}^2 = 0$. There are two bound states where this occurs: the photon A and the photon-gluon $\tilde{A}$. In both cases, the two strands are in a single orbital, and so there is no binding energy, $\delta = 0$. Therefore the relation (20) reduces to

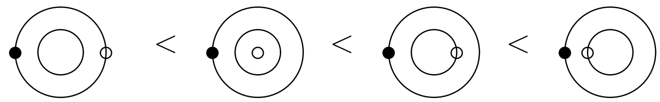
$$p^a = \sum_{\alpha} p^a_{\alpha}.$$

We say such a bound state is on-shell if the common phase velocity $v = \omega_\alpha |\boldsymbol{k}_\alpha|^{-1}$ of the two strands is lightlike, $v = 1$, and off-shell otherwise.

Although our model cannot retrodict the experimental values of the lepton, quark, and gauge boson masses, our model does exhibit surprising qualitative patterns that agree with experiment. Specifically, the masses of the bound states with fixed spatial group respect the following three orderings:

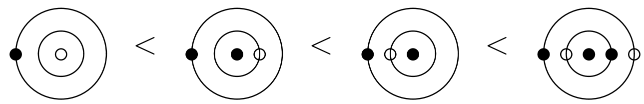
(1) If a bound state without a central strand has at most one strand (resp. two strands) of charge $w^\pm$, then adding a central strand lowers (resp. raises) the total mass of the bound state. This agrees with the experimental mass values:

bound states of two strands:



$O(3)$:	$0 = m(A)$	$<$	$m(\nu_e)$	$<$	$m(\nu_\mu)$	$<$	$m(\nu_\tau)$
$O(2)$:	$0 = m(\tilde{A})$	$<$	$m(d)$	$<$	$m(c)$	$<$	$m(t)$

with a central strand:



$O(3)$:	$m(\nu_e)$	$<$	$m(\mu)$	$<$	$m(\tau)$	$<$	$m(W)$
$O(2)$:	$m(d)$	$<$	$m(s)$	$<$	$m(b)$	$<$	$m(\tilde{W})$

FIGURE 9. Orderings of the particle masses based on the configurations of strands within the bound states, independent of the spatial group $O(3)$ or $O(2)$.

0 or 1 strand of charge w	2 strands of charge w
$m(e) > m(\nu_e)$	$m(\nu_\mu) < m(\mu)$
$m(d) > m(u)$	$m(\nu_\tau) < m(\tau)$
$m(c) > m(s)$	
$m(t) > m(b)$	

(2) The masses of the bound states (with fixed spatial group) consisting of precisely two strands follow the ordering given in Figure 9 (upper).
(3) The masses of the bound states (with fixed spatial group) containing a central strand follow the ordering given in Figure 9 (lower).

We emphasize that these orderings hold independently of the choice of fixed spatial group $O(3)$ (leptons) or $O(2)$ (quarks).

5. Wave-particle complementarity: a thought experiment

Imagine waking up in a large, completely black, empty room, so that there are no visible features that distinguish the different directions before you. If you stand up and walk in a straight line, any direction you choose to walk is indistinguishable from any other direction. In particular, walking five steps in one direction is indistinguishable from walking five steps in another direction. Thus, by the indistinguishable principle, regardless of what direction you choose to walk, *you will at arrive at the same location after five steps.* Therefore the circle of radius five steps, centered at your initial location O, becomes identified as a single 1-dimensional point.

In your simple act of walking, *you have unwittingly changed the topology of the flat floor you are walking on.* Suppose you walk a distance $r > 0$. Then the indistinguishable principle implies that the circle C of radius r, centered at O, becomes a single point. Consequently, the closed disc D (on the floor) with boundary $\partial D = C$ becomes topologically a 2-sphere. On this sphere, the point O and the boundary circle C are antipodal points; if we take these points to be the north and south poles, then the circle of radius $\frac{r}{2}$ centered at O may be mapped to the equator. Furthermore, the Gaussian curvature of this sphere will be concentrated at the point C: the curvature will be zero in the interior of D and infinite at the point C. The integral of the curvature over the entire disk D will be the curvature of the sphere itself, namely

$$\kappa = \frac{\pi^2}{r^2}.$$

Now suppose there are small tables placed throughout the room, each with a unique bell on it. You are not able to see these tables, however, since the room is completely black.

If, in your forward walking, you happen to stumble into a table, then its bell will ring. The bell will thus specify your location in the room. (This is analogous to wavefunction collapse.) At such a moment, the 1-dimensional circular point you are standing on will vanish, and you will once again be standing on a 0-dimensional point. Furthermore, the topology of the floor will resume its initial state of being perfectly flat.

If instead you happen to miss the table (unbeknownst to you), then the arc along the circle C that encounters the table will vanish from the 1-dimensional point you are standing on. If you miss $n \geq 2$ tables, say, then C will be reduced to a disjoint union of n circular arcs, all of which will remain identified as a one single point on the floor of the room. (This is analogous to partial wavefunction collapse, due to partial which-way information.)

Of course, this thought experiment would not actually hold on our macroscopic scale, because the atoms in the floor would detect your footsteps, and thus distinguish the direction you had chosen to walk. But on the level of classical fundamental particles moving about in a manifold, there would be nothing present to detect the direction of travel, and therefore a single particle would propagate as a wave that is 'spreading out'.

Now suppose a classical point particle has a unique position p in a fixed inertial frame of $\tilde{M}$. By the indistinguishable principle, the particle evolves in space as an expanding 2-sphere S_t centered at $\boldsymbol{p}$ (or more generally, the image of the exponential map of an expanding 2-sphere in the tangent space at $\boldsymbol{p}$). Each image $\pi(S_t)$ is a single point in space, and the image $\pi(\cup_{t>0} S_t)$ is a single point in spacetime M. Therefore the π-image of the ball $B_t \subset \tilde{M}$ with boundary S_t is a 3-sphere in M. Just as in the 2-dimensional case, the scalar curvature of the 3-sphere (namely, $R = 6\pi^2/r^2$ if $\tilde{M}$ is flat) resides entirely at the point $\pi(S_t) \in M$.

Thus, by the indistinguishable principle, a point particle propagates as an expanding spherical wave. The wave also has an internal phase $e^{i\theta}$ if the particle is a circular strand particle; see Section 3.9.

6. The quantum-to-classical transition: uniqueness of stationary paths

The underlying principle of classical mechanics may be stated thus:

Nature takes a single stationary path.

This principle follows from the Euler-Lagrange equations of motion.

Based on the Huygen-Fresnel principle in optics, Feynman interpreted Dirac's path integral for a free scalar field in $0+3$ dimensions (that is, for quantum mechanics) to fundamentally imply that [F]

Nature takes all possible paths.

However, we have shown that the indistinguishable principle implies wave-like propagation of fundamental classical particles, where each particle[4] follows a geodesic in spacetime unless acted upon by an external force. We thus propose:

Nature takes a set of all indistinguishable stationary paths,
and therefore
Nature takes a single stationary path.

The indistinguishable principle implies that our assumption reduces to the underlying principle of classical mechanics, namely, that nature takes a single stationary path. It also brings us closer to the original spirit of general relativity, where the geodesic hypothesis of particle motion is assumed to hold. Furthermore, our assumption allows the possibility that quantum field theory may be an approximation of a deeper underlying theory that is mathematically well-defined, since the statement 'Nature takes every possible path' is, at the outset, an ill-defined notion. Finally, the quantum-to-classical transition is a consequence of our assumption if the indistinguishability of stationary paths is a common feature of the microscopic scale, and rare at the macroscopic, or emergent, scale.

7. Quantum state reduction is the endpoint of a strand

7.1. Quantum state reduction.

Recall our assumption from Section 2 that quantum state reduction occurs whenever two or more states which were indistinguishable become distinguishable. In this section we address the question of what constitutes distinguishability in the context of strands.

[4]Or rather, its central worldline if it is a circular strand particle; see Section 3.1.

Since time does not flow along a strand, a strand particle cannot detect change, whence undergo any fundamental interaction, without simultaneously being annihilated. Therefore distinguishability occurs at a depicted spacetime point $p \in \tilde{M}$ if and only if p is the endpoint of a strand. Consequently,

Quantum state reduction occurs at a depicted spacetime point $p \in \tilde{M}$
if and only if p is the endpoint of a strand.

7.2. **Scattering and on-shell apexes.**

We assume that the standard Feynman rules hold for propagators and vertices in computing cross sections and decay rates, with two additional constraints arising from apexes and intersecting strands.

Definition 7.1. An apex is *on-shell* if both strands at the apex are on-shell, and otherwise the apex is *off-shell*.

In a scattering event with fixed incoming and outgoing particles, we propose that *only Feynman diagrams that share the same on-shell apexes can occur in superposition.* (The reason for this assumption is given in Section 8.) Therefore, certain scattering amplitudes obtained from strands will differ from those obtained from the full path integral $Z = \int \mathcal{D}\psi e^{i\hbar S[\psi]}$.

Tree-level electroweak Møller scattering ($e^-e^- \to e^-e^-$) and Bhabha scattering ($e^-e^+ \to e^-e^+$) are shown in Figure 10. In each diagram, the internal boson is necessarily off-shell since the legs are on-shell. Thus, there are no on-shell apexes in any of the diagrams (regardless of the signs of the strands). Therefore the on-shell apexes in the four diagrams trivially agree, and so the diagrams may exist in superposition.

In contrast, the electron-photon scattering diagram shown in Figure 11.i has two on-shell apexes.

New constraints are also expected to arise in the integration $\int d^4x$ of the Lagrangian density of the path integral due to the assumption that intersecting strands are the same point in spacetime M. These constraints could potentially eliminate certain ultraviolet divergences in quantum field theory. We leave these questions for future work.

7.3. **A photon at a beam splitter.**

Recall our description of electrons and photons from Section 4: an electron is a negative strand, a positron is a positive strand, and a photon is a bound state of the two.[5]

An on-shell electron absorbing and emitting an on-shell photon is shown in Figure 11.i. The electron first meets the positive strand of the photon, and the two strands annihilate at an on-shell apex. At this precise point, the quantum state of the system is reduced. This may constitute, for example, the transfer of which-way information

[5] An electron and positron may also form the unstable hydrogen-like bound state of positronium.

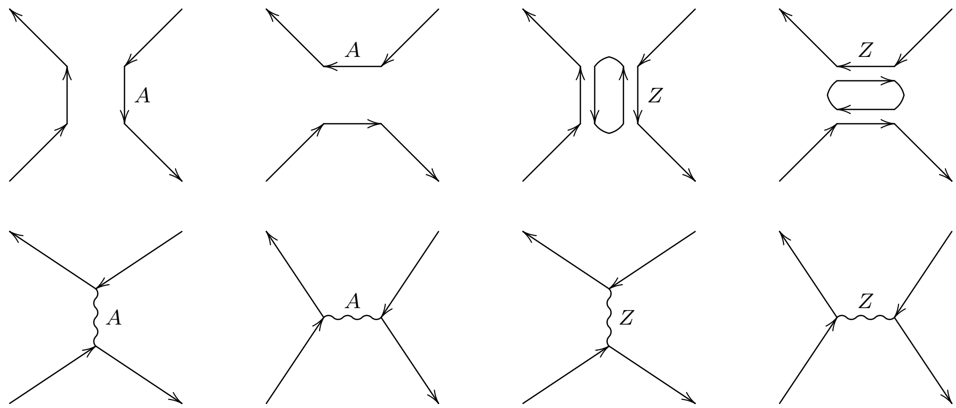


FIGURE 10. Tree-level electroweak electron-electron (Møller) scattering, or electron-positron (Bhabha) scattering, drawn topologically with strand diagrams (top) and Feynman diagrams (bottom).

from the photon to the surviving electron, or a measurement of the initial electron's position. After annihilation, the surviving negative strand propagates as a free electron. A strand pair is then created at a second on-shell apex; the new positive strand binds with the electron to form a new photon, and the negative strand escapes as a free electron.

In Figure 11.ii, a similar electron-photon scattering takes place, but no strand terminates. This strand configuration therefore shows how an electron and photon can interact, and thus influence each other, without reducing the quantum state of the system. The configuration could describe, for example, the passage of a photon through a polarizer or beam splitter, where the photon's quantum state does not collapse during the scattering.

The case of a photon meeting a beam splitter is shown in Figure 12. Suppose the photons location is put in superposition through its interaction with an electron in the beam splitter. Upon meeting the electron, the positive and negative strands of the photon each branch into two indistinguishable strands: one transmitted and one reflected. These two strands trace out distinct paths in the depiction $\tilde{M}$ of spacetime M, and thus the photon appears to take two paths. However, these two paths are the same path in spacetime itself.

8. Spacetime curvature from strand apexes

Suppose a photon passes through a beam splitter, as shown in Figure 12. In the framework of strands, the two copies of the photon are both physically 'real'. We may even suppose that the energy of the two copies are equal, and equal to the initial photon. This does not pose a problem with respect to energy-momentum conservation: the two copies are really one and the same photon sitting at the same

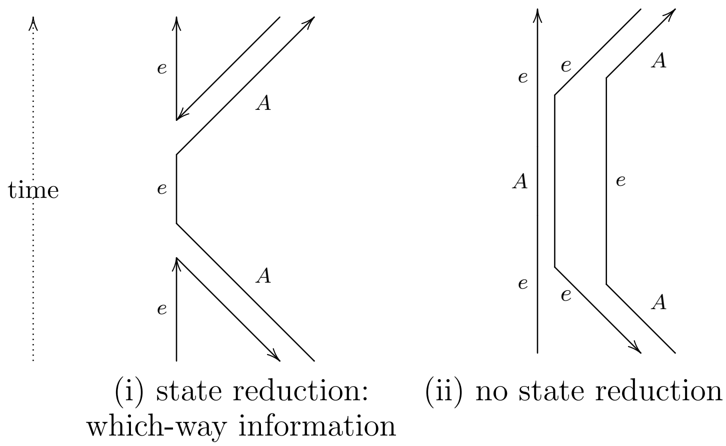


FIGURE 11. How quantum state reduction may arise in the interaction of a photon and an electron. The scattering (i) contains two on-shell apexes, and the scattering (ii) contains no apexes.

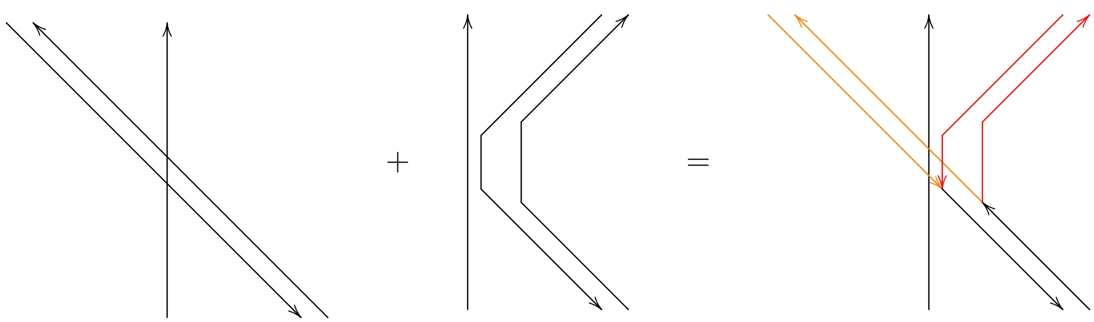


FIGURE 12. Superposition of a photon passing through a beam splitter (here a single electron).

point in spacetime M (although in different points in depicted spacetime $\tilde{M}$), and so the total photon energy is not doubled when the photon passes through the beam splitter.

However, there is an irreconcilable problem of our two physically real copies of the photon with Einstein's equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ given by Bohr's gedankenexperiment: each copy of the photon has energy, and thus produces gravitational radiation as it propagates. But this gravitational radiation transmits which-way information, in contradiction to our assumption that superposition arises from indistinguishability.

To remedy this problem, we propose that spacetime curvature is caused only by on-shell apexes. The more thermal energy a massive object has, the more interactions takes place among its constituent elementary particles, and thus the greater the number of on-shell apexes occur in its interior.

To be precise, consider an on-shell apex $q \in \tilde{M}$ where two strands α, $\tilde{\alpha}$ are created or annihilated. Let β, $\tilde{\beta}$ be their respective central wordlines, and let

$$p^a = \omega \dot{\beta}^a, \qquad \tilde{p}^{\tilde{a}} = \tilde{\omega} \dot{\tilde{\beta}}^{\tilde{a}}$$

be their four-momenta. Recall that the strands at an apex have opposite charge, $q(\alpha) = -q(\tilde{\alpha})$. However, the strands need not have equal energies. Indeed, energy-momentum conservation always holds at a splitting of bound states (that is, at Feynman vertices), but need not hold at an individual apex involved in a splitting.

Definition 8.1. Set

$$d^\mu := \operatorname{sgn}(\alpha) p^\mu + \operatorname{sgn}(\tilde{\alpha}) \tilde{p}^\mu.$$

We define the energy-momentum tensor at the on-shell apex q to be

$$T_q^{\mu\nu} = |d^0|^{-1} d^\mu d^\nu \delta^{(4)}(x - q).$$

Einstein's equation is thus modified to

$$G_{\mu\nu} = 8\pi \sum_q (T_q)_{\mu\nu}.$$

Spacetime curvature (of $\tilde{M}$) is thus classical, and in particular there are no gravitons.[6]

Remark 8.2. Informally, this idea unifies electromagnetism with gravity, in the sense that if we turn the strength of the electric charge e to zero, so that the positive and negative strands are no longer attracted to each other, then spacetime curvature, and thus gravity, would disappear.

9. Conclusion

Nonnoetherian spacetime is a spacetime where the worldlines of particles may be single one-dimensional points, called strands. Such a geometry is obtained by incorporating Aristotle's notion that time passes only if something changes into general relativity. The purpose of this geometry is to describe quantum nonlocality in a spacetime framework. We introduced a model, generalizing 't Hooft's double line formalism, where leptons, quarks, and gauge bosons are bound states of strands. Our model establishes a new symmetry between leptons and quarks, and predicts the existence of charged and neutral massive gluons that are cousins of the $W^\pm$ and Z bosons. We also presented a thought experiment to show how a classical particle may propagate as a wave in nonnoetherian spacetime. Finally, we proposed a slight modification of Einstein's equation in which it is not energy, but the creation and annihilation of on-shell strands, that causes the curvature of spacetime. The framework we have presented offers a new interpretation of quantum theory that is closely modeled on the principles of general relativity.

[6] For different theories of classical(-like) gravity in a quantum setting, see for example [P1, P2, D, O].

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Institut für Mathematik und Wissenschaftliches Rechnen, Universität Graz, Heinrichstrasse 36, 8010 Graz, Austria.
Email address: charles.beil@uni-graz.at