Identification with External Instruments in Structural VARs under Partial Invertibility

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IV in Macroeconomics

- **New and increasingly** popular method for **Macroeconometrics**

$$z_t = \alpha u_t^i + \eta_t \qquad \eta_t \sim \mathcal{WN}(0, \sigma_\eta^2)$$

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- Wealth on **new instruments** and expanding Macro literature:
  - **oil shocks** (e.g. Hamilton, 2003; Kilian, 2008; Känzig, 2018)
  - **fiscal spending shocks** (e.g. Ramey, 2011; Ricco et al., 2016; Ramey and Zubairy, 2018)
  - **tax shocks** (e.g. Romer and Romer, 2010; Leeper et al., 2013; Mertens and Ravn, 2012; Mertens and Montiel-Olea, 2018)
  - **conventional monetary policy shocks** (e.g. Romer and Romer, 2004; Gürkaynak et al., 2005; Gertler and Karadi, 2015; Zanetti and Li 2016; Miranda-Agrippino and Ricco, 2017; Jarocinski and Karadi 2017)
  - **unconventional monetary policy shock** (e.g. Miranda-Agrippino and Ricco, 2018)
  - **government asset purchases** (Fieldhouse et al., 2017; Fieldhouse et al., 2018)
  - **confidence shocks** (Lagerborg et al., 2018)
  - **technology news** (e.g. Miranda-Agrippino et al., 2018; Cascaldi-Garcia, 2019)

IV in Macroeconomics

- **New and increasingly** popular method for **Macroeconometrics**

$$z_t = \alpha u_t^i + \underbrace{(\ldots \qquad \ldots)}_{\text{contamination}} + \eta_t \qquad \eta_t \sim \mathcal{WN}(0, \sigma_\eta^2)$$

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Usual Conditions for SVAR-IV

Stock (2008), Stock and Watson (2012, 2018) and Mertens and Ravn (2013)

Reduced-Form VAR

$$A(L)Y_t = \nu_t$$

Conditions – Identification in SVAR-IV

(i) $\mathbb{E}[u_t^1 z_t] = \alpha$ *(Relevance)*

(ii) $\mathbb{E}[u_t^{2:n} z_t] = 0$ *(Contemporaneous Exogeneity)*

(iii) $u_t = Proj(u_t | Y_t, Y_{t-1}, \ldots)$ *(Fundamentalness/Invertibility)*

What if a SVAR is not Fully Invertible?

1. Is identification possible in SVAR-IV
2. Under which conditions?

This Paper

1. Relax full invertibility conditions $\Longrightarrow$ IV/Proxy-SVAR under **partial invertibility**

$$u_t^i \propto \lambda' \nu_t$$

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   $\Longrightarrow$ **Intuition**: No contamination at lags/and leads from non-invertible shocks

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- **Simulation**
- **Empirical application** to **MP shocks**

IV Identification & Invertibility

Wold Representation & Identification

- For any weakly stationary process Y_t the **Wold Representation Theorem** guarantees that

$$Y_t = \eta_t + C(L)\nu_t \qquad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu)$$

  - $C(L)$ is a causal, time-independent, square summable filter and η_t a deterministic term
  - ν_t is the **Wold innovation** that belongs to the space of present and past values Y_t

$$\nu_t = Y_t - Proj(Y_t | Y_{t-1}, Y_{t-2}, \dots)$$

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- If all structural shocks u_t are Y_t–fundamental

$$u_t \simeq Proj(u_t | Y_t, Y_{t-1}, \dots)$$

$\Longrightarrow$ u_t and ν_t generate the same space ($\mathcal{H}_t^u \equiv \mathcal{H}_t^\nu \ \forall t$)

$$\boxed{\nu_t = \Theta_0 u_t}$$

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- **Impulse Response Functions** (can be estimated by a **SVAR**)

$$Y_t = \eta_t + C(L)\Theta_0 u_t \qquad u_t \sim \mathcal{WN}(0, \mathbb{I}_n)$$

Partial Invertibility

- A shock u_t^1 is Y_t–fundamental and invertible if

$$\boxed{u_t^1 = Proj(u_t^1 | Y_t, Y_{t-1}, \dots)}$$

- Linear combination of the innovations ν_t

$$\boxed{u_t^1 = \lambda' \nu_t}$$

Semi-structural MA representation

Proposition – Semi-structural Moving Average Representation

Let the Wold representation of a covariance stationary vector process Y_t *be*

$$\boxed{Y_t = C(L)\nu_t \qquad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu)} \tag{1}$$

where Σ_ν *is the positive definite variance-covariance matrix of Wold innovations. If the system is partially invertible in the shocks* u_t^i*, for* $i = 1, \ldots, m$*, i.e. there exist* m *vectors* λ_i *such that* $\lambda_i'\nu_t = u_t^i$*, then* Y_t *admits a semi-structural moving average representation of the form*

$$\boxed{Y_t = C(L)\Sigma_\nu \sum_{i=1}^{m} \lambda_i u_t^i + C(L)\Sigma_\nu \tilde{\lambda}\xi_t} \tag{2}$$

where ξ_t *is an* $(n - m) \times 1$ *vector of linear combinations of Wold innovations that is orthogonal to all* u_t^i *for* $i = 1, \ldots, m$*, i.e.* $\mathbb{E}(u_t^i \xi_t') = 0$*.*

Proof

Semi-structural MA representation

- The Wold representation factorise in two **orthogonal** terms:
  - the first depends only on the **partially-invertible** shocks at **time** t
  - the second on **past, current and future** other **non-invertible shocks**

- **Intuition:**

$$\nu_t = \Theta_0 B(L) u_t = \widetilde{B}(L) u_t = \left(b_1 \;\; b_2(L)\right) u_t$$

- The IRFs of a **partially identified** SVARs are the the dynamic causal responses to **partially invertible** shocks

Identification in SVAR-IV under Partial Invertibility

Conditions – Identification in SVAR-IV under Partial Invertibility

Let $u_t^{1:m}$ denote the m invertible structural shocks in the system, and $u_t^{m+1:n}$ the remaining $n - m$ non-invertible shocks. Let z_t be an instrument for the shock of interest u_t^1, and define $z_t^{\perp} = z_t - Proj\left(z_t | \mathcal{H}_{t-1}^{Y}\right)$.

The impact effects of u_t^1 onto Y_t and the (relative) IRFs are correctly identified in a SVAR-IV if z_t satisfies the following conditions:

(i) $\mathbb{E}[u_t^1 z_t] = \alpha$ *(Relevance)*

(ii) $\mathbb{E}[u_t^{2:n} z_t^{\perp}] = 0$ *(Contemporaneous Exogeneity)*

(iii) $\mathbb{E}[u_{t-j}^{m+1:n} z_t^{\perp}] = 0$ *for all* $j \neq 0$ *for which* $\mathbb{E}[u_{t-j}^{m+1:n} \nu_t'] \neq 0$. *(Limited Lead-Lag Exogeneity)*

Proof

Identification in SVAR-IV under Partial Invertibility

Condition (iii) arises because of the dynamics

- If **all the shocks are invertible**
  $\Longrightarrow$ Condition (iii) is trivially satisfied

- If **some** of the shocks are **non-invertible**
  $\Longrightarrow$ The instrument can 'safely' incorporate past and future invertible shocks only

- If **all of the other** shocks are **non-invertible**
  $\Longrightarrow$ The instrument can 'safely' incorporate only past/future of the shock of interest

Identification in SVAR-IV under Partial Invertibility

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What if **Condition (iii)** is **violated**?

Identification in SVAR-IV under Partial Invertibility

Remark – Violation of the Exogeneity Conditions

Let z_t be an instrument that satisfies Condition (i) but possibly fails Condition (ii) and Condition (iii), i.e.

$$z_t = \alpha u_t^1 + \sum_k \beta_k u_{t-k}^{\not{1}} \; , \tag{3}$$

where $u_t^{\not{1}}$ is a non-invertible shock, for $k \in \mathbb{Z}$. The Wold representation can be mapped into the structural shocks employing Blaschke factors

$$\nu_t = \left(b_1 \;\; b_2(L) \right) u_t \; , \tag{4}$$

The estimated IRFs for variable i, to shock 1, at horizon h, are biased and of the form

$$\widetilde{IRF}_{i1}^{h} = IRF_{i1}^{h} + \left[C_h \sum_{j \in J} \sum_{k \in K} b_{2,j,\not{1}} \frac{\beta_k}{\alpha} \delta_{jk} \right]_i \; , \tag{5}$$

Identification in SVAR-IV and LP-IV

Proposition – Relation between SVAR-IV under Partial Invertibility and LP-IV$^{\perp}$

Let Z be the set of scalar stochastic processes z_t that satisfy LP-IV Conditions (i) and (ii) – i.e. $\mathbb{E}[u_t^1 z_t] = \alpha$ and $\mathbb{E}[u_t^{2:n} z_t] = 0$ –, but satisfy Condition LP-IV (iii) $\mathbb{E}[u_{t-j} z_t] = 0$ only for $j < 0$ and not for $j > 0$. Let us also assume that $Proj(u_t | \mathcal{H}_{t-1}^{Y}) = 0$. Let $\widetilde{Z} \subseteq Z$ be such that any $z_t \in \widetilde{Z}$ satisfies the LP-IV$^{\perp}$ conditions for $\mathcal{W}_t \equiv \mathcal{H}_{t-1}^{Y}$. z_t is an element of $\widetilde{Z}$ if and only if it identifies the shock of interest in a Structural VAR in Y_t.

Identification in SVAR-IV and LP-IV

1. If u_t^i is invertible & z_t is not contaminated by **leads of invertible shocks** or **non-invertible shocks**
   $\Longrightarrow$ **LP-IV** and **SVAR with external/internal instrument** correctly identifies IRFs

Identification in SVAR-IV and LP-IV

1. If u_t^i is invertible & z_t is not contaminated by **leads of invertible shocks** or **non-invertible shocks**
   $\Longrightarrow$ **LP-IV** and **SVAR with external/internal instrument** correctly identifies IRFs

2. If u_t^i is **non-invertible** & z_t is **not contaminated** by **non-invertible shocks**
   $\Longrightarrow$ only **LP-IV** and **SVAR with internal instrument** correctly identifies IRFs

3. If u_t^i is **invertible** & z_t is **contaminated** by **leads of invertible shocks**
   $\Longrightarrow$ only **SVAR-IV** correctly identifies IRFs

An Observation on VAR Misspecifications

Let us consider a **stationary VARMA(p,q) process** $Y_t = (y_{1,t}' \;\; y_{2,t}')'$

$$\begin{pmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{pmatrix} \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix}.$$

Fitting a VAR(k) to $y_{1,t}$ corresponds to imposing some or all of the **following restrictions**

$$\begin{aligned} \Phi_{11,i} &= 0, & i &= k+1, k+2, \dots p, \\ \Phi_{12,i} &= 0, & i &= 1, 2, \dots p, \\ \Psi_{11,i} &= 0, & i &= 1, 2, \dots q, \\ \Psi_{12,i} &= 0, & i &= 1, 2, \dots q. \end{aligned}$$

Some of these restrictions may not be not reflected in the DGP $\Longrightarrow$ **Misspecification**!

An Observation on VAR Misspecifications

- The Wold Representation Theorem guarantees the **existence of an ARMA representation** of the form

$$\widetilde{\Phi}_1(L) y_{1,t} = \widetilde{\Psi}_1(L) \nu_t \ .$$

- The **shocks** u_t are **trivially non-invertible** in $y_{1,t}$ ($m < n$ innovations ν_t)

- If the system is **partially invertible** in a shock u_t^i,

$$\lambda' \nu = \kappa u_t^i$$

$\Longrightarrow$ **retrieve impact effects** of the shock u_t^i onto $y_{1,t}$

- Dynamics of the system **asymptotically approximated** by infinitely many lags of $y_{1,t}$ only

Partial Invertibility in a Simulated System

Partial Invertibility in a Simulated System

- Standard **NK model**
- **Technology news** shocks

$$a_t = \rho_a \, a_{t-1} + \sigma_a u^a_{t|t-4} \ ,$$

- Gov't spending with **fiscal foresight**

$$g_t = \rho_g \, g_{t-1} + u^g_{t-4} \ ,$$

- Taylor rule with smoothing

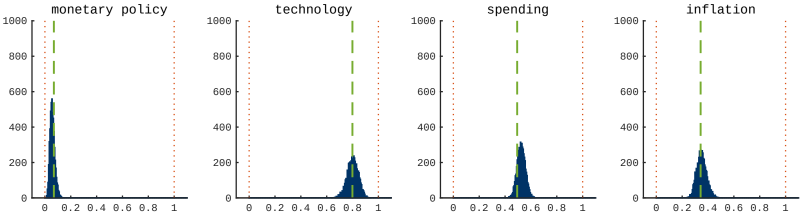
$$r_t = \rho_r \, r_{t-1} + (1 - \rho_r) \left(\phi_\pi \overline{\pi_t} + \phi_y \overline{\Delta y_t} \right) + u^r_t$$

- We **simulate 5000 economies** each of sample size $T = 300$ periods

Partial Invertibility in a Simulated System

- Estimate a **VAR(4)** in four observables (y_t, g_t, π_t, i_t)
- Model **fails the 'poor man's invertibility condition'** (Fernandez-Villaverde et al., 2007)
- **Degree of invertibility** of structural shocks (Sims and Zha 2006, Forni et al., 2018)

$$\delta_i = \mathrm{var}[u_t^i - Proj(u_t^i | \mathcal{H}_t^Y)]/\sigma_i^2,$$



- System is **partially invertible** in the monetary policy shock

Partial Invertibility in a Simulated System

Table 1: Variance Decomposition

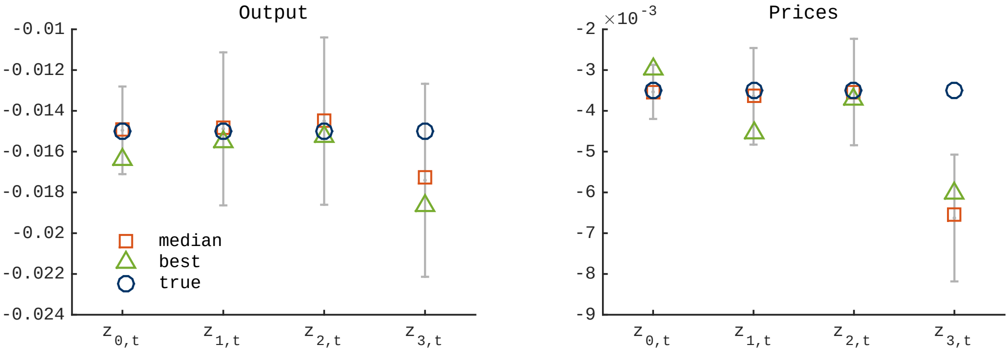
		u_t^r	u_t^a	u_t^g	u_t^π
output	y_t	16.45	77.01	0.98	12.95
spending	g_t	0.00	0.00	61.91	0.00
inflation	π_t	9.07	51.34	0.01	67.03
policy rate	r_t	25.32	19.81	0.15	14.47

Partial Invertibility in a Simulated System

- Four different external instruments:

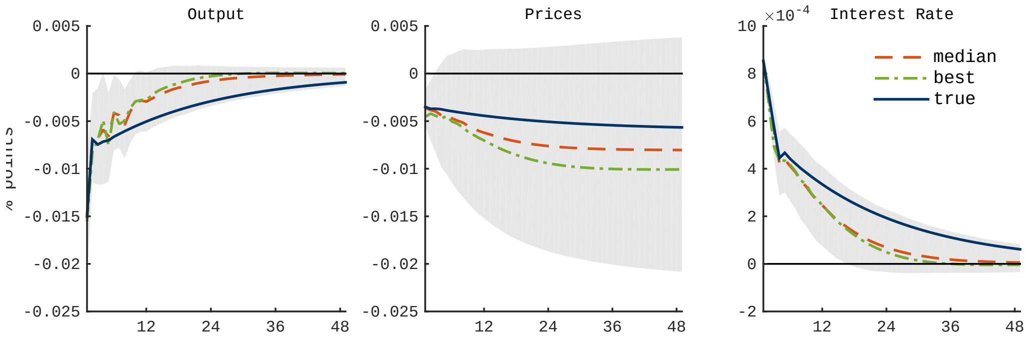
$$
\begin{aligned}
z_{0,t} &= u_t^r \\
z_{1,t} &= 0.7u_t^r - 0.5u_{t-2}^r + \varsigma_t \\
z_{2,t} &= 0.7u_t^r - 0.5\left(u_{t-1}^g + u_{t-2}^g + u_{t-3}^g\right) + \varsigma_t \\
z_{3,t} &= 0.7u_t^r + 0.5\left(u_{t-1}^a + u_{t-2}^a + u_{t-3}^a\right) + \varsigma_t
\end{aligned}
$$

Estimated Impact Coefficients



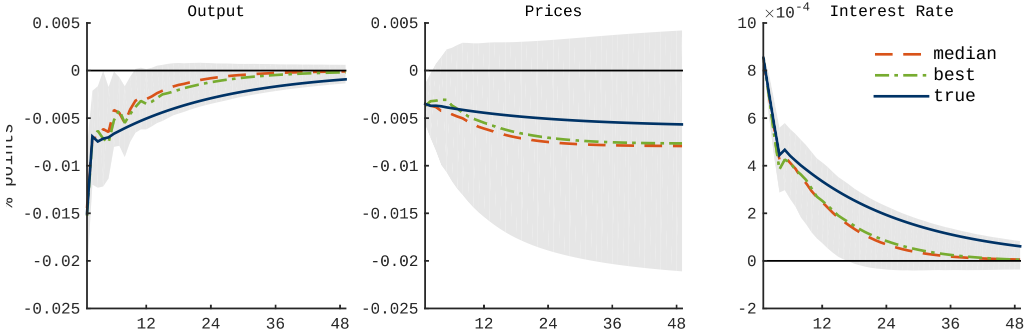
Estimated IRFs

$z_{1,t}$ – external instrument correlates with monetary policy shock only



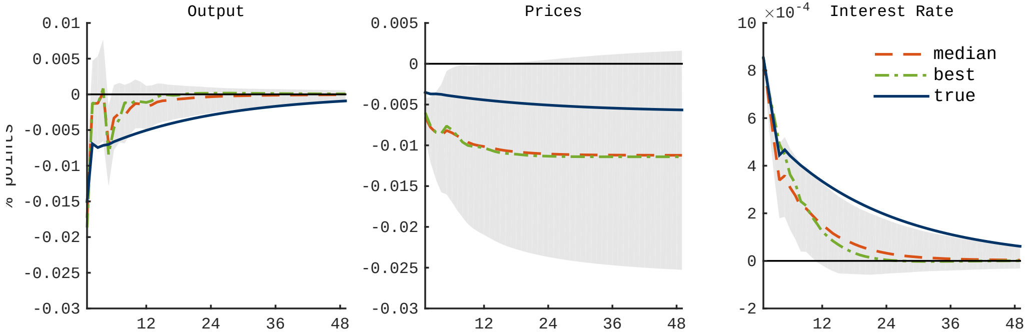
Estimated IRFs

$z_{2,t}$ – external instrument also correlates with lagged spending shocks



Estimated IRFs

$z_{3,t}$ – external instrument also correlates with lagged technology shocks



An Application to HFI of MP Shocks

High Frequency Identification of MP Shocks

- **High-Frequency Instruments**
  - $z_{A,t}$ HF surprises at scheduled meetings only (Stock and Watson 2018, Caldara and Herbst 2019)
  - $z_{B,t}$ moving average of HF surprises within the month (Gertler and Karadi 2015)
  - $z_{C,t} = z_{A,t}$ corrected for Information Effects (Miranda-Agrippino, Ricco 2018)
- **Empirical Models**
  - VAR(12) in 5 variables
  - VAR(2) in 3 variables

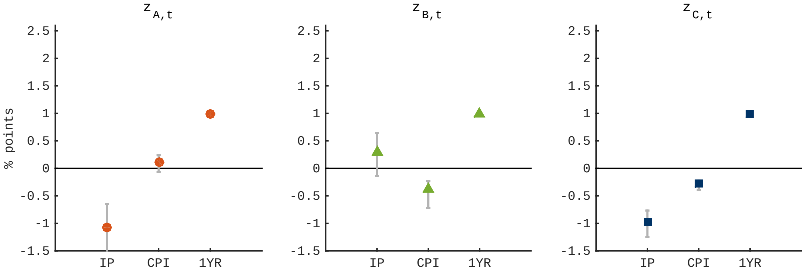
Information in instruments for MP

Table 2: Contamination of Monetary Policy Instruments

$H_0 : \beta_{f_{1,t-1}} = \beta_{f_{2,t-1}} = \ldots = \beta_{f_{10,t-1}} = 0$			
	$z_{\mathrm{A},t}$	$z_{\mathrm{B},t}$	$z_{\mathrm{C},t}$
$F_{(10,227)}$	2.12 (0.0240)		
$F_{(10,226)}$		3.52 (0.0002)	
$F_{(10,215)}$			1.77 (0.0669)
N	239	238	227

Responses to MP

Impact Responses to Monetary Policy Shocks – 1979:2012



Responses to MP

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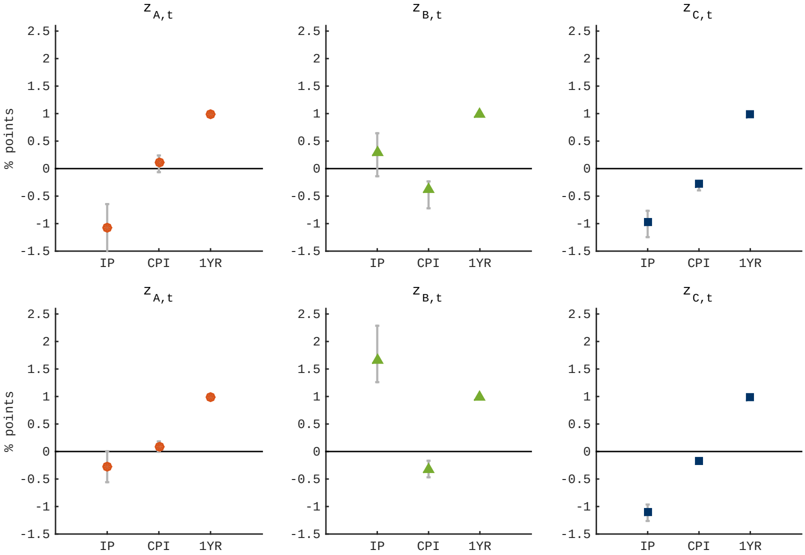
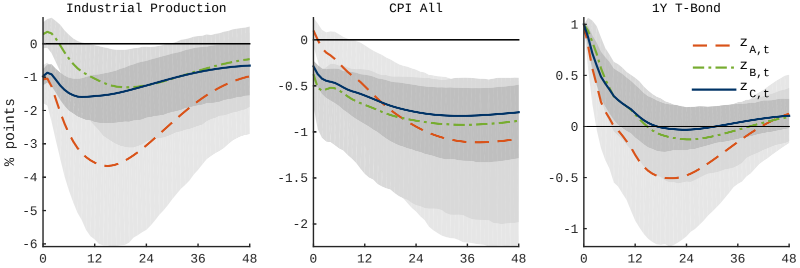


Responses to MP: Misspecified VAR

IRFs to Monetary Policy Shocks – 1979:2012



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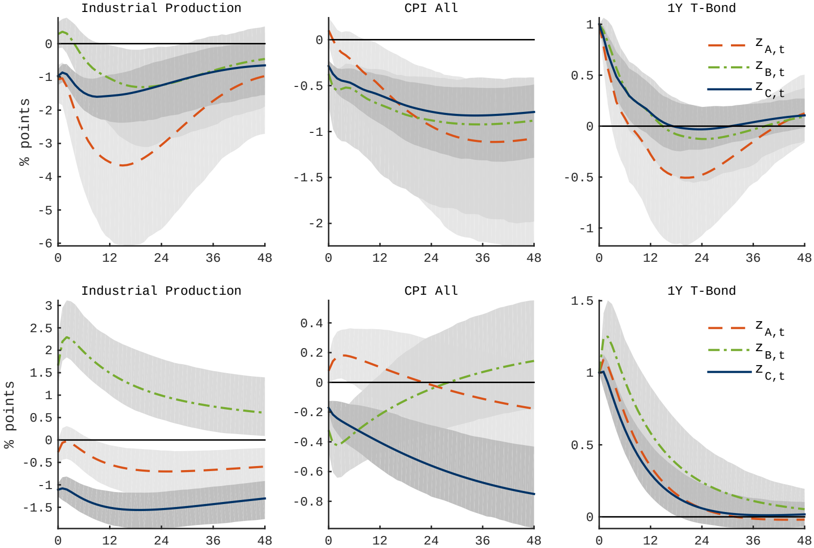

Conclusions

- Identification with IV in SVARs under partial invertibility
- **Partial invertibility** is a general and **not very stringent condition**
- SVAR-IVs (and LP-IV with controls) require
  - standard **relevance** and **contemporaneous exogeneity** conditions
  - a **limited lead-lag exogeneity** condition
- **SVAR-IV** and LP-IV with controls provide **robust results** in a variety of settings

Semi-structural MA representation

Proof.

- $\exists$ Nonsingular $n \times n$ matrix $\Lambda = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix}$, where λ is an $n \times m$ matrix, and $\tilde{\lambda}$ is an $n \times (n-m)$ matrix such that

$$\Lambda' \nu_t = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \nu_t = \begin{pmatrix} u_t^{1:m} \\ \xi_t \end{pmatrix} \quad (6)$$

- and

$$\Lambda' \Sigma_\nu \Lambda = \Lambda' \mathbb{E}\left[\nu_t \nu_t'\right] \Lambda = \mathbb{E}\left[\begin{pmatrix} u_t^{1:m} \\ \xi_t \end{pmatrix} \begin{pmatrix} u_t^{1:m\prime} & \xi_t' \end{pmatrix}\right] = \mathbb{I}_n \ . \quad (7)$$

- Since $\Sigma_\nu = (\Lambda\Lambda')^{-1}$, it follows that $\Sigma_\nu \Lambda\Lambda' = \mathbb{I}_n$
- Using this identity, it is possible to write

$$Y_t = C(L)\nu_t = C(L)\Sigma_\nu \Lambda\Lambda' \nu_t = C(L)\Sigma_\nu \lambda u_t^1 + C(L)\Sigma_\nu \tilde{\lambda} \xi_t \ ,$$

that is the representation in Eq. (2). Back...

Identification in SVAR-IV under Partial Invertibility

Proof.

- u_t^1 a partially invertible structural shock

$$Y_t = C(L)\Sigma_\nu \sum_{i=1}^{m} \lambda_i u_t^i + C(L)\Sigma_\nu \tilde{\lambda}\xi_t \ ,$$

where ξ_t is a linear combination of leads and lags of the remaining $(n-1)$ structural shocks $u_t^{2:n}$, some of which may be non-invertible

- Conditions (i) to (iii) imply that

$$\mathbb{E}[\nu_t z_t] = \mathbb{E}\left[\Sigma_\nu \Lambda\Lambda' \nu_t z_t\right] = \Sigma_\nu \Lambda \mathbb{E}\left[\begin{pmatrix} u_t^1 \\ u_t^{2:m} \\ \xi_t \end{pmatrix} z_t\right] = \Sigma_\nu \begin{pmatrix} \lambda_1 & \dots & \lambda_m & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \mathbb{E}[u_t^1 z_t] \\ \mathbb{E}[u_t^{2:m} z_t] \\ \mathbb{E}[\xi_t z_t] \end{pmatrix} = \alpha \Sigma_\nu \lambda_1$$

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