A Fluid Model of Recombination and Ionization for Deduction of Flow Velocity in Magnetized Plasmas

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Received 21 September 2007, accepted 23 January 2008
Published online 13 June 2008

Key words Recombination, ionization, plasma flow, Probes, edge plasma.
PACS 52.70.-m, 52.70.Nc, 52.30.-q, 52.40.Hf

For the analysis of atomic processes such as recombination and ionization in the divertor plasmas, a new fluid model on the ion collection to a probing object in magnetized plasma is developed including electron-ion recombination, molecular activated recombination and ionization. Density profiles and ratio of ion saturation currents are presented with ionization and recombination for the application to the Mach probe.

1 Introduction

Although significant ion drift resulting from scrape-off flow plays an important role in impurity transport and fluctuation levels, it is difficult to accurately measure the flows [1]. As for the collisionless models, one-dimensional kinetic analysis has been performed for the free pre-sheath with drift flow [2]. A fluid model is presented and solved numerically for the bounded pre-sheath in terms of normalized viscosity [3]. Analytic solutions for both the bounded and free pre-sheaths are obtained from a fluid model with different approximation for the cross-field contribution of viscous force [4]. Along the magnetized presheath formed by the probes, not only the diffusive ion source in the magnetized pre-sheath with Boltzmann electrons [4], but also other sources such as charge exchange [5], ionization [6] [7], and recombination [8] are important in the scrape-off layer of divertor-type tokamaks. Recombination and charge exchange processes in the fusion edge plasmas are important for the reduction of particle and heat fluxes onto the divertor targets. Molecular activated recombination (MAR) processes induced by the hydrogen or hydrocarbon puffing recently have shown the capability of contributing to the volume recombination [8, 9], along with electron-ion recombination processes characterized by radiative and three-body recombinations. Especially, hydrogen-MAR process can play a major role in cooling the plasma for low electron temperature of 1-3 eV [10]. The conditions for each process to be effective strongly depend on electron temperature and electron density. The goal of this paper is to develop a fluid model of ion collection including recombination and ionization in order to apply to deduce the Mach numbers in strong magnetic field.

2 Model

Using the Mach probe, composed of two opposite directional probes, the ratio ($R$) of ion saturation currents are measured in the upstream and downstream directions with strongly negative bias modes (e.g., $V_b = -150 \, V$), then the relevant Mach numbers ($\hat{M}_\infty$), is deduced using appropriate theories of recombining and ionizing plasmas in the magnetized plasma. For this, consider a Boltzmann transport equation of ions without volumetric sources such as ionization and recombination, assuming $\vec{E} = E_z \hat{z}$ and $\vec{B} = B \hat{k}$

$$v_x \frac{\partial f_i}{\partial z} + \frac{q}{m} E_z \frac{\partial f_i}{\partial v_z} = -v_x \frac{\partial f_i}{\partial x},$$

(1)

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where the cross-field transport source can be approximated as [2]

\[
S_t \equiv -v_z \frac{\partial f_i}{\partial \xi} \approx W \left[ \alpha (f_{i\infty} - f_i) + (1 - \alpha)(1 - \frac{n}{n_{i\infty}})f_{i\infty} \right],
\]

(2)

where \(n_i, n_{\infty}, f_i, f_{i\infty}, V_z, W\) and \(\alpha\) are ion density along (or within) the pre-sheath, the pre-sheath, atomic ion distribution along the pre-sheath, atomic ion distribution outside the pre-sheath with the drift velocity \(V_d\), flow velocity, cross-field frequency (\(\equiv D_\perp/a^2, D_\perp = \text{anomalous cross-field diffusivity, } a = \text{probe radius}\) and normalized shear viscosity (\(\equiv \eta_\perp/n_i m D_\perp\), respectively. One can add ionization, recombination and charge exchange to the above equations for the general cases. The ionization source, which is dominant for \(T_e < 2\text{ eV}\) in hydrogen plasma, can be either (a) \(S_i = <\sigma v>_\text{ion} n_e f_{i\infty}(v)\) from Bissell and Johnson (BJ) [11], or (b) \(S_i = <\sigma v>_\text{ion} n_e f_{i\infty}(v)|v|/C_e\) from Emmert, et al. (EE) [12], where \(f_{i\infty}(v)\) is the distribution function of atomic neutrals. BJ’s case seems to be reasonable, but their model cannot recover the Maxwellian distribution as \(z \to \infty\), while EE’s case can become Maxwellian although their ionization term does not seem to reasonable, because there is no source particle with zero velocity. We adopt BJ’s case along with cross-field transport source, then we can recover \(f_{i\infty}\) as \(z \to \infty\), and obtain the similar sheath values as those by a kinetic model [2].

As for the recombination, there are two cases: \(S_r = -<\sigma v>_\text{rec} n_e f_i(v)\) for electron-ion recombination (EIR), which is dominant for low temperature (\(T_e < 2\text{ eV}\) in hydrogen plasma, and \(S_r = -<\sigma v>_\text{rec} n_e f_M(v)\) for molecular-activated recombination (MAR), which exists over the wide range of electron temperature in hydrogen plasma. Here \(f_M(v)\) is the distribution function of relevant molecular ions. Dominant hydrogen-MAR processes are (a) dissociative attachment (DA: \(H_2 + e \to (H_2^+) \to H^+ + H\)) followed by mutual neutralization (MN: \(H^- + H^+ \to H^+ + H\)), (b) ion conversion (IC: \(H_2 + A^+ \to (AH)^+\)) followed by dissociative recombination (DR: \((AH)^+ + e \to A + H^*\)), and (c) charge exchange ionization (CX: \(H_2 + A^+ \to H_2^+ + A\)) followed by dissociative recombination (DR: \(H_2^+ + e \to H + H^*\)). With negligible negative hydrogen ions (neglecting channel (a)), DR is prevailing, yet channel (b) is dominant than channel (c) [10]. Hence the reaction rate of hydrogen MAR can be approximated as

\[
<\sigma v>_\text{MAR} \approx <\sigma v>_\text{MN-DA} + <\sigma v>_\text{DR-IC} + <\sigma v>_\text{DR-CX} \approx <\sigma v>_\text{DR-IC},
\]

where \(A\) is a neutral atom such as H, He or Ar, since some portion of \((AH)^+\) are dissociated into \(A\) and \(H^*\) after recombination with electron. Here the second term of the left hand side is larger than the third, although for hydrogen plasma they are not distinguishable [10]. Since molecular ion density \((n_M \approx n_i((AH)^+))\) is increased by IC, while it is decreased by DR, the contribution of DR to MAR can further be approximated as

\[
<\sigma v>_\text{DR-IC} n_e f_M \approx (1 - \delta) <\sigma v>_\text{DR} ((AH)^+ + e \to A + H^*) n_e f_M
\]

where \(\delta (\equiv <\sigma v>_\text{non-IC}/<\sigma v>_\text{IC})\) is the ratio of non-IC process among IC, so that \(1 - \delta\) is the probability that DR occurs after IC. For example, the rate of MAR for hydrogen molecules, \(<\sigma v>_\text{MAR} \approx 3 \times 10^{-16}\text{ m}^3/\text{sec}\), assuming \(n(H_2)/n_e \approx 0.1\) and \(T_e \approx T(\text{heavy particles}) \approx 1\text{ eV}, (T_e < 4 \sim 5\text{ eV})\) and \(n_N \approx n_i\), indicating that main channel of the plasma recombination is the dissociative recombination of the molecular ions for \(10^{20} < n_e < 10^{21}\text{ m}^{-3}\) [8, 13]. So plasma volume source of atomic processes becomes

\[
S_a = -K_M n_e f_M(v) - K_N n_e f_N(v) + K_T f_N(v) - K_f f_N(v),
\]

(3)

where \(f_M, f_i, f_N, K_M, K_N, K_T\) and \(K_f\) are the distributions and reaction rates of molecular ions, atomic ions, atomic neutrals, the reaction rates of molecular-activated recombination (\(\equiv <\sigma v>_\text{MAR}\)), electron-ion recombination (\(\equiv <\sigma v>_\text{EIR}\)) and ionization (\(\equiv <\sigma v>_\text{ION}\)), respectively. Then the Boltzmann equation with cross-field transport, recombinant and ionization sources becomes

\[
v_z \frac{\partial f_i}{\partial z} + \frac{q}{m} E_z \frac{\partial f_i}{\partial v} = S_i + S_a
\]

\[
\approx W[\alpha (f_{i\infty} - f_i) + (1 - \alpha)(1 - \frac{n}{n_{i\infty}})f_{i\infty}] + n_e [-K_M f_M - K_N f_N + K_T f_N].
\]

(4)
Assuming the same temperature of atomic neutrals as that of molecular ions, i.e., \( T_N = T_M \equiv \tau T_i \), and \( m_N = m_i \), the unperturbed distribution functions of ions \( f_{i\infty} \) and atomic neutrals \( f_N \equiv f_{N\infty} \) are given as

\[
f_{i\infty}(v) = n_\infty \sqrt{\frac{m_i}{2\pi T_i}} \exp \left[ -\frac{m_i(v - V_d)^2}{2T_i} \right],
\]

\[
f_N(v) = \nu_1 n_\infty \sqrt{\frac{m_i}{2\pi T_i}} \exp \left[ -\frac{m_i v^2}{2T_i} \right],
\]

where \( \nu_1 \equiv n_N/n_{\infty} \). By taking moments of Eq. (4) and using the dimensionless parameters such as \( L \equiv C_s/W(z), y \equiv \int [W(z)/C_s] dz \), or \( y \equiv zW/C_s, M \equiv V_z/C_s, \ n \equiv n_i/n_{\infty}, \) one can get the following dimensionless equations:

\[
M \frac{dn}{dy} + n \frac{dM}{dy} = 1 - n + k_i n - k_r n^2,
\]

\[
\frac{dn}{dy} + nM \frac{dM}{dy} = (M_\infty - M)[1 - (1 - \alpha)n] - k_i n M,
\]

where \( k_i, k_m, k_e \), and \( k_r \) are the normalized ratios of ionization \( \equiv (K_i Z n_{Na}/C_s)(L/a) \), molecular-activated recombination \( \equiv (K_m Z n_{Ma}/C_s)(L/a) \), electron-ion recombination \( \equiv (K_e Z n_{e\infty}/C_s)(L/a) \) and total recombination \( \equiv k_m + k_e \) with respect to the cross-field transport contribution, respectively, and \( n_M \) is the density of relevant molecular ions. Here \( C_s \) is the ion acoustic speed \( \equiv \sqrt{(T_e + Z T_i)/m_i} \). \( n_{\infty} \) is the atomic neutral density and quasi-neutrality \( (en_{\infty} = Ze n_i) \) is used. After some arrangements, one can get the following equations for \( \frac{dn}{dy} \) and \( \frac{dM}{dy} \):

\[
\frac{dn}{dy} = \frac{M_\infty - 2M - (M_\infty - M)(1 - \alpha)n + (1 + k_r)n M}{1 - M^2},
\]

\[
\frac{dM}{dy} = \frac{1 - n - M(M_\infty - M)[1 - (1 - \alpha) n] + k_i n (1 - M^2) - k_r n^2}{n(1 - M^2)}.
\]

Dividing Eq. (7) by Eq. (8) leads to

\[
\frac{1}{n} \frac{dn}{dM} = \frac{M_\infty - 2M - (M_\infty - M)(1 - \alpha)n + (1 + k_i) n M}{1 - n - M(M_\infty - M)[1 - (1 - \alpha) n] + k_i n (1 - M^2) - k_r n^2}.
\]

### 3 Analysis

One can recover the same form of non-viscous Stangeby model [15] by putting \( \alpha = 0, k_i = k_r = 0 \), and that of strong viscous Hutchinson model [16] by putting \( \alpha = 1, k_i = k_r = 0 \), which confirms the validity of the fluid equation (Eq. (9)). General solutions of Eqs. (7) and (8) should be given numerically instead of solving Eq. (9) to avoid the possible problem of singularity [7]. Figure 1 shows normalized density \( n(\alpha) \) profiles in terms of normalized fluid velocity \( \alpha = 1 \) and \( M_\infty = 0.4 \) with different ionization contribution \( (k_i = 0.001, 0.1, 1.0) \) with respect to the cross-field transport contribution. Both results by solving Eqs. (7) and (8), and Eq. (9) are exactly matched for this case. However, there are cases with \( n(M) > 1 \), which seems to be an indication of singularity [16], for the larger ionization \( (k_i \geq 0.1) \), indicating that even Hutchinson model can produce un-physical solutions with larger ionization, while kinetic model does not [17]. Since this may be too early to maintain this argument, we need to explore further the singularity problem in one-dimensional fluid approximation for the flowing magnetized plasmas.

Figure 2 shows the ratio of ion saturation currents \( R \equiv J_{sat}/J_{dn} \) in terms of ionization and recombination as an application of our model. For the small contribution, say less than 1% for ionization and much less than 0.1% for recombination, there is no visible change in \( R \). Larger ionization contribution \( (k_i \geq 0.01) \) makes \( R \) be decreased with the same Mach number \( M_\infty \) indicating the decrease of the calibration factor \( (K = \ln[R]/M_\infty) \), which leads to the possibility of under-estimation of Mach number with the same measured ratio \( (R) \), while \( R \) increases with recombination, although we cannot handle the case somewhat larger contribution of recombination, say, larger than \( k_r \geq 10^{-3} \) due to singularity of our fluid model or due to un-known numerical instability.
in our numerical code written by Visual C++ and LabView program. There might be a cure for this if one would adopt the atomic/molecular volume source of Eq. (3) with

\[ S_0 = -K_M n_e f_M(\nu) - K_E n_e f_E(\nu) + K_I n_e f_N(\nu), \]

where \( \infty \) indicates the un-perturbed values.

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**Fig. 1** Normalized density \((n)\) profiles in terms of normalized fluid velocity \((M)\) for \(\alpha = 1\) and \(M_\infty = 0.4\) with different normalized ionization contribution \((k_i = 0.001 (\text{line 3}), 0.1 (\text{line 2}), 1.0 (\text{line 1}))\) with respect to the cross-field transport contribution. (Online colour: www.cpp-journal.org).

**Fig. 2** Ratio of ion saturation currents \((R \equiv J_{up}/J_{dn})\) in terms of normalized ionization rate \((k_i)\) and recombination \((k_r)\). (Online colour: www.cpp-journal.org).

### 4 Conclusion

One-dimensional fluid models for the ion collection are developed and calculated the density in terms of Mach number by solving both \(dn/dM\) and \(dn/dy\) and \(dM/dy\), separately, and both methods produces the same results for \(\alpha = 1\) with very small contribution of recombination. Calibration factor decreases with ionization, leading…
to the higher Mach numbers for the same ratio of ion saturation currents, while it increases with recombination. With high neutral pressure and high electron temperature ($\geq 10\ eV$) or larger connection length of the flux tube ($L$) of existing and future toroidal divertor-type machines, contributions from ionization and recombination would become larger due to strong magnetic field ($L/a \gg 1$) and neutral pressure (strong e-n and i-n collisions) comparing linear machines, then it will strongly affect the deduction of the Mach numbers.

Acknowledgements This work is supported by the National Research Laboratory(NRL) Project of the Korea Science and Engineering Foundation(KOSEF) under the Korea Ministry of Science and Technology(MOST). This is also supported by the Brain Korea 21 (BK21) program under the Korea Ministry of Education.

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