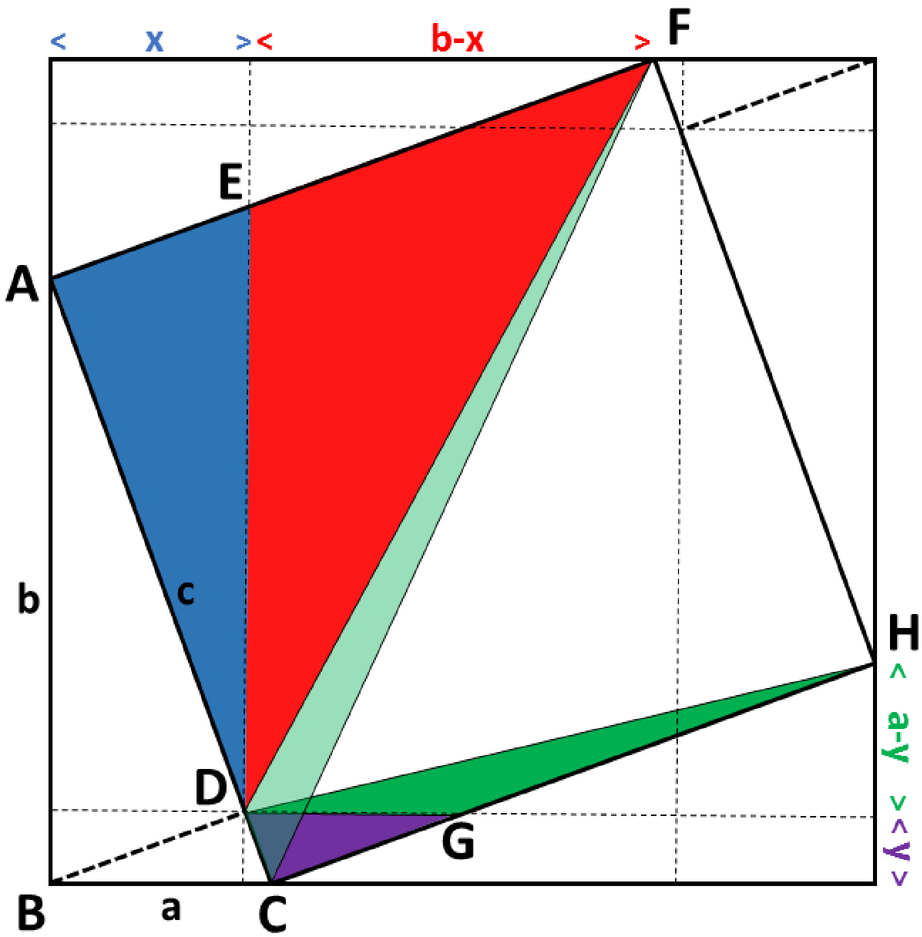
Pythagorean Theorem Proof #1 | Gil Brand, 2015



Parallelogram ABDE: AB = DE = b

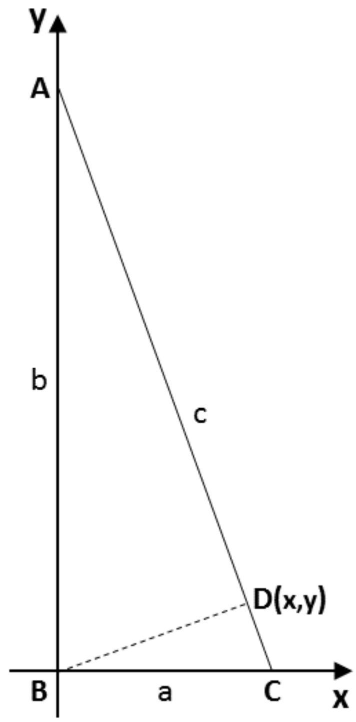
Area(ADF) = Area(ADE) + Area(EDF) = bx/2 + b(b-x)/2 = $\mathbf{b^2/2}$

Parallelogram BCGD: BC = DG = a

Area(CDH) = Area(CDG) + Area(GDH) = ay/2 + a(a-y)/2 = $\mathbf{a^2/2}$ = **Area(DCF)**

Area(ACF) = Area(ADF) + Area(DCF) = $a^2/2 + b^2/2$ = $\mathbf{c^2/2}$

Pythagorean Theorem Proof #2 | Gil Brand, 2015



$$AC \perp BD$$

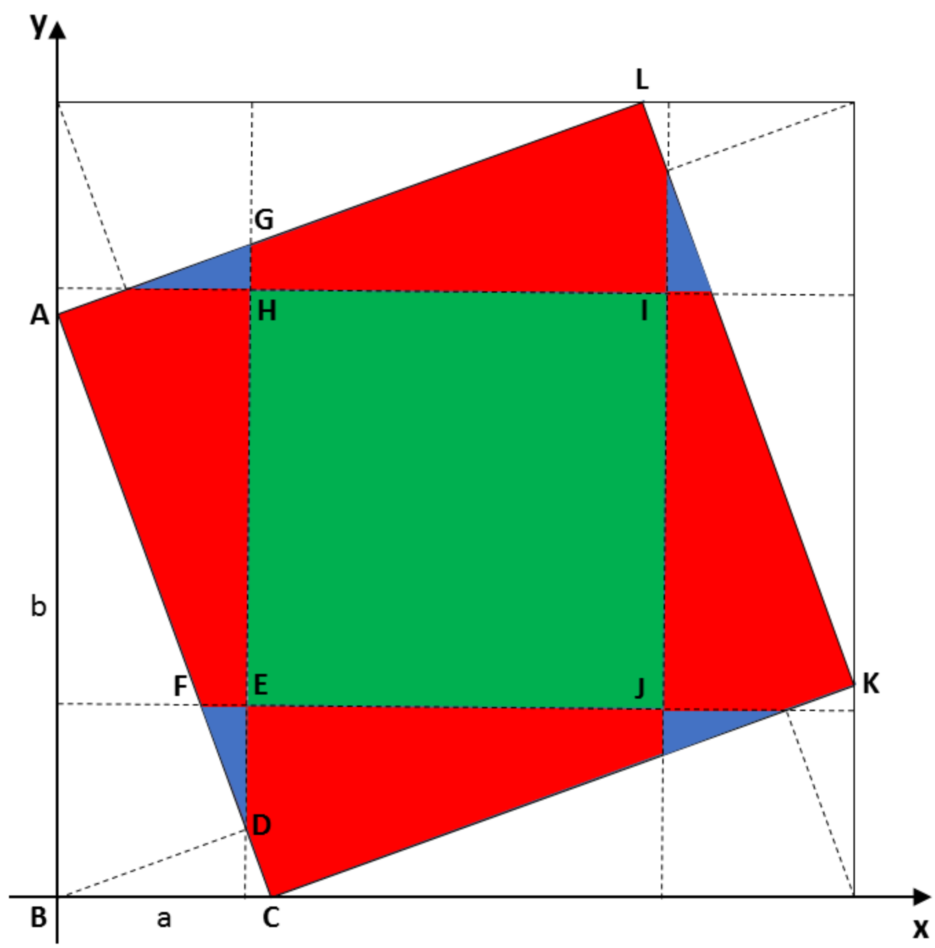
$$AC: \quad y = -\frac{b}{a}x + b$$

$$BD: \quad y = \frac{a}{b}x$$

Intersection Point D:

$$-\frac{b}{a}x + b = \frac{a}{b}x$$

$$\boldsymbol{x = \frac{ab^2}{a^2 + b^2}} \quad ; \quad \boldsymbol{y = \frac{a^2 b}{a^2 + b^2}}$$



$\underline{\Delta DAG}$

Parallelogram ABDG: $DG = b$

$$\boldsymbol{S_{DAG}} = \frac{DG \cdot x}{2} = \frac{bx}{2} = \boldsymbol{\frac{ab^3}{2(a^2+b^2)}}$$

$\underline{\Delta ABC \sim \Delta DEF}$

$$\frac{S_{ABC}}{S_{DEF}} = \left(\frac{b}{DE}\right)^2 \rightarrow \frac{\frac{ab}{2}}{S_{DEF}} = \left(\frac{b}{x-y}\right)^2$$

$$\boldsymbol{S_{DEF}} = \boldsymbol{\frac{a(ab^2-a^2b)^2}{2b(a^2+b^2)^2}}$$

Square EJIH:

$$EJ = a + b - 2x = a + b - \frac{2ab^2}{a^2+b^2}$$

$$\boldsymbol{S_{EJIH}} = EJ^2 = \boldsymbol{\left(a + b - \frac{2ab^2}{a^2+b^2}\right)^2}$$

Square ACKL:

$$\boldsymbol{S_{ACKL}} = c^2 = 4(S_{DAG} - S_{DEF}) + S_{EJIH}$$

$$c^2 = 4\left(\frac{ab^3}{2(a^2+b^2)} - \frac{a(ab^2-a^2b)^2}{2b(a^2+b^2)^2}\right) + \left(a + b - \frac{2ab^2}{a^2+b^2}\right)^2$$

$$\boldsymbol{c^2} = \frac{(a^2+b^2)^3}{(a^2+b^2)^2} = \boldsymbol{a^2+b^2}$$