



## Chapter 3

# Memory

These slides support chapter 3 of the book

*The Elements of Computing Systems*

By Noam Nisan and Shimon Schocken

MIT Press

# Chapter 3: Memory

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## Time matters

- Sequential logic
- Flip Flops
- Memory units
- Counters
- Project 3 overview

# Time-independent logic

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- So far we ignored the issue of *time*
- The chip's inputs were just “sitting there” – fixed and unchanging
- The chip's output was a pure function of the current inputs, and did not depend on anything that happened previously
- The output was computed “instantaneously”
- This style of gate logic is sometimes called:
  - *time-independent logic*
  - *combinational logic*.

# Hello, time

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## Abstraction issues:

- The hardware must support maintaining “state”
- The hardware must support computations over time

example:

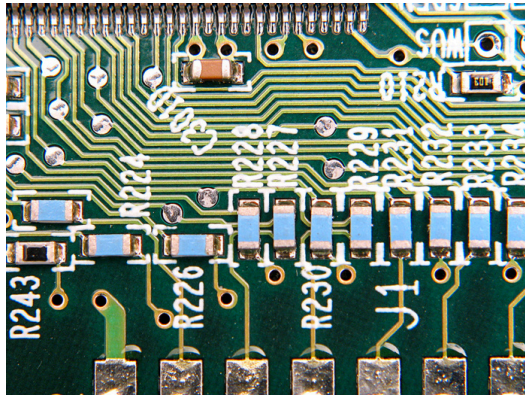
```
x = 17
```

example:

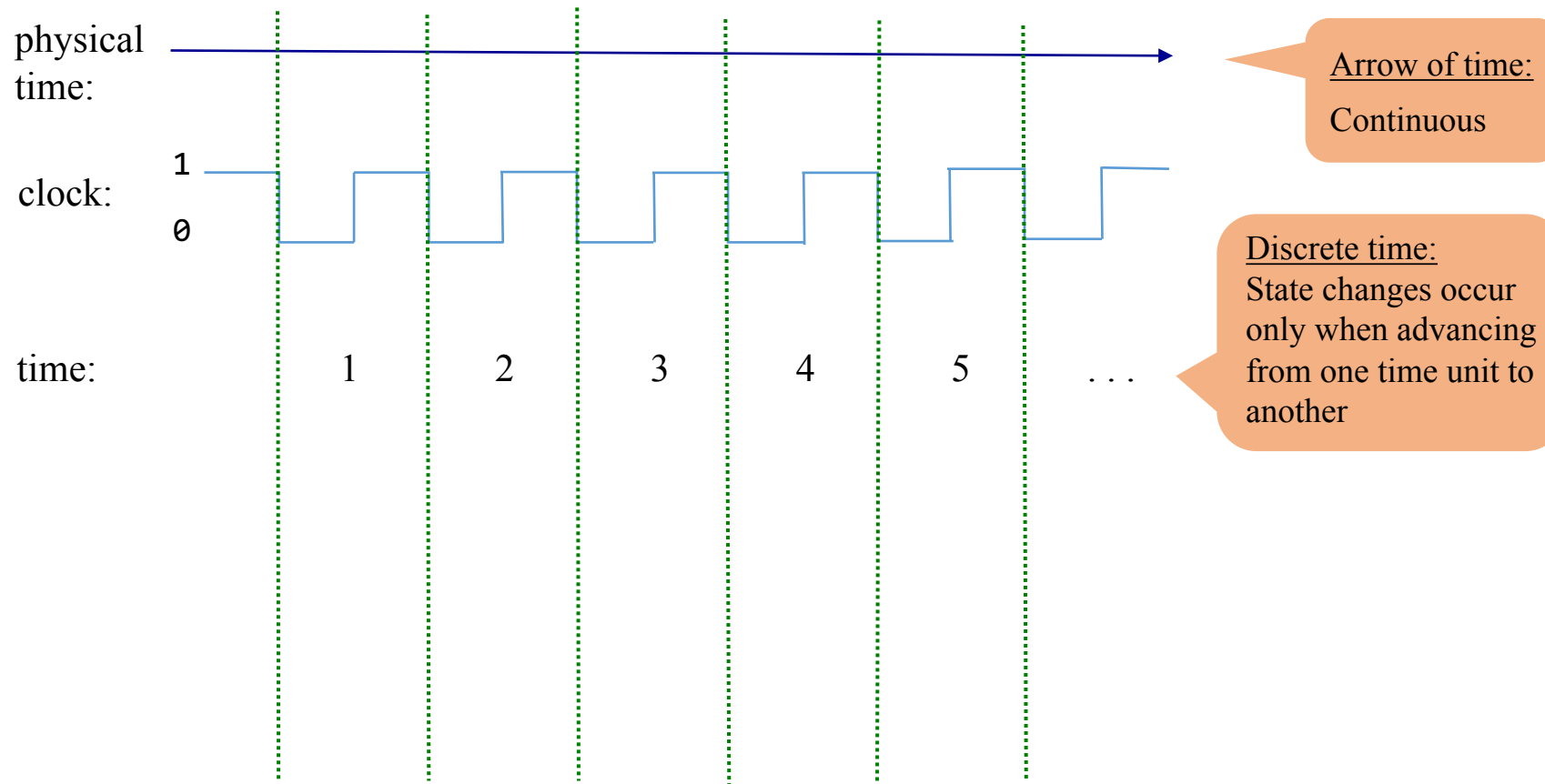
```
for i = 0 ... 99:  
    sum = sum + a[i]
```

## Implementation issues:

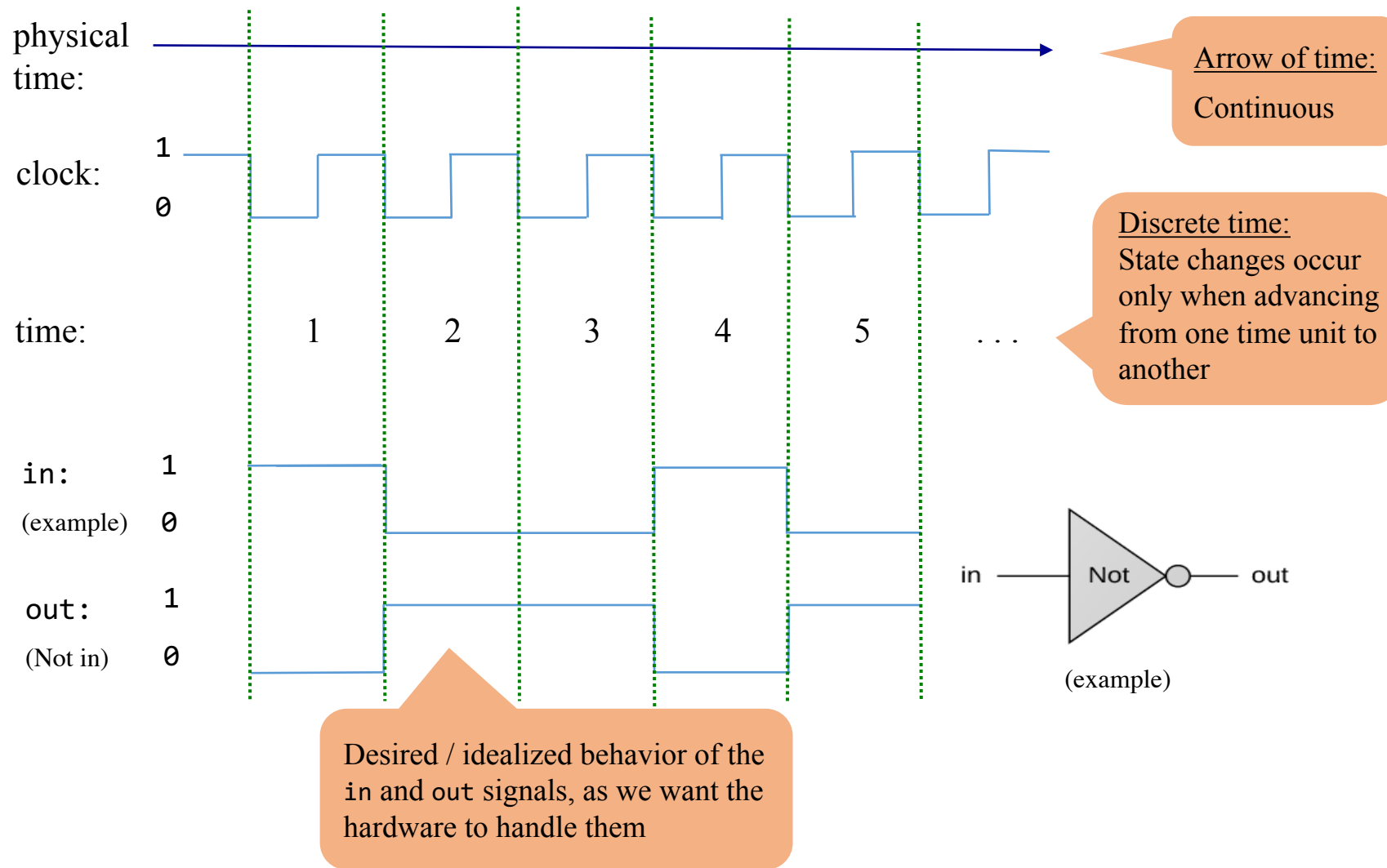
- The hardware must handle the *physical time delays* associated with calculating and moving data from one chip to another.



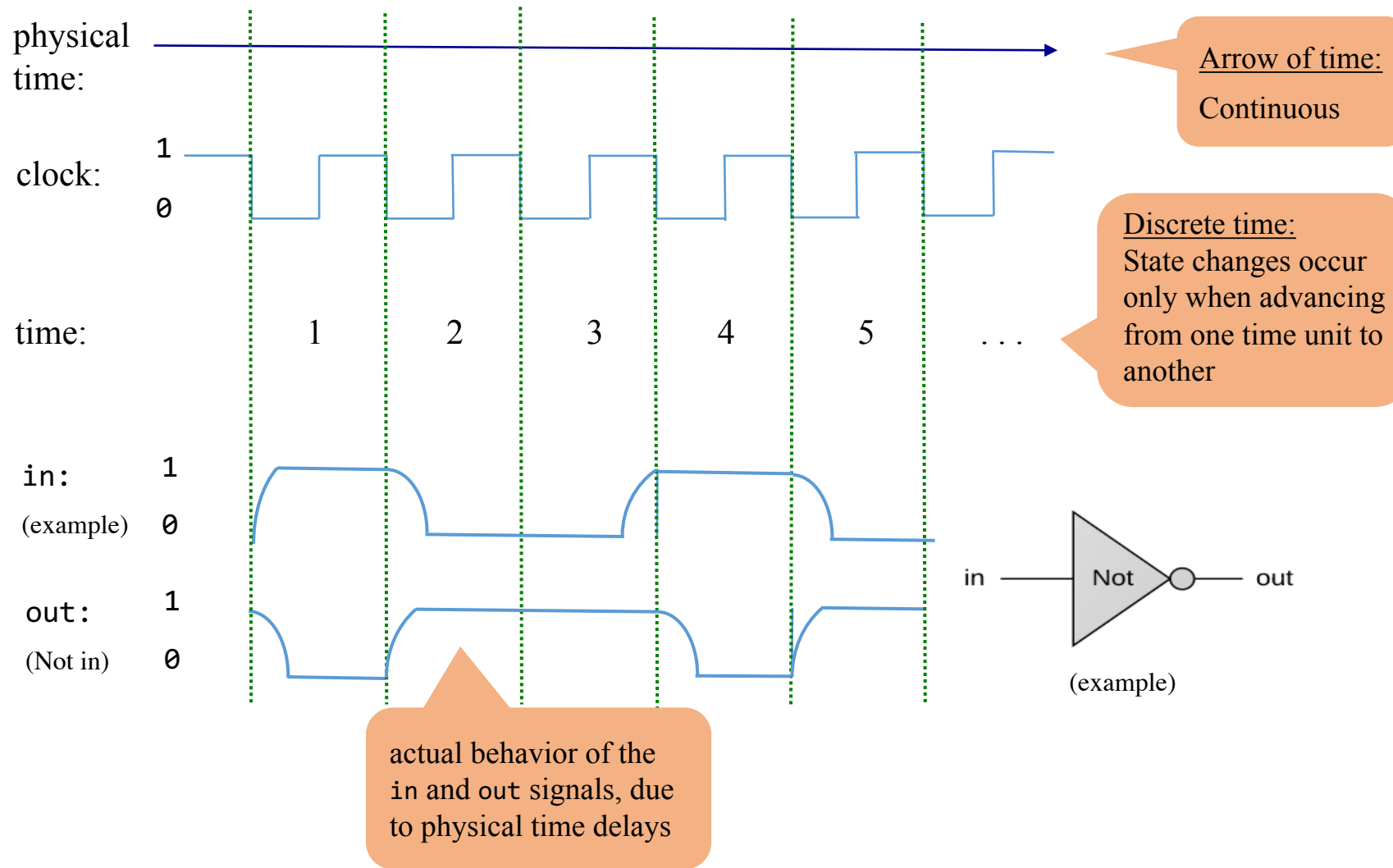
# Physical time / clock time



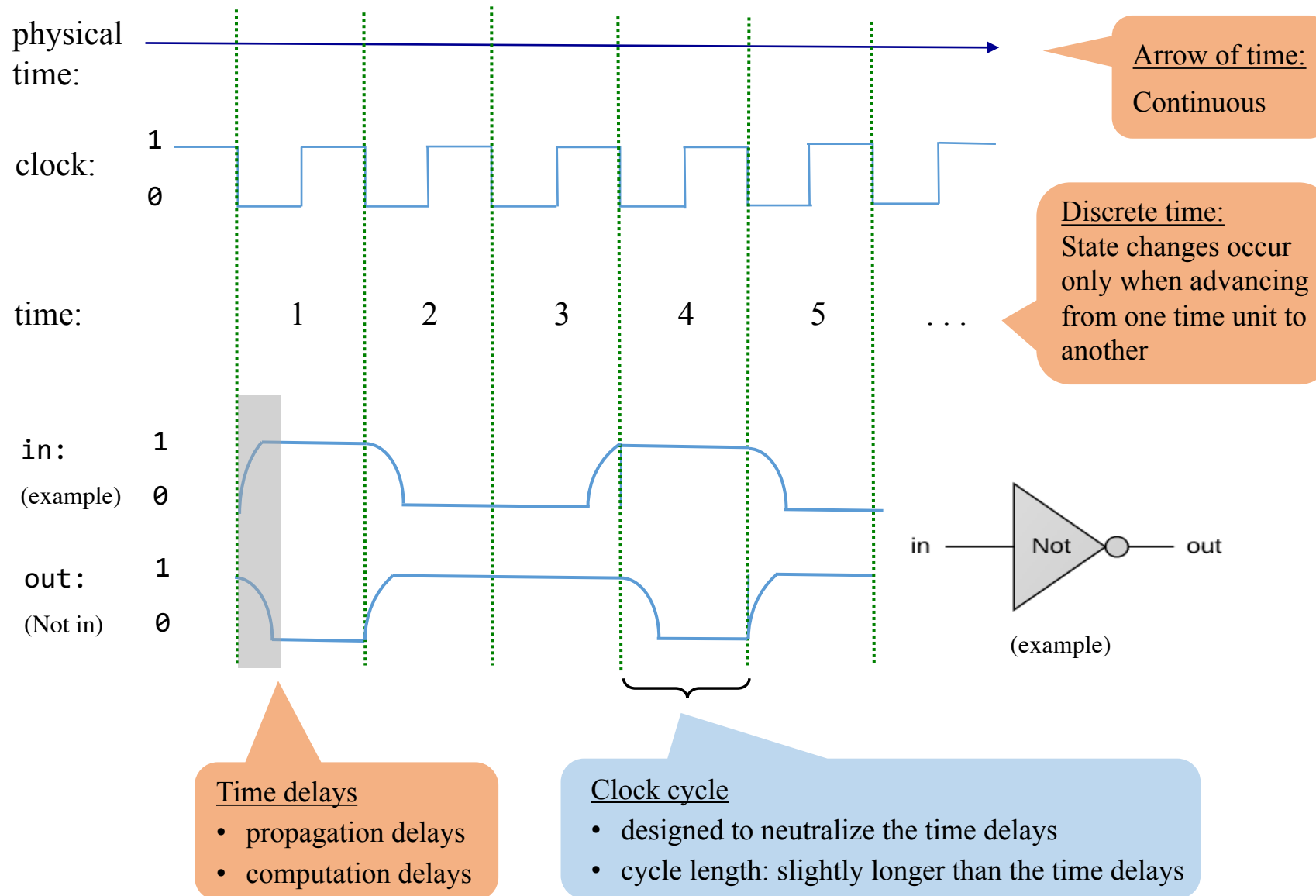
# Chip behavior over time (example)



# Chip behavior over time (example)

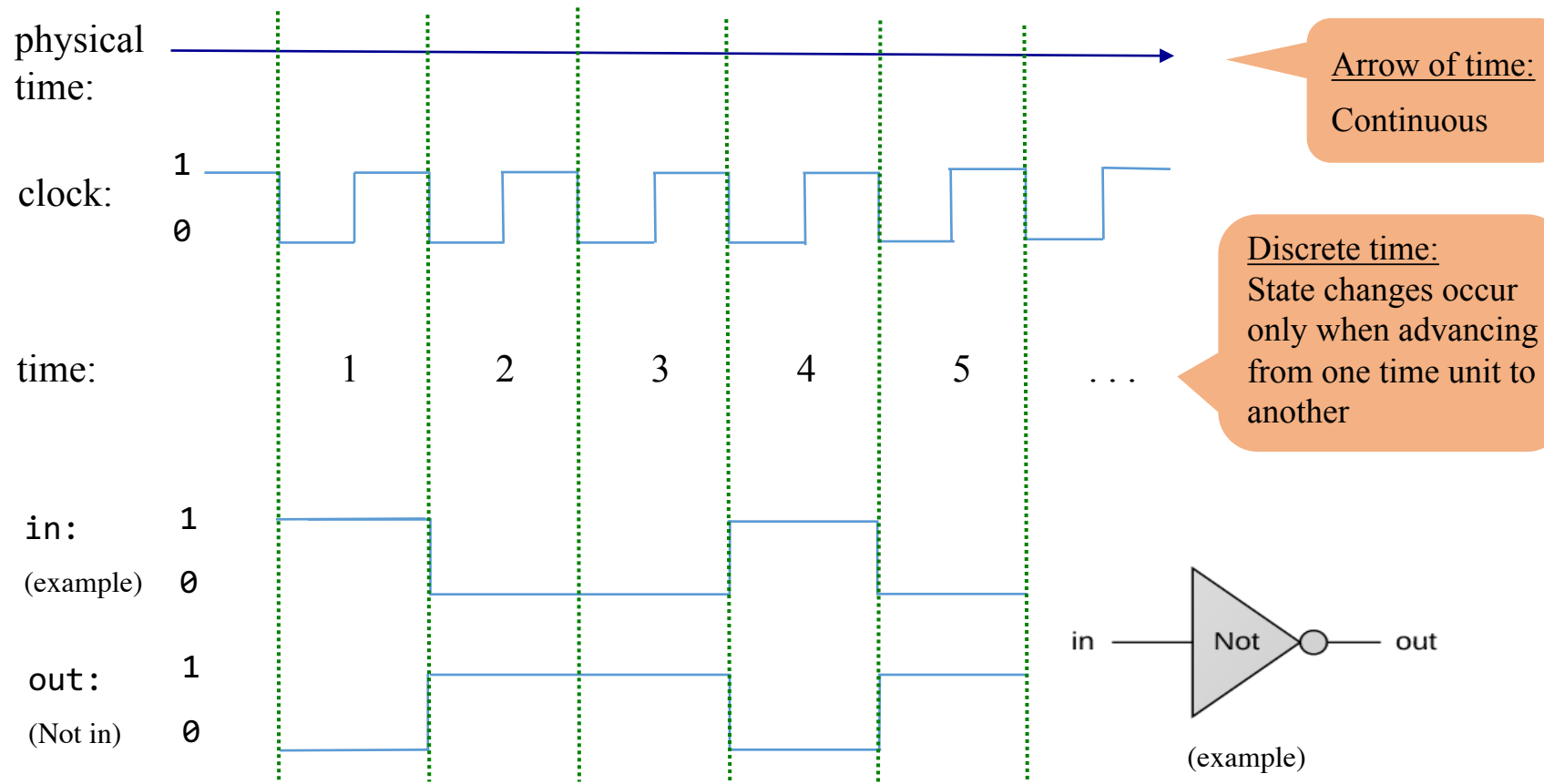


# Chip behavior over time (example)





# Chip behavior over time (example)



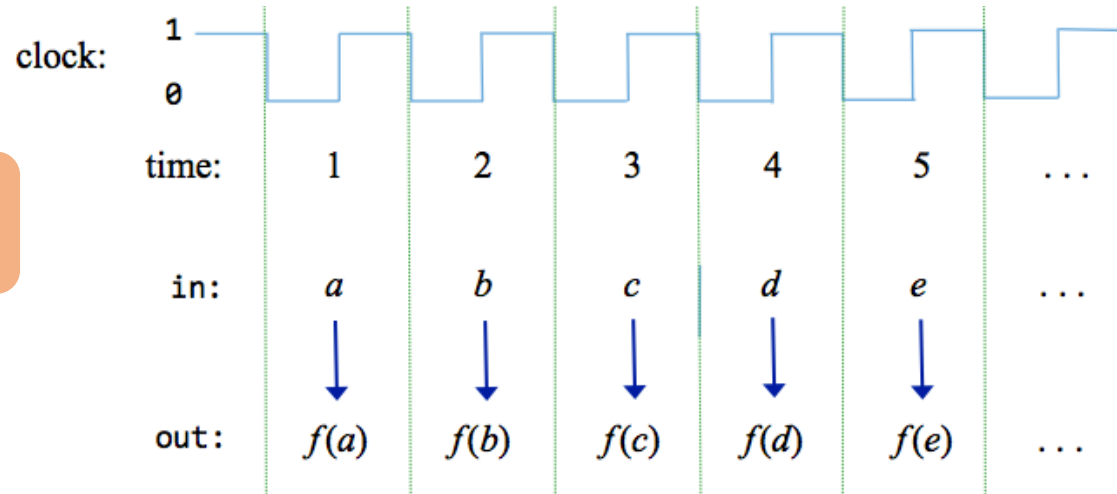
Not: an example of a *combinational chip*:

- The gate reacts “immediately” to the inputs
- Well, not really, but the clock’s behavior creates this effect.

# Combinational logic / sequential logic

## Combinational logic:

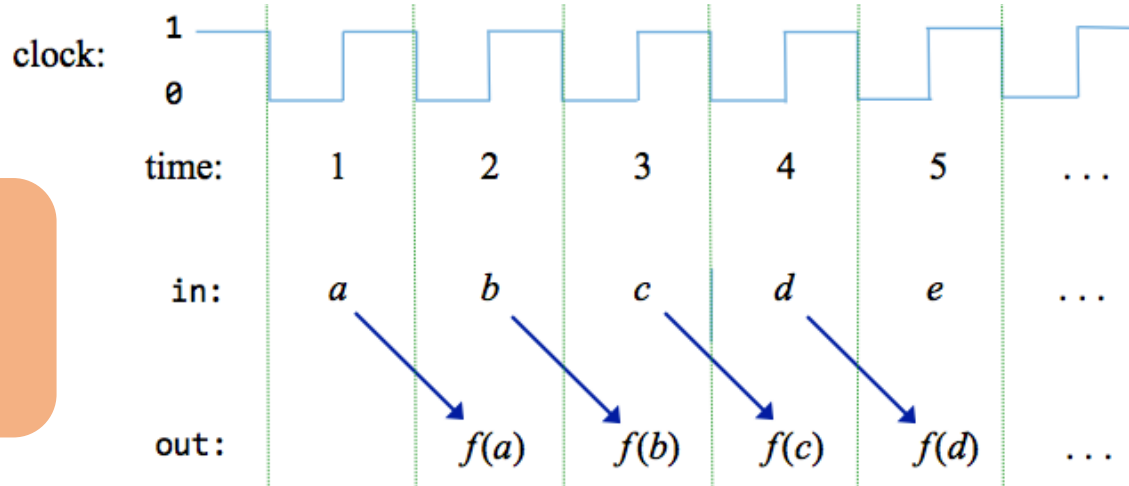
The output is a pure function of the present input only



## Sequential logic:

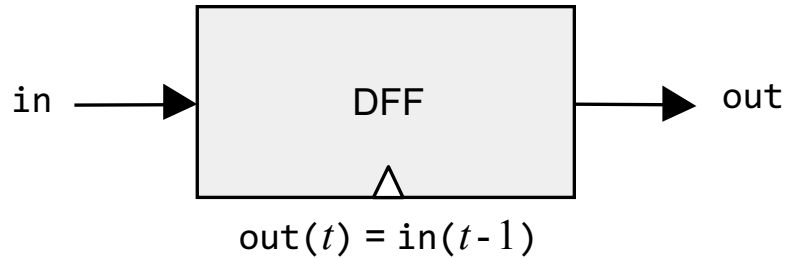
The output depends on:

- the present input (optionally)
- the history of the input (creates a memory effect).



# Flip-Flop

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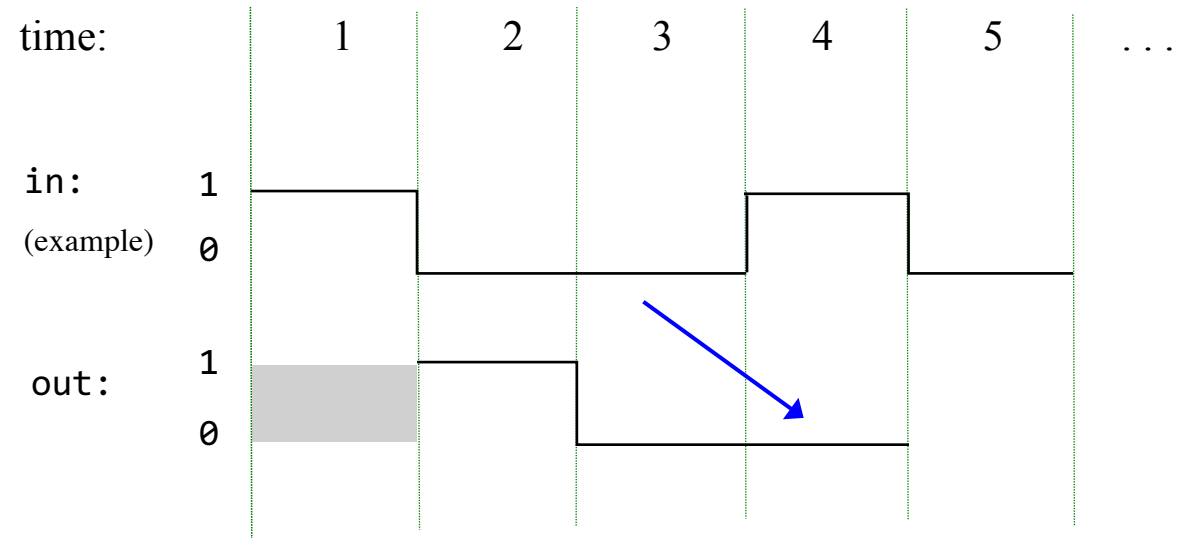
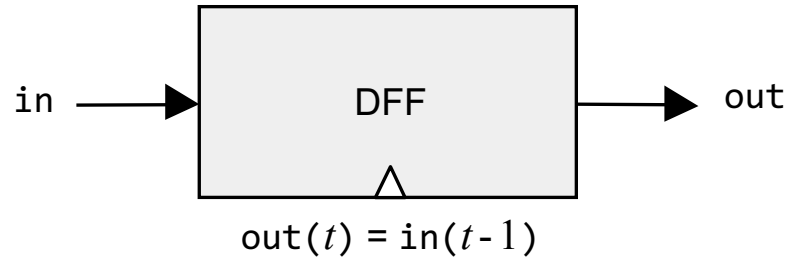


The simplest state-keeping gate:

- 1-bit input, 1-bit output
- The gate outputs its previous input:  $out(t) = in(t-1)$
- Implementation: a gate that can flip between two stable states:  
“remembering 0”, or “remembering 1”
- Gates that feature this behavior are called *data flip-flops*.

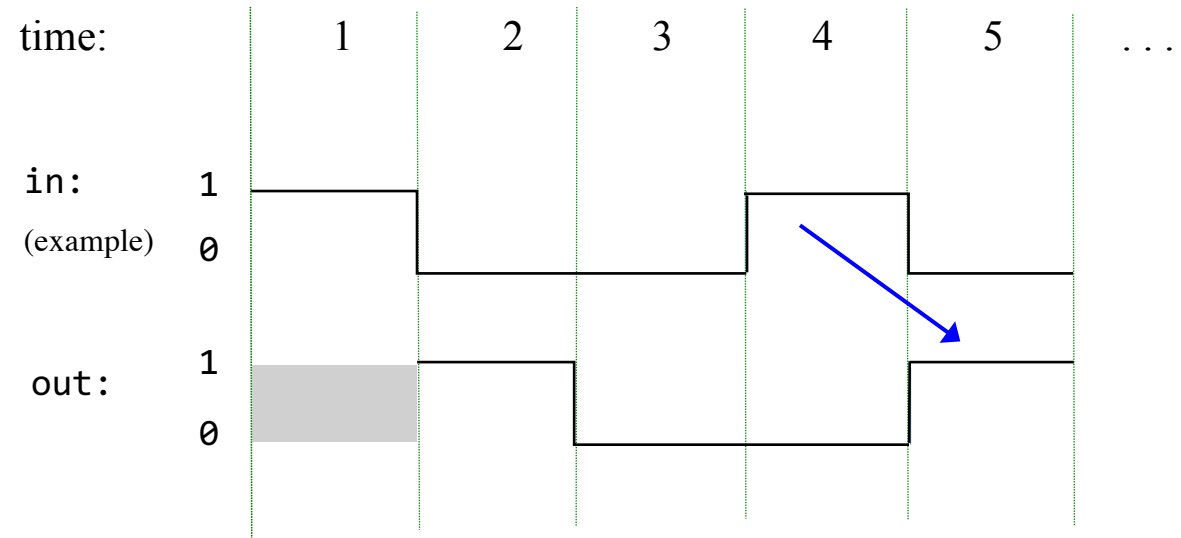
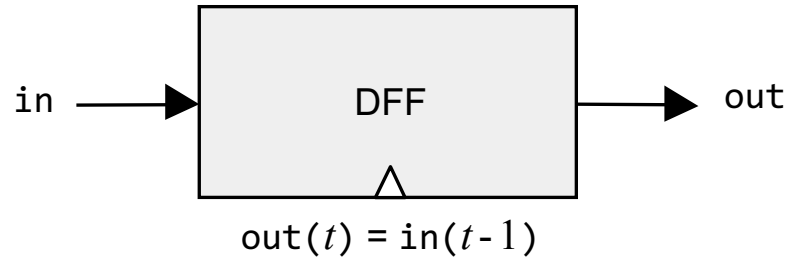
# Flip-Flop

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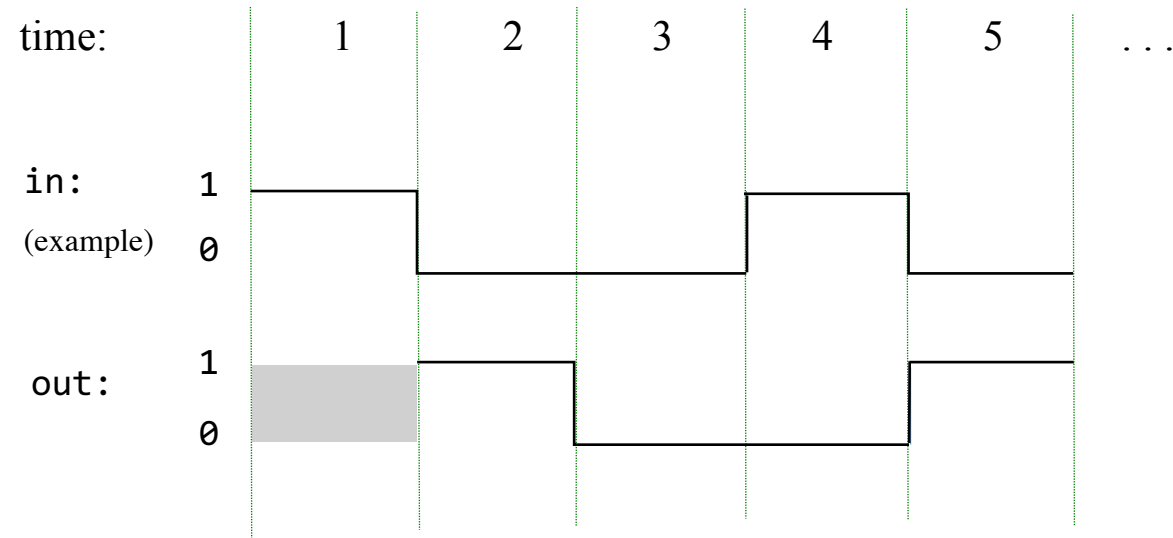
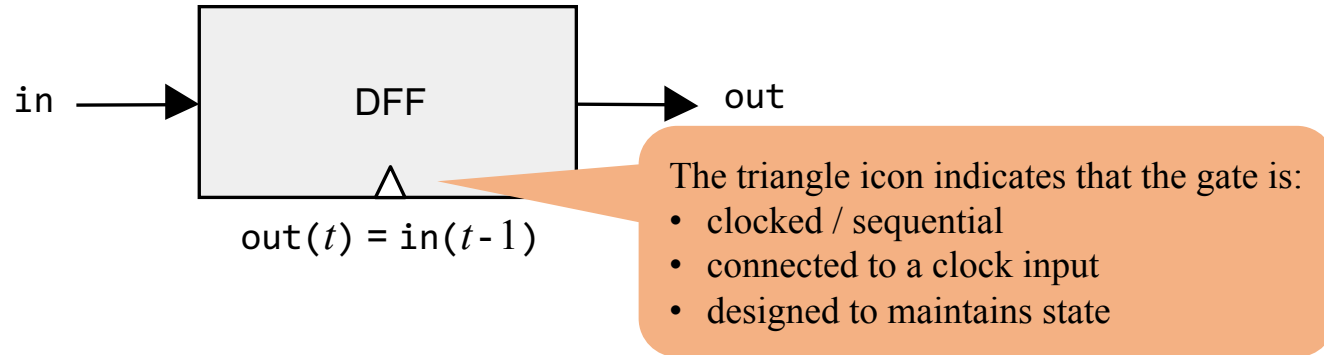


# Flip-Flop

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# Flip-Flop



# DFF implementation notes

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A DFF bi-state architecture can be built from Nand gates:

- step 1: create an input-output loop,  
achieving a combinational (un-clocked) flip-flop
  - step 2: isolate across time steps using a “master-slave” architecture
- The resulting implementation is elegant, but conceptually confusing

## Technical note

The implementation described above is impossible in our hardware simulator, since:

- The supplied simulator does not permit combinational loops
- A cycle in hardware connections is allowed only if the cycle passes through a sequential (“clocked”) gate

## Implementing sequential chips

- The supplied simulator features a built-in DFF gate
- Sequential chips are implemented by using built-in DFF chip parts.

# Sequential chips

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Sequential chips are capable of:

- maintaining state, and, optionally,
  - acting on the state, and on the current inputs
- $$\left. \vphantom{\begin{matrix} \bullet \\ \bullet \end{matrix}} \right\} \text{state}(t) = f(\text{state}(t-1), \text{input}(t))$$

Example: DFF

- The DFF state: the value of the input from the previous time unit
- The simplest, most elementary sequential chip

Example: RAM

- The RAM state: the current values of all its registers
- given some address (input), the RAM emits the value of the selected register

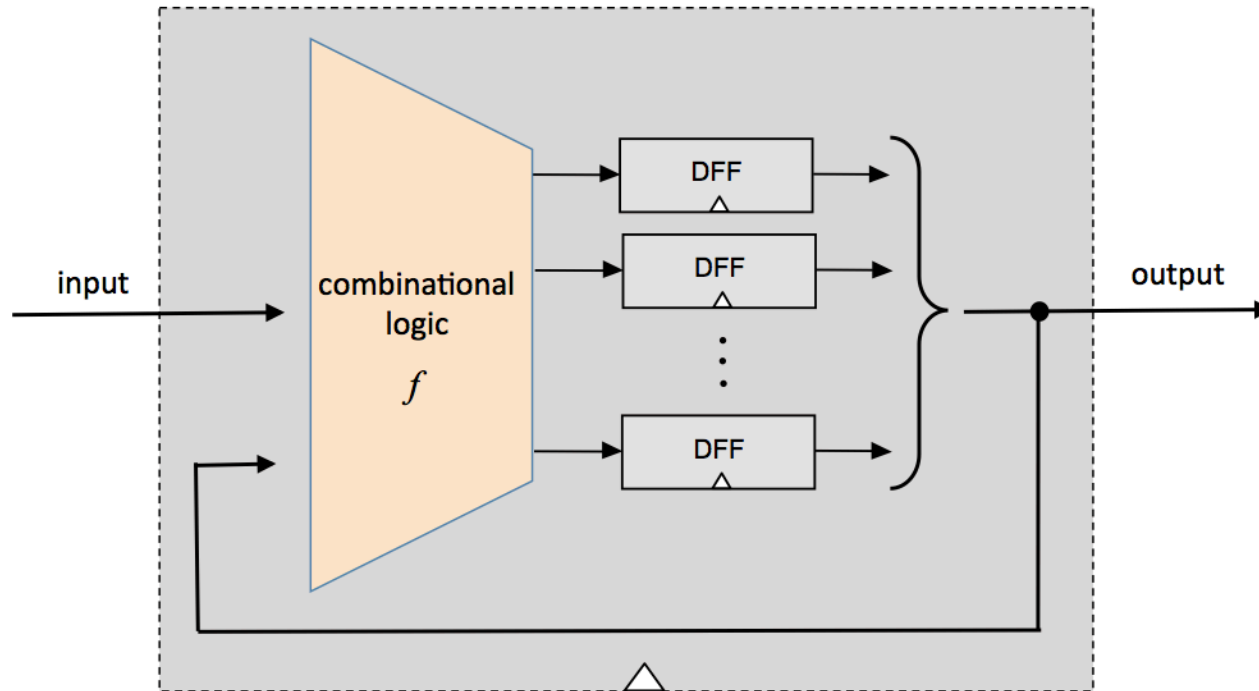
Implementation note

- All combinational chips are constructed from Nand gates
- All sequential chips are constructed from DFF gates, and combinational chips.



# Sequential chips

$$\text{state}(t) = f(\text{state}(t-1), \text{input}(t))$$

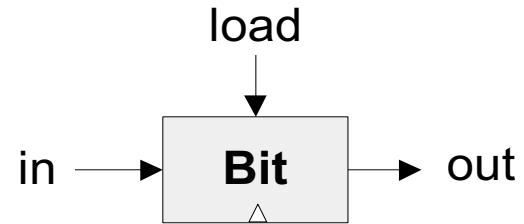


## Implementation note

- All combinational chips are constructed from Nand gates
- All sequential chips are constructed from DFF gates, and combinational chips.

# Sequential chip: 1-bit register

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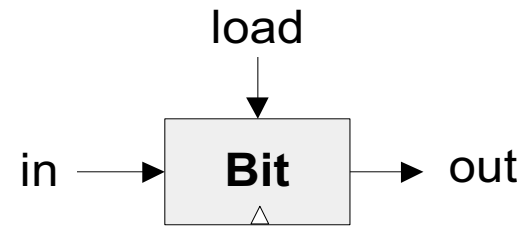


if  $\text{load}(t)$  then  $\text{out}(t+1) = \text{in}(t)$   
else  $\text{out}(t+1) = \text{out}(t)$

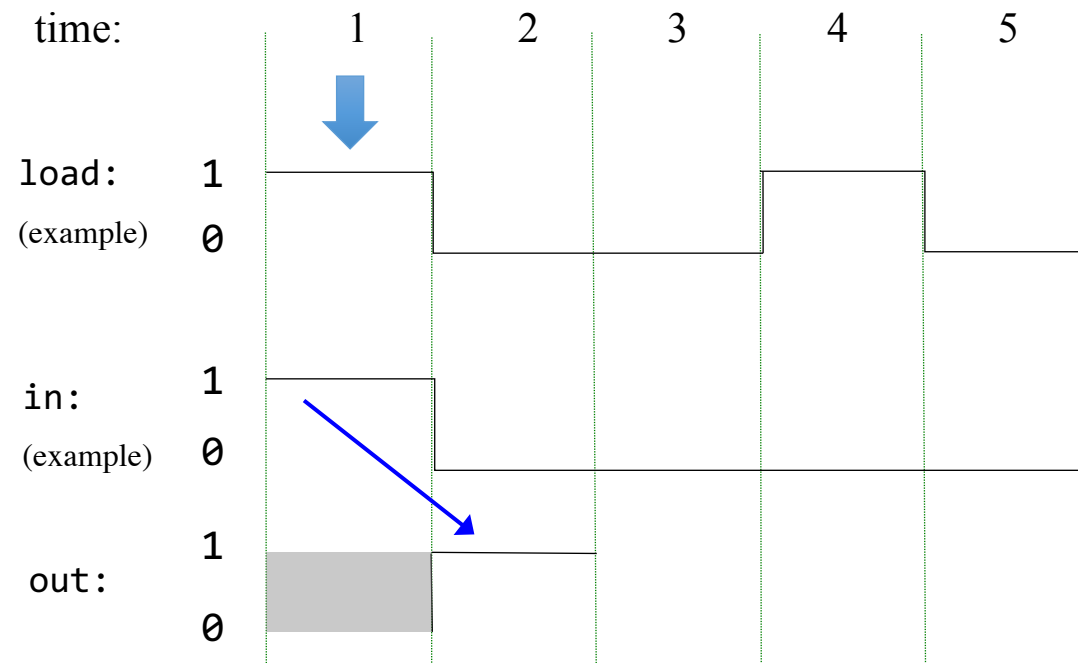
- Designed to “remember”, or “store”, a single bit
- More accurately:
  - Stores a bit until...
  - Instructed to load, and store, another bit.

# 1-bit register

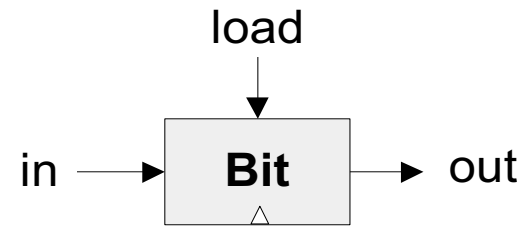
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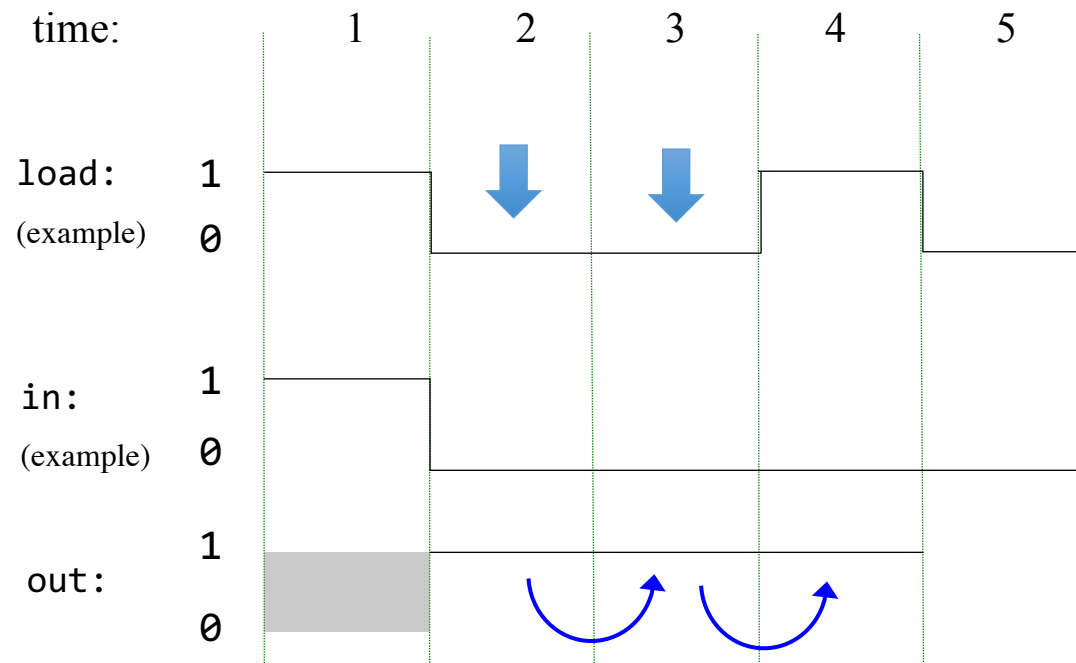
if  $\text{load}(t)$  then  $\text{out}(t+1) = \text{in}(t)$   
else  $\text{out}(t+1) = \text{out}(t)$



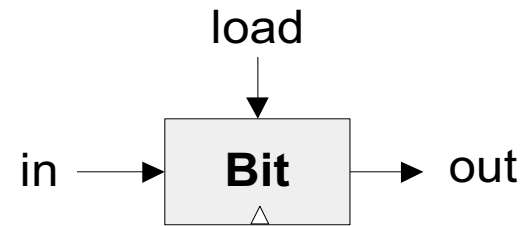
# 1-bit register



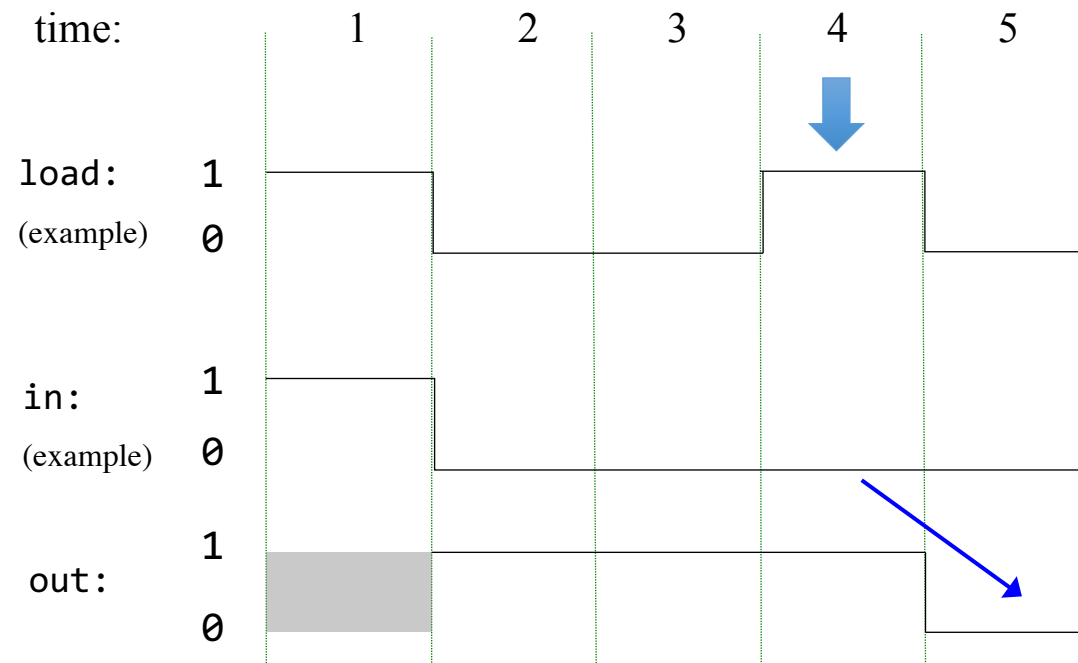
if  $\text{load}(t)$  then  $\text{out}(t+1) = \text{in}(t)$   
else  $\text{out}(t+1) = \text{out}(t)$



# 1-bit register



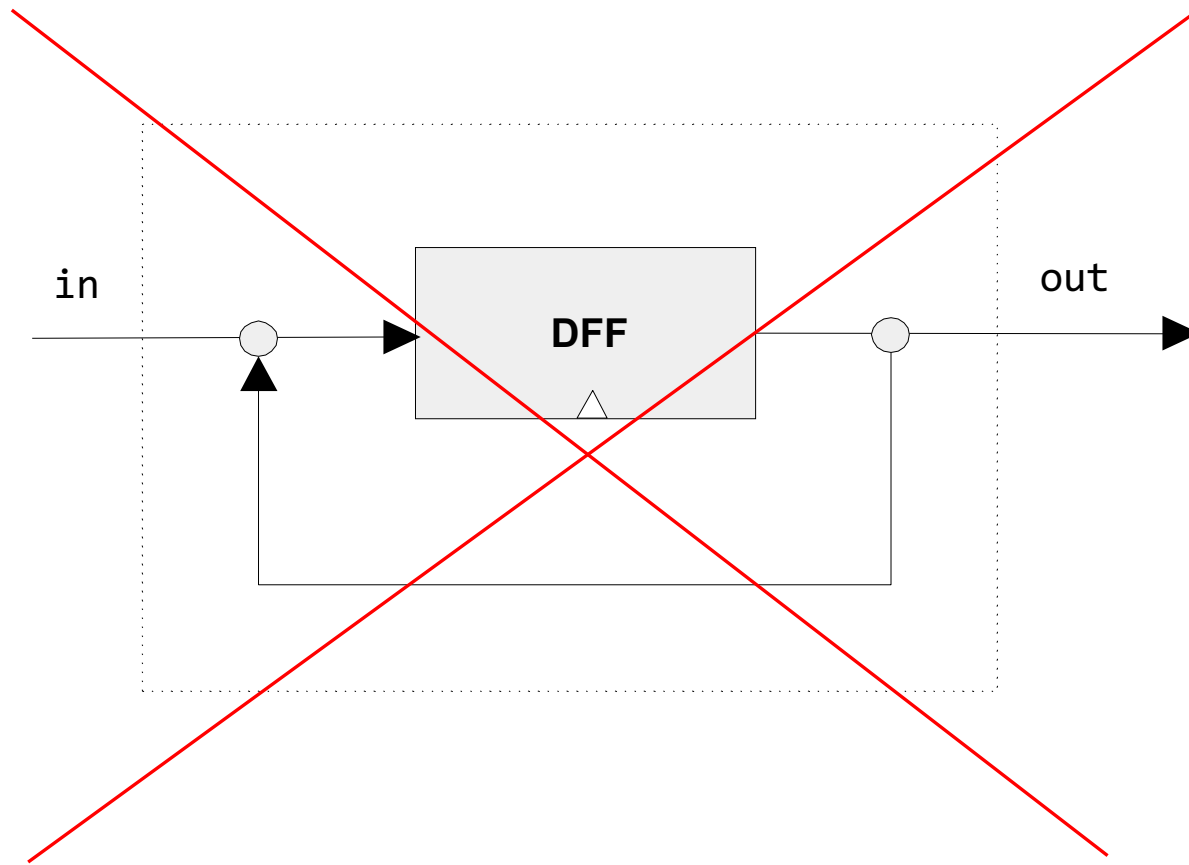
if  $\text{load}(t)$  then  $\text{out}(t+1) = \text{in}(t)$   
else  $\text{out}(t+1) = \text{out}(t)$



Resulting behavior:  
Stores and emits a value, until instructed to load (and store) a new value

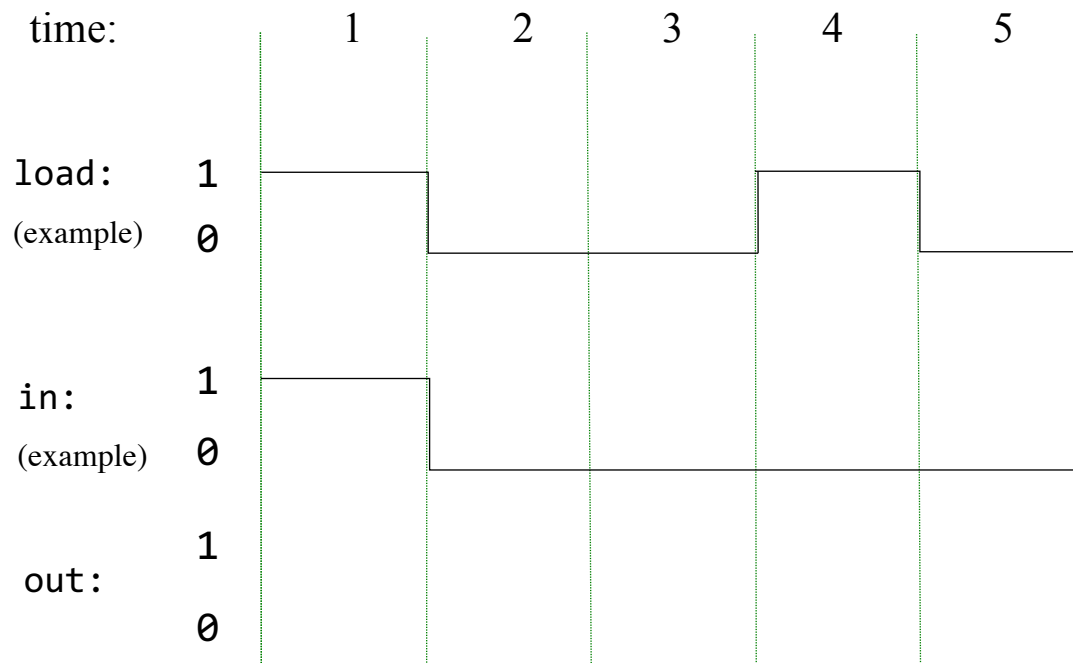
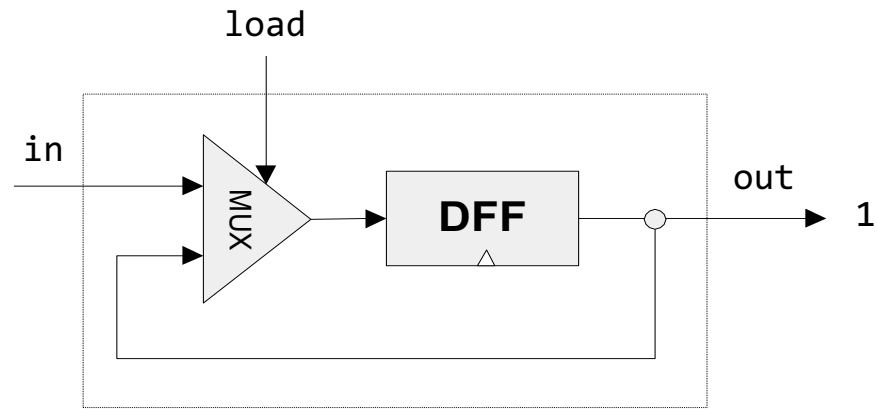
# 1-bit register implementation – first attempt

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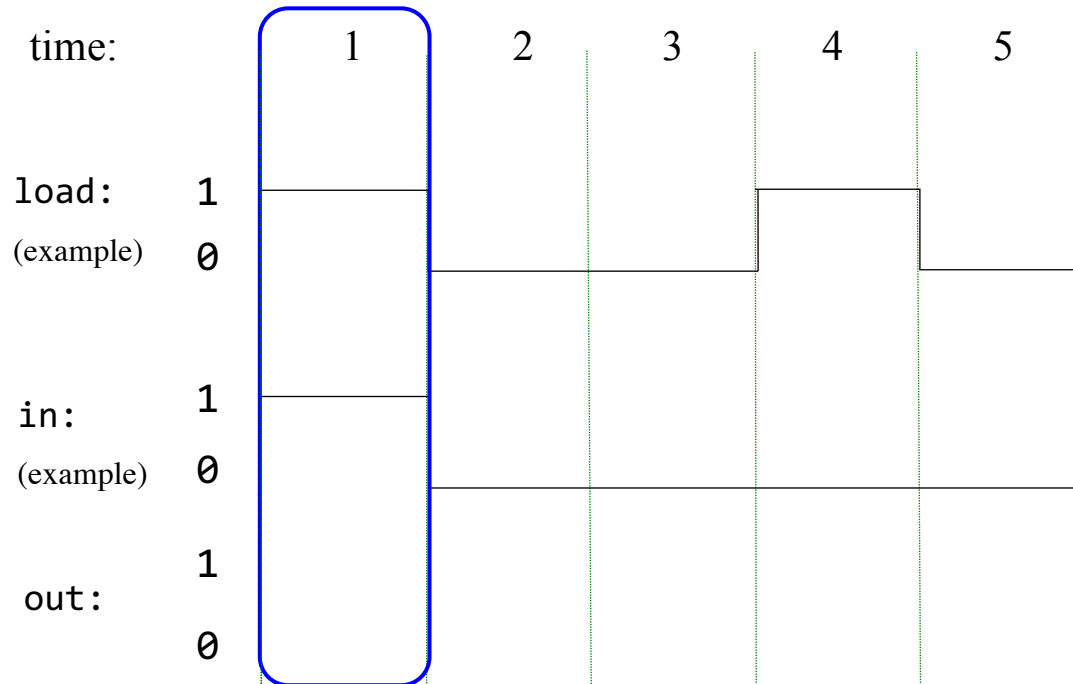
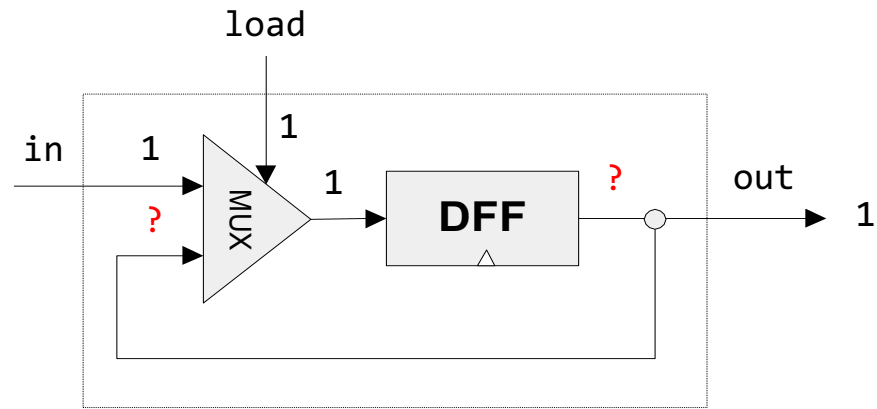


# 1-bit register implementation

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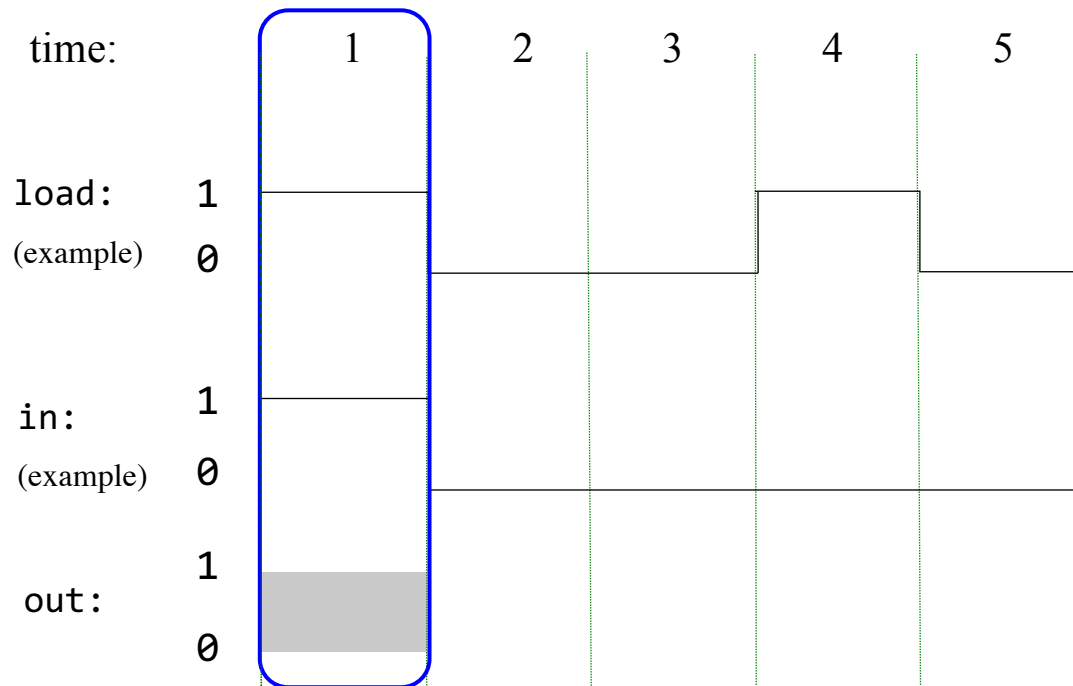
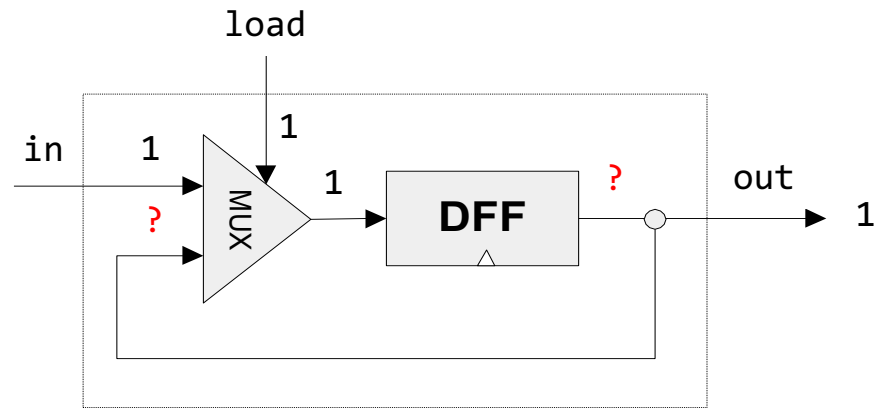


# 1-bit register implementation

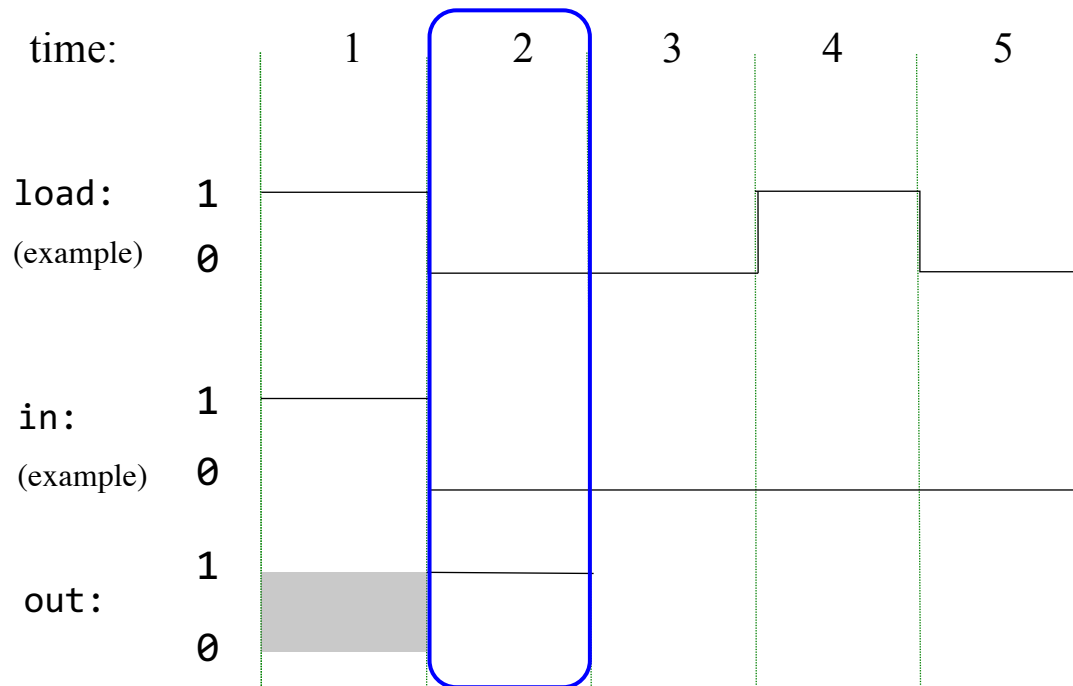
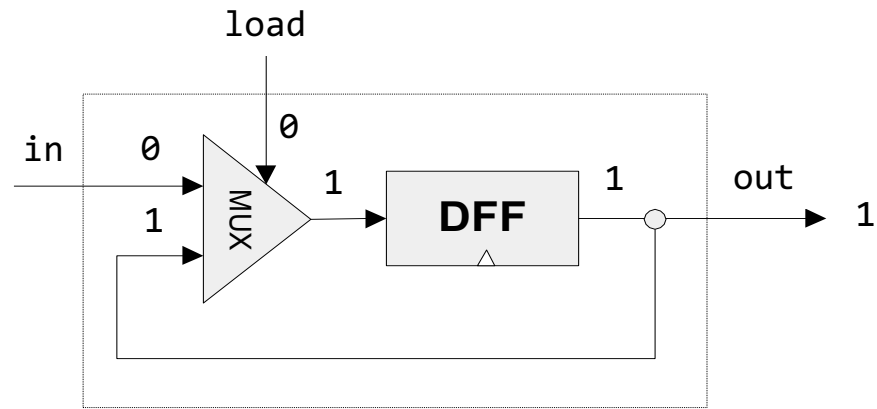




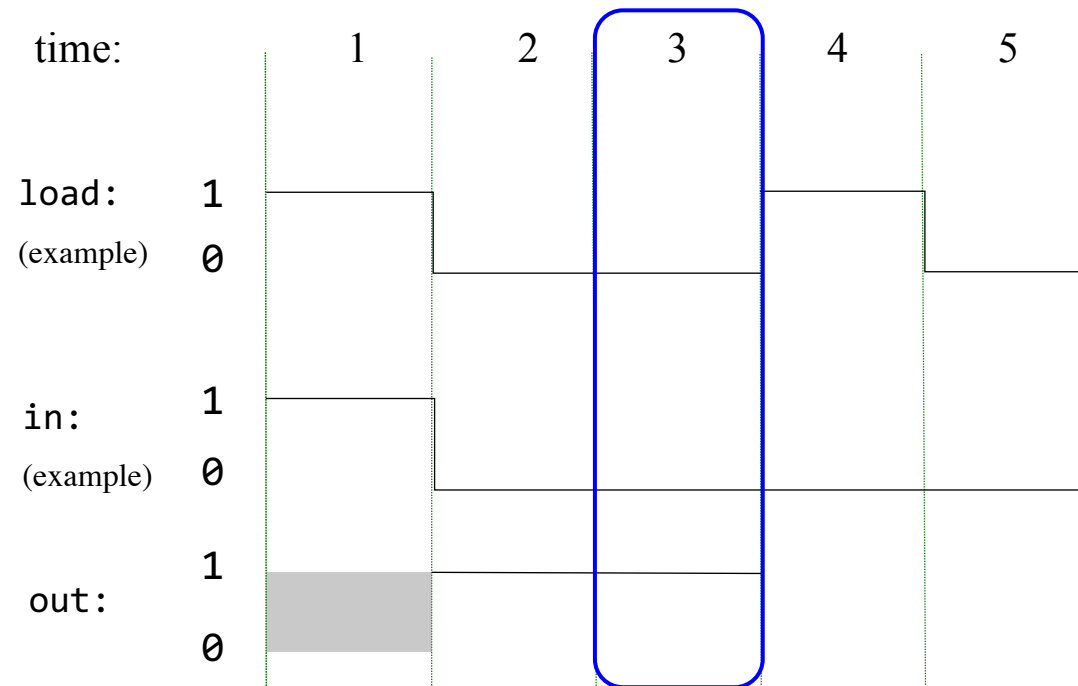
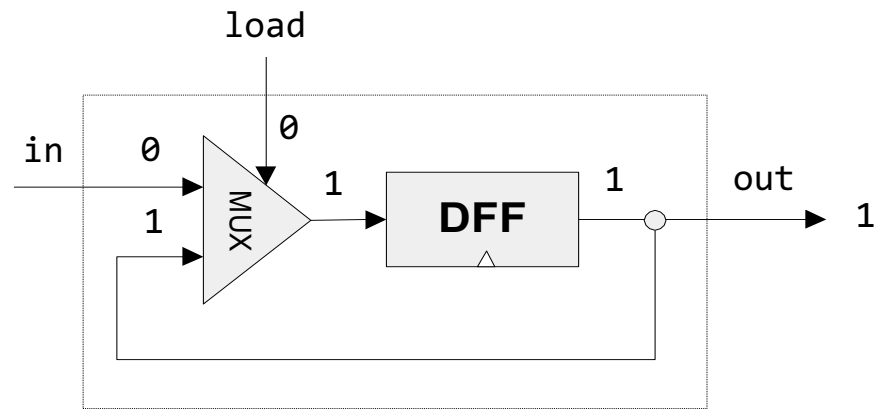
# 1-bit register implementation



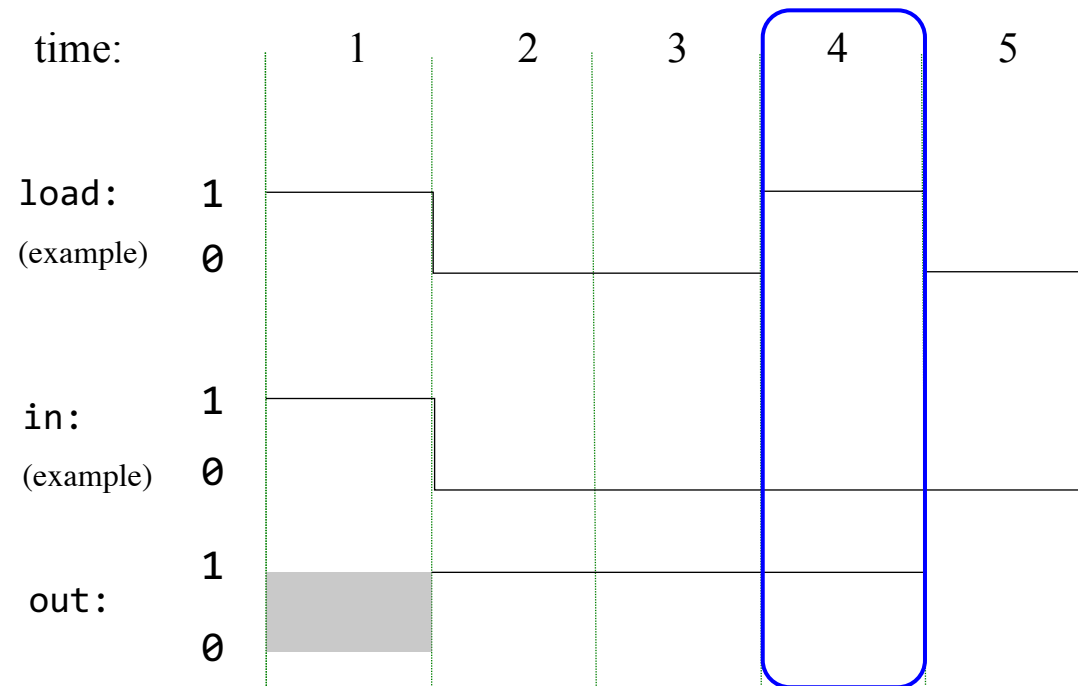
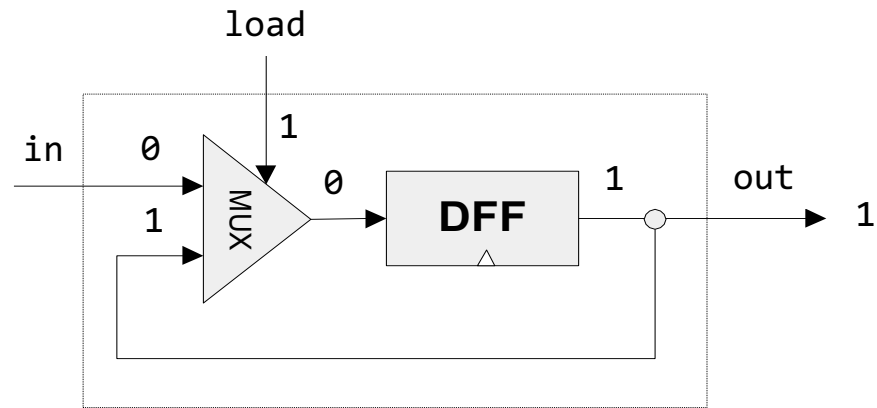
# 1-bit register implementation



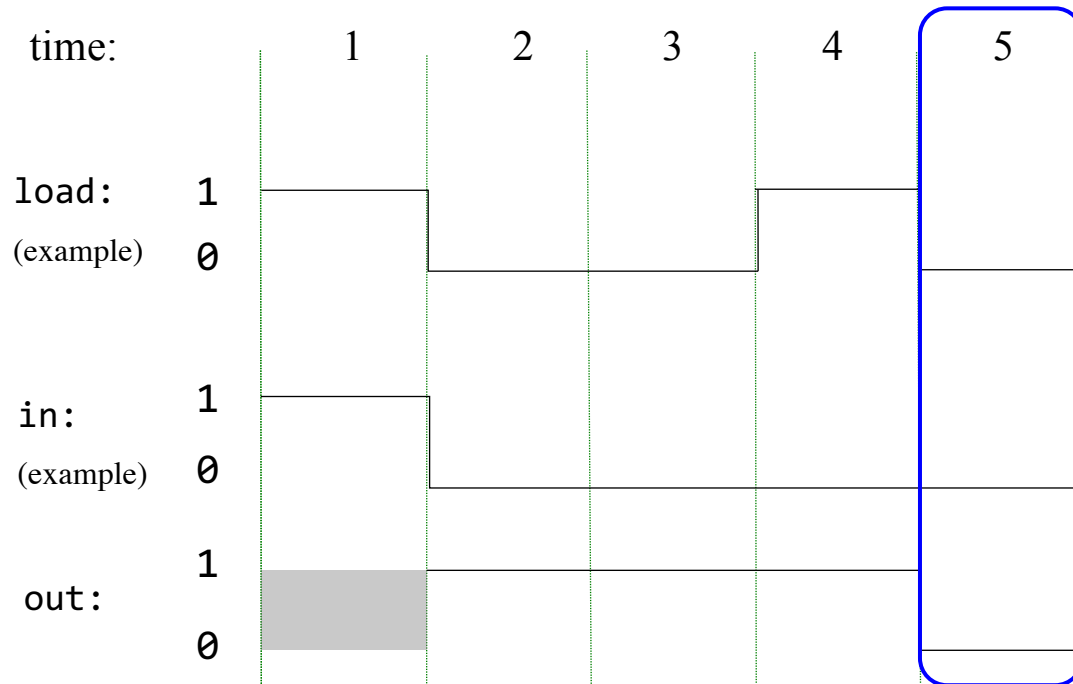
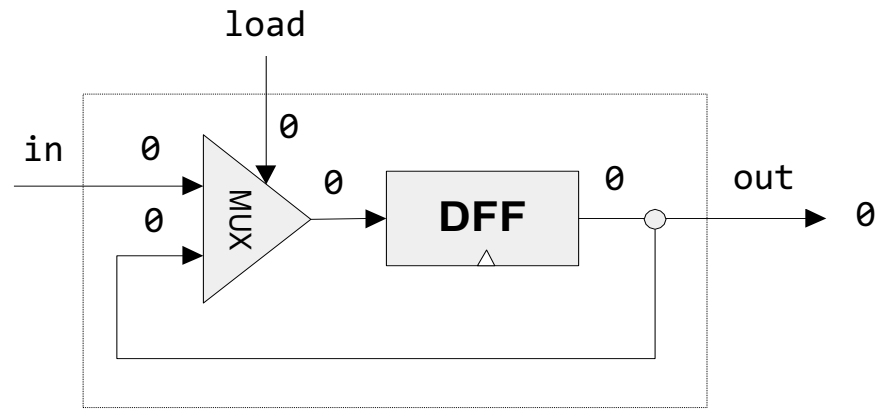
# 1-bit register implementation



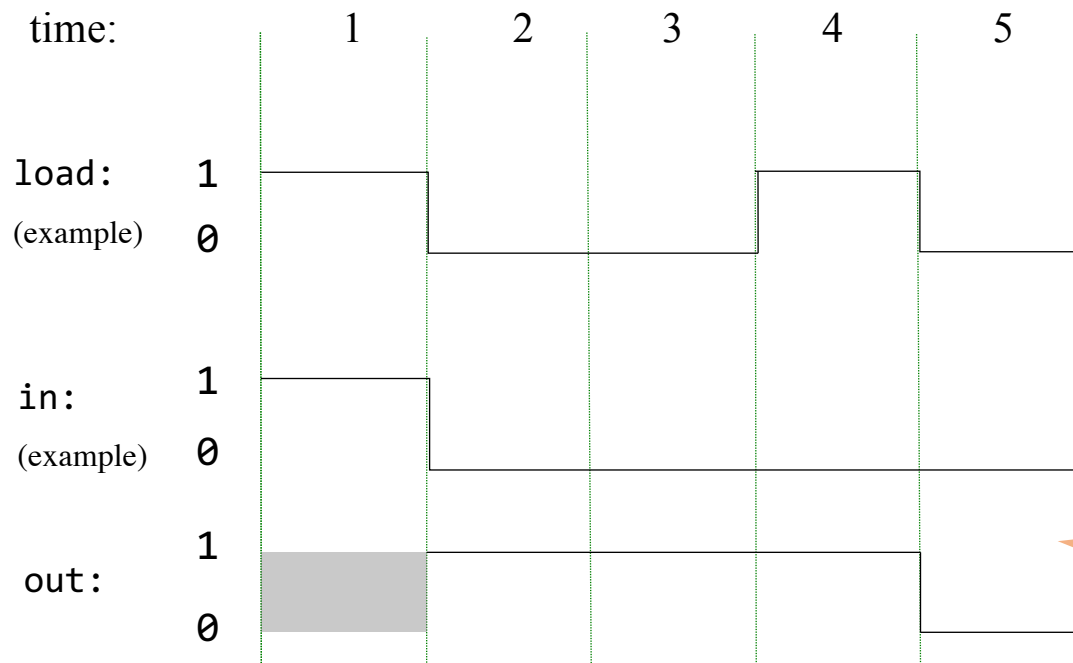
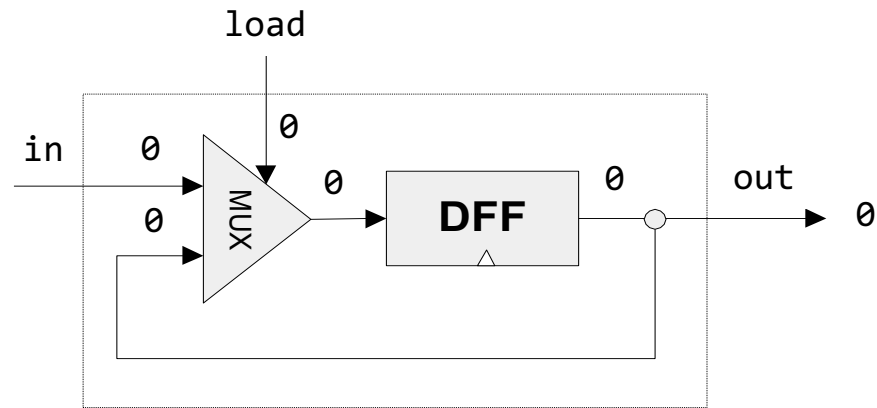
# 1-bit register implementation



# 1-bit register implementation



# 1-bit register implementation



Resulting behavior:  
Stores and emits a value, until instructed to load (and store) a new value

# Chapter 3: Memory

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Time matters



Sequential logic



Flip Flops



Memory units

- Counters
- Project 3 overview

# Memory units

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We'll describe (and build) a progression of memory units:



1-bit register:

Designed to store a single bit



Multi-bit register:

Designed to store, say, a 16-bit value

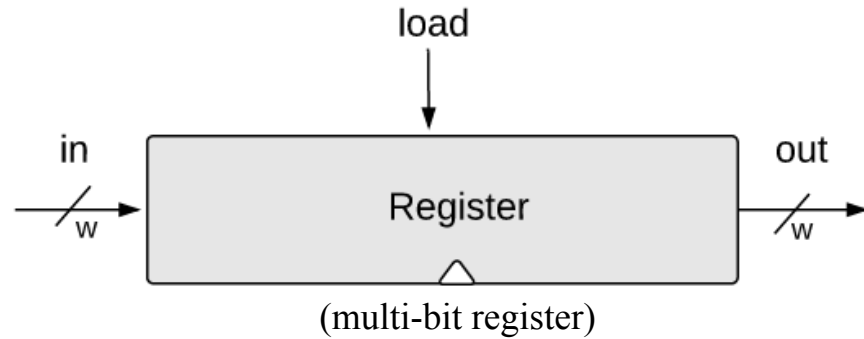
- Random Access Memory (RAM):

Designed to store  $n$  addressable 16-bit values,  
each having a unique *index*, or *address*, ranging from 0 to  $n-1$ .



# Multi-bit register (also known as “register”)

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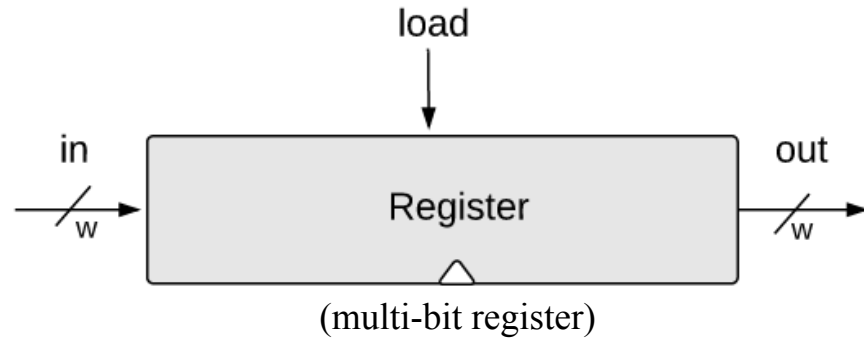


Word width ( $w$ ):

- 16-bit, 32-bit, 64-bit, ...
- We will focus on 16-bit registers, without loss of generality.

# Register: abstraction

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## To read a Register:

probe out

### Result:

out emits the Register's state

## To set Register = $v$

set in =  $v$

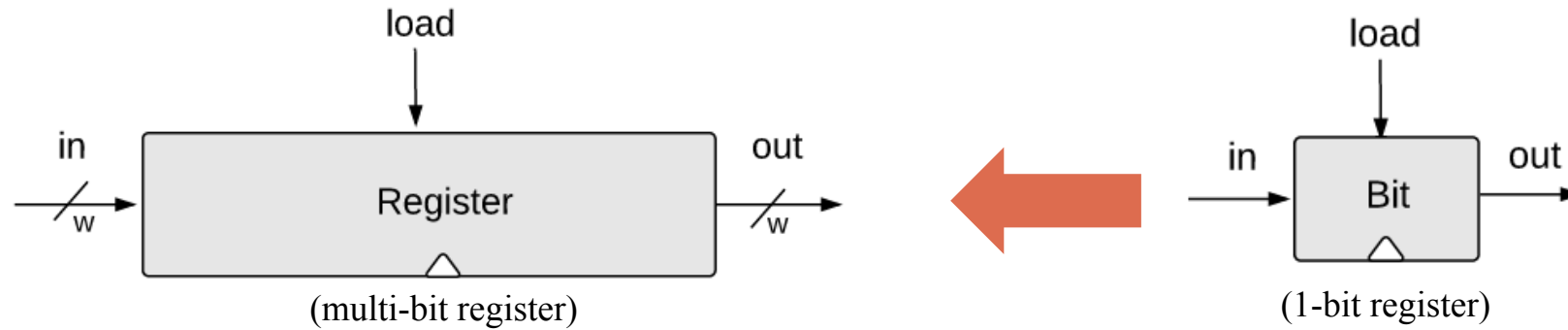
set load = 1

### Result:

- The Register's state becomes  $v$ ;
- From the next cycle onward, out emits  $v$

# Register: implementation

---



A  $w$ -bit register can be created from an array of  $w$  1-bit registers.

# Register chip in action

Run the clock

Chip Name: DRegister (Clocked) Time: 0

Input pins		Output pins	
Name	Value	Name	Value
in[16]	17	out[16]	0
load	0		

Set in to 17

Inspect the register's output

Inspect the register's contents

D: 0

For the demo, we use a built-in 16-bit register from the Hack chipset, named Dregister, or simply D.

```
HDL
* This built-in chip implements a 16-bit register
* providing a GUI representation of the register
* called "D register" (typically used in the Hack chip)
*/

CHIP DRegister {
    IN in[16], load;
    OUT out[16];

    BUILTIN DRegister;
    CLOCKED in, load;
}
```

# Register chip in action

Run the clock

Chip Name: DRegister (Clocked) Time: 12

Input pins	
Name	Value
in[16]	17
load	0

Output pins	
Name	Value
out[16]	0

**HDL**

```
* This built-in chip implements a D register,
* providing a GUI representation of the chip.
* called "D register" (typically used for
*/
CHIP DRegister {
    IN in[16], load;
    OUT out[16];

    BUILTIN DRegister;
    CLOCKED in, load;
}
```

Inspect the register's output

Inspect the register's contents

D: 0

# Register chip in action

Run the clock

Chip Name: DRegister (Clocked) Time: 12+

Input pins		Output pins	
Name	Value	Name	Value
in[16]	17	out[16]	0
load	1		

Set in to 17  
Set load to 1

Inspect the register's output

Inspect the register's contents

D: 17

```
CHIP DRegister {  
    IN in[16], load;  
    OUT out[16];  
  
    BUILTIN DRegister;  
    CLOCKED in, load;  
}
```

# Register chip in action

Run the clock

Chip Name: DRegister (Clocked) Time: 13

Input pins		Output pins	
Name	Value	Name	Value
in[16]	17	out[16]	17
load	1		

Inspect the register's output

Inspect the register's contents

```
HDL
/* This built-in chip implements a D register,
 * providing a GUI representation of the register's
 * state. The register is called "D register" (typically
 * used to store the current state of the game).
 */
CHIP DRegister {
    IN in[16], load;
    OUT out[16];

    BUILTIN DRegister;
    CLOCKED in, load;
}
```

D: 17

# Memory units

---

We'll describe (and build) a progression of memory units:



1-bit register:



Multi-bit register:

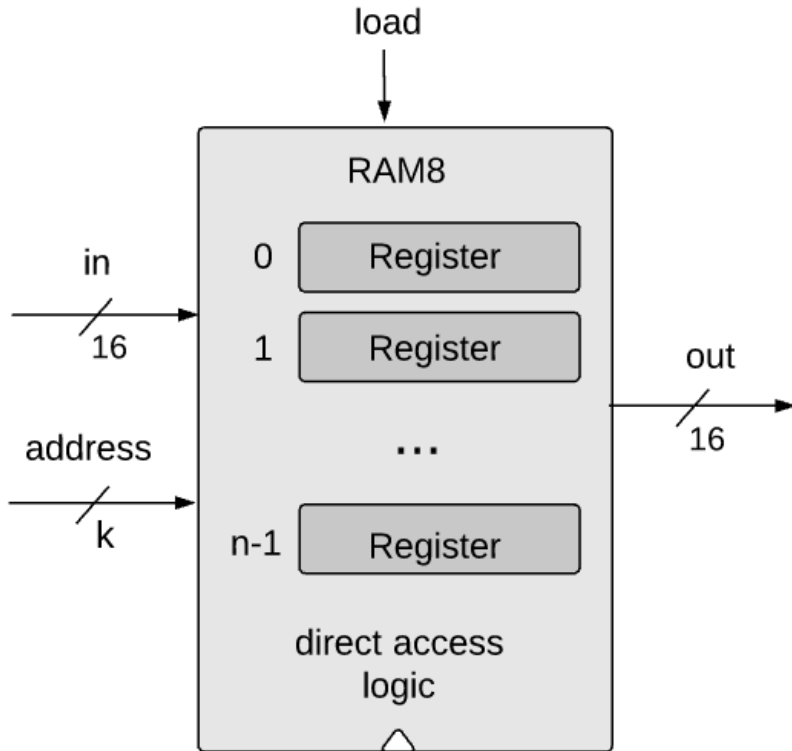


Random Access Memory (RAM)



# RAM

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## Architecture:

A sequence of  $n$  addressable registers,  
with addresses 0 to  $n-1$

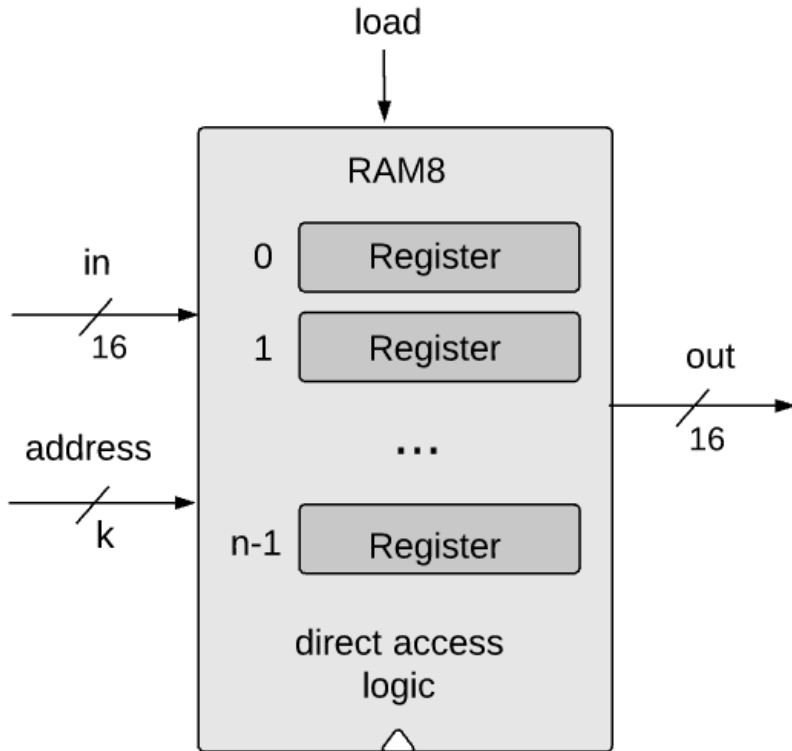
## Address width:

$$k = \log_2 n$$

## Word width:

No impact on the RAM logic  
(Hack computer:  $w = 16$ )

# RAM: abstraction



At any given point of time:

- ❑ *one* register in the RAM is selected
- ❑ all the other registers are irrelevant

**To read Register  $i$  :**

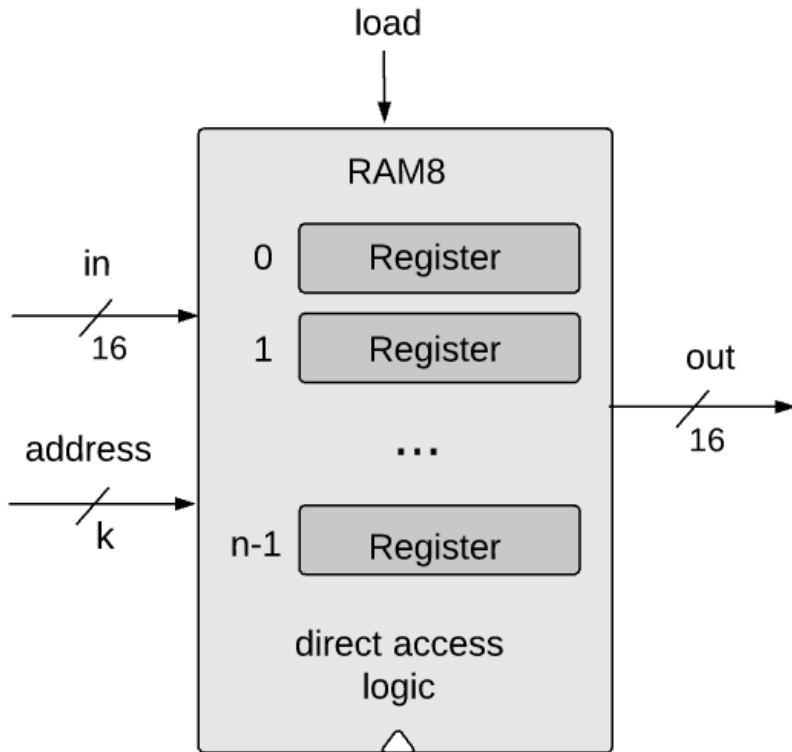
set address =  $i$

probe out

**Result:**

out emits the value of Register  $i$

# RAM: abstraction



At any given point of time:

- ❑ *one* register in the RAM is selected
- ❑ all the other registers are irrelevant

**To set Register  $i$  to  $v$  :**

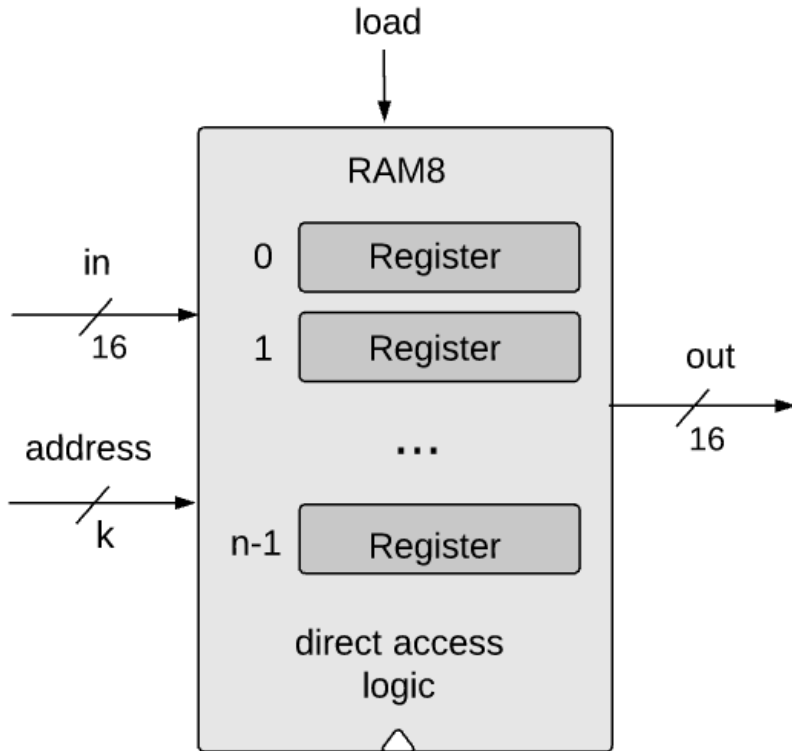
set address =  $i$   
set in =  $v$   
set load = 1

**Result:**

- The state of Register  $i$  becomes  $v$
- From the next cycle onward, out emits  $v$

# RAM: abstraction

---

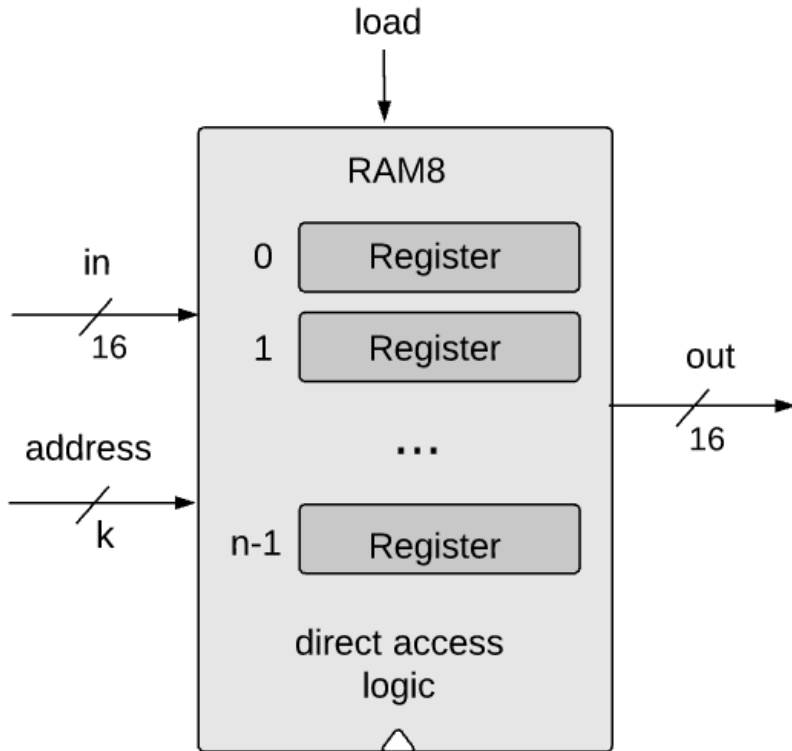


## Why “Random Access Memory”?

Irrespective of the RAM size ( $n$ ), every randomly selected register can be accessed “instantaneously”, at more or less the same time.

# A family of 16-bit RAM chips

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chip name	$n$	$k$
RAM8	8	3
RAM64	64	6
RAM512	512	9
RAM4K	4096	12
RAM16K	16384	14

Why these particular RAM chips?

Because that's what we need for building the Hack computer.

# RAM chip in action

---



# Chapter 3: Memory

---

 Time matters

 Sequential logic

 Flip Flops

 Memory units

 Counters

- Project 3 overview

# Where counters come to play

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- The computer must keep track of which instruction should be fetched and executed next
- This control mechanism can be realized by a register called Program Counter
- The PC contains the address of the instruction that will be fetched and executed next
- The PC is designed to support three possible control operations:

Reset: fetch the first instruction

PC = 0

Next: fetch the next instruction

PC++

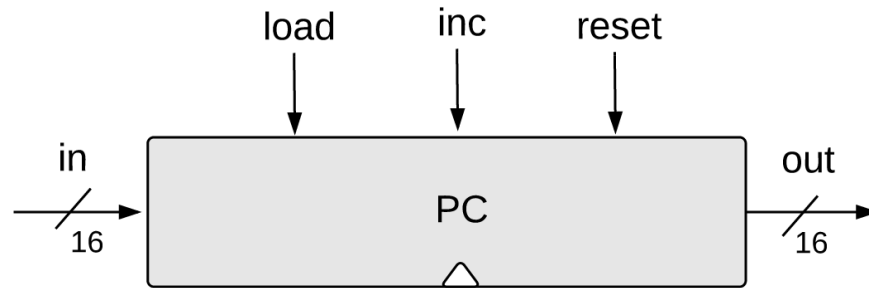
Goto: fetch instruction  $n$

PC =  $n$



# Program Counter

---



```
/**
 * A 16-bit counter with load and reset control bits.
 * if      reset(t) out(t+1) = 0
 * else if load(t)  out(t+1) = in(t)
 * else if inc(t)   out(t+1) = out(t) + 1 (integer addition)
 * else             out(t+1) = out(t)
 */

CHIP PC {
    IN in[16], load, inc, reset;
    OUT out[16];

    PARTS:
    // Put your code here:
}
```

# Counter chip in action

---



# Chapter 3: Memory

---

 Time matters

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 Project 3 overview

# Project 3

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## Given:

- ❑ All the chips built in Projects 1 and 2
- ❑ Flip-Flop (built-in DFF gate)

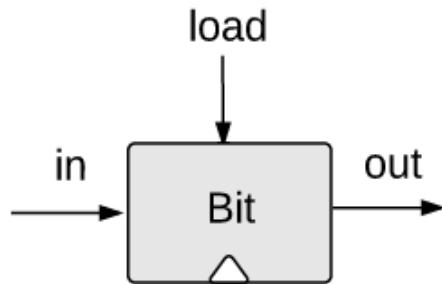
## Goal: Build the following chips:

- ❑ Bit
- ❑ Register
- ❑ RAM8
- ❑ RAM64
- ❑ RAM512
- ❑ RAM4K
- ❑ RAM16K
- ❑ PC

A family of sequential chips, from  
a 1-bit register to a 16K RAM unit.

# 1-bit register

---



Bit.hdl

```
/**
 * 1-bit register:
 * If load(t) then out(t+1) = in(t)
 * else          out(t+1) = out(t)
 */

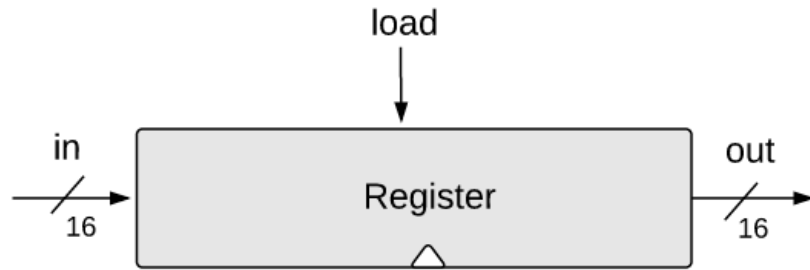
CHIP Bit {
    IN in, load;
    OUT out;

    PARTS:
    // Put your code here:
}
```

Implementation tip:  
Can be built from a DFF  
and a multiplexor.

# 16-bit Register

---



Register.hdl

```
/**
 * 16-bit register:
 * If load(t) then out(t+1) = in(t)
 * else out(t+1) = out(t)
 */

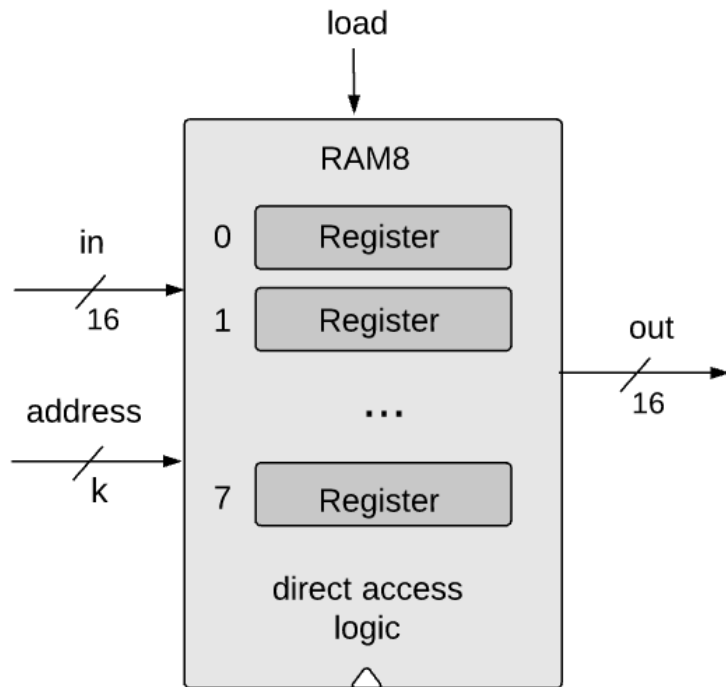
CHIP Register {
    IN in[16], load;
    OUT out[16];

    PARTS:
    // Put your code here:
}
```

Implementation tip:

Can be built from an array  
of sixteen 1-bit registers.

# 8-Register RAM



RAM8.hdl

```
/*
 * Let M stand for the state of the
 * register selected by address.
 * if load(t) then {M=in(t), out(t+1)=M}
 * else
 *     out(t+1)=M
 */

CHIP RAM8 {
    IN in[16], load, address[3];
    OUT out[16];

    PARTS:
        // Put your code here:
}
```

## Implementation tips:

- ❑ Feed the in value to all the registers, simultaneously
- ❑ Use mux / demux chips to select the register specified by address.

# Project 3

---

## Given:

- ❑ All the chips built in Projects 1 and 2
- ❑ Flip-Flop (DFF gate)

## Goal: Build the following chips:

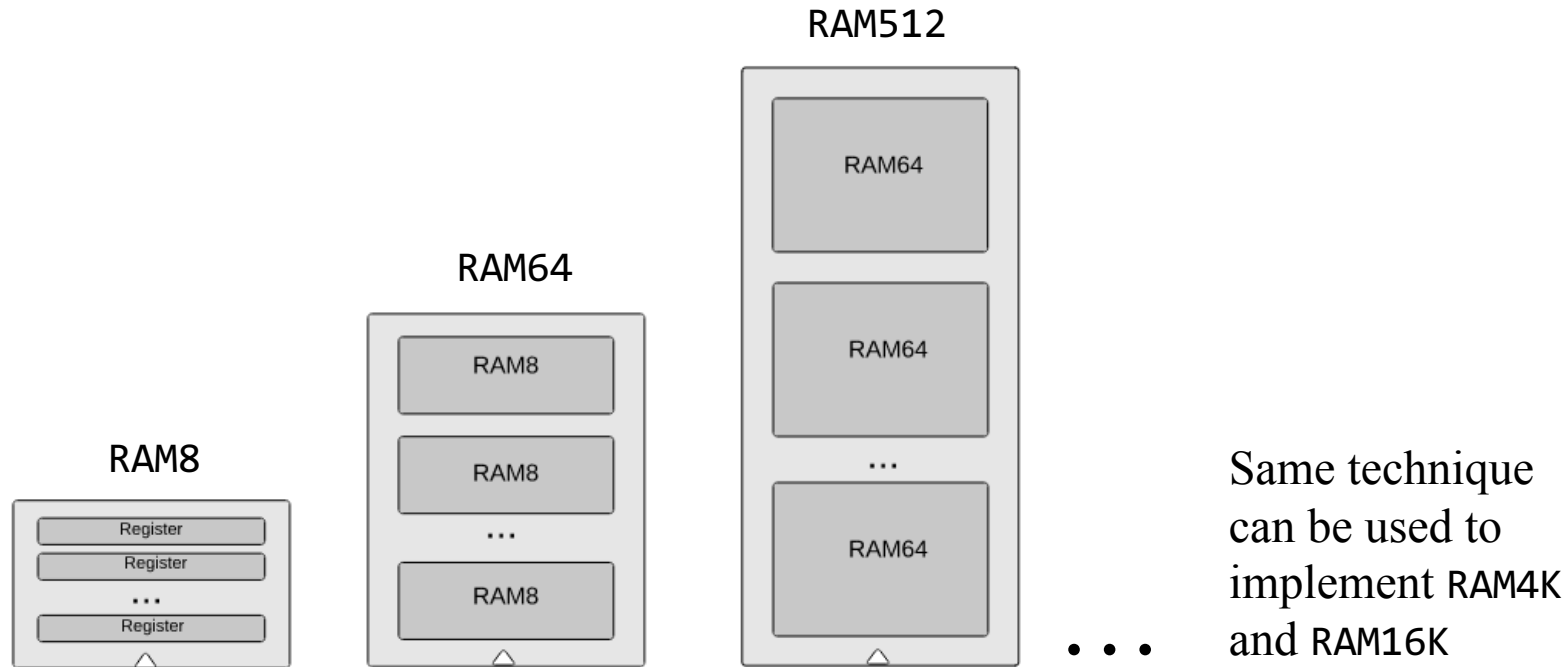
- ✓ ❑ Bit
- ✓ ❑ Register
- ✓ ❑ RAM8
- ❑ RAM64
- ❑ RAM512
- ❑ RAM4K
- ❑ RAM16K
- ❑ PC

} Our next task



# RAM8, RAM64, ... RAM16K

---



Same technique  
can be used to  
implement RAM4K  
and RAM16K

## Implementation tips

- A RAM unit can be built by grouping smaller RAM-parts together
- Think about the RAM's address input as consisting of two fields:
  - one field can be used to select a RAM-part;
  - the other field can be used to select a register within that RAM-part
- Use mux/demux logic to effect this addressing scheme.

# Project 3

---

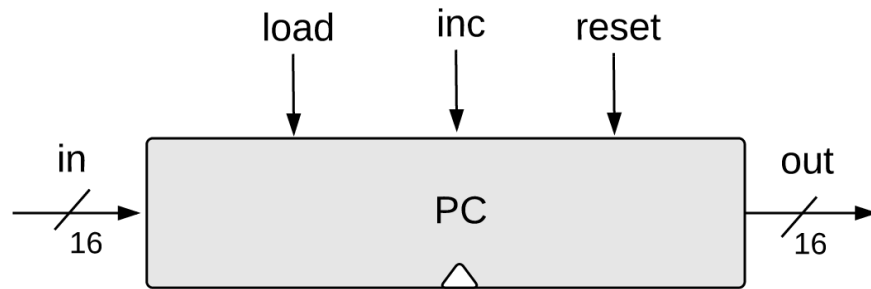
## Given:

- ❑ All the chips built in Projects 1 and 2
- ❑ Flip-Flop (DFF gate)

## Goal: Build the following chips:

- ✓ ❑ Bit
- ✓ ❑ Register
- ✓ ❑ RAM8
- ✓ ❑ RAM64
- ✓ ❑ RAM512
- ✓ ❑ RAM4K
- ✓ ❑ RAM16K
- ❑ PC

# Program Counter



## Implementation tip:

Can be built from a register, an incrementor, and some logic gates.

```
/**
 * A 16-bit counter with load, inc, and reset control bits.
 *
 * if reset(t)    out(t+1)=0           // resetting: counter = 0
 * else if load(t) out(t+1)=in(t)       // setting counter = value
 * else if inc(t)  out(t+1)=out(t)+1    // incrementing: counter++
 * else           out(t+1)=out(t)       // counter does not change
 */

CHIP PC {
    IN in[16], load, inc, reset;
    OUT out[16];

    PARTS:
        // Implementation comes here.
}
```

# Project 2 resources

From NAND to Tetris  
Building a Modern Computer From First Principles  
www.nand2tetris.org

Home

Prerequisites

Syllabus

Course

Book

Software

Terms

Papers

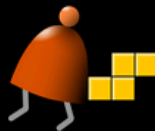
Talks

Cool Stuff

About

Team

Q&A



## Project 3: Sequential Chips

### Background

The computer's main memory, also called *Random Access Memory*, or *RAM*, is an addressable sequence of  $n$ -bit registers, each designed to hold an  $n$ -bit value. In this project you will gradually build a RAM unit. This involves two main issues: (i) how to use gate logic to store bits persistently, over time, and (ii) how to use gate logic to locate ("address") the memory register on which we wish to operate.

### Objective

Build all the chips described in Chapter 3 (see list below), leading up to a *Random Access Memory* (RAM) unit. The only building blocks that you can use are primitive DFF gates, chips that you will build on top of them, and chips described in previous chapters.

### Chips

Chip (HDL)	Description	Test script	Compare file
DFF	Data Flip-Flop (primitive)		
Bit	1-bit register	Bit.tst	Bit.cmp
Register	16-bit register	Register.tst	Register.cmp
RAM8	16-bit / 8-register memory	RAM8.tst	RAM8.cmp
RAM64	16-bit / 64-register memory	RAM64.tst	RAM64.cmp
RAM512	16-bit / 512-register memory	RAM512.tst	RAM512.cmp
RAM4K	16-bit / 4096-register memory	RAM4K.tst	RAM4K.cmp
RAM16K	16-bit / 16384-register memory	RAM16K.tst	RAM16K.cmp
PC	16-bit program counter	PC.tst	PC.cmp

All the necessary project 3 files are available in:  
nand2tetris / projects / 03

## More resources

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- HDL Survival Guide
- Hardware Simulator Tutorial
- nand2tetris Q&A forum



All available in: [www.nand2tetris.org](http://www.nand2tetris.org)

# Best practice advice

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- Try to implement the chips in the given order
- If you don't implement some of the chips required in project 3, you can still use them as chip-parts in other chips. Just rename the given stub-files; this will cause the simulator to use the built-in versions of these chips
- You can invent new, “helper chips”; however, this is not required: you can build any chip using previously-built chips only
- Strive to use as few chip-parts as possible.
- You will have to use chips from Projects 1 and 2
- Best practice: use their built-in versions
- For technical reasons, the HDL files of this project are organized in two directories named `a` and `b`
- This directory structure should remain as is.

# Chapter 3: Memory

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- ✓ Time matters
- ✓ Sequential logic
- ✓ Flip Flops
- ✓ Memory units
- ✓ Counters
- ✓ Project 3 overview



## Chapter 3

# Memory

These slides support chapter 3 of the book

*The Elements of Computing Systems*

By Noam Nisan and Shimon Schocken

MIT Press