# **Building and Sustaining Professional Collaborations**

# Using Japanese Lesson Study to Improve the Teaching and Learning of Mathematics

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This chapter has a double focus. One is the chronicling of the evolution of a professional learning community engaged in lesson study. This story includes the challenges the group faced, its recommitment to the process despite these challenges, and the innovations the group forged that sustained it over the next six years. The second focus is an account of the design process of a single, representative lesson developed by the members of this learning community. The process and results of the design and implementation of this grade 2 lesson, which concerned the addition of positive and negative numbers on a number line, illustrates how the group's innovations in the lesson study process were successful both in fostering students' understanding of integers and in giving the teachers important insights into mathematics for teaching and thus sustaining their ongoing interest in the lesson study approach. This lesson study case addresses some of the issues central to professional learning communities. These issues include facing challenges in professional development and related opportunities for learning; gaining and nurturing teachers' ownership of, and engagement in, collaborations; and employing structures that can support and sustain teachers' active participation over time.

We, the authors, are, respectively, a mathematics education researcher, a classroom teacher, an elementary school principal, and a doctoral student in math education. We are all affiliated with the Ontario Institute for Studies in Education at the University of Toronto, and we have all, in different roles, been involved with this lesson study project. We begin our chapter with a brief discussion of Japanese Lesson Study in North America, and the challenges it has faced in this setting. Then we document how the staff in a

university elementary laboratory school took up the lesson study process. We highlight three significant adaptations the group forged that appear to have contributed to the sustainability of the project. Finally, we discuss the design, implementation, and results of a representative lesson designed by members of the lesson study group.

### **Lesson Study and Professional Collaboration**

Recent years have seen a dramatic increase in North American settings in the practice of lesson study (Perry and Lewis 2009), a practice that involves collaborative planning, teaching, and reflecting on classroom lessons. The cycle of the four steps that constitute lesson study are: (1) goal setting/investigation; (2) planning; (3) implementation/research lesson; and (4) debriefing/reflection (Lewis, Perry, and Murata 2006). Credited with substantial gains in mathematics learning in Japan (e.g., Fernandez, Cannon, and Chokshi [2003]; Fernandez and Cannon [2005]; Lewis [2002a]), lesson study has gained the attention of researchers and educators as a powerful professional development model because of its classroom context, its ongoing collaborative process, its focus on students, and its attention to teachers' concerns and questions (Bruce and Ladky 2010). Additionally, Stigler and Hiebert (1999), in describing the benefits/potential of lesson study, assert that it provides a unique way for teachers to look at their own practice with new eyes.

Research has found that, despite the known benefits of lesson study and educators' increasing familiarity with the process, there appear to be considerable obstacles to the success of lesson study in North America (e.g., Fernandez and Cannon [2005]; Lewis [2002a]). One obstacle is the sustainability of lesson study over time. In particular, Murata has noted that in most cultures outside of Japan (such as in the United States), professional development is usually experienced as discrete and separate programs implemented from one year to another. In contrast, lesson study in Japan operates as a sustained effort that continues over years and becomes rooted in the teaching culture as an integral process in teachers' professional lives (Murata, in press). Although the challenge of sustainability besets many professional development initiatives, the particular challenge for lesson study in North America is that it requires a shift in mindset from discrete, year-to-year programs to an ongoing process.

## Experimenting with Lesson Study: Troubled Beginnings

The schoolwide lesson study project began in fall of 2004 with the teaching staff in a university laboratory school in downtown Toronto. The school is composed of nine classes of twenty-two students each, from preschool to grade 6, with each class representing a wide range of abilities. Each class has one teacher and, for part of the year, an intern student teacher. The school also has specialty teachers including art, French, physical education, and special education. Having heard reports of the early North American adaptations of lesson study, in particular from the initiatives of the Mills College Lesson Study Group

(e.g., Lewis [2002b]; Lewis and Tsuchida [1998]) and the work of the Fernandez and colleagues group at Teachers' College at Columbia University (e.g., Fernandez and Yoshida [2004]), several teachers and the principal proposed lesson study for the laboratory school.

A number of features of lesson study appealed to the teaching community at the school, particularly the collaborative nature of the process and the focus on observations of children's reasoning. However, once the school community began to engage with the lesson study process, what became immediately apparent was the tension that emerged between the teacher's customary discourse practices—typically focused on general pedagogy—and the discourse in lesson study, which is only intended to address the specific aspects of specific lesson goals and design elements.

Furthermore, as the teachers moved towards the planning stages, many became skeptical about some of the major components of lesson study—a situation that led to significant alterations to some of its fundamental principles. One aspect was a matter of philosophy: for many of the teachers in the school, the focus on the design of a *single* lesson seemed to contradict the school's implicit philosophy that the focus should be on a full unit of study. Thus, when the members of the school community first undertook lesson design, rather than following the customary lesson study practice of concentrating on a single lesson, they planned a longer unit of study.

A second concern was that teachers felt they were developing a closed, highly specific lesson script. Consequently, as the group engaged in their lesson planning, the teachers designed the lessons based on more open-ended ideas of how the lesson might be taught rather than on specific trajectories.

A third and very important issue with the process was the lesson study's inclusion of live classroom observations by other teachers and outside guests. The teachers felt this might be too intrusive for the students. To address this concern, it was decided that instead of observers in the classrooms with children, the lessons would be videotaped and reviewed later by the participants and discussant.

These concerns, and the consequent alterations to the lesson study process by the school staff, exemplify Fullan's (1993) caveat that educational terms travel well, but the underlying principles often do not. By the time the teachers had modified the process to suit their levels of comfort and familiarity, they had departed so substantially from the lesson study process as to render it nearly unrecognizable. In the end, this first-year effort was unsuccessful, and the teachers were disheartened. They found the lesson study process time-consuming and burdensome, with little that had been learned by either the teachers or the students.

### A Second (Successful) Try at Lesson Study

Although the group still had questions about the need for all of the steps and formalities of the lesson study approach, they still saw merit in the *goals* of lesson study and therefore were willing to learn more about the process. In particular, teachers were interested in how the structures of lesson study could be applied in their own setting and help them, as teachers, to become more knowledgeable about the teaching and learning of mathematics.

#### Learning about Lesson Study

In the fall of the second year the school invited a well-regarded lesson study researcher/practitioner, Aki Murata, to come to Toronto to lead the school staff in a one-day in-depth introduction to the philosophy and practices of lesson study. As a result of the teachers acquiring a much deeper sense of the potential of the lesson study process, the school staff committed to a new start and agreed to follow the detailed protocol of the full lesson study cycle. At this time they invited a mathematics education researcher (the first author of this chapter) to establish a database to track the development and use of lesson study in their school community. At the time of writing, the school is in the beginning of its sixth year of continuous lesson study—the first pilot year discussed above, followed by five successful years.

### **Establishing New Lesson Study Routines and Schedules**

In that year following Murata's visit the group established a number of protocols/practices and schedules for lesson study practice that continue to the present time. First, it was decided to have two lesson study groups (LSGs) with all of the teaching staff in the school belonging to one or the other. As for the schedule: each year, in the fall and in the spring, each LSG participates in a full lesson study cycle and designs and implements a new research lesson. The school staff set aside two full days per year (fall and spring) when these lessons are publicly presented. These public lesson days follow a specific protocol: in the morning members of the two LSGs distribute lesson plans and observation guides that they create for the visiting teachers and discussants. Then the two research lessons are presented consecutively, with one member of each LSG teaching the specially designed lesson to a class of students while being observed by the rest of the LSG and guests and discussant. The morning lessons are followed by an afternoon colloquium session in which members of the LSG reflect on the lessons, and then other observers, followed by a discussant, draw out implications for lesson study and for teaching learning more broadly. (For more detail on the process, see Moss [2007].)

### Researching Our Practice: Design Research Framework

Through this process, the school staff has designed and implemented more than twenty math research lessons covering a wide range of mathematical topics for the full span of age groups in the school. We have also been carrying out an ongoing and extensive data collection encompassing video data from teachers' planning meetings, the public research lessons, and the debrief sessions; artifacts produced by the teachers such as lesson plans, written rationales for the lessons, research gathered on the selected topic, observation checklists, and email correspondence; and student work (collected for selected studies). We have employed the Design Research approach of Kelly and Lesh (2000) and the analytic methods of Collins et al. (2004) to trace the evolution of the lesson study process and to focus on the "products" of this work (Wood and Berry 2003). These "products" include not only materials such as lessons for dissemination, presentations, and publications, but

also products of a more theoretical nature, such as the innovations and adaptations to the lesson study process that we describe in this chapter. We chose the Design Research methodology because it allows us to codify and analyze teaching and learning in complex classroom contexts over an extended time. Collins, Joseph, and Bielaczyc (2004) make explicit connections from the Design Research framework to lesson study, describing the iterative processes of testing and refining inherent in both.

At the outset, our research focused on change and growth among the school staff in their level of comfort with and knowledge of mathematics for teaching (Ball et al. 2008). To do this, we examined changes in the teachers' use of mathematical language both in planning and debrief meetings, as well as in the teachers' production of lesson plans and written materials. In the written materials the teachers produced and in the transcripts of the planning and debrief sessions, we found notable shifts from general language to a more specialized use of mathematical vocabulary in the use of mathematical language.

Our research focused on tracking the innovations and adaptations that the group forged over the years. Our goal has been to contribute to Lewis et al.'s (2006) call for more research to identify mechanisms in the lesson study process that may support its sustainability and promote teacher learning. In the remainder of this chapter, we present our findings on the adaptations and provide a detailed discussion of a specific lesson to highlight these adaptations in practice.

## Findings: Sustaining Lesson Study through Adaptations

Three adaptations evolved over the five years, and, unlike the adaptations in the initial year, these informed adaptations significantly advanced the function of the group and therefore the sustainability of the lesson study. Given that sustaining change in general (and the lesson study process in particular) is challenging, and given the ambivalence with which the teachers initially began this project, these adaptations that led to sustained lesson study are significant. The three adaptations can be summarized as: (1) broadening the composition of the LSGs; (2) expanding the criteria for selecting mathematics topics; and (3) inserting a practice lesson, an extra step in the usual lesson study process.

### First Adaptation: Broadening the Composition of the Lesson Study Groups

As we mentioned above, when the school staff recommenced their lesson study work in the second year, they chose to establish two LSGs that matched the grade levels of the students. In keeping with common lesson study protocol, the two LSGs represented different school divisions. One LSG was composed of primary teachers (pre-kindergarten to second grade) and the other, the junior grade teachers (third to sixth grade). The school specialty teachers (such as art, French, and physical education) also moved into the groups congruent with the ages of the children they were teaching. In year 3, the teachers decided

to expand the grade-level span so that each LSG had teachers spanning pre-K to grade 6. One very clear benefit of this new grouping of teachers was the diverse range of knowledge that was brought to their lesson design process.

## Second Adaptation: Expanding the Criteria for Selecting Mathematics Topics

With the broadened composition of the LSGs came a shift away from selecting mathematics topics specific to the curriculum, and regarded as challenging, in favor of topics that crossed grade levels and were inspired by shared teacher interests. Initially, the LSGs chose topics that aligned with specific curriculum topics. Over time, however, teachers considered broader topics (e.g., proportional reasoning, understanding mathematical relationships though music, and, as we elaborate below, integers on a number line). This expanded perspective opened up the possibility for the teachers to explore diverse mathematical ideas that they might not have otherwise encountered. This teacher-initiated adaptation enriched the lesson study process for the teachers (and for the students).

### Third Adaptation: Insertion of the Practice Lesson

As noted earlier, in their second year of lesson study teachers committed to follow the four-step lesson study process. In step 1, goal setting/investigation, the teachers select two topics and align goals for the lesson study. Next, in step 2, planning, the LSG examines samples of students' work on the chosen topic, and studies different textbooks' treatments of the topic, which members then use or adopt for their lesson. The third and fourth steps—conduct research and debrief—involve the LSG in the public teaching and debriefing of the specially designed lessons, with guests and members of the LSG collecting data on students' learning.

As the teachers gained more experience with lesson study, they decided to expand the process by adding an intermediate step between steps 2 and 3. This new step became known as the practice lessons—a series of lessons carried out in different grades in advance of the public lesson. Members of the LSG began the practice of experimenting with variations of the research lesson in their own classrooms. These experimental practice lessons gave members of the LSGs the opportunity to determine the appropriate grade level for the final research lesson, to explore the effectiveness of various representations, and to decide on appropriate revisions and refinements. Indeed, our review of our data over the five years revealed that the practice lessons not only led to significant revisions of the final lessons, but also, in the majority of cases, to the LSGs reconsidering the appropriate grade level for the lesson. A particular advantage of the iterative nature of the practice lessons was that it led the teachers to a higher level of analysis of the mathematical topic itself and of its pedagogical implications, and thus a more enriched debriefing session on the public lesson days.

### Integers in Grade 2 on a Life-Size Number Line: The Adaptations in Practice

As an example of what these adaptations/innovations looked like in practice, we present the case of the Integer Lesson—one of the two lessons created in the eleventh cycle of lesson study. We summarize each of the phases of lesson study, including the additional practice-lessons process.

#### Step 1: Goal Setting

At the outset of this lesson study cycle the school staff met to select the topics for the term (fall 2009). In the case of the integer lesson, the selection process began with a suggestion from the physical education teacher to explore the connections between mathematics learning and movement. This idea appealed to others in the group. One teacher mentioned her interest in Howard Gardner's ideas of multiple intelligences, kinesthetic learners, and the implications for mathematics learning. The grade 5 and 6 teacher added that he had heard about research suggesting that gesture may play an important role in mathematics learning. The grade 2 teacher also mentioned a movement game, a form of hopscotch, that she was using with her students as they consolidated addition and subtraction learning. The foregoing were some of the comments that led the group to the decision to explore number relationships through body movement and location in space. The idea of integers came up when the grade 1 teacher recounted a story of a routine classroom calendar activity in which one of her students proposed including a negative number as a creative solution strategy. As the discussion of these ideas went on, the teachers became intrigued with a central question: What could primary students understand about integers, a topic normally introduced to much older students, if mathematics learning were to be integrated with movement? Based on these discussions, an LSG was formed. It included the pre-kindergarten teacher, art teacher, librarian, school principal, several grade-level teachers, and the physical education teacher, and was charged with exploring and designing an introductory lesson on integers and movement.

#### Step 2: Planning

Once the LSG had decided on movement and integers, the next step was to find resources offering theories and teaching approaches to support students' introduction to integers in grade 2. Generally, at this stage of the design process, the LSGs would consult a variety of math textbooks to find published lessons to draw from. In this case, however, the LSG, though unable to find appropriate resources for grade 2 students, did find lessons for older students that, among other representations, used a thermometer metaphor as an approach to teaching integers. The librarian noted that, because this lesson was to be taught in the fall, the students would be accustomed to the November temperature fluctuations above and below zero Celsius. After considerable discussion, the LSG decided on the context of weather and a thermometer representation because of the real-world connections. A large

thermometer was constructed with two sections, red for the positive integers above zero, and blue for negative integers below zero. The general plan for the lesson involved students in moving along the thermometer in directions and increments randomly assigned by the throw of color-coded dice—red for positive and blue for negative.

### Step 2.5: Practice Lessons

**Practice lesson 1.** To test the effectiveness of their lesson plan, the LSG presented a practice lesson in a grade 3 classroom—the targeted grade level for the lesson. As the teachers embarked on teaching this practice lesson, they immediately found it was not working as expected. There were two problems. The first was the context of weather: the students appeared to be more interested in discussing weather fluctuations (and storms they had experienced) than in trying to make sense of the properties or concepts of integers. The second problem was the use of color in both the die and the thermometer: instead of considering directionality on the number line, the students become preoccupied with color as a indicator of movement.

The experience of this practice lesson provided the members of the LSG with a broader, more nuanced understanding of the issues arising from the use of real-world contexts. The issue of real-world applications in mathematics teaching then became a central focus of the LSG's subsequent discussions as they worked on revisions to this lesson. The teachers had assumed that embedding mathematics in real-world contexts would naturally make the mathematics meaningful. Forced by the results of the first practice lesson to reconsider this assumption, the teachers decided to eliminate the thermometer and to try a more abstract context, a number line.

Additional practice lessons. In a subsequent practice lesson the LSG discovered that this new number-line context did indeed support the students in gaining an initial understanding of integers. And, based on further evidence gleaned from a third practice lesson conducted with grade 2 students, they refined the lesson further and added an additional component. In our view, the planning process for the integer lesson was enriched by the diversity of the LSG's interests and the breadth of their collective experiences and mathematical knowledge—all of which were deepened and strengthened by the additional iterations of the practice lessons.

### Step 3: Implementation/Research Lesson

The public lesson was taught in second grade by the pre-kindergarten teacher with the help of the physical education teacher. The scope of this paper does not permit an account of the full lesson design, but briefly, the lesson focused on the addition of integers through movement on a floor-sized number line in response to challenges on prompt cards. First in the sequence, a student was given a card (the Start Card) with a number that indicated a starting position on the number line. A second card (the Add/Moving Card) was given next. The student then had to determine the sum and move to the appropriate space on the number line. To begin the lesson, the students sat on the

floor around a number line calibrated from 0 to +8 and the teacher led the students in a specific series of challenges. The initial prompts in the game were sequenced so that after two warm-up rounds, where the students were asked only to move to sums of positive integers, a negative integer card was introduced as a Move Card. Specifically, this challenge was to determine and move to a number on the line that corresponded to the sum of positive 2 and negative 4. The conjecture was that, although the students had not been formally introduced to number lines with a negative extension, through the specially sequenced activities, and because of the students' comfort with the operation of addition, the students would naturally see how the 0 to +8 number line would need to be extended to include negative integers. (In anticipation of this, the LSG had already constructed another number line, calibrated from 0 to -8, which they placed out of sight of the students.)

As the transcript below reveals, the students did recognize a need for an extension of the number line:

Teacher:

[Holding up a card for the children to see] What is our start number?

Children:

[In chorus] Two . . .

Teacher:

[Holding up a second card and pointing to it] What is the moving

number?

Children:

[At once] Minus four, no, negative four, minus four ...

Teacher:

So then what will the end number be? Who will try? [One of the students, Sam (a pseudonym) stands on the number line on positive 2 as the other children make suggestions.]

Student 1:

Maybe go back to the starting.

Sam:

[Moving two steps back to stand on zero; he looks puzzled] But we don't have enough numbers. We need more numbers. We need more number line. There would have to be minuses.

Student 2:

We need minuses.

Student 3:

[Pointing to the floor just below the start of the 0-8 number line] They would go here.

Teacher:

Well ... I just happen to have more number line. [Teacher places this extension on the floor, and Sam continues to walk backwards and stands at negative 2.]

Student 5:

Now you can see that the answer is negative two.

Overall, the children appeared to be happy to participate and engaged with the lesson. As a final piece of the lesson, the teachers gathered the students back to the central number line (students had worked in pairs on individual number lines -8 to +8) and posed the following question:

Teacher: If my number was a negative number, any negative number, and I wanted to get to positive six, what could my moving number be?

Many students put up their hand to answer with suggestions; below are the responses of three of the students.

Student 5: I am imagining it [the negative integer] as minus three, so the answer is nine.

Student 6: I am imagining it as negative 100, so positive 106.

Student 7: The negatives go on and on and never stop, just like the positives.

#### Step 4: Debriefing/Reflection

Both the members of the LSG and the visiting observers agreed on the success of the flow of the lesson, the sequencing of the activities, and generally that the lesson design and particular challenges had supported the students' understanding. As one visiting teacher commented: "One thing that I loved about the lesson is that negative numbers were introduced in such a natural way, almost as a by-product of moving on the number line. It was if you are moving that far in that direction on the number line, there have to be numbers down there."

The discussant for this public lesson, a research mathematician, offered several positive comments on the choice of the number line representation. He noted that this representation not only supported the students' understanding, but also allowed them to communicate this understanding physically, as shown by the way the students naturally oriented themselves on the number line as they responded to the challenges: "I was so intrigued to see them walking forward going up the number line for positive numbers and walking backwards down for negative numbers."

The discussant also noted how the final probes designed by the LSG enabled students to express their understandings and insights, thereby serving to inform the teachers. As a wrap-up at the end of the lesson and as a way to encourage the students to reflect, the teacher (from the pre-kindergarten), asked the (grade 2) students to consider how this same lesson might work for her very young students. The children agreed that the first part of the lesson (adding positive integers walking on a number line) would be fine for her pre-kindergarten students, but the second part of the lesson, the addition of negative numbers, would not work because, in the words of the grade 2 students: "JK children don't know subtraction." This response impressed the discussant, indicating to him that the grade 2 students had made a significant mathematical connection between negative numbers and subtraction.

### Reflections on Sustaining Lesson Study

The evolution of the lesson study process and the innovations that were generated and adopted by the teachers themselves—group composition, topic selection, and practice lesson—seems to have afforded the teachers with diverse and frequent authentic learning opportunities that were engaging and served to support the sustainability of this five-year project.

Broadening the composition of the LSGs. When the teachers began that first year of committing to practice the full cycles of the lesson study, they spontaneously moved into two LSGs that represented similar grade levels and school divisions. As time went on they began the practice of sorting themselves into one of two groups based on their individual interests—a process that continues to this day. This new way of grouping themselves appears to have created a cross-fertilization of ideas, experiences, and knowledge; this, in our view, has served to increase the staff's enthusiasm and learning, and to have sustained their engagement in the project. Davis and Simmt (2003, 2006) make compelling arguments grounded in complexity theory regarding the affordances of diverse groupings: in their words, "Each learner has access to a diversity of interpretations and strategies to make sense of the concept or task at hand."

Expanding the criteria for topics. Along with the new mix of LSG membership, and afforded by this new diversity, came the natural broadening of potential subject areas and mathematical topics for the teachers to explore. The effect of this change has been profound. As we look at the evolution of the staff's mathematical discourse, as analyzed through reviewing the videotapes of their discussions in planning and debriefing meetings, it is clear that because of the broad range of topics selected over the years, many of them challenging, and because of the depth at which the LSGs come to know these, there has been significant overall growth in the teachers' Mathematical Knowledge for Teaching (MKT).

The practice lessons. The extra step of the practice lessons has also played an important role in supporting the growth of the MKT. Considering the integer lesson alone, the addition of practice lessons resulted in discussions that not only involved analyses of learning trajectories and developmental issues for students' understanding of integers, but also the effectiveness of different representations for understanding integers and the role of contexts in learning mathematics.

This case study, while focused on a particular group in a particular situation, provides general insights and implications for both lesson study and other professional development. In a recent paper, Perry and Lewis (2009) present a case study of a successful sustained district-wide lesson study initiative. Their findings point to factors that contributed to the longevity and sustainability of that lesson study project. Amongst their findings was the crucial importance of providing teachers with diverse and frequent learning opportunities, or in Fullan's words, with continuous learning (Fullan 1993). In this case study, having the capacity to adapt the lesson study process after it was known and effective provided diverse and intellectually engaging learning opportunities for teachers and for students.

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