

# Tangencies

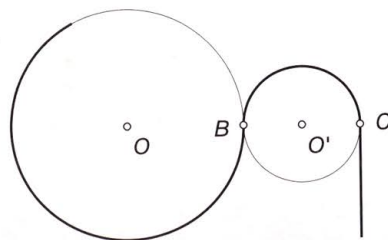
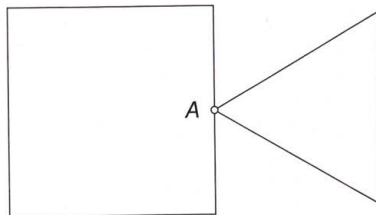


Look at this window. The design of the bars is based on straight lines, circles and arcs which join together to form shapes linked at points of tangency.

Two shapes are **tangents** when they meet at only one point, known as the point of tangency.

A smooth join between two curves or between a curve and a straight line is called **blending**, and this kind of join depends on tangency.

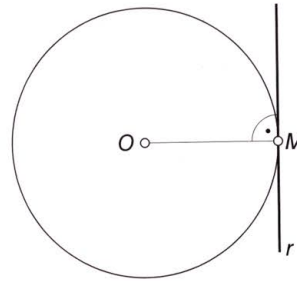
Tangency can occur between arcs and straight lines, between polygons and straight lines, between arcs and polygons and so on. However, the tangents used most in technical drawing are between straight lines and circles and between two circles.



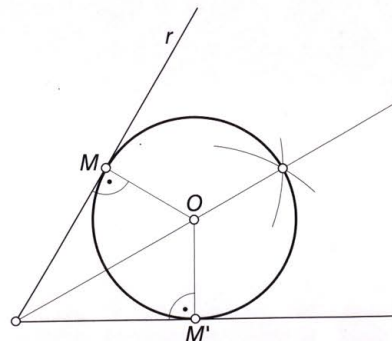
## 3.1 Basic properties of tangents

To work out exactly where tangent lines go, we use the following theorems:

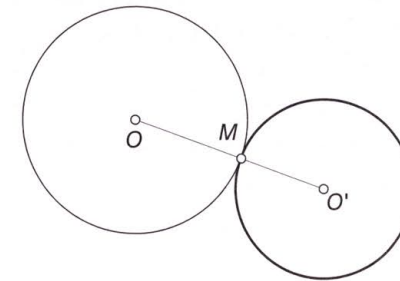
- **First theorem:** A straight line is tangent to a circle if there is a single point of contact between them ( $M$ ), and the straight line is perpendicular to the circle's radius at point  $M$ .



- **Second theorem:** A circle is tangent to two straight lines that meet if the circle's centre is located along the angle bisector of the angle formed by the straight lines.



- **Third theorem:** Two circles are tangents if they have a single point of contact ( $M$ ) on a straight line between the two circles' centres.

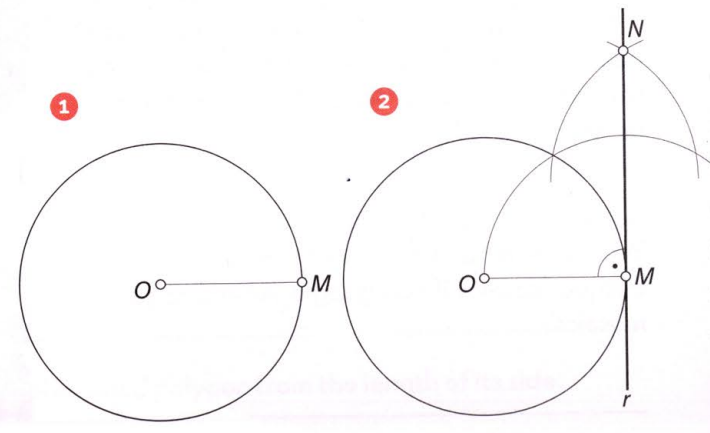


## 3.2 Tangency between straight lines and circles

We will now look at how to draw some of the most common tangents used in technical drawing.

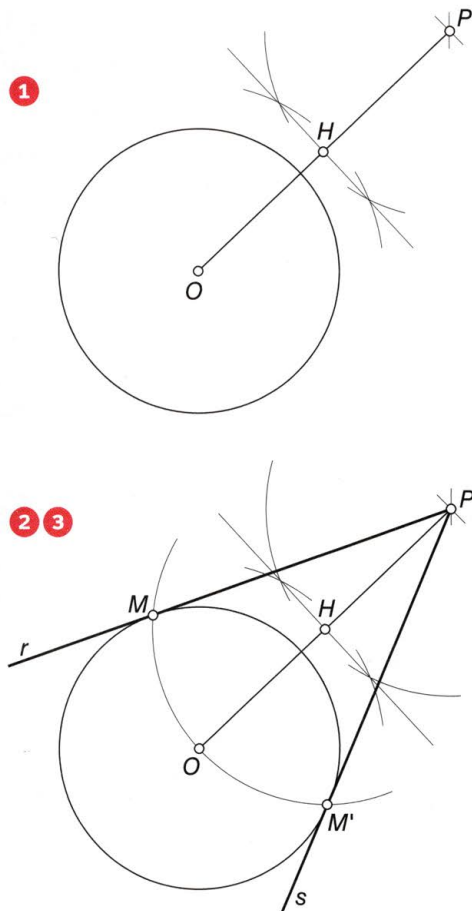
### A. Drawing a straight line tangent to a circle with a point of contact at $M$ .

1. Draw the radius between points  $O$  and  $P$ .
2. Then, at point  $M$ , draw a line perpendicular to the radius. This is the tangent line,  $r$ .



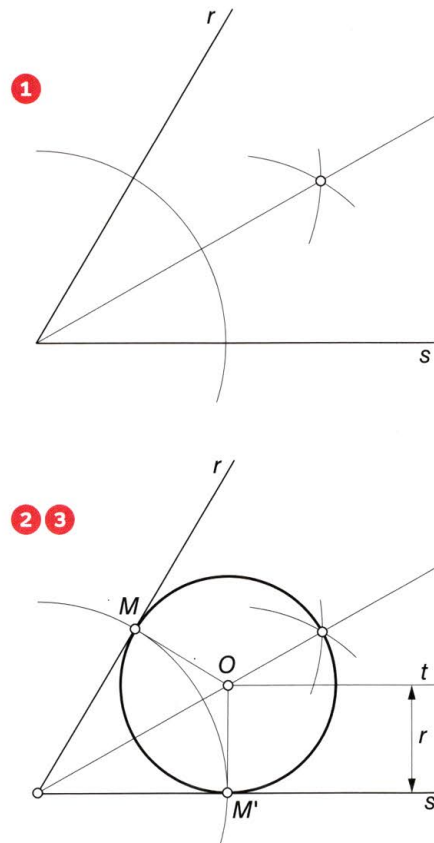
### B. Drawing tangents to a circle from a point, $P$ , outside the circle

1. Join point  $P$  with the centre of the circle,  $O$ , and draw its segment bisector to find point  $H$ .
2. Using point  $H$  as the centre, and a radius of length  $HO$ , draw an arc that crosses the circle at points  $M$  and  $M'$ , which form the points of tangency.
3. Join point  $P$  with points  $M$  and  $M'$  to find the tangent lines  $r$  and  $s$ .



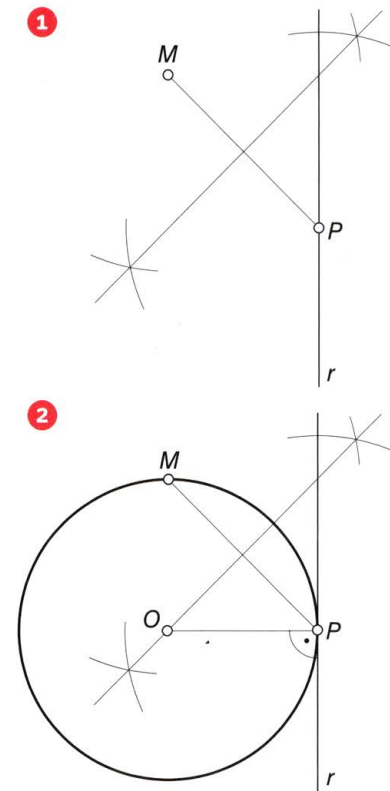
### C. Drawing a circle with a known radius $r$ and $s$ tangent to two convergent lines, $r$ and $s$

1. Draw the angle bisector of the angle formed by the two straight lines.
2. Draw a straight line,  $t$ , parallel to one of the given straight lines at a distance equal to the length of the known radius,  $r$ . The point where  $t$  crosses the bisector is the centre of the circle we want to draw, point  $O$ .
3. Draw the perpendicular radius to  $r$  and  $s$ . The intersection between the radius and the lines are points of tangency  $M$  and  $M'$ .



### D. Drawing a circle that passes through point $M$ and is tangent to the straight line $r$ at point $P$

1. As  $M$  and  $P$  have to be points on the circle that we want to draw, the circle's centre must be somewhere along the perpendicular bisector of segment  $MP$ .
2.  $P$  has to be the point of tangency on line  $r$ , and the circle's centre,  $O$ , must be where the line perpendicular to  $r$  at point  $P$  crosses the perpendicular bisector of the line  $MP$ .



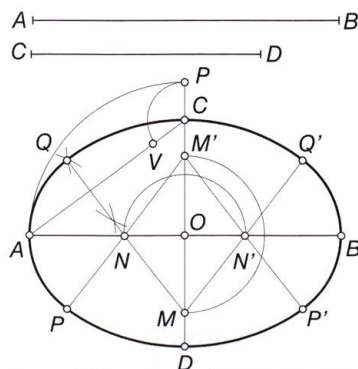
# Blending

## 5.1 Ovals and ovoids

Ovals and ovoids are flat, closed, symmetrical curves. These curves are structured by two axes, a major one and a minor one. Ovals are symmetrical along their two perpendicular axes, while ovoids are only symmetrical along the long axis. In both cases, the shapes are formed by four arcs.

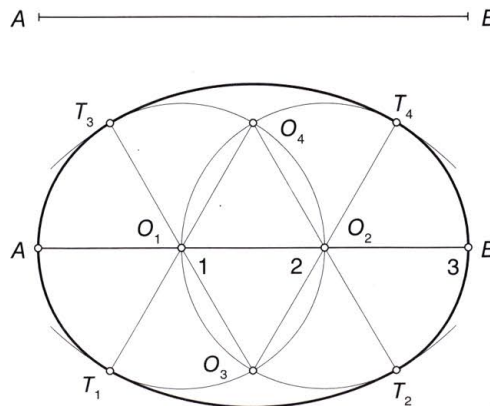
### A. Drawing an oval knowing its axes

1. Draw an arc, using centre  $O$  and radius length  $OA$ , that crosses the extension of  $CD$  at point  $P$ . Join  $A$  and  $C$ .
2. Draw another arc with centre  $C$  and radius length  $CP$  to cross the  $AC$  segment at point  $V$ .
3. Draw the segment bisector of  $AV$ , which crosses the extension of  $OD$  at point  $M$  or inside the segment, and the long axis at point  $N$ .
4. Mark the symmetrical points of  $M$  and  $N$  in relation to the oval's axes,  $M'$  and  $N'$ .
5. Join  $M$  with  $N$  and  $M'$  with  $N'$ , and draw arcs with centres at  $M$  and  $M'$ , with radius lengths  $M'D$  and  $MC$  respectively, to obtain points  $Q$  and  $Q'$ , and  $P$  and  $P'$  (tangency points).
6. Finally, draw arcs with centres  $N$  and  $N'$  and radius lengths  $NA$  and  $N'B$  to join the points of tangency marked previously,  $Q$  and  $P$ ,  $Q'$  and  $P'$ , to complete the oval.



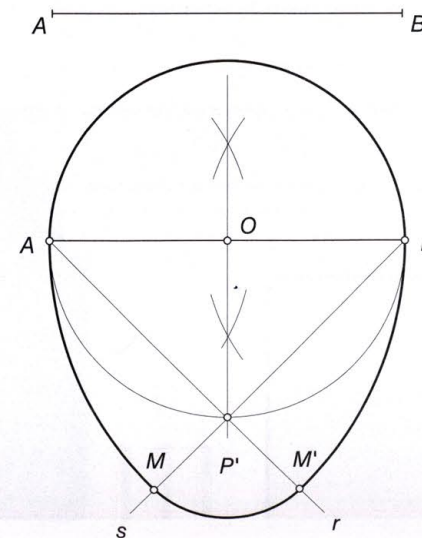
### B. Draw an oval knowing its major axis AB

1. Divide the major axis  $AB$  into three equal parts in order to find points  $O_1$ ,  $O_2$  and  $O_3$ .
2. With the centre at  $O_1$  and the radius  $O_1-O_2$ , draw a circumference. Do the same thing with the centre at  $O_2$  and a radius  $O_2-O_3$ . This last circumference crosses the previous one at points  $O_3$  and  $O_4$ .
3. Draw lines from  $O_3$  to  $O_1$  and  $O_2$ , and from  $O_4$  to  $O_1$  and  $O_2$ . The points where these lines cross the circumference are tangency points  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$ .
4. Finally, with the centre at  $O_3$ , join  $T_3$  and  $T_4$ , and with the centre at  $O_4$ , join points  $T_1$  and  $T_2$ .



### C. Drawing an ovoid with a known minor axis

1. Get point  $O$  by drawing the perpendicular bisector of the known axis  $AB$ .
2. With its centre at  $O$  and radius  $OA$ , draw a circumference that intersects the perpendicular bisector at point  $P$ .
3. Join points  $A$  and  $B$  to  $P$  to give us the lines  $r$  and  $s$ .
4. Draw two arcs with radius  $AB$  and centres at points  $A$  and  $B$  to get tangency points  $M$  and  $M'$ .
5. With  $P$  as its centre and radius  $PM$  or  $PM'$ , draw the last arc to finish our ovoid.



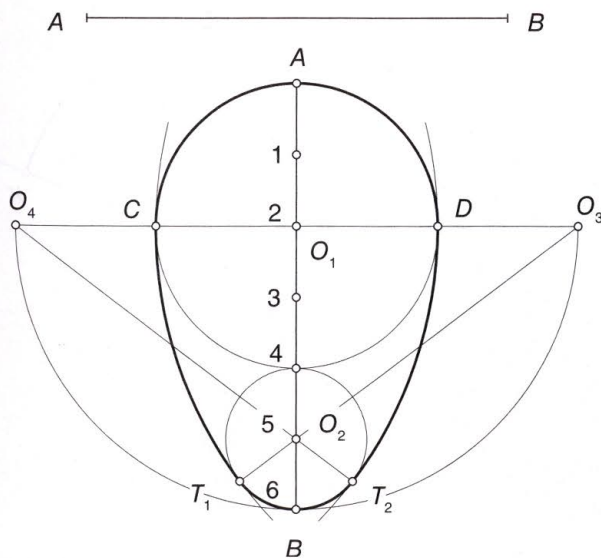


## 5.2 Spirals

Spirals are flat, open and continuous curves that use arcs to expand from a central, linear or polygonal nucleus. In a spiral, the locus is the distance travelled by one of its points when it moves the whole way around.

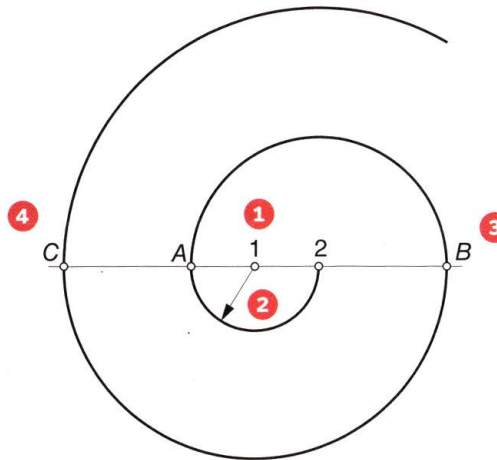
### D. Draw an ovoid knowing its major axis $AB$

1. Divide  $AB$  into six equal parts. Draw a perpendicular to  $AB$  through point 2. With the centre at 2 and a radius  $2A$ , draw the circumference to find points  $C$  and  $D$  at the intersection with the perpendicular.
2. Transfer the length  $A$  from  $C$  and  $D$  to find points  $O_3$  and  $O_4$  on the perpendicular line. From  $O_3$  and  $O_4$ , draw two lines that cross point 5.
3. With the centre at  $O_3$  and radius  $O_3C$ , and with the centre at  $O_4$  and radius  $O_4D$ , draw arcs until they cross the previous lines at points  $T_1$  and  $T_2$ .
4. Finally, with the centre at 5, draw a circumference that joins tangency points  $T_1$  and  $T_2$ .



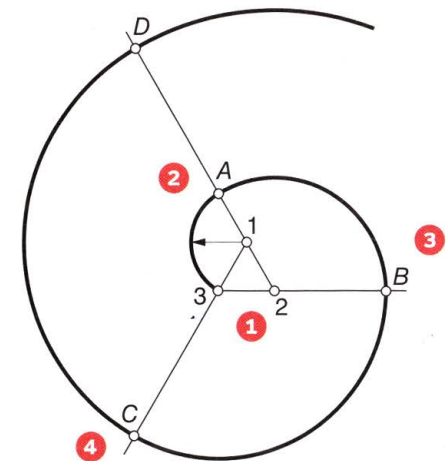
### A. Drawing a two-centred spiral

1. Draw a straight line and define two points on it, 1 and 2.
2. With point 1 as your centre, and using a radius of the segment 1-2, draw a semi circle (which will give us point A at its intersection with the straight line).
3. Using point 2 as the centre and radius  $2A$ , draw another semi circle to get point B on the straight line.
4. Draw another semi circle using 1 as its centre and with a radius of  $1B$  to get point C on the straight line.
5. This way, if we continue drawing semi circles by successively alternating between centres 1 and 2, we will get our spiral.



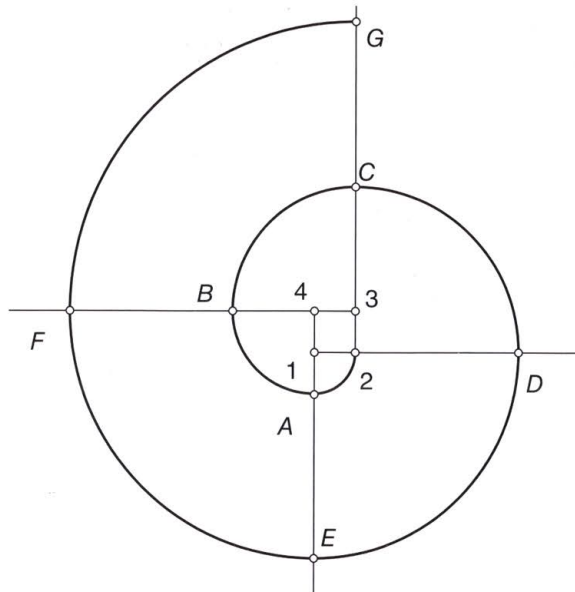
### B. Drawing a three-centred spiral with the centres on the vertices of an equilateral triangle

1. Draw an equilateral triangle, extend one end of each of its sides and number its vertices: 1, 2 and 3.
2. With radius 1-3 and its centre at point 1, draw an arc until we define point A on the extension of side 2-1 of the triangle.
3. Repeat this process with radius  $2A$  and centre at point 2 (making an arc that gives us point B on the extension of side 3-2 of the triangle).
4. Alternate this procedure successively (centre at 3 and radius  $3B$  to define point C, etc.) to draw out the spiral.



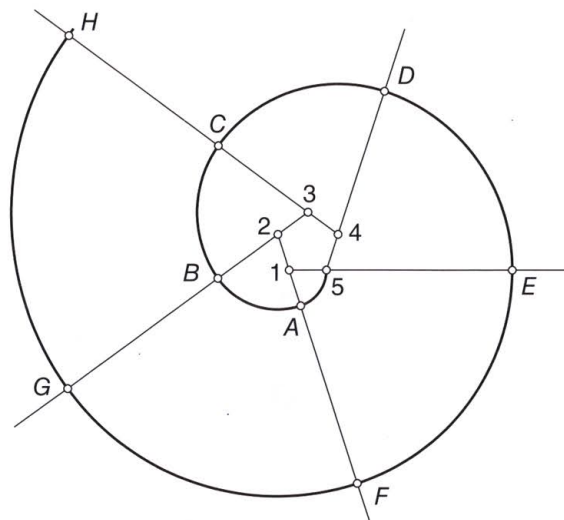
### C. Drawing a four-centred spiral

1. Draw a square and extend the four sides. Number the corners: 1, 2, 3 and 4.
2. Using point 1 as the centre and radius length 1-2, draw an arc to obtain point A on the extension of side 1-4.
3. Follow the same method with point 4 as the centre and radius length 4-A to draw another arc to obtain point B on the extension of side 1-2.
4. Now, with point 3 as the centre and radius 3B, draw another arc to obtain point C on the extended side.
5. Finally, with point 2 as the centre and radius 2C, draw an arc to obtain point D.
6. If you want to obtain points E, F and G, repeat the cycle to continue constructing the spiral.



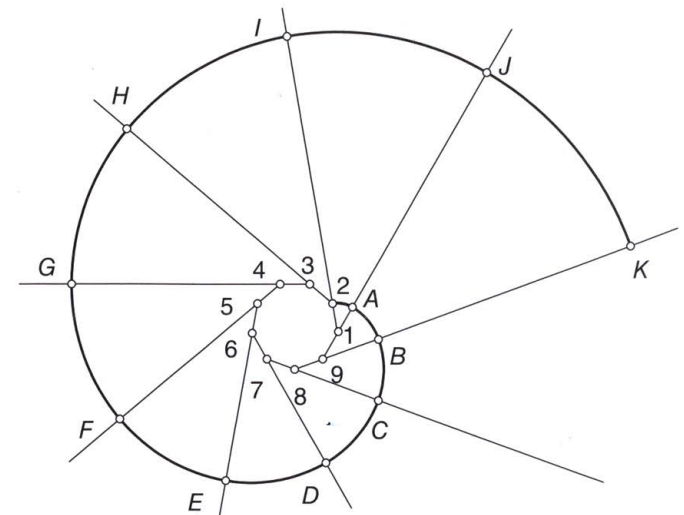
### D. Drawing a five-centred spiral

1. Draw a regular pentagon, extend its sides and number the corners 1, 2, 3, 4 and 5. Extend the sides, as in the previous spiral, to find the tangency points.
2. Using point 1 as the centre and radius length 1-5, draw an arc to obtain point A on the extended side 1-2 of the pentagon.
3. Now look at the diagram to see how the spiral has been constructed. You can obtain the remaining points in the same way.  
With 2 as the centre, draw arc 2A and obtain point B. With 3 as the centre, draw arc 3B and obtain point C. With 4 as the centre, draw arc 4C and obtain point D. With 5 as the centre, draw arc 5D and obtain point E.
4. Repeat the centres to draw any more arcs you require.



### E. Drawing a spiral with any number of centres

1. To draw a spiral with  $n$  number of centres, divide a circumference by  $n$ . We will use a nine-centred spiral as an example. Follow this method to find the centres of the arcs, 1 to 9.
2. Draw tangents to the circumference for each of the points 1 to 9, in which the arcs determine the tangency points.
3. Take a close look at the diagram to see how it has been drawn.
  - Using 1 as the centre, draw arc 1-2 to obtain point A on the tangent to the circumference.
  - Using 2 as the centre, draw arc 2A and obtain point B on the next tangent.
  - Using 3 as the centre, draw arc 3B and obtain point C. Follow this procedure to draw the remaining arcs.





# Structure

At first glance, the objects in the images below don't seem to have anything in common. The shapes and colours are completely different: Some are natural forms but others are man-made. However, if we look closely, we can see that the thing they have in common is structure. By **structure** we mean the interior lines that separate and organise the shapes that make up the image. This is a concrete example of internal and external spaces, and the smooth, sequential repetition of shapes.

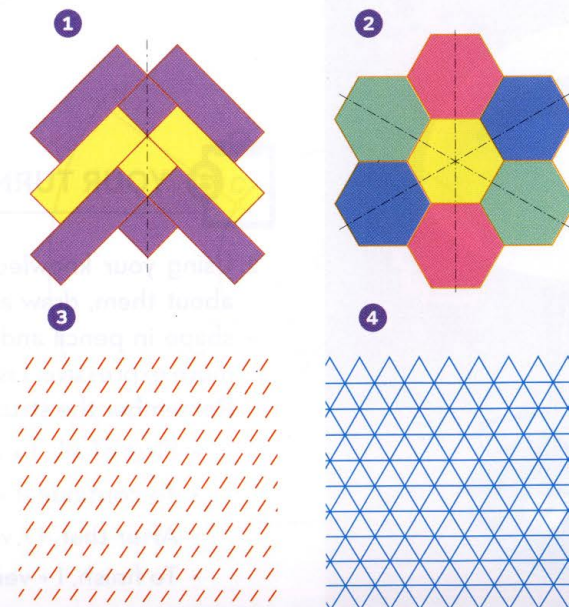
Even though some of the shapes are very irregular, they have an internal order that structures them and determines their outward appearance. The order and proportions of shapes are often determined by their function. Shapes in nature follow specific structures based on mathematical relationships. Examples include mineral crystallisation and the veins in a leaf.



## 6.1 Regular structures

Regular structures are made up of elements that are the same and that are distributed regularly. Regular structures may be:

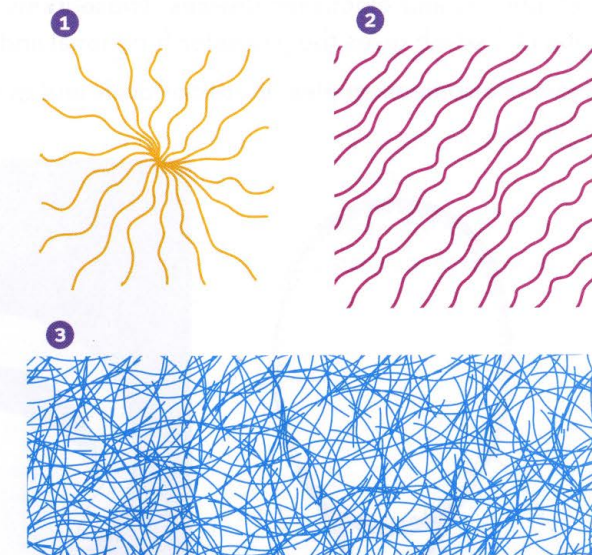
1. **Symmetrical about an axis:** The elements that make up the structure are repeated and inverted on either side of an axis of symmetry.
2. **Radial:** The elements start from a centre point and are repeated and distributed regularly in the space.
3. **Unidirectional:** A set of parallel lines that order and organise the elements that compose the structure.
4. **Basic or complex nets:** The basis of the structures are equilateral triangles or squares. Combining basic nets creates complex nets.



## 6.2 Irregular structures

Irregular structures may be:

1. **Radial:** Characterised by their circular shape and, as a result, the linear elements that compose their structure originate at the centre.
2. **Unidirectional:** In these structures, all the elements face the same direction.
3. **Complex:** The linear elements of this kind of structure are not in a regular order.



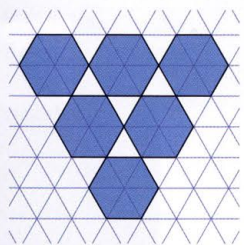
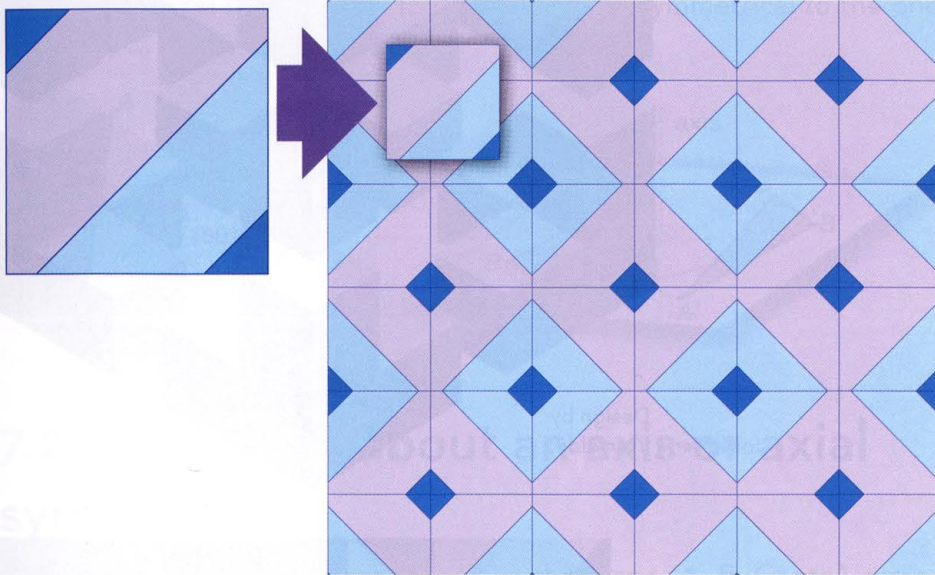
### TWO-DIMENSIONAL STRUCTURES

REGULAR	All the elements that compose them are the same and they follow a regular order. The most typical ones are symmetrical with axial, radial, unidirectional, basic and complex structures.
IRREGULAR	The elements that compose them are not the same and there is no regular order. There are three types: radial, unidirectional and complex structures.

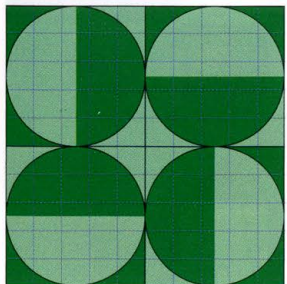


## 6.3 Modules

A **module** is a shape repeated a certain number of times to form a net. It may be regular or irregular. Modules can be used to create regular and irregular structures in both two and three dimensions. For example, the equilateral triangles and squares that form basic flat nets are basic modules.



For the moment, we will focus on analysing two-dimensional modules, so the examples given here are all flat. It is possible to create other kinds of modules, formed by combining two, three or more modules. These are known as compound modules.

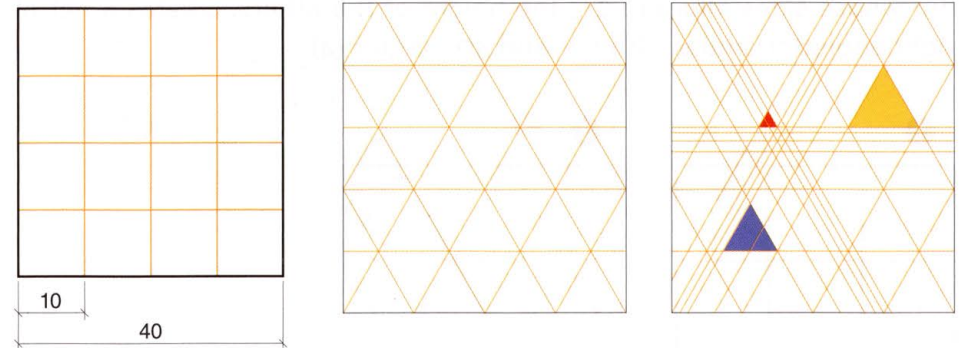


Basic nets are also often used as the basis for more complex compositions. For example, hexagons can be drawn in a triangular net, and octagons in a square net.

Then, both nets can act as the basis for much more complex compositions by using diagonal lines, semi-circles, circles, etc.

## 6.4 Submodules

A **submodule** is part of a module that has the same shape as the module and is contained within it an exact number of times. Have a look at the images on this page. If, for example, we draw a square with 40 mm sides, and we take it as a module, then, the sixteen 10 mm squares drawn inside that square are, by definition, its submodules. As you can see, modules and submodules with the same shape can be formed in many different ways, creating structures that can be anything from very simple to very complex.



One of the artistic possibilities for using submodules is that you can create a new structure or modular net with smaller dimensions, and even combine different sizes of submodules. This capacity for manipulation lets us create really beautiful artistic compositions.

The drawings below were created by people of your age. As you can see, they started their designs using submodular nets based on squares and equilateral triangles.

