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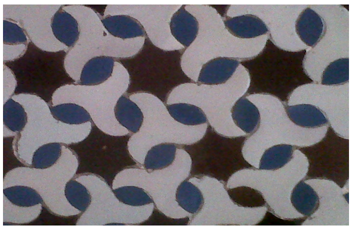

GOF DAYS 2015:

2nd Workshop on Goodness-of-Fit and Change-point problems

Athens, 4–6 September 2015

Book of Abstracts

Kostis Palamas building,
workshop venue

Preface

This volume contains abstracts of the papers presented during the workshop

GOF DAYS 2015
2nd WORKSHOP ON GOODNESS-OF-FIT AND CHANGE-POINT PROBLEMS
Department of Economics, National & Kapodistrian University of Athens

The workshop took place in the historical building KOSTIS PALAMAS of the University of Athens, 4-6 September 2015. It was the second of its kind. The first, essentially a Spanish event, took place on $17th$ November 2012 in the University of Sevilla, Spain, and was restricted to goodness-of-fit problems.

We take this opportunity to thank all participants for their contributions. They are listed below in alphabetic order.

We would also like to thank the Dean of the School of Economics and Political Science Professor Michalis Spourdalakis and the Governor of the Bank of Greece Professor Yiannis Stournaras for the financial support.

The scientific part of the organization, and the local details and logistics of this event were carried out by the Scientific Program Committee (SPC) and the Local Organizing Committee (LOC), respectively. Their names are listed below. We feel indebted for their fine work.

In closing we wish to express our wish that this event becomes a regular international meeting. This goal may be achieved by involving more members of the statistical community that work in the areas covered by the workshop. To this end we intend to mobilize additional members of the Computational & Methodological Statistics (CMStatistics) specialized team for GOODNESS-OF-FIT and CHANGE-POINT problems to be involved in future workshops of this kind.

Simos G. Meintanis
Chair of SPC & LOC

List of Participants

Allison, James (South Africa)
Bassiakos, Yiannis (Greece)
Basu, Ayanendranath (India)
Brodsky, Boris (Russia)
Corradi, Valentina (UK)
Dalla, Violetta (Greece)
Darkhovsky, Boris (Russia)
Delgado, Miguel A. (Spain)
Dufour, Jean-Marie (Canada)
Francq, Christian (France)
Fried, Roland (Germany)
Genton, Marc G. (Saudi Arabia)
Giraitis, Liudas (UK)
González-Manteiga, Wenceslao (Spain)
Hallin, Marc (Belgium)
Henze, Norbert (Germany)
Horvath, Lajos (USA)
Hušková, Marie (Czech Republic)
Janssen, Arnold (Germany)
Jiménez-Gamero, M. Dolores (Spain)
Karagrigoriou, Alex (Greece)
Khmaladze, Estate (New Zealand)
Koul, Hira (USA)
Ledwina, Teresa (Poland)
Lee, Sangyeol (Korea)
Lockhart, Richard (Canada)
Meintanis, Simos G. (Greece)
Michailidis, George (USA)
Nikitin, Yakov (Russia)
Oja, Hannu (Finland)
Paparoditis, Stathis (Cyprus)
Pardo-Fernández, Juan C. (Spain)
Peskir, Goran (UK)
Politis, Dimitris (USA)
Remillard, Bruno (Canada)
Santana, Leonard (South Africa)
Sun, Ying (Saudi Arabia)
Thas, Oliver (Belgium)
Van Keilegom, Ingrid (Belgium)
Zhu, Lixing (Hong Kong)
Zografos, Konstantinos (Greece)

Local Organizing Committee

Simos Meintanis (U. Athens), Chair
Alex Karagrigoriou (U. Aegean)
Yiannis Bassiakos (U. Athens)

Scientific Program Committee

Simos Meintanis (U. Athens), Chair
Marie Hušková (Charles U. Prague)
M. Dolores Jiménez-Gamero (U. Sevilla)

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Abstracts

Multinomial Goodness-of-fit Tests: Power Adjustment by Modification of Disparities

Ayanendranath Basu[1]

ayanbasu@isical.ac.in

[1]Interdisciplinary Statistical Research Unit, Indian Statistical Institute

Multinomial goodness of fit tests represent one of the oldest paradigms in the application of statistics. Pearson's goodness-of-fit test criterion has been heavily used by researchers in all disciplines of science to check for multinomial or other patterns in real data, and is by far one of the most popular tools in applied statistics.

Pearson's goodness-of-fit chi-square checks for patterns in real data by "matching" the observed frequencies with the expected frequencies through the Pearson's chi-square discrepancy. The measure quantifies the closeness of the observed and expected (under the null hypothesis) data and when the measure exceeds the appropriate threshold, we reject the null hypothesis.

Although most practitioners, even today, routinely use the Pearson's chi-square (and sometimes the likelihood ratio chi-square) in multinomial goodness-of-fit problems, it is by no means the only available measure of discrepancy in our disposal. There are a large number of chi-square type discrepancies any of which might also be used to quantify the amount of dissimilarity between the vector of observed frequencies and that of expected frequencies. Why this particular fixation on the Pearson's chi-square (and, to a lesser degree, on the likelihood ratio chi-square) then?

To be fair, the Pearson's chi-square does enjoy certain optimality properties as some of our predecessors have observed, and has a reasonably fast convergence to the limiting chi-square distribution. Yet there are well known cases where this test may have very low power at certain alternatives, although possibly it does well in others. In the former situation it can often be substantially poorer than the most powerful alternative for that situation.

Cressie and Read (1984) and Read and Cressie (1988) highlighted this issue through their description of the power divergence family. This family provides the user with several alternative chi-square type goodness-of-fit test statistics indexed by a real parameter λ. These authors demonstrated that when testing the equiprobable null against bump alternatives, the power of the test increased with λ, while the power decreased with λ under dip alternatives. However these authors did not look at the particular geometric structure of the divergences which led to this kind of behavior.

In this talk we will try to present a systematic investigation of the geometric structure of the divergences and provide some insight on how the choices affect the power for alternatives of different kinds. Equipped with this understanding, we will discuss new classes of divergences which provide very good compromise tests in the multinomial goodness-of-fit problem. Many possible motivations for such constructions will be provided in this talk. Some illustrations of the general phenomenon will be described in terms of tests of hypothesis involving the kappa statistic.

The review and (in part) the development of the techniques will borrow from some older and some (more) recent work, including Basu and Sarkar (1994), Basu et al. (2002), Basu et al. (2011) and Mandal and Basu (2011).

Keywords: multinomial goodness-of-fit tests, power divergence, kappa statistics.

AMS: 62G10, 62E17.

References

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Splitting Mixtures of Probabilistic Distributions: A Change-Point Approach

Boris Brodsky[1], Boris Darkhovsky [2]

[1]Central Institute for Mathematics and Economics, RAS
[2]Institute for Systems Analysis, RAS

Classification problems for univariate and multivariate observations are often encountered in statistics and economics. However, all existing approaches to solving these problems have several essential drawbacks:

1. All these methods cannot help in testing the null hypothesis of no different classes;

2. The number of classes is assumed to be known a priori;

3. Theoretical justification of performance effectiveness of these methods is lacking.

The method based upon the change-point analysis and proposed in this report can help us to solve these problems.

Consider the simplest case of a binary mixture of probabilistic distributions. Let $X = \{x_n\}_{n=1}^{\infty}$ be a random sequence and $X^N = \{x_1, \ldots, x_N\}$ be an observed sample. Suppose that the density function (d.f.) of observations is described by a model of a binary mixture:

$$f(x) = (1-\epsilon)f_0(x) + \epsilon f_1(x),$$

where the parameter $0 < \epsilon < 1/2$ is unknown.

Let $\int x f_0(x)dx = h_1$, $\int x f_1(x)dx = h_2$, and $h = |h_1 - h_2| \neq 0$ is unknown.

In words, we suppose that with a certain small probability ϵ some outliers can emerge in the sample. The problem is to classify the sample $X^N = \{x_1, \ldots, x_N\}$ into sub-samples of ordinary observations and outliers.

In our report we discuss and analyze the nonparametric method designed for detection and estimation of nonstationarities in such models. The method is as follows:

1) For each value of the parameter $b \in \mathbb{B}$, where $\mathbb{B} = [\kappa, B]$, $0 < \kappa < B$, is a certain known segment, the method classifies the sample X^N into sub-samples $X_1^N = \{\tilde{x}_i\}_1^{N_1}$ and $X_2^N = \{\hat{x}_i\}_1^{N_2}$ of supposedly usual observations and outliers with the sample sizes $N_1(b)$ and $N_2(b)$, respectively.

2) Calculate the following statistic:

$$\Psi_N(b) = \frac{1}{N^2}\left(N_2\sum_{i=1}^{N_1}\tilde{x}_i - N_1\sum_{i=1}^{N_2}\hat{x}_i\right).$$

This statistic is one of our basic statistics in change-point problems (see Brodsky and Darkhovsky (2000)).

3) For chosen threshold $C > 0$ calculate the value $J = \sup_{b\in\mathbb{B}} |\Psi_N(b)|$.

Our main results are as follows.

Theorem 1. Let $\epsilon = 0$. Suppose the d.f. $f_0(\cdot)$ is symmetric w.r.t. zero and bounded. Assume also that Cramer's and ψ-mixing conditions are satisfied for the sequence X.

Then for any $C > 0$ the following estimate holds:

$$\mathbf{P}\{\sup_{b\in\mathbb{B}} |\Psi_N(b)| > C\} \leq L_1 \exp(-L_2(C)N),$$

where the constants $L_1, L_2 > 0$ do not depend on N.

Theorem 2. Let $0 < \epsilon < 1/2$ and fixed. Suppose all assumptions of theorem 1 are satisfied. Then for any $0 < C < \sup_{b\in\mathbb{B}} |\Psi(b)|$ the following estimate holds:

$$\mathbf{P}\{\sup_{b\in\mathbb{B}} |\Psi_N(b)| \leq C\} \leq L_3 \exp(-L_4(\delta)N),$$

where the constants $L_3, L_4 > 0$ do not depend on N, $\delta = \sup_{b\in\mathbb{B}} |\Psi(b)| - C > 0$.

Results of Monte Carlo testing of the proposed method witness about its high efficiency.

We also present results of comparative analysis of the proposed method with other well-known parametric and nonparametric methods of classification: the 'Maximum Likelihood' method and the 'k-means' method.

Keywords: classification, change-point analysis, nonparametrics

AMS: 62L10.

References

B.E. Brodsky, B.S. Darkhovsky (2000). *Nonparametric Statistical Diagnosis.* Kluwer Academic Publishers, Dorddrecht/Boston /London.

Testing for Jumps Intensity and Path Dependence

V. Corradi[1], M.J. Silvapulle[2], N.R. Swanson[3]

[1]University of Surrey, [2] Monash University, [3] Rutgers University

Jump diffusions are widely used in the financial econometrics literature when analyzing returns or exchange rates. The common practice is to jointly estimate the parameters characterizing the drift, variance, jump intensity and jump size probability density. However, the parameters characterizing the jump size density are not identified when the jump intensity is identically zero. Clearly, when one estimates a jump diffusion with jump intensity equal to zero, a subset of the parameters is not identified. This in turn precludes consistent estimation of other parameters in the model.

The above estimation problem serves to underscore the importance of pretesting for jumps. In the extant literature, there is a large variety of tests for the null of no jumps versus the alternative of jumps. These tests are consistent against "realized" jumps. In fact, by making use only of observations over a finite time span, they can only detect whether during that period jumps have occurred. While this is hardly a weakness of the existing tests, there are may be situations in which interest lies on testing for the existence of jumps in the data generating process or within a class of models.

Heuristically, if instead of using observations on a finite time span, we use an increasing time span, then if there are jumps, they will be eventually detected. Hence, in the case of long span asymptotics, we can test for the probability of jumps occurring, rather than testing for realized jumps.

This paper makes two key contributions to the literature. First, we introduce a (almost) model free "jump" test for the null of zero intensity. Second, under the maintained assumption of strictly positive intensity, we introduce a "self excitement" test for the null of constant intensity against the alternative of path dependent intensity. The test has power against autocorrelation in the jump component. We provide in detail a direct test for Hawkes diffusions (see Ait-Sahalia, Cacho-Diaz and Laeven 2014) in which jump intensity is modeled as a mean-reverting diffusion process. When the tests are implemented prior to model specification, standard estimation of jump diffusions can be subsequently carried out, avoiding the identification problems discussed above.

The jump test is based on realized third moments, or, as commonly called, tricity. Realized tricity over finite time span has been already used for different purposes: to detect jumps realization, as in Jacod (2012), to study the contribution of realized skewness to predict the cross-section of equity returns, as in Amaya, Christoffersen, Jacobs and Vasquez (2014), to test for the endogeneity of sampling times, Li, Mykland, Renault, Zhang and Zheng (2014). What distinguishes our test is that it is based on both in-fill and long-span asymptotics. Importantly, our test is robust to the presence of leverage. Under the null hypothesis of zero intensity, the statistics is characterized by normal limiting distribution. Under the alternative, it is necessary to distinguish between the case in which the density of the jumps have third and/or first moment different from zero, or whether is symmetric around zero. In the former case, the proposed tests have a well defined Pitman drift and have power against $\sqrt{T}/\Delta-$local alternatives, whenever the third moment is non-zero, and power against $\sqrt{T}-$local alternatives, whenever the first moment is non-zero, where T denotes the time span and Δ the discrete interval. In the latter case, the sample third moment approaches zero, but the probability order of the statistics is larger than that which obtains under the null, since the jump component does not contribute to the mean, while it does contribute to the variance. We introduce a thresholded estimator for the variance, which is consistent under the null of zero intensity, and bounded in probability under the alternative. Thus, inference can be performed via a simple t-statistic.

The test for (no) path dependent intensity, is based on the autocorrelation function of the jump component. A necessary condition for the power of the test is that the mean of the jump size is non-zero. Thus, we suggest a simple (t-statistic type) test for the null of zero jump mean. If we reject both the null of zero intensity, as well as that of zero mean jumps, we can then turnin now to the self excitement test, in order to ascertain whether jump intensity is a constant, or it is path dependent.

As none of the tests proposed in this paper are robust to microstructure noise, one might choose to build a dataset consisting of observations at the highest frequency for which the noise is not binding.

Testing Mean Stability of Heteroskedastic Time Series

V. Dalla[1], L. Giraitis[2], P.C.B. Phillips[3]

vidalla@econ.uoa.gr, l.giraitis@qmul.ac.uk, peter.phillips@yale.edu

[1]Department of Economic Sciences, National and Kapodistrian University of Athens
[2]School of Economics and Finance, Queen Mary University of London
[3]Department of Economics, Yale University

Diagnostic checks relating to the properties of data to be used in time series modeling are now routinely implemented in empirical research. Nonetheless, in various applications with time series data, stationarity is often presumed with no preliminary checks concerning such fundamental properties as stability of the mean, the unconditional variance, or the higher moments. Time constancy of the mean and variance is unlikely to hold for much economic and financial data over long periods, even without concerns over other forms of nonstationarity such as random wandering behavior and the presence of unit roots. The issue of general structural instabilities in macroeconomic time series has been frequently raised in modern empirical research and affects estimation, inference, forecasting, and policy analysis.

Time series dynamics are particularly vulnerable to shifts that occur in the mean and variance of the series. Neglecting such shifts therefore has many potential implications because model dynamics adjust to compensate for the omission of structural changes, leading to the fitting of spurious models and drawing controversial conclusions on the time forms of dependence and policy assessments concerning the impact of unanticipated shocks. Variance changes in the data may still allow investigators to extract time series dynamics but these changes typically invalidate standard errors, confidence intervals, inference and forecast intervals. More seriously disruptive is the presence of time varying means, which makes stationary time series modeling implausible, at least until the source of the time variation is extracted from the data.

Stability checks on the moments are equally important in analyzing uncorrelated data. For example, although series of financial returns r_t may reasonably be assumed to have constant mean and be serially uncorrelated, constancy of the unconditional variance of returns may well be unrealistic, particularly over long historical periods. As a result, a strategy like fitting absolute or squared returns using a stationary form of GARCH model may be questionable when the data may be better modeled as independent random variables with a time-varying mean. In spite of the apparently obvious differences between a time series of heteroskadastic independent variables, and a time series generated by a stationary GARCH process with a constant mean that can reproduce persistent dynamic patterns, such processes may be hard to distinguish in practical work using basic time series diagnostic tools.

We study some practical and easy to implement statistical procedures for testing for stability of the mean $\mu_t = \mathbb{E}(x_t)$ of a time series $x_t = \mu_t + u_t$, where u_t is a heteroskedastic uncorrelated process of martingale differences. We discuss the equally important but harder task of testing for changes in the mean of a weakly dependent time series $x_t = \mu_t + y_t$ where y_t is a dependent zero mean process. Finally, if the time series x_t has constant mean, tests for the stability of the variance of x_t reduces to a test for mean stability in the transformed data, such as absolute or squared centered values. We apply our methods to tests of stability of the variance of daily S&P and IBM stock market returns. Our findings provide evidence against both stationarity and conditional heteroskedastic ARCH effects in returns, thereby corroborating the somewhat surprising claims in Stărică and Granger (2005) that most of the dynamics of such time series are "concentrated in shifts of the unconditional variance".

Keywords: Heteroskedasticity, KPSS test, Mean stability, Variance stability, VS test.

AMS: 62G10, 62M10, 62P20.

References

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Off-line change-points detection in time series of arbitrary nature

Boris Darkhovsky[1]

[1]Institute for Systems Analysis, Russian Academy of Sciences; Higher School of Economics, Moscow, Russia

The talk consists of two parts.

Part one: *Main ideas of our approach to off-line change-points detection in random time series* (joint works with Boris Brodsky, Central Institute for Mathematics and Economics, Moscow, Russia).

Part two: *The ϵ-complexity of continuous functions and its application to off-line change points detection in time serise of arbitrary nature* (joint works with Alexandra Piryatinska, San Francisco State University, USA).

Part one. Our approach to off-line change-points detection in random time series is based upon two main ideas (see Brodsky and Darkhovsky).

The *first* idea consists in the property that detection of changes in any density function or some other probabilistic characteristics of random sequence can be (with an arbitrary degree of accuracy) reduced to detection of changes in the mean value of some new sequence – so called *diagnostic sequence*– constructed from the initial one. For example, if the correlation function of an observed sequence changes, then considering for every fixed $\tau = 0, 1, 2, ...$ new sequences $V_t(\tau) = x_t x_{t+\tau}$, we reduce the problem to detection of changes in the mathematical expectation of one of sequences $V_t(\tau)$.

The *second* idea of our approach consists in the use of the following family of statistics for detection of change-points (generalized version of Kolmogorov-Smirnov statistics)

$$Y_N(n, \delta) = \left[\frac{(N-n)n}{N^2}\right]^{\delta} \left(n^{-1} \sum_{k=1}^{n} x^N(k) - (N-n)^{-1} \sum_{k=n+1}^{N} x^N(k)\right), \quad (1)$$

where δ takes the values $1, 1/2, 0,\ 1 \le n \le N-1$, $X^N = \{x^N(k)\}_{k=1}^N$ is an observed realization (or a diagnostic sequence).

The 3-stage procedure for off-line change-points detection will be described. Prior lower bounds in retrospective change-point problems will be given. The asymptotic minimaxity of the change-points estimates received by means of basic algorithm (1) will be established.

Part two.

The novel concept of the ϵ-complexity of continuous functions is proposed (see Darkhovsky and Piryatinska). This concept is in line with the Kolmogorov's idea on "complexity" of an object. Roughly speaking, the ϵ-complexity of an individual continuous function on a compact is the (logarithmic) number of the function values on a uniform grid that must be retained to reconstruct the function via a certain fixed family of approximation methods with a given relative error ϵ.

In other words, the ϵ-complexity can be called a *shortest description* (up to ϵ) of the function by its values on a unifrom grid with the help of given set of approximation methods.

It can be shown that the ϵ-complexity of "almost any" function satisfying Hölder condition is effectively characterized by couple of real numbers. We call these numbers the *ϵ-complexity coefficients.*

Based on this result, we offer the *model-free methodology* of change-points detection in time series of *arbitrary nature* (stochastic, deterministic or mixed). This methodology is reduced to creation of special diagnostic sequences – the sequences of ϵ-complexity coefficients — and processing these sequences by basic algorithm (1). In such processing we assume that the mean values of the ϵ-complexity coefficients remain constant on the homogeneity intervals, and changes in the generating mechanisms of the time series lead to the changes of these mean values.

Keywords: Kolmogorov-Smirnov statisitcs, the ϵ-complexity.

AMS: 62M10, 68Q30.

References

B.E. Brodsky, B.S. Darkhovsky (2000) *Non-Parametric Statistical Diagnosis. Problems and Methods.* Kluwer Academic Publishers, Dorddrecht/Boston /London.

B.S. Darkhovsky, A. Piryatinska (2014) New Approach to the Segmentation Problem for Time Series of Arbitrary Nature, *Proceedings of the Steklov Institute of Mathematics*, 287, 54–67.

A Nonparametric Test of Conditional Symmetry

Miguel A. Delgado[1] and Xiaojun Song[2]

[1]Department of Economics, Universidad Carlos III de Madrid
[2] Guanghua School of Management, Peking University

Consider a $\mathbb{R}^{1+d}-$valued strictly stationary absolutely regular time series process $\{Y_t, X_t\}_{t\in\mathbb{Z}}$, which satisfies the Markov property

$$\mathbb{P}\left(Y_t \leq y|\,\{Y_{s-1}, X_s\}_{s=-\infty}^{t}\right) = \mathbb{P}\left(Y_t \leq y|\,\{Y_{s-1}, X_s\}_{s=t-k+1}^{t}\right),\ \forall y \in \mathbb{R}\ a.s.$$

We propose a nonparametric test for the hypothesis that the conditional distribution of Y_t is symmetric about the regression function, *i.e.*

$$H_0 : \mathbb{P}\left(U_0 \leq u|\, I_0\right) = \mathbb{P}\left(-U_0 \leq u|\, I_0\right) \text{ for all } u \in \mathbb{R}\ a.s.,$$

where $I_t = (Y_{t-1}, X_t,\ Y_{t-2}, X_{t-1}, ..., Y_{t-k}, X_{t-k+1})' = (I_{1t}, ..., I_{pt})'$, with $p = k(d+1)$, $U_t = Y_t - r(I_t)$, and $r(I_t) = \mathbb{E}\left(Y_t|\, I_t\right)$. Our proposal is an alternative to existing tests for symmetry of marginal distributions of nonparametric regression errors in scale-location models using i.i.d. observations, e.g. Dette, Kusi-Appiah and Neumeyer (2002), Neumeyer and Dette (2007) or Huskova and Meintanis (2012), to a time series context where $U_t/\sqrt{Var\left(Y_t|\, I_t\right)}$ may depend on I_t.

The conditional symmetry restriction can be expressed in terms of the joint characteristic function $\psi\left(u, v\right) = \mathbb{E}\left[\exp\left\{i\left(I_0'v + U_0 u\right)\right\}\right]$, $i^2 = -1$, as,

$$H_0 : \psi\left(u, v\right) - \psi\left(-u, v\right) = 0, \forall\left(u, v\right) \in \mathbb{R}^{1+p}.$$

Given a sample $\{Y_t, X_t\}_{t=1}^{T}$, the test statistic is $T\hat{\eta}_T$, with

$$\hat{\eta}_T = \int_{\mathbb{R}^{1+p}} \left[\hat{\psi}_T\left(u, v\right) - \hat{\psi}_T\left(-u, v\right)\right]^2 w(u, v) du dv,$$

for a suitable weight functions $w : \mathbb{R}^{1+p} \to \mathbb{R}^{+}$, and

$$\hat{\psi}_T\left(u, v\right) = \frac{1}{T}\sum_{t=2}^{T} \exp\left\{i\left(I_t'v + \hat{U}_t u\right)\right\}$$

with nonparametric residuals $\hat{U}_t = Y_t - \hat{r}(I_t)$ based on a higher order kernel estimator $\hat{r}$ of r. The asymptotic distribution of the test statistic $T\hat{\eta}_T$ under H_0 is derived under regularity conditions that combine restrictions on the moments of Y_0, the smoothness of the underlying density of I_0 and r, the order of the high order kernel and the rates of convergence of the bandwidth and the parameter governing the serial dependence of the absolutely regular time series. Since the limiting distribution of the test statistic under H_0 is non-pivotal, the test is implemented using a bootstrap technique. We show that the test is able to detect alternatives converging to H_0 at the rate $T^{-1/2}$.

Interestingly, when $w(u,v) = \bar{k}^2(au)\prod_{j=1}^{p}\bar{k}^2(av_k)$, where $\bar{k}$ is the Fourier transform of some kernel k symmetric around zero and integrating to one, i.e. $\bar{k}(t) = \int_{\mathbb{R}} \exp(ixt)k(x)dx$, and a is a positive number,

$$\hat{\eta}_T = (2\pi)^{(1+p)} \int_{\mathbb{R}^{1+p}} \left[\hat{f}_a(u,v) - \hat{f}_a(-u,v)\right]^2 dudv,$$

where

$$\hat{f}_a(u,v) = \frac{1}{Ta^{1+p}} \sum_{t=2}^{T} k\left(\frac{\hat{U}_t - u}{a}\right) \prod_{j=1}^{p} k\left(\frac{I_{jt} - v_j}{a}\right)$$

is a smooth kernel estimator of the joint probability density of (Y_t, I_t) with a kernel k and a fixed bandwidth number a.

The finite sample performance of the test is studied by means of Monte Carlo experiments and an application using stock markets data.

Keywords: Nonparametric testing; Conditional symmetry; Time series data; Smoothing.

AMS: 62G10, 62G7, 62G8, 62G9.

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Improved exact nonparametric confidence bands for distribution functions with application to poverty measures

Jean-Marie Dufour[1], Mame Astou Diouf[2]

[1]Department of Economics, McGill University
[2]International Monetary Fund

Goodness-of-fit tests are of great interest in econometrics. In many procedures, especially in parametric ones, determining the distribution from which the sample comes from may be an important step. The Kolmogorov-Smirnov (KS, henceforth) test is one of the most popular nonparametric goodness-of-fit tests. However, it does not allow to discriminating a lot between distributions that differ mostly through their tails. Weighted KS statistics have been proposed by Anderson and Darling (1952) and Eicker (1979) to improve the performance of the test in the tails but they suffer from important drawbacks. We propose improved weighted KS statistics to correct these limits. These statistics are obtained by adding a regularization term in the denominator of the Anderson-Darling and the Eicker statistics. They retain the advantages of the weighted KS statistics but their denominators do not become close to 0 in the tails of distributions as it is the case for the original statistics. We derive exact nonparametric confidence bands (CBs, henceforth) for distribution functions using the weighted and regularized KS statistics. We show that in the continuous case, these CBs are independent of the distribution assumed under the null hypothesis and are conservative for noncontinuous distributions. In the noncontinuous case, we derive monotonicity properties that exploit embeddedness of the image sets of different distributions to narrow the CBs without altering their reliability. Monte Carlo simulations are performed to study the relative performance of the inference methods and illustrate how to choose the regularization parameter. The results show that the regularized statistics yield more powerful goodness-of-fit tests than the existing ones when applied to distributions with more discrepancy in the tails. Likewise, the CBs for distribution functions based on these regularized statistics are of better performance.

These results are then applied to inference on poverty measures in economics. We observe that the poverty indicators can be interpreted as the

expectation of a bounded random variable which is itself a functional of a distribution function. Using projection techniques, we derive finite-sample nonparametric confidence intervals for the mean from confidence bands for the distribution of the underlying variable. We investigate methods based on improved standardized Kolmogorov-Smirnov statistics and a likelihood-ratio criterion. We then apply these procedures to the FGT poverty measures.

Monte Carlo simulations show that asymptotic and bootstrap confidence intervals can fail to provide reliable inference, while the proposed methods are robust and yield shorter confidence intervals. As an illustration, we analyze the profile of poverty of Mexico in 1998. The results show that the widths of the asymptotic confidence intervals are often too small to be realistic while those of the bootstrap can be ten times larger than the widths delivered by exact meth- ods. The study shows that the poverty profile of Mexican households depends greatly on the type of household head: poverty levels among households with a male head or an educated head is much smaller than poverty levels among other households. Hence, policies aimed at reducing illiteracy and at securing the income of households with a female head could help reduce poverty in rural Mexico

Keywords: Nonparametric inference; Komolgorov-Smirnov statistic; Anderson-Darling statistic ; Eicker statistic; Poverty measure.

AMS: 62G30, 62G10, 62G15, 62G09, 62P20, 62P25.

Exponential or Log GARCH ?

C. Francq[1], O. Wintenberger[2], J-M. Zakoian[1]

[1]CREST and University Lille 3
[2]Universities of Paris 6 and Copenhaguen

A Lagrange-Multiplyer (LM) test is derived for testing the null assumption of a log-GARCH against more general formulations including the Exponential GARCH (EGARCH). The null assumption of an EGARCH is also tested. Portmanteau goodness-of-fit tests are developed for the log-GARCH and EGARCH models. Simulations illustrating the theoretical results and an application to real financial data are proposed.

For our testing problem, we introduce the general model

$$\begin{cases} \epsilon_t & = & \sigma_t \eta_t, \\ \log \sigma_t^2 & = & \omega_0 + \sum_{i=1}^q \omega_{0,i-} 1_{\{\epsilon_{t-i}<0\}} \\ & & + \sum_{i=1}^q \left(\alpha_{0,i+} 1_{\{\epsilon_{t-i}>0\}} + \alpha_{0,i-} 1_{\{\epsilon_{t-i}<0\}} \right) \log \epsilon_{t-i}^2 \\ & & + \sum_{j=1}^p \beta_{0j} \log \sigma_{t-j}^2 + \sum_{k=1}^{\ell} \gamma_{0,k+} \eta_{t-k}^+ + \gamma_{0,k-} \eta_{t-k}^-, \end{cases}$$

where the coefficients are not *a priori* subject to positivity constraints, and (η_t) is a sequence of independent and identically distributed (iid) variables such that $E\eta_1 = 0$ and $E\eta_1^2 = 1$. We wish to test the null hypothesis of an asymmetric log-GARCH, *i.e.* the hypothesis

$$H_0 : (\gamma_{01,+}, \gamma_{01,-}, \dots, \gamma_{0\ell,-}) = 0.$$

In the time series literature, similar testing problems are solved by a standard test, using for example the Wald, Lagrange-Mutiplier (LM) or Likelihood-Ratio (LR) principle. See among others Luukkonen, Saikkonen and Tersvirta (1988), Francq, Horváth and Zakoian (2010).

A difficulty, in the present framework, is that we do not have a consistent estimator of the parameter of the general model. Two difficulties arise to prove that the QMLE is consistent. First, the stationarity conditions of the model are unknown. Second, due to the presence of the $|\eta_{t-k}|$'s, it seems extremely difficult to obtain invertibility conditions allowing to write $\log \sigma_t^2$ as a function of the observations (see Wintenberger, 2013).

To circumvent this difficulty, we propose a LM approach which uses the asymptotic results given in Francq, Wintenberger and Zakoian (2013).

Keywords: EGARCH, LM tests, Invertibility of time series models, log-GARCH, Portmanteau tests, Quasi-Maximum Likelihood.

AMS: 62M10.

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Robust and nonparametric detection of shifts using two-sample U-statistics and U-quantiles

R. Fried[1], H. Dehling[2], M. Wendler[3]

[1]Department of Statistics, TU Dortmund University, Germany
[2] Department of Mathematics, Ruhr-Universität Bochum, Germany
[3] Institut für Mathematik und Informatik, Universität Greifswald, Germany

We study tests for detecting level shifts in time series. Specifically, we assume that the sequence of observations $(X_t)_{t\geq 1}$ is generated by the model

$$X_t = \mu_t + Y_t,$$

where $(\mu_t)_{t\geq 1}$ is a sequence of unknown constants and $(Y_t)_{t\geq 1}$ is a stationary process with mean zero. We will focus on the case when $(Y_t)_{t\geq 1}$ is near epoch dependent. As examples, we will be able to treat most standard models of time series analysis, such as ARMA and GARCH processes.

Given observations $X_1, \ldots, X_n$, we want to test the null hypothesis that the process is stationary, i.e.

$$H_0 : \mu_1 = \ldots = \mu_n,$$

against the alternative that the level changes at some unknown point in time,

$$H_1 : \text{ there exists } \tau \in \{1, \ldots, n-1\} \text{ such that } \mu_\tau \neq \mu_{\tau+1}.$$

The CUSUM test statistic for this change-point problem reads

$$\hat{\sigma}^{-1} \max_{k=1,\ldots,n} \frac{1}{\sqrt{n}} \left| \sum_{i=1}^{k} X_i - \frac{k}{n} \sum_{i=1}^{n} X_i \right|,$$

where $\hat{\sigma}^2$ is a consistent estimator of the long-run variance $\sigma^2 = Var(Y_1) + 2\sum_{k=2}^{\infty} Cov(Y_1, Y_k)$. Asymptotical critical values for the CUSUM test can be calculated from tables of the Kolmogorov-Smirnov distribution, i.e. the distribution of the supremum of the Brownian bridge process $|W(\lambda) - \lambda W(1)|_{0\leq\lambda\leq 1}$.

The CUSUM test statistic is not robust to outliers and can be improved in case of non-normal data, particularly for heavy-tails. Dehling, Fried and

Wendler (2015) propose a modification of the CUSUM test based on the Hodges-Lehmann two sample estimator, which is the median of all pairwise differences between the two samples. It is highly robust and has a high efficiency in the case of Gaussian observations. Like for a related test based on the two-sample Wilcoxon statistic, the asymptotics of the Hodges-Lehmann change-point test can be established under general conditions without any moment assumptions. Both tests offer similar power against a shift in the center of the data, but the test based on the Hodges-Lehmann estimator performs superior if the shift is far from the center. This can be explained by the well-known discretization problems of the Wilcoxon statistic in small samples.

CUSUM-type tests comparing all observations before and after each candidate change-point are designed under the idea that there is only one change. MOSUM-type tests restrict the comparison to data in two subsequent moving time windows. This may overcome possible masking effects due to several shifts into different directions. Mielke (2015) investigates MOSUM-type tests based on the two sample Wilcoxon statistic or Hodges-Lehmann estimator in case of independent data. While versions of the tests relying on asymptotic critical values turn out to be conservative if n is not very large, versions deriving critical values using the permutation principle provide improvements particularly in the presence of multiple shifts. The study of tests based on the idea of comparing many instead of only two subsamples is ongoing work (jointly also with Simos Meintanis and Max Wornowizki).

Keywords: change-point, heavy tails, outliers, time series.

AMS: 62M10.

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Tukey g-and-h Random Fields

Ganggang Xu[1], **Marc G. Genton**[2]

[1]Department of Mathematical Sciences, Binghamton University
[2]CEMSE Division, King Abdullah University of Science and Technology

We propose a new class of trans-Gaussian random fields named Tukey g-and-h (TGH) random fields to model non-Gaussian spatial data. The proposed TGH random fields have extremely flexible marginal distributions, possibly skewed and/or heavy-tailed, and, therefore, have a wide range of applications. The special formulation of the TGH random field enables an automatic search for the most suitable transformation for the dataset of interest while estimating model parameters. An efficient estimation procedure, based on maximum approximated likelihood, is proposed and an extreme spatial outlier detection algorithm is formulated. The probabilistic properties of the TGH random fields, such as second-order moments, are investigated. Kriging and probabilistic prediction with TGH random fields are developed along with prediction confidence intervals. The predictive performance of TGH random fields is demonstrated through extensive simulation studies and an application to a dataset of total precipitation in the south east of the United States.

Keywords: CRPS, heavy tails, kriging, log-Gaussian random field, non-Gaussian random field, PIT, probabilistic prediction, skewness, spatial outliers, spatial statistics, Tukey g-and-h distribution.

AMS: 62M30, 62M40.

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Smoothing-based goodness-of-fit tests for the regression with directional data

W. González-Manteiga[1], E. García-Portugués[2,3], I. Van Keilegom[4], R. M. Crujeiras[1]

[1]Department of Statistics and OR, University of Santiago de Compostela
[2]Department of Mathematical Sciences, University of Copenhagen
[3]The Bioinformatics Centre, University of Copenhagen
[4]Institute of Statistics, Catholic University of Louvain

In this paper we propose a goodness-of-fit test for parametric regression models with scalar response and *directional* predictor, that is, a random variable (rv) with support in the hypersphere $\Omega_q = \{\mathbf{x} \in \mathbb{R}^{q+1} : ||\mathbf{x}|| = 1\}$. The framework of our contribution is a location-scale model of the form

$$Y = m(\mathbf{X}) + \sigma(\mathbf{X})\varepsilon,$$

where $m(\mathbf{x}) = \mathbb{E}[Y|\mathbf{X} = \mathbf{x}]$, $\sigma^2(\mathbf{x}) = \mathbb{V}\mathrm{ar}[Y|\mathbf{X} = \mathbf{x}]$, Y is a rv with support in $\mathbb{R}$ and $\mathbf{X}$ is a directional rv with density f. See Mardia and Jupp (2000) for a review on directional statistics.

The testing of the composite hypothesis

$$H_0 : m \in \mathcal{M}_\Theta = \{m_{\boldsymbol{\theta}} : \boldsymbol{\theta} \in \Theta \subset \mathbb{R}^s\}$$

is achieved by means of a smoothed-based test statistic, in the spirit of the proposal of Hardle and Mammen (1993). Specifically, we consider the weighted squared distance between a nonparametric estimator of m and a smoothed parametric regression estimator

$$T_n = \int_{\Omega_q} \left(\hat{m}_{h,p}(\mathbf{x}) - \mathcal{L}_{h,p} m_{\hat{\theta}}(\mathbf{x})\right)^2 \hat{f}_h(\mathbf{x}) w(\mathbf{x}) \omega_q(d\mathbf{x}),$$

where $\hat{f}_h$ is a kernel estimator of the density f, $w : \Omega_q \to [0, \infty)$ is an optional weight function and $\omega_q(d\mathbf{x})$ denotes the integration with respect to the Lebesgue measure on Ω_q. The nonparametric estimator on which the test relies is constructed using a local linear *projected estimator* of the form $\hat{m}_{h,p}(\mathbf{x}) = \sum_{i=1}^n W_n^p(\mathbf{x}, \mathbf{X}_i) Y_i$. Its weights are employed to smooth the parametric estimate in the fashion $\mathcal{L}_{h,p} m_{\hat{\boldsymbol{\theta}}}(\mathbf{x}) = \sum_{i=1}^n W_n^p(\mathbf{x}, \mathbf{X}_i)\, m_{\hat{\boldsymbol{\theta}}}(\mathbf{X}_i)$. It is worth recall that the smoothing approach for testing, despite its natural disadvantage on the dependence of the bandwidth h, is able to bypass the non existence of a natural order in Ω_q and therefore the difficulty of considering suitable empirical processes in this setting.

The main result of this work is the asymptotic distribution of the statistic under certain regularity conditions and under $H_0 : m \in \mathcal{M}_\Theta$ (that is, $m(\mathbf{x}) = m_{\boldsymbol{\theta}_0}(\mathbf{x})$, for all $\mathbf{x} \in \Omega_q$),

$$nh^{\frac{q}{2}}\left(T_n - \frac{\lambda_q(L^2)\lambda_q(L)^{-2}}{nh^q}\int_{\Omega_q}\sigma^2_{\boldsymbol{\theta}_0}(\mathbf{x})w(\mathbf{x})\,\omega_q(d\mathbf{x})\right)\xrightarrow{d}\mathcal{N}\left(0,2\nu^2_{\boldsymbol{\theta}_0}\right),$$

where $\sigma^2_{\boldsymbol{\theta}_0}(\mathbf{x}) = \mathbb{E}[(Y - m_{\boldsymbol{\theta}_0}(\mathbf{X}))^2|\mathbf{X}=\mathbf{x}]$, $2\nu^2_{\boldsymbol{\theta}_0}$ is the asymptotic variance and $\lambda_q(L^2)\lambda_q(L)^{-2}$ is the directional analogue of the integral of the squared kernel in the euclidean setting. The power under local alternatives is also obtained and a bootstrap resampling procedure, proved to be consistent, is developed at sight of the slow convergence towards the asymptotic distribution.

The performance of the test in finite samples is illustrated in an extensive simulation study for a variety of scenarios, sample sizes, dimensions and estimators. Finally, the test is applied to study whether the mean length of the pseudo-bonds in the C_α representation of the protein backbone is constant, a common simplifying assumption used in bioinformatics.

We recall finally that this work is based on García-Portugués et al. (2015).

Keywords: directional, smoothing, regression, nonparametric, local linear.

AMS: 62G10, 62H11, 62G08, 62G09.

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Monge-Kantorovich Ranks and Signs

M. Hallin

mhallin@ulb.ac.be

ECARES, Université libre de Bruxelles

Unlike the real line, the real space $\mathbb{R}^d$, $K \geq 2$ is not "naturally" ordered. As a consequence, such fundamentals univariate concepts as quantile and distribution functions, ranks, signs, all order-related, do not straightforwardly extend to the multivariate context. Since no universal pre-existing order exists, each distribution, each data set, has to generate its own—the rankings behind sensible concepts of multivariate quantile, ranks, or signs, inherently will be distribution-specific and, in empirical situations, data-driven. Many proposals have been made in the literature for such orderings—all extending some aspects of the univariate concepts, but failing to preserve the essential properties that make classical rank-based inference a major inferential tool in the analysis of semiparametric models where the density of some underlying noise remains unspecified: (i) exact distribution-freeness, and (ii) asymptotic semiparametric efficiency, see Hallin and Werker (2006).

Starting from very practical problems, Monge, with his 1781 *Mémoire sur la Théorie des Déblais et des Remblais*, initiated a profound mathematical theory anticipating different areas of differential geometry, linear programming, nonlinear partial differential equations, and probability. In modern terms, the simplest and most intuitive formulation of the problem is as follows. Let P_1 and P_2 denote two probability measures over $(\mathbb{R}^d, \mathcal{B}^d)$, and let $L : \mathbb{R}^{2d} \to [0, \infty]$ be a Borel-measurable loss function: find a measurable transport map $T_{\mathrm{P}_1;\mathrm{P}_2} : \mathbb{R}^d \to \mathbb{R}^d$ that achieves the infimum

$$\inf_T \int_{\mathbb{R}^d} L(\mathbf{x}, T(\mathbf{x}))\mathrm{dP}_1 \qquad \text{subject to} \quad T * \mathrm{P}_1 = \mathrm{P}_2$$

where $T*\mathrm{P}_1$ denotes the "*push forward of* P_1 *by* T"—more classical statistical notation for this would be $\mathrm{P}_1^{T\mathbf{X}} = \mathrm{P}_2$. A map $T_{\mathrm{P}_1;\mathrm{P}_2}$ that attains this infimum is called an "*optimal transport*", of P_1 to P_2. In the sequel, we restrict to a L^2 loss function. Monge's problem was revisited in the 1940s by Kantorovich, who showed that, for L^2 loss, if P_1 and P_2 are absolutely continuous with finite second-order moments, the solution exists, is a.e. unique, and the gradient of a convex function—a form of multivariate monotonicity.

Kantorovich's contribution is further enhanced by a most remarkable result by McCann (1995) which implies that, for any given absolutely continuous P_2, there exists a P_1-essentially unique element in the class of gradients of convex functions mapping P_1 to P_2. Under the existence, for P_1, of finite moments of order two, that mapping moreover coincides with the L^2-optimal transport of P_1 to P_2. That fundamental result is the basis of Carlier et al. (2015)'s concept of *vector quantile regression*, and Chernozhukov et al. (2015)'s concept of *Monge-Kantorovich depth*. In close connection with the latter, it also serves as the starting point for the *Monge-Kantorovich ranks and signs* described here.

Ranks and signs, and the resulting inference methods, are well understood and well developed, essentially, in two cases: one-dimensional observations, and elliptically symmetric ones. We start by establishing the close connection, in those two cases, between classical ranks and signs and measure transportation results, showing that the rank transformation actually reduces to an empirical version of the unique gradient of convex function mapping a distribution to the uniform over the unit ball. That fact is then exploited to define fully general concepts of ranks and signs—called the *Monge-Kantorovich ranks and signs*—coinciding, in the univariate and elliptical settings, with the traditional concepts, and enjoying under completely unspecified (absolutely continuous) d-dimensional distributions, the essential properties that make traditional rank-bsd inference an essential part of the semiparametric inference toolkit.

Keywords: Ranks, Signs, Measure transportation.

AMS: 62G99, 62H99

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Two-sample change-point

Z. Hlávka[1], M. Hušková[1]

hlavka@karlin.mff.cuni.cz, huskova@karlin.mff.cuni.cz

[1]Department of Statistics, Charles University in Prague

Although the change-point analysis dates back to 1950s, it remains a very active area of research and change-point methods are increasingly often applied to practical problems. One of the simplest formulations assumes that $Y_1, \dots, Y_n$ are independent observations with distribution functions $F_1, \dots, F_n$ and a change-point problem concerns the test of the hypothesis:

$$H_0 : F_1 = F_2 = \cdots = F_n \quad \text{against} \quad H_1 : \exists k_0 \text{ such that } F_{k_0} \neq F_{k_0+1},$$

where k_0 is the unknown change-point. The distribution functions $F_1, \dots, F_n$ can be unknown or known up to a finite number of parameters. Various test procedures were developed and studied; for basic information see, e.g., Csörgö and L. Horváth (1997). We note that, to the best of our knowledge, change-point problems are typically investigated in the one-sample setup.

Two-sample change-point Let us assume that we observe two groups of independent observations $Y_{1,1}, \dots, Y_{1,N_1}$ and $Y_{2,1}, \dots, Y_{2,N_2}$ at the ordered time points $t_1 < \cdots < t_n$ so that $n_{1,i}$ observations from the first group and $n_{2,i}$ observations from the second group are observed at each t_i, for $i = 1, \dots, n$ (i.e., $\sum_i n_{1,i} = N_1$ and $\sum_i n_{2,i} = N_2$). Denoting by $F_{j,i}$ the distribution function of the observations at time t_i in j-th group ($i = 1, \dots, n$, $j = 1, 2$), we introduce the *general two-sample change-point problem* as a test of the null hypothesis:

$$H_0 : F_{1,i} = F_{2,i} \text{ for all } i = 1, \dots, n$$

against the alternative

$$H_1 : \exists k_0 \text{ such that } F_{1,k} \neq F_{2,k}, \text{ for } k \geq k_0.$$

Such testing problems occur naturally whenever we are interested in differences between two populations (e.g., males vs. females or treatment vs. control group) observed in several ordered categories (given, e.g., by age or drug dosage). Experiments of this type are usually evaluated through multiple two-sample t-tests leading to indirect and imprecise estimators of the change-point k_0.

Two-sample gradual change analysis A simple example is the gradual change model (Hinkley, 1971; Hušková, 1998) applied to mean differences:

$$E(\overline{Y}_{1i} - \overline{Y}_{2i}) = \delta \left(\frac{i - k_0}{n} \right)_+, \quad i = 1, \dots, n, \tag{1}$$

where $\overline{Y}_{1i}$ and $\overline{Y}_{2i}$ are sample means observed in the i-th category, $(x)_+ = \max(0, x)$, k_0 denotes the change-point, and δ is the speed of the change.

Hlávka and Hušková (2015) introduce and study the two-sample gradual change model (1) and show that it can be used to test hypotheses concerning the the location of the change-point k_0. Interestingly, the change-point approach is more powerful than standard two-sample t-tests.

Keywords: change point; two-sample test, gradual change.

AMS: 62F10, 62F25, 62F40, 62F03.

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Testing for changes in the means of panels

Lajos Horváth[1] and Gregory Rice[2]

[1]Department of Mathematics, University of Utah
[2] Department of Statistics, University of Waterloo

In this talk we consider the model

$$X_{i,t} = (\mu_i + \alpha_i I\{t \geq t^*\}) + \gamma_i \eta_t + e_{j,t}, \;\; 1 \leq i \leq N, 1 \leq t \leq T. \qquad (2)$$

The observation $X_{i,t}$ denotes the i^{th} panel at time t. As usual, μ_i denotes the initial mean of the i^{th} panel which can change to $\mu_i + \alpha_i$ at the unknown time t^*. Under the null hypothesis the system is stable during the observation period $1 \leq t \leq T$, i.e. $H_0: \; t^* > T$. Under the null hypothesis the model of (2) reduces to

$$X_{i,t} = \mu_i + \gamma_i \eta_t + e_{i,t}, \;\; 1 \leq i \leq N, 1 \leq t \leq T.$$

Let $\cdot^\top$ denote the transpose of vectors and matrices and define the vectors $\mathbf{X}_t = (X_{1,t}, X_{2,t}, \ldots, X_{N,t})^\top \in R^N$. Several authors considered to detect changes in the means of vectors but , these results might not be used when N is large or $N = N(T) \to \infty$. If N is large, the analysis of the $N-$dimensional mean vectors and the $N \times N$ covariance matrix of $\mathbf{X}_t$ might not be feasible. However, the first K largest eigenvalues of the covariance matrix of $\mathbf{X}_t$ might capture the changes in the means of the coordinates of $\mathbf{X}_t$. The largest eigenvalues also play in important in the Markowitz portfolio optimization problem, it is used to model co–movements of markets, stocks and as a barometer for risks. Let

$$\hat{\mathbf{C}}_{N,T}(u) = \frac{1}{\lfloor Tu \rfloor} \sum_{t=1}^{\lfloor Tu \rfloor} (\mathbf{X}_t - \bar{\mathbf{X}}_T)(\mathbf{X}_t - \bar{\mathbf{X}}_T)^\top, \;\; \text{with } \bar{\mathbf{X}}_T = \frac{1}{T} \sum_{t=1}^{T} \mathbf{X}_t.$$

The first K largest eigenvalues of $\hat{\mathbf{C}}_{N,T}(u)$ are denoted by $\hat{\lambda}_{N,T,1}(u) = \hat{\lambda}_1(u) \geq \hat{\lambda}_{N,T,2}(u) = \hat{\lambda}_2(u) \geq \ldots \geq \hat{\lambda}_{N,T,K}(u) = \hat{\lambda}_K(u)$. We are interested in the behavior of these eigenvalues as a function of time u, i.e. the way they are evolving with time. Assuming that $\mathbf{X}_t$ is a stationary sequence, $\mathbf{C} = \text{cov}(\mathbf{X}_t)$ does not depend on t and define $\lambda_1 > \lambda_2 > \ldots > \lambda_K$, the first K largest eigenvalues of $\mathbf{C}$. The main goal of our paper is to establish the weak convergence of the K–dimensional process

$$\mathbf{A}_{N,T}(u) = (A_{N,T,1}(u), A_{N,T,2}(u), \ldots, A_{N,T,K}(u))^\top,$$

where for any $1 \le i \le K$

$$A_{N,T,i}(u) = T^{1/2}u(\hat{\lambda}_i(u) - \lambda_i), \;\; 1/T \le u \le 1 \;\text{ and }\; A_{N,T,i}(u) = 0, \;\; 0 \le u < 1/T.$$

We provide several limit results for $\mathbf{A}_{N,T}(u)$ under various conditions. The limit is determined by the size of $\|\boldsymbol{\gamma}\|$, where $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_N)^\top$. The power of the suggested test depends on $\|\boldsymbol{\gamma}\|$ and $\|\boldsymbol{\alpha}\|$, where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_N)^\top$.

Keywords: panel data, loadings, change in the mean

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Testing the adequacy of semiparametric transformation models

M. Hušková[1], S. Meintanis[2,3], J. Allison[3]

huskova@karlin.mff.cuni.cz, simosmei@econ.uoa.gr, james.allison@nwu.ac.za

[1] Department of Statistics, Charles University in Prague
[2] Department of Economics, National and Kapodistrian University of Athens
[3] Unit for Business Mathematics and Informatics, North-West University

We consider a semiparametric model where the response variable following a transformation can be expressed by means of a nonparametric regression model. In this model the form of the transformation is specified analytically but incorporates an unknown transformation parameter. We develop testing procedures for the null hypothesis that this semiparametric model (parametric in the transformation function but nonparametric in the regression function) adequately describes the data at hand.

More rigorously, it can be formulated as follows: Let Y denote a univariate response, $\mathbf{X} = (X_1, \dots, X_p)$ be a covariate vector, and consider the transformation $\Upsilon := \mathcal{T}_\vartheta(Y)$, where $\mathcal{T}_\vartheta(\cdot)$ is indexed by the parameter ϑ. It is assumed that the transformation is strictly increasing so that $\mathbb{E}(Y)$ is uniquely determined as a function of $\mathbf{X}$, and that $\mathcal{T}_\vartheta(\cdot)$ belongs to a fixed (known) parametric family of transformations. The parameter ϑ is assumed to belong to a compact subset Θ of $\mathbb{R}^q$, but it is otherwise considered unknown. We will consider the problem of testing the adequacy of regressors for this transformation model. Specifically for a fixed $p_0 < p$, write $\mathbf{X}_0 = (X_1, ..., X_{p_0})'$. Then on the basis of data $\{Y_j, \mathbf{X}_j\},\ j = 1, \dots, n$, on $\{Y, \mathbf{X}\} \in \mathbb{R}^{1+p}$, we wish to test the null hypothesis

$$\mathcal{H}_0 : \mathbb{E}\left[\Upsilon - m(\mathbf{X}_0)|\mathbf{X}\right] = 0,\ a.s.,$$

for some $\vartheta \in \mathbb{R}^q$, and some function $m = m_\vartheta : \mathbb{R}^{p_0} \mapsto \mathbb{R}$, against general alternatives.

Similar problems have been considered in Linton et al. (2008) and Neymeyer et al. (2015) but they applied a different approach.

Our test statistic is based on Fourier-type conditional expectations. This idea was put forward by Bierens (1982). The asymptotic distribution of the

test statistic is obtained under the null as well as under contiguous alternative hypotheses. Since the limit null distribution is nonstandard, a bootstrap version is suggested in order to actually carry-out the test procedure. Monte Carlo as well as results with real data are included that illustrate the finite sample properties of the new method.

Keywords: Transformation model; Goodness-of-fit test; Nonparametric regression; Bootstrap test.

AMS: class 62E20, 62F12, 62G05.

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Preferences of goodness-of-fit tests: A survey about the analysis of nonparametric power functions

Arnold Janssen[1]

[1]Mathematical Institute, Heinrich-Heine-University of Duesseldorf

Goodness of fit tests are usually constistent for nonparametric models. However, they do not meet the power of oracle tests (Neyman Person tests) for local alternatives when the distributions would be known.The statistician likes to distinguish and to compare the power of different competing tests. It is shown that under certain circumstances every test has a preference for a finite dimensional space of alternatives. Apart from this space, the power function is almost flat on balls of alternatives. There exists no test which pays equal attention to an infinite number of orthogonal alternatives. The results are not surprising. Every statistician knows that it is impossible to seperate an infinite sequence of different parameters simultaneously if only a finite number of observations is available.
The conclusions of the results are two-fold.

1. The statistician should analyze the goodness of fit tests of his computer package in order to get some knowledge and an impression about their preferences.

2. A well-reflected choice of tests requires some knowledge about preferences concerning alternatives which may come from the practical experiment. A guide to the construction of tests is given. A principle component decomposition of goodness of fit tests has been studied in Janssen (1995, 2000, 2003).

Global power functions of one-sided Kolmogorov-Smirnov tests for a restricted class of alternatives were obtained by Anděl (1967) and Hájek and Šidák (1967). We refer also to the early work of Neuhaus (1976), Milbrodt and Strasser (1990) and Rahnenfuehrer (2003).

Keywords: Goodness of fit test, Kolmogorov-Smirnov test, power function, envelope power function, curvature of power functions, level points.

AMS: 62G10, 62G20

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On the estimation of the characteristic function in finite populations with applications

M.D. Jiménez-Gamero[1], J.L. Moreno-Rebollo[1], J.A. Mayor-Gallego[1]

dolores@us.es, jlmoreno@us.es, jmayor@us.es

[1]Departamento de Estadística e I.O., Universidad de Sevilla, Spain

Let $U = \{1, ..., N\}$ be a finite population of size N. Let $y_U = (y_1, ..., y_N)'$, where y_j is the value of the study variable $Y \in \mathbb{R}$ for the jth population element. For simplicity in notation, it will assumed that Y is scalar, nevertheless all methods and results presented here are valid for $Y \in \mathbb{R}^d$, for any fixed $d \in \mathbb{N}$.

The basic question in survey sampling is to estimate a linear parameter $\theta_\gamma = \gamma_U' y_U$, with $\gamma_U' = (\gamma_1, ..., \gamma_N)$ a known parameter vector. Usually, survey sampling focuses on the estimation of the finite population total or the mean, on the whole population or on a domain, or the population distribution function (df), at a point or at all points,

$$F_N(y) = \frac{1}{N} \sum_{j \in U} \Delta(y_j \le y), \quad y \in \mathbb{R},$$

where $\Delta(\cdot)$ stands for the indicator function. The estimation of the df is of interest per se and because it provides a useful tool for making inferences on the population (see, for example, Wang, 2012, and Conti, 2014).

In the context of sampling from a random variable (sfarv), in addition to the df, the characteristic function (cf) is another helpful device for inferential purposes. To the best of our knowledge, no finite population version of the cf has been defined. Proceeding as in the sfarv context, we define the cf associated to the study variable Y in the finite population U as

$$C_N(t) = \int \exp(\mathrm{i}yt) dF_N(y) = \frac{1}{N} \sum_{j \in U} \exp(\mathrm{i}y_j t) = R_N(t) + \mathrm{i}I_N(t), \quad t \in \mathbb{R},$$

with $R_N(t) = \frac{1}{N} \sum_{j \in U} \cos(ty_j)$, $I_N(t) = \frac{1}{N} \sum_{j \in U} \sin(ty_j)$. $C_N(t)$ can be estimated by replacing in the above expression $F_N(y)$ by its Horvitz-Thompson estimator

$$\hat{F}_{N,\pi}(y) = \frac{1}{N} \sum_{j \in s} \frac{\Delta(y_j \leq y)}{\pi_j}, \quad y \in \mathbb{R},$$

where $\{\pi_j = P(j \in s),\, j \in U\}$ are the first order inclusion probabilities and s is a sample, or its Hájek estimator

$$\hat{F}_{N,H}(y) = \frac{1}{\hat{N}_\pi} \sum_{j \in s} \frac{\Delta(y_j \leq y)}{\pi_j}, \quad y \in \mathbb{R},$$

with $\hat{N}_\pi = \sum_{j \in s} \pi_j^{-1}$, obtaining $\hat{C}_{N,\pi}(t)$ and $\hat{C}_{N,H}(t)$, respectively.

The estimation of the population cf at a single point has little (or no) concern since $C_N(t)$ is seldom an interesting parameter. The objective of this work is to study the performance of $\hat{C}_{N,\pi}(t)$ and $\hat{C}_{N,H}(t)$ as estimators of the whole cf for general sample designs. We will see that, under certain conditions on the sample design, the asymptotic behavior of the process $\{\sqrt{n}(\hat{C}_{N,H}(t) - C_N(t)),\, t \in \mathbb{R}\}$ is quite similar to that obtained in the sfarv, where n stands for the sample size. This fact let us propose procedures for testing several hypotheses of interest, in parallel to some well-established cf-based procedures in the sfarv context such as tests for the two-sample problem, testing for independence of two variables or testing for symmetry. All inferences are made under a design-based framework.

Keywords: finite population, characteristic function, tests.

AMS: 62D05, 62G10.

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Distribution-free Tests for Discrete Distributions

E. Khmaladze,

School of Mathematics, Statistics and OR, Victoria University of Wellington

The main driver for this work was the need for a class of distribution free tests for discrete distributions. The basic step, reported below, could have been made long ago, maybe even soon after the publication of the classical papers of K. Pearson (1900) and R. Fisher (1922, 1924). However, the tradition of using the chi-square goodness of fit statistic became so widely spread and the point of view that, for discrete distributions, other statistics "have to" have their asymptotic distributions dependent on the underlying probabilities, became so predominant and "evident", that it required quite a few years before we asked ourselves again: "why is the theory of distribution free tests for discrete distributions so much more narrow than for continuous distributions?" We will see that it does not have to be.

In attempting the broader theory, we hope to demonstrate that the geometric insight behind K. Pearson (1900) or R. Fisher (1924) goes considerably further than one goodness of fit statistic.

In obvious notations, consider the vector Y_n of components of the chi-square statistic

$$Y_{in} = \frac{\nu_{in} - np_i}{\sqrt{np_i}}, \ i = 1, \dots, m.$$

Let $X = (X_1, \dots, X_m)^T$ denote a vector of m independent $N(0,1)$ random variables. As $n \to \infty$, the vector Y_n has a limit distribution of the zero-mean Gaussian vector $Y = (Y_1, \dots, Y_m)^T$ such that

$$Y = X - \langle X, \sqrt{p} \rangle \sqrt{p}, \tag{3}$$

where $\sqrt{p}$ denotes the vector $\sqrt{p} = (\sqrt{p}_1, \dots, \sqrt{p}_m)^T$. and $\langle a, b \rangle = \sum_{i=1}^m a_i b_i$.

This vector is an orthogonal projection of X parallel to $\sqrt{p}$. Of course its distribution depends on $\sqrt{p}$ – it is only the sum of squares $\langle Y, Y \rangle$ which is chi-square distributed and hence has a distribution free from $\sqrt{p}$.

Now choose a vector r, $\|r\| = 1$, and transform Y_n into a new vector

$$Z_n = Y_n - \langle Y_n, r\rangle \frac{1}{1 - \langle\sqrt{p}, r\rangle}(r - \sqrt{p}). \qquad (4)$$

Proposition. *(Khm, 2013) The asymptotic distribution of Z_n is that of another, "standard", orthogonal projection*

$$Z \stackrel{d}{=} X - \langle X, r\rangle\, r,$$

and therefore any statistic based on Z_n has asymptotic distribution dependent on r and not on $\sqrt{p}$. The transformation of Y_n to Z_n is one-to-one.

Thus the problem of testing p is translated into the problem of testing for the distribution $q = \{r_1^2, \ldots, r_m^2\}$, corresponding to r, which we can choose the way *we wish*: the same q (or r) for lots of different p. The approach generalizes to the parametric families of distributions.

It also opens new facts in the theory of empirical processes in continuous time (Khm, 2015).

Keywords: distribution-free test, discrete distribution, unitary operator

AMS: 62D05, 62E20, 62E05, 62F10.

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Goodness-of-Fit Testing in the Linear Regression Errors-in-Variables Model

Xiaoqing Zhu[1], Weixing Song[2], **Hira L. Koul**[1]

[1]Department of Statistics & Probability, Michigan State University
[2]Department of Statistics, Kansas State University

Here we consider the linear errors-in-variables model

$$Y = \alpha + \beta' X + \varepsilon, \quad Z = X + u, \quad \text{Or} \quad Y = \alpha + \beta' Z + e, \quad e = \varepsilon - \beta' u,$$

where Y is the response variable, X is the d-dimensional vector of unobserved predictors, and Z is the observed surrogate vector. The variables X, u and ε are assumed to be mutually independent. For model identifiability, we assume the density g of u is known. Numerous practical examples of this model and relevant theoretical results can be found in the monographs of Fuller (1987), Cheng and Van Ness (1999), and Carroll *et al.* (2006).

Let f denote density of ε, and $f_\theta, \theta \in \Theta \subseteq \mathbb{R}^q$, be a family of known parametric densities with zero mean. The problem of interest is to test the hypothesis $H_0 : f = f_{\theta_0}$, for some $\theta_0 \in \Theta$, versus the alternative $H_1 : H_0$ is not true, based on a random sample (Y_i, Z_i), $1 \leq i \leq n$ from the above model. This problem has been well studied when there is no measurement error in the above model as is evidenced in Khmaladze and Koul (2004, 2009), Gonzlez-Manteiga and Crujeiras (2013), and the references there in.

The independence of u and ε imply that the density of e is $h(v) = \int f(v + \beta' u) g(u) \mathrm{d}u, v \in \mathbb{R}$. Because f is involved in the convolution h, it is natural to construct tests of H_0 based on deconvolution density estimators. Let Φ_γ denote the characteristic function of a density γ. In this talk we shall present certain L_2 distance type tests for testing H_0 based on deconvolution density estimators

$$\hat{f}_n(x) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{-itx} \Phi_K(bt) \frac{\hat{\Psi}_n(t)}{\Phi_g(-\hat{\beta} t)} \, \mathrm{d}t, \quad \hat{\Psi}_n(t) := n^{-1} \sum_{j=1}^{n} e^{it(Y_j - \hat{\alpha} - \hat{\beta}' Z_j)},$$

where b and K are the window width and kernel used to estimate h, and $\hat{\alpha}, \hat{\beta}$ are some $n^{1/2}$-consistent estimators of α, β, under H_0. The proposed class o tests, one for each K and b, of H_0 is based on

$$\hat{T}_n = \int_{\mathbb{R}} \big(\hat{f}_n(x) - K_b * f_{\hat{\theta}}(x)\big)^2 \,\mathrm{d}x, \;\; K_b * f_{\hat{\theta}}(x) = b^{-1} \int K((x-y)b^{-1}) f_{\hat{\theta}}(y) \mathrm{d}y,$$

where $\hat{\theta}$ is a $n^{1/2}$-consistent estimator of θ_0 under H_0.

This talk will describe the asymptotic distributions of $\hat{T}_n$ under H_0 and under some alternatives, under fairly general conditions on the underlying entities. The findings of a finite sample simulation that compares a member of the proposed class of tests with that of the Kolmogorov–Smirnov, Cramér–von Mises tests based on the empirical d.f. of $\{Y_j - \hat{\alpha} - \hat{\beta}' Z_j, 1 \leq j \leq n\}$, and a Koul and Song (2012) test will be also reported.

Keywords: Deconvolution density estimators. L_2-distance tests.

AMS: 62G08, 62G10.

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Validation of positive quadrant dependence

T. Ledwina[1]

[1]Institute of Mathematics, Polish Academy of Sciences

Quadrant dependence is a useful notion of dependence of two random variables, widely applied in reliability, insurance and actuarial sciences. In recent years renewed interest in this notion is noticeable. It has been stimulated by an evident need to account for positive dependence in analysis of risk measures and other economic indexes as well as by justified call for going beyond simple linear correlation and classical models to characterize the dependence properties of studied phenomena. See Denuit and Scaillet (2004) for related discussion.

In this talk, we focus on testing for positive quadrant dependence and measuring its strength. Our approach is nonparametric and naturally linked to bivariate copulas.

We shall present two new rank tests for verifying positive quadrant dependence recently introduced in Ledwina and Wyłupek (2014). We shall also discuss an interesting result on finite sample behavior of power function of one of the proposed tests. Namely, we shall show that the new test statistic possesses order preserving property saying that stronger positive quadrant dependence makes it to be stochastically larger. More precisely, the concordance ordering for population distributions implies stochastic ordering of finite sample distributions of the rank statistic. It is very useful but scarcely known property of bivariate rank statistics.

Next, we shall summarize simulation study performed to evaluate and compare the two new solutions with the best existing ones, including a recent construction described and studied in Gijbels et al. (2010) and related papers. These comparisons demonstrate that the new solutions are slightly weaker in detecting positive quadrant dependence modeled by classical bivariate distributions such as Gaussian, Clayton or Gumbel, for example, and significantly outperform the best existing solutions when some mixtures, regression and heavy-tailed models have to be detected.

The new methods, introduced in the paper, shall be applied to real life insurance data, to assess the dependence and test them for positive quadrant dependence.

The proposed approach to construction of one of the new tests for positive quadrant dependence naturally leads to a new function valued measure of dependence of two random variables; cf. Ledwina (2014, 2015). The measure allows to study and visualize underlying dependence structure, both in some theoretical models and empirically, without prior model assumptions. This provides a comprehensive view of association structure and makes possible much more detailed inference than the one based on standard numeric measures of association. In this contribution, we focus on copula-based variant of the measure. It is defined on $(0, 1) \times (0, 1)$, takes values in $[-1, 1]$ and is non-negative if and only if positive quadrant dependence occurs. The measure possesses many properties advocated as desirable in the literature on the subject. It is also clearly linked to some tail dependence indexes. Moreover, it coincides with bivariate quantilogram defined by Linton and Whang (2007), when applied to our setup.

Keywords: concordance ordering, copula, dependence measure, rank test.

AMS: 62G10 , 62H20.

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Parameter Change Test for Bivariate Poisson Autoregressive Models

Y. Lee[1], **S. Lee**[1]

[1]Department of Statistics, Seoul National University, Seoul, Korea

Suppose that $\mathbf{Y}_t = (Y_{t,1}, Y_{t,2})^T$ is a two dimensional vector of counts at time t, where $\{Y_{t,1}, t \geq 1\}$ and $\{Y_{t,2}, t \geq 1\}$ conditionally follow a Poisson distribution with mean $\lambda_{t,1}$ and $\lambda_{t,2}$, respectively. Suppose that $\{\mathbf{Y}_t\}$ follows a bivariate Poisson INGARCH(1,1) model:

$$\mathbf{Y}_t|\mathcal{F}_{t-1} \sim BP(\lambda_{t,1}, \lambda_{t,2}, \phi), \;\; \boldsymbol{\lambda}_t = (\lambda_{t,1}, \lambda_{t,2})^T = \boldsymbol{\delta} + \boldsymbol{A}\boldsymbol{\lambda}_{t-1} + \boldsymbol{B}\mathbf{Y}_{t-1},$$

where $\mathcal{F}_t$ is the σ-field generated by $\boldsymbol{\lambda}_1, \mathbf{Y}_1, \ldots, \mathbf{Y}_t, \phi \geq 0, \boldsymbol{\delta} = (\delta_1, \delta_2)^T \in \mathbb{R}^2_+$ and $\boldsymbol{A} = \{\alpha_{ij}\}_{i,j=1,2}$ and $\boldsymbol{B} = \{\beta_{ij}\}_{i,j=1,2}$ are 2×2 matrices with nonnegative entries, and further,

$$\begin{aligned} &P(Y_{t,1} = m, Y_{t,2} = n|\mathcal{F}_{t-1}) \\ &= e^{-(\lambda_{t,1}+\lambda_{t,2}-\phi)} \frac{(\lambda_{t,1}-\phi)^m}{m!} \frac{(\lambda_{t,2}-\phi)^n}{n!} \sum_{s=0}^{m\wedge n} \binom{m}{s}\binom{n}{s} s! \left\{ \frac{\phi}{(\lambda_{t,1}-\phi)(\lambda_{t,2}-\phi)} \right\}^s \end{aligned}$$

with $m \wedge n = \min\{m, n\}$ and $\phi = Cov(Y_{t,1}, Y_{t,2}|\mathcal{F}_{t-1}) \in [0, \lambda_{t,1} \wedge \lambda_{t,2})$.

We assume that $\mathbf{A}$ is diagonal. Let $\theta = (\theta_1^T, \theta_2^T, \phi)^T$, where $\theta_1 = (\delta_1, \alpha_1, \beta_{11}, \beta_{12})$ and $\theta_2 = (\delta_2, \alpha_2, \beta_{21}, \beta_{22})$, $\boldsymbol{\delta} = (\delta_1, \delta_2)^T$, $\boldsymbol{A} = diag(\alpha_1, \alpha_2)^T$, and $\boldsymbol{B} = \{\beta_{ij}\}_{i,j=1,2}$. For estimating the true parameter θ_0, we recursively define $\tilde{\boldsymbol{\lambda}}_t$, $t \geq 2$, by using an arbitrary chosen initial value $\tilde{\boldsymbol{\lambda}}_1$ and the equations $\tilde{\boldsymbol{\lambda}}_t = \boldsymbol{\delta} + \boldsymbol{A}\tilde{\boldsymbol{\lambda}}_{t-1} + \boldsymbol{B}\mathbf{Y}_{t-1}$, where $\boldsymbol{\delta}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are expressed as $\boldsymbol{\delta}(\theta)$, $\boldsymbol{A}(\theta)$ and $\boldsymbol{B}(\theta)$ when θ is emphasized. Then, constructing the conditional log-likelihood function based on the observation $\mathbf{Y}_1, \ldots, \mathbf{Y}_n$, we obtain the the conditional maximum likelihood estimator (CMLE) of θ_0 by $\hat{\theta}_n = \arg\max_{\theta\in\Theta} \sum_{t=1}^n \tilde{\ell}_t(\theta)$, where $\tilde{\ell}_t(\theta) = \log p_\theta(\mathbf{Y}_t \,|\, \tilde{\boldsymbol{\lambda}}_t)$.

Below, we assume that the following conditions hold:

(A1) $\theta_0 \in \Theta$ and Θ_0 is compact.

(A2) $\delta(\theta), \mathbf{A}(\theta)$ and $\mathbf{B}(\theta)$ have non-negative entries and $\mathbf{B}(\theta)$ is full rank for all $\theta \in \Theta$.

(A3) $\phi(\theta) < \min(a_1, a_2)$ where $(a_1, a_2)^T = (\mathbf{I} - \mathbf{A}(\theta))^{-1}\delta(\theta)$ for all $\theta \in \Theta$.

(A4) There exists a $p \in [1, \infty]$ such that $\|\mathbf{A}(\theta)\|_p + 2^{1-(1/p)}\|\mathbf{B}(\theta)\|_p < 1$ for all $\theta \in \Theta$, where $\|\mathbf{A}\|_p = \max_{\mathbf{x}\neq 0}\{\|\mathbf{A}\mathbf{x}\|_p/\|\mathbf{x}\|_p : \mathbf{x} \in \mathbb{C}^n\}$.

Theorem 1. *Under (A1)-(A4), as $n \to \infty$,*

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \xrightarrow{w} N(0, I(\theta_0)^{-1}),$$

where $I(\theta_0) := E\left(\frac{\partial \ell_t(\theta_0)}{\partial \theta}\frac{\partial \ell_t(\theta_0)}{\partial \theta^T}\right) = -E\left(\frac{\partial^2 \ell_t(\theta_0)}{\partial \theta \partial \theta^T}\right)$ *and* $\ell_t(\theta) = \log p_\theta(\mathbf{Y}_t \,|\, \boldsymbol{\lambda}_t)$

Now, we consider the problem of testing the null and alternative hypotheses:

$$H_0 : \theta \text{ does not change over } \mathbf{Y}_1, \cdots, \mathbf{Y}_n \;\; vs. \;\; H_1 : \text{not } H_0.$$

The estimates-based CUSUM test is given as

$$T_n^{est} = \max_{1\leq k\leq n} \frac{k^2}{n}(\hat{\theta}_k - \hat{\theta}_n)^T \hat{I}_n(\hat{\theta}_k - \hat{\theta}_n),$$

where $\hat{\theta}_k$ is the CMLE of θ_0 based on $x_1, \ldots, x_k$ and $\hat{I}_n = -\frac{1}{n}\sum_{t=1}^n \frac{\partial \tilde{\ell}_t(\hat{\theta}_n)}{\partial \theta \partial \theta^T}$.

Theorem 2. *Suppose that H_0 and (A1)-(A4) hold. Then,*

$$T_n^{est} \xrightarrow{w} \sup_{0\leq s\leq 1} \|\mathbf{B}_9^\circ(s)\|^2,$$

where $\{\mathbf{B}_9^\circ(s), 0 < s < 1\}$ is a 9-dimensional Brownian bridge.

Let $\boldsymbol{\epsilon}_t = (\epsilon_{t,1}, \epsilon_{t,2})^T = \mathbf{Y}_t - \boldsymbol{\lambda}_t(\theta_0)$ with $\epsilon_{t,i} = Y_{t,i} - \lambda_{t,i}(\theta_{i0})$ for $i = 1, 2$. Since $\boldsymbol{\epsilon}_t$ are not observable, we use the estimated residuals $\hat{\boldsymbol{\epsilon}}_t$, $\hat{\boldsymbol{\epsilon}}_t = (\hat{\epsilon}_{t,1}, \hat{\epsilon}_{t,2})^T = \mathbf{Y}_t - \hat{\boldsymbol{\lambda}}_t$, $\hat{\boldsymbol{\lambda}}_t = \hat{\boldsymbol{\delta}}_n + \hat{\mathbf{A}}_n \hat{\boldsymbol{\lambda}}_{t-1} + \hat{\mathbf{B}}_n \mathbf{Y}_{t-1}$, $t \geq 2$, where $\hat{\boldsymbol{\delta}}_n = (\hat{\delta}_{1,n}, \hat{\delta}_{2,n})^T$, $\hat{\mathbf{A}}_n = diag(\hat{\alpha}_{1,n}, \hat{\alpha}_{2,n})$, $\hat{\mathbf{B}}_n = \{\hat{\beta}_{ij,n}\}_{i,j=1,2}$ and $\hat{\boldsymbol{\lambda}}_1$ is an arbitrarily chosen initial random variable. Then, the residual-based CUSUM test is given as

$$T_n^{res} = \max_{1\leq k\leq n} \frac{1}{\sqrt{n}} \left\| \hat{\boldsymbol{\Gamma}}^{-1/2} \left(\sum_{t=1}^{[ns]} \hat{\boldsymbol{\epsilon}}_t - \frac{[ns]}{n} \sum_{t=1}^{n} \hat{\boldsymbol{\epsilon}}_t \right) \right\|,$$

where $\hat{\boldsymbol{\Gamma}}$ is a consistent estimator of $\boldsymbol{\Gamma}$.

Theorem 3. *Under the assumption (A1)-(A4) and H_0, we have*

$$T_n^{res} \xrightarrow{w} \sup_{0\leq s\leq 1} \|\mathbf{B}_2^0(s)\|,$$

where $\{\mathbf{B}_2^\circ(s), 0 < s < 1\}$ is a 2-dimensional Brownian bridge.

Bayesian optimality for goodness-of-fit

R. Lockhart[1], Z. Sun[1], M. A. Stephens[1], A. Contreras Cristan[2]

[1]Statistics and Actuarial Science, Simon Fraser University
[2]IMAS, UNAM

The goodness-of-fit literature has many papers with the following general structure. A new test is proposed for some distributional assumption, say, that some sample is from the normal distribution. The test is studied theoretically and shown to be consistent against a wide range of alternatives and its asymptotic distribution under the null distribution is characterized or computed. The paper concludes with a Monte Carlo study, first studying the quality of the asymptotics for the null distribution and then studying the power of the test. In the power study several alternative distributions are chosen and power estimated by simulation for each alternative. The authors argue the alternatives in question represent the collection of realistic alternatives. Usually the proposed test works well for some alternatives and not quite as well for others. Typically the paper tries to describe the alternatives for which the proposed procedure is good saying something like “particularly sensitive to skewed alternatives” or “good for heavy tails” or some such.

Our goal is to provide insight into this process. We will consider several priors on the alternatives for goodness-of-fit problems. We:

1. try to choose “realistic” priors and deduce which tests are optimal; these tests might then be recommended.

2. try to evaluate the extent to which non-optimal tests are inferior to optimal tests and perhaps show that simple non-optimal tests are as almost as good as more complex optimal ones.

3. try, for particular tests, to deduce the sort of alternatives the tests are good for; having found a prior for which a test is Bayes optimal we can look at how realistic the prior is by examining samples from the prior.

4. try to compare two or more non-optimal tests by comparing their Bayes powers for “realistic” priors.

When the null hypothesis is a parametric family of possible joint distributions for the data we generate a prior on the omnibus alternative (“not the null") in two stages: by putting first a prior on the null and then second conditional on a point in the null modelling the log-likelihood ratio of the alternative to that null point as a Gaussian process. We consider two approximation regimes: a fixed sample size regime where we study local alternatives and an asymptotic regime in which the priors focus on contiguous tubes around the null hypothesis. We learn:

- that putting priors on alternative densities leads to statistics generally of the empirical distribution function type or perhaps more accurately to U-statistics.
- that composite null hypotheses generally lead to tests with the following structure: assume you know which point in the null is relevant and find the test statistic appropriate to that parameter value, then average over parameter values with respect to the posterior on the null.
- that tests for latent variables or more generally involving any unobserved quantities should be carried out as if you had observed those quantities and then averaging the results over all unknowns.

I will focus on a big picture presentation of the thoughts illustrated perhaps by either a directional regression model or by a latent variable example.

Keywords: Calibrated Bayes, contiguity, Empirical Distribution Function, frailties, random effects.

AMS: 62G30, 62G20.

Change-Point Analysis for High Dimensional Vector Autoregressive Models

George Michailidis

Department of Statistics, University of Florida

Vector autoregressive(VAR) models have been extensively used to discern the dynamic causal chain (causality in terms of *predictability* rather than control) among variables. Such models have also been intimately connected to the notion of Granger causality and have been widely used in macroeconomics and financial econometrics (see discussion in Basu et al. 2015a), and more recently in genomic studies (see discussion in Michailidis et al. 2013).

Next, we define a VAR model; let $X_1, \ldots, X_p$ be p stationary stochastic processes and denote by $\mathbf{X}$ the rearrangement of these stochastic processes into a vector time series, i.e. $\mathbf{X}^t = (X_1^t, \ldots, X_p^t)^\intercal$. We consider Vector Autoregressive (VAR) models of order d of the form

$$\mathbf{X}^T = A^1\mathbf{X}^{T-1} + \cdots + A^d\mathbf{X}^{T-d} + \varepsilon^T. \tag{5}$$

The connection to Granger causality comes as follows: X_j^{T-t} is said to be Granger-causal for X_i^T if the corresponding coefficient, $A_{i,j}^t$ is statistically significant. In that case, there exists an edge $X_j^{T-t} \to X_i^T$ in the underlying graphical model with $T \times p$ nodes (for details see Basu et al. 2015a).

Under an assumption of *sparsity* on the transition matrices $A^1, \cdots, A^d$, a Granger causal network (i.e. the entries of the transition matrices A) can be estimated by solving the following collection of p distinct ℓ_1 penalized least squares problems (Basu et al. 2015b) of the form

$$\text{argmin}_{\theta^t \in \mathbb{R}^p} n^{-1} \|\mathcal{X}_i^T - \sum_{t=1}^{d} \mathcal{X}^{T-t}\theta^t\|_2^2 + \lambda \sum_{t=1}^{d}\sum_{j=1}^{p} |\theta_j^t| \tag{6}$$

where $\mathcal{X}^t$ denotes the $n \times p$ matrix of observations at time t, and $\mathcal{X}_i^t$ denotes the i^{th} column of $\mathcal{X}^t$.

In this study, we consider the following change-point problem related to VAR models and corresponding Granger causal networks. The simplest VAR model to consider is that of order 1: let

$$X(t) = A(t)\,X(t-1) + \epsilon(t)\,,$$

where $A(t)$ is a matrix that can change in time, and time is specified through a discrete grid. In the simplest case, the innovations $\epsilon(t)$ may be taken to be $N_p(0, \sigma^2 I_p)$ and independent across time and $X_0 \equiv \epsilon_0$. We assume that at some point in time $\tau_\star$, the parameter $A(t)$ takes two different values A_l and A_r to the left and right of $\tau_\star$, respectively.

The change-point estimate is obtained by maximizing the corresponding Gaussian likelihood under a sparsity assumption and appropriate technical conditions on the sample size, restricted strong convexity and on the signal-to-noise ratio, so that the problem becomes identifiable. We also provide a tight bound for the estimate of the change-point for settings where the number of parameters in the transition matrices A_l and A_r exceed the sample size (time points). The methodology is illustrated on synthetic data and on log-returns of selected financial companies.

Keywords high-dimensional, VAR models, change-point analysis, consistency.

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New Tests of Symmetry Based on Extremal Order Statistics and Their Efficiencies

Ya. Yu. Nikitin[1,2]

[1]Department of Probability and Statistics, University of Saint-Petersburg, Russia
[2] Supported by grants of RFBR 13-01-00172 and SPbGU 6.38.672.2013

Testing of symmetry is one of most known and important problems of non-parametric statistics which is very close by ideas and techniques to goodness-of-fit testing. The promising but almost unexplored approach for testing of symmetry is based on various characterizations of symmetry. One of rare example of such approach is the paper by Baringhaus and Henze (1992). Later Nikitin (1996) found the efficiencies of tests from this paper which turned out to be rather high.

In this talk we build and explore new tests of symmetry based on the following characterization of symmetry from Ahsanullah (1992):

Let $X_{(1)}$ and $X_{(k)}$ be two extremal order statistics of the sample of i.i.d. observations $X_1,\dots,X_k, k \geq 2$, from continuous d.f. F. Then $|X_{(1)}|$ and $|X_{(k)}|$ are identically distributed iff F is symmetric with respect to 0.

Consider the simplest case $k = 2$. We build two V-empirical df's

$$G_n(t) = n^{-2}\sum_{1\leq i,j\leq n} \mathbf{1}\{|\min(X_i,X_j)| < t\}, t \in R^1,$$
$$H_n(t) = n^{-2}\sum_{1\leq i,j\leq n} \mathbf{1}\{|\max(X_i,X_j)| < t\}, t \in R^1,$$

and let Q_n be the empirical df corresponding to the sample $|X_i|, i = 1,...,n$.

For testing of symmetry we introduce two distribution-free statistics: the integral one

$$J_n := \int_{R^1}[G_n(t) - H_n(t)]dQ_n(t),$$

and the statistic of Kolmogorov type

$$D_n := \sup_{t\in R^1} \mid G_n(t) - H_n(t) \mid .$$

The statistic J_n is equivalent to V-statistic with the kernel Ψ such that

$$\begin{aligned}&3\Psi(x,y,z) = \\ &= \mathbf{1}\{|\min(x,y)| < |z|\} + \mathbf{1}\{|\min(x,z)| < |y|\} + \mathbf{1}\{|\min(y,z)| < |x|\} \\ &-\mathbf{1}\{|\max(x,y)| < |z|\} - \mathbf{1}\{|\max(x,z)| < |y|\} - \mathbf{1}\{|\max(y,z)| < |x|\},\end{aligned}$$

while D_n is the supremum in t of a family of V-statistics with the kernels

$$\Xi(x, y, t) = \mathbf{1}\{|\min(x, y)| < t\} - \mathbf{1}\{|\max(x, y)| < t\},\ 0 \leq t \leq 1.$$

It can be proved that J_n is asymptotically normal

$$\sqrt{n} J_n \xrightarrow{d} N(0,\ 1/5).$$

The limiting distribution of $\sqrt{n} D_n$ is unknown, but the critical values can be found via simulation.We study asymptotic properties of new statistics including large deviations and calculate their local Bahadur efficiency.

Similar reasoning is valid for statistics of higher orders $k \geq 3$ with more complicated kernels depending on k. The efficiencies of new tests for location and skew alternatives are often high in comparison with other nonparametric tests of symmetry. We also discuss the conditions of their local optimality. Detailed exposition and proofs can be found in Nikitin and Ahsanullah (2015).

Keywords: symmetry, characterization, efficiency, alternative.

AMS: 62F05, 62G10, 62G30.

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Goodness of Fit for Dimension Reduction

H. Oja[1], K. Nordhausen[1], D. Tyler[2]

[1]Department of Mathematics and Statistics, University of Turku
[2] Department of Statistics and Biostatistics, The State University of New Jersey

In linear dimension reduction for a p-variate random vector $\mathbf{x}$, the general idea is to find a $k \times p$ matrix $\mathbf{B}$ with $k << p$ such that $\mathbf{Bx}$ carries all or most of the information. In unsupervised dimension reduction this means that $\mathbf{x}|\mathbf{Bx}$ just presents (uninteresting) noise. In supervised dimension reduction $\mathbf{x} \perp\!\!\!\perp y|\mathbf{Bx}$ for an interesting response variable y, that is, the dependence of $\mathbf{x}$ on y is only through $\mathbf{Bx}$. In this talk we propose new bootstrap tests to check the validity of these general assumptions.

Most of the linear dimension reduction methods found in the literature are based on a simultaneous diagonalization of two matrices as shown in the following examples. To simplify notations assume that $E(\mathbf{x}) = \mathbf{0}$. (i) In *principal component analysis (PCA)*, for example, one finds a $p \times p$ transformation matrix $\mathbf{W}$ such that $\mathbf{WW}' = \mathbf{I}_p$ and $\mathbf{W}E(\mathbf{xx}')\mathbf{W}' = \mathbf{D}$ where $\mathbf{D}$ is a diagonal matrix (with diagonal elements in a decreasing order). (ii) In the *independent component analysis (ICA)*, the so called FOBI solution finds $\mathbf{W}$ such that $\mathbf{W}E(\mathbf{xx}')\mathbf{W}' = \mathbf{I}_p$ and $\mathbf{W}E(\mathbf{xx}'E(\mathbf{xx}')^{-1}\mathbf{xx}')\mathbf{W}' = \mathbf{D}$ where the rows of $\mathbf{W}$ are ordered so that the diagonal elements $\mathbf{D}$ are in a decreasing order of $|d_i - (p+2)|$ (measuring deviation from normality). (iii) The *sliced inverse regression (SIR)* uses a dependent variable $\mathbf{y}$, and $\mathbf{W}$ satisfies $\mathbf{W}E(\mathbf{xx}')\mathbf{W}' = \mathbf{I}_p$ and $\mathbf{W}E(E(\mathbf{x}|\mathbf{y})E(\mathbf{x}|\mathbf{y})')\mathbf{W}' = \mathbf{D}$ where the rows of $\mathbf{W}$ are ordered so that the diagonal elements $\mathbf{D}$ are in a decreasing order of $|d_i|$(measuring deviation from linear independence). In dimension reduction $\mathbf{W} = (\mathbf{W}_1', \mathbf{W}_2')'$ and k-variate $\mathbf{W}_1\mathbf{x}$ is hoped to carry the relevant information. In PCA $\mathbf{W}_2\mathbf{x}$ is then often thought to be spherical. In ICA, the components of $\mathbf{Wx}$ are independent and $\mathbf{W}_2\mathbf{x} \sim N_{p-k}(\mathbf{0}, \mathbf{I}_{p-k})$. In SIR, $\mathbf{W}_2\mathbf{x}$ is thought to be independent on $\mathbf{W}_1\mathbf{x}$ and y. Unconventional semiparametric bootstrapping can be used to test for these assumptions.

Let $\mathbf{X} = (\mathbf{x}_1, ..., \mathbf{x}_n)'$ be a random sample for the distribution of $\mathbf{x}$ and let $\hat{\mu}$, $\widehat{\mathbf{W}}$ and $\widehat{\mathbf{D}}$ be the natural estimates of population values μ (location centre), $\mathbf{W}$ and $\mathbf{D}$, respectively. Natural goodness-of-fit test statistics $T(\mathbf{X}) \in \mathbb{R}_+$ are then based on diagonal elements $\hat{d}_{k+1}, ..., \hat{d}_p$ and measure their mutual deviations, deviations from one, deviations from $p+2$, or deviations from zero depending on the problem at hand. We next propose bootstrap tests based on a chosen $T(\mathbf{X})$. Write $\widehat{\mathbf{Z}}_i = (\mathbf{X} - \mathbf{1}_n\hat{\mu}')\widehat{\mathbf{W}}_i'$, $i = 1, 2$, and $\widehat{\mathbf{Z}} = (\mathbf{X} - \mathbf{1}_n\hat{\mu}')\widehat{\mathbf{W}}'$. Then $\widehat{\mathbf{Z}} = (\widehat{\mathbf{Z}}_1, \widehat{\mathbf{Z}}_2)$. Bootstrap samples $\mathbf{X}^*$ under different null models are then obtained as follows.

1. Find $\widetilde{\mathbf{Z}} = (\widehat{\mathbf{Z}}_1, \widetilde{\mathbf{Z}}_2)'$ where
   - $\widetilde{\mathbf{Z}}_2 = (\tilde{z}_{21}, ..., \tilde{z}_{2n})'$ is a random sample of size n from $N_{p-k}(\mathbf{0}, \mathbf{I}_{p-k})$ (subgaussianity; ICA).
   - $\widetilde{\mathbf{Z}}_2 = (\mathbf{U}_1\hat{z}_{21}, ..., \mathbf{U}_n\hat{z}_{2n})'$ for n independent random orthogonal $(p-k) \times (p-k)$ matrices $\mathbf{U}_1, ..., \mathbf{U}_n$ (subsphericity; PCA)
   - $\widetilde{\mathbf{Z}}_2 = \mathbf{P}\hat{\mathbf{Z}}_2$ for a random $n \times n$ permutation matrix $\mathbf{P}$ (independence of subvectors; SIR)
2. Write $\mathbf{Z}^*$ for a bootstrap sample of size n from $\{\tilde{\mathbf{z}}_1, ..., \tilde{\mathbf{z}}_n\}$. (In the IC model the bootstrap samples are taken separately componentwise.)
3. Write $\mathbf{X}^* = \widehat{\mathbf{Z}}^*(\widehat{\mathbf{W}}')^{-1} + \mathbf{1}_n\hat{\mu}'$.

An estimated p-value for a bootstrap test with the test statistic $T(\mathbf{X})$ is then obtained as $M^{-1}\#\{T(\mathbf{X}_j^*) \geq T(\mathbf{X})\}$ where $\mathbf{X}_1^*, ..., \mathbf{X}_M^*$ are M independent bootstrap samples. A small simulation study is used to show that the proposed procedure performs in a reliable way. Estimation of k is also discussed briefly.

Keywords: bootstrapping, independent component analysis, principal component analysis, sliced inverse regression, semiparametric.

AMS: 62F40, 62G10, 62H99.

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Frequency Domain Based Tests of Stationarity

E. Paparoditis[1]

stathisp@ucy.ac.cy

[1]Department of Mathematics and Statistics, University of Cyprus

Parts joint with **Philip Preuss**

Statistical inference for stochastic processes with time-varying spectral characteristics has received considerable attention in recent decades.We review nonparametric, frequency domain based tests of stationarity against the alternative of a smoothly time-varying spectral structure. L^2-type tests are considered which are based on a comparison of a spectral density calculated locally on a moving window of data and a global spectral density estimator using the whole stretch of observations. Such a comparison is the basic idea of some other tests proposed in the literature too, using integrated deviations or Kolmogorov-Smirnov type criteria. Asymptotic properties of the test statistic considered under the null hypothesis of stationarity are discussed.

As an explorative tool, a simple and powerful procedure is proposed to validate the assumption of weak stationarity and to identify reasons for rejecting the null hypothesis of stationarity. The procedure evaluates the supremum over time of the L^2-distance between the local sample spectral density (local periodogram) calculated using a segment of observations falling within a rolling window and an estimator of the spectral density obtained using the entire time series at hand. Real data examples demonstrate the ability of the procedure to identify and detect (possible) changes in the autocovariance structure of a time series at hand and to understand their nature.

Power properties under fixed alternatives of a time-varying spectral structure are discussed and the behavior of the test for such type of alternatives is investigated. Finally, a framework for the asymptotic analysis of local power properties of tests of stationarity is developed. Appropriate sequences of locally stationary processes are defined that converge at a controlled rate to a limiting stationary process as the length of the time series increases. Different interesting classes of local alternatives to the null hypothesis of stationarity can be considered and the local power properties of recently proposed, frequency domain-based tests for stationarity are investigated. Some simulations illustrate our theoretical findings.

Keywords: Local Periodogram, Stationarity,Testing Hypotheses, Local Alternatives.

AMS: 62G09, 62M10.

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Asymptotic distribution-free tests for semiparametric regressions

J.C. Pardo-Fernández[1], J.C. Escanciano[2], Ingrid Van Keilegom[3]

[1] Departamento de Estatística e IO, Universidade de Vigo
[2] Department of Economics, Indiana University
[3] Institute de Statistique, Université catholique de Louvain

In this talk, we will present a new general methodology for constructing nonparametric asymptotic distribution-free tests for semiparametric hypotheses in regression models.

Let Y_t be a response variable and X_t a d-dimensional explanatory variable. Assume that the process (X_t, Y_t), $t = 0, 1, 2, \ldots$, is strictly stationary and ergodic, and that $E[Y_t^2] < \infty$. We assume Y_t is related to X_t through the heteroscedastic regression model $Y_t = \mu(X_t) + \sigma(X_t)\varepsilon_t$, where $\mu(x) = E(Y_t \mid X_t = x)$ and $\sigma^2(x) = Var(Y_t \mid X_t = x)$ are the conditional mean and the conditional variance of Y_t given $X_t = x$, respectively, and ε_t is an error term, which is assumed to be independent of X_t. We are interested in testing

$$H_0 : \mu \in \mathcal{M} \qquad \text{versus} \qquad H_0 : \mu \notin \mathcal{M},$$

where

$$\mathcal{M} = \left\{\mu : \mathbb{R}^d \to \mathbb{R} \text{ such that } \mu(x) = g(\theta, \eta(x), x),\ \theta \in \Theta \subset \mathbb{R}^r,\ \eta \in \mathcal{H}\right\}$$

is a class of parametric or semiparametric models defined in terms of a known function g, a finite-dimensional unknown parameter θ and a possibly infinite-dimensional unknown parameter η. This general formulation covers many testing problems in semiparametric regression. In particular, if $g(\theta, \eta(x), x) = g(\theta, x)$, then the problem reduces to testing for a parametric model for the regression function μ, which is a classical problem in statistics. Special cases of semiparametric null hypotheses include the partially linear model with $g(\theta, \eta(x), x) = \theta' x_1 + \eta(x_2)$, $x = (x_1, x_2)$, additive models with $g(\theta, \eta(x), x) = \eta_1(x_1) + \eta_2(x_2)$, $\eta = (\eta_1, \eta_2)$, and single-index models with $g(\theta, \eta(x), x) = \eta(\theta' x)$, among many others. We also cover constrained mean-variance models when $\eta(x) = \sigma(x)$. For example, the choice $g(\theta, \sigma(x), x) = \sigma(x) g_1(\theta, x)$ leads to tests for a particular parametric model given by $g_1(\theta, x)$ for the standardized first moment $\mu(x)/\sigma(x)$, which is an interesting problem in the financial literature.

Our tests are based on the comparison of two estimators of the standardized error distribution. Define the restricted model for the regression function as $\mu_0(x) = g(\theta_0, \eta_0(x), x)$, where

$$(\theta_0, \eta_0) = \arg \min_{\theta \in \Theta, \eta \in \mathcal{H}} E[(\mu(X_t) - g(\theta, \eta(X_t), X_t))^2].$$

Then, define the standardized errors

$$\varepsilon_{t0} = \frac{Y_t - \mu_0(X_t)}{\sigma(X_t)} \qquad \text{and} \qquad \varepsilon_t = \frac{Y_t - \mu(X_t)}{\sigma(X_t)},$$

with cumulative distribution functions $F_{\varepsilon 0}(y) = P(\varepsilon_{t0} \leq y)$ and $F_\varepsilon(y) = P(\varepsilon_t \leq y)$, respectively. Our testing procedure will be based on the integrated difference of the distribution functions of ε_{t0} and ε_t. The following theorem justifies the testing procedure.

Theorem. *The following statements are equivalent:*

(i) H_0 *is true.*

(ii) ε_{t0} *and* ε_t *have the same distribution.*

(iii) $D(y) = \int_{-\infty}^{y} (F_{\varepsilon 0}(s) - F_\varepsilon(s))ds = 0$ *for all* $y \in \mathbb{R}$.

The equivalence between statements (i) and (ii) has been extensively used in recent literature to construct tests for H_0. In this talk we will exploit the equivalence between (i) and (iii), which will allow us to obtain asymptotic distribution-free tests. We will explain the construction of the test statistics and the derivation of the corresponding asymptotic null distribution. Monte Carlo experiments will show a satisfactory finite sample performance for the proposed tests in several.

Keywords: asymptotics, distribution-free, goodness-of-fit, nonparametric regression.

AMS: 62G08, 62G10, 62G20.

Quickest Detection Problems for Bessel Processes

Goran Peskir[1], Peter Johnson[2]

[1]School of Mathematics, The University of Manchester
[2]School of Mathematics, The University of Manchester

Imagine the motion of a Brownian particle that initially takes place in a two-dimensional plane and then after some random/unobservable time continues in the three-dimensional space. Assuming that only the distance of the particle to the origin is being observed, the problem is to detect the time at which the particle departs from the plane as accurately as possible. I will present some recent results on this problem and discuss its solution.

Keywords: Quickest detection, Brownian motion, Bessel process, optimal stopping, parabolic partial differential equation, free-boundary problem, smooth fit, entrance boundary, nonlinear Fredholm integral equation of second kind, the change-of-variable formula with local time on curves/surfaces.

Time-varying NoVaS vs. GARCH: robustness against structural breaks in financial returns

Dimitris N. Politis[1]

[1]Department of Mathematics, University of California at San Diego

Consider financial returns data $Y_1, \ldots, Y_n$ satisfying $EY_t = 0$ for all t. The NoVaS transformation of Politis (2007) is defined as

$$W_{t,a} = \frac{Y_t}{\sqrt{\alpha s_{t-1}^2 + a_0 Y_t^2 + \sum_{i=1}^p a_i Y_{t-i}^2}} \quad \text{for} \quad t = p+1, p+2, \ldots, n \qquad (7)$$

where $s_{t-1}^2 = (t-1)^{-1} \sum_{k=1}^{t-1} Y_k^2$, and the coefficients $\alpha, a_0, a_1, \ldots, a_p$ (and the order p) are selected such that the series $\{W_{t,a} \text{ for } t = p+1, p+2, \ldots\}$ is (approximately) i.i.d. $N(0,1)$. The transformation can be inverted to yield

$$Y_t = \frac{W_{t,a}}{\sqrt{1 - a_0 W_{t,a}^2}} \sqrt{\alpha s_{t-1}^2 + \sum_{i=1}^p a_i Y_{t-i}^2} \quad \text{for} \quad t = p+1, \ldots, n. \qquad (8)$$

Eq. (8) is a model-like equation that can be used in lieu of a model for prediction. In fact, the NoVaS transformation is just an application of the Model-free Prediction Principle of Politis (2015) to financial returns data.

Financial returns are often assumed to be strictly stationary. Nevertheless, if the data $Y_1, \ldots, Y_n$ span a long time interval, e.g. daily financial returns spanning several years, it may be unrealistic to assume that the stochastic structure of time series $\{Y_t, t \in \mathbf{Z}\}$ has stayed invariant over such a long stretch of time. Instead, one can assume a slowly-changing stochastic structure, i.e., a locally stationary model. Indeed, the theory of time–varying ARCH (TV-ARCH) processes was developed to capture such a phenomenon; see Dahlhaus and Subba Rao (2006).

Let $g(\cdot)$ denote a function of interest; in order to predict $g(Y_{t+1})$ based on data $\{Y_s,\ s \le t\}$ via a TV-GARCH(1,1) model, we can simply fit a standard GARCH(1,1) model using as data the subseries $Y_{t-b+1}, \ldots, Y_t$. Here, the window size b should be large enough so that accurate estimation of the

GARCH parameters is possible based on the subseries $Y_{t-b+1}, \ldots, Y_t$ but small enough so that such a subseries can plausibly be considered stationary.

In a similar vein, we can predict $g(Y_{t+1})$ by fitting one of the NoVaS algorithms (Simple vs. Exponential, etc.) just using the 'windowed' data $Y_{t-b+1}, \ldots, Y_t$. In so doing, we are constructing a **time–varying NoVaS** (TV-NoVaS) transformation. In numerical work, Politis and Thomakos (2012) showed that NoVaS fitting can be done more efficiently than GARCH fitting by (numerical) MLE. Thus, it is expected that TV-NoVaS may be able to capture a changing stochastic structure in a more flexible manner.

We confirm this conjecture via simulation with data from a TV-GARCH model. Note, however, that an alternative form of nonstationarity may be due to the possible presence of structural breaks, i.e., change points. Mikosch and Starica (2004) show the interesting effects that an undetected change point may have on our interpretation and analysis of ARCH/GARCH modeling. Hence, in our simulation we also include a structural break model, and indeed witness the instability of TV-GARCH predictors. By contrast, TV-NoVaS predictors appear robust, adapting effortlessly to the new regime; see Politis (2015) for more details.

Keywords: Financial returns, local stationarity, prediction.

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Serial independence tests for time series

Bruno Rémillard[1], Kilani Ghoudi[2]

[1]Department of Decision Sciences, HEC Montréal
[2]Department of Statistics, United Arab Emirates University

In what follows, one considers stochastic volatility models given by

$$Y_i = \mu_i(\theta) + \sigma_i(\theta)\varepsilon_i, \quad i \geq 1,$$

where the innovations ε_i are i.i.d, with mean 0 and variance 1 random variables with common distribution F, and for any $i \geq 1$, μ_i, σ_i are $\mathcal{F}_{i-1}$ measurable functions depending on unknown parameters $\theta \in \mathbb{R}^m$, where $\mathcal{F}_{i-1}$ is the σ-algebra containing past information on the observations, and any past or current information about exogenous variables. A famous example of a volatility model is the well-known GARCH(1,1) model, where $\mu_i \equiv \mu$, and $\sigma_i^2 = \omega + \alpha(Y_{i-1} - \mu)^2 + \beta\sigma_{i-1}^2$, $i \geq 2$. Other popular models are the so-called AR-GARCH models, where $\mu_i = \mu + \sum_{j=1}^{P} \phi_j Y_{i-j}$.

Checking the adequacy of these times series models can involve testing for

- a given specification of the distribution of the innovations (e.g., F is Gaussian or Student);
- change-point in the distribution of the innovations;
- serial independence, i.e., the model captured the serial dependence in the observations.

The latter is the problem one wants to tackle. Instead of restricting the scope to tests based on autocorrelations and other measures of serial dependence, one takes a more general approach which consists in studying test statistics that can be written as functionals of the joint empirical distribution function of consecutive residuals. As a by-product of the convergence results, one also obtains the convergence of empirical processes based on consecutive squared residuals. Note in passing that the asymptotic behavior of the proposed Cramér-von Mises type randomness tests, as well as tests based on autocorrelations of residuals, absolute residuals and squared residuals, and their rank-based counterparts, can be easily derived from the asymptotic behavior of the empirical processes considered here.

In testing for randomness, it is often desired to treat the marginal distribution as a nuisance parameter and develop tests based on the associated empirical serial copula process. Its asymptotic behavior is also studied, using residuals or squared residuals. Even in the simplest case where the innovations would be observed, tests of randomness based on the empirical copula process are not very tractable and not as powerful as one should like. However, Ghoudi, Kulperger & Rémillard (2001) and Genest & Rémillard (2004) showed that easier to handle and more powerful tests of independence are obtained if one uses the so-called Möbius decomposition. Their asymptotic behavior is also obtained.

Unfortunately, the limiting distributions of the empirical processes considered here depend on unknown parameters. However, it is shown that a Monte Carlo method based on the so-called multipliers can be applied in order to estimate the P-values of the proposed test statistics.

The finite sample properties of the proposed tests are studied through numerical experiments, in order to demonstrate the effectiveness and power of the proposed tests.

Finally, the behavior of the empirical processes under contiguous alternatives is also studied, shedding some light on the results of the numerical experiments.

Keywords: Independence tests, GARCH models, empirical processes, multipliers.

AMS: 60F05, 62G09, 62G30.

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Detecting Change-points in Extremes

Ying Sun[1]

[1]King Abdullah University of Science and Technology (KAUST)

Even though most work on change-point estimation focuses on changes in the mean, changes in the variance or in the tail distribution can lead to more extreme events. In this paper, we develop a new method of detecting and estimating the change-points in the tail of multiple time series data. In addition, we adapt existing tail change-point detection methods to our specific problem and conduct a thorough comparison of different methods in terms of performance on the estimation of change-points and computational time. We also examine three locations on the U.S. northeast coast and demonstrate that the methods are useful for identifying changes in seasonally extreme warm temperatures. This research is based on joint work with Debbie Dupuis and Judy Wang.

Keywords: exceedances over threshold, extreme value theory, tail behavior, quantile methods, return level.

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Goodness-of-Fit Tests Utilising L-Moments

O. Thas[1,2], J.C.W. Rayner[2,3], J. De Neve[1]

[1]Department of Mathematical Modelling, Statistics and Bioinformatics, Ghent University

[2]National Institute for Applied Statistics Research Australia, University of Wollongong

[2]School of Mathematics and Physical Sciences, University of Newcastle

We are conserned with the one-sample goodness-of-fit problem in the presence of nuisance parameters. The null hypothesis is formulated as

$$H_0^{\mathrm{P}} : f \in \mathcal{F}_P = \{f(\cdot;\beta) : \beta \in B \subseteq \mathbb{R}^p\},$$

where $f(x)$ denotes the density function of X and β is a nuisance parameter.

Many goodness-of-fit tests have been shown to measure deviations between the moments of the hypothesised distribution and the sample moments (e.g. Anderson-Darling, Neyman smooth tests, ...). The dependence on moments may be explicit, but sometimes it is implicit and the test statistic is expressed as a functional of the empirical distribution function. See Thas (2010) for an overview of goodness-of-fit tests.

In this paper we propose goodness-of-fit tests based on L-moments. A unified treatment of L-moments was first presented by Hosking (1990). There are two important properties of L-moments that make them suitable for goodness of fit testing. First, all L-moments of a random variable X exist if and only if the mean of X exists and is finite. Second, a distribution whose mean exists is uniquely characterised by its L-moments (Hosking, 1990; Hosking, 2007). There have been previous uses of L-moments in goodness of fit procedures particularly in applications arising in hydrology and meteorology.

If F denotes the cumulative distribution function of a random variable X with support $\mathcal{S} \subseteq \mathbb{R}$, then the quantile function of X is defined as $Q(u) = \inf\{x \in \mathcal{S} : F(x) \geq u\}$. We limit the discussion here to continuous distributions. Let $X_1, \ldots, X_n$ denote n sample observations that are i.i.d. with cumulative distribution function F, and let $X_{i:n}$ denote the ith order statistic of the sample. The rth L-moment is defined as

$$\lambda_r = \frac{1}{r}\sum_{s=0}^{r-1}(-1)^s\binom{r-1}{s}\mathrm{E}\{X_{r-s:r}\} = (2r-1)^{-1/2}\int_0^1 Q(u)h_{r-1}(u)\mathrm{d}u,$$

in which $h_r(u)$ represents the rth order normalised shifted Legendre polynomial. Sample L-moments are denoted by $\hat{\lambda}_r$.

Our test statistics are based on the statistics $\hat{\theta}_r = \hat{\lambda}_r - \lambda_{0r}(\hat{\beta})$ ($r = 1, 2, \ldots, k > p$), where $\lambda_{0r}(\hat{\beta})$ is the rth L-moment of the hypothesised distribution with β replaced by its Method of L-Moments Estimator (MLME) $\hat{\beta}$. The statistic $\hat{\theta}_r$ measures the deviation between the rth empirical L-moment and the rth L-moment of $f(.;\hat{\beta})$. Because of the weak existence property of L-moments and the expression of L-moments in terms of the quantile function, L-moments goodness-of-fit tests are particularly useful for assessing the fit of distributions that have explicit expressions for their quantile function and that have existence issues with their classical moments. The Generalised Pareto Distribution (GPD) is one such example.

The joint asymptotic null distribution of $\sqrt{n}\hat{\theta}_r$ ($r = 1, \ldots, p$) is obtained, and quadratic forms of $\hat{\theta}_r$ are suggested as test statistics. In this paper we demonstrate the construction of the test for the GPD and for the logistic distribution. We empirically evaluate the new tests in a simulation study. Finally, we show how the new test statistics are related to the Wasserstein distance and to QQ-plots.

Keywords: generalised pareto distribution, logistic distribution, quantile function.

AMS: 62G10, 62G20, 62G30.

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Wilks' Phenomenon in Two-Step Semiparametric Empirical Likelihood Inference

I. Van Keilegom[1], F. Bravo[2], J.C. Escanciano[3]

[1]Institute of Statistics, Université catholique de Louvain
[2]Department of Economics, University of York
[3]Department of Economics, Indiana University

In both parametric and certain nonparametric statistical models, the empirical likelihood ratio satisfies a nonparametric version of Wilks' theorem. For many semiparametric models, however, the commonly used two-step (plug-in) empirical likelihood ratio (see Hjort, McKeague and Van Keilegom, 2009) is not asymptotically distribution free, that is, Wilks' phenomenon breaks down. In this paper we suggest a general approach to restore Wilks' phenomenon in two-step semiparametric empirical likelihood inferences. The main insight consists in using as the moment function in the estimating equation the influence function of the plug-in sample moment. The proposed method is general, and leads to distribution-free inference, namely to a χ^2-limit. Hence, Wilks' phenomenon is valid. Moreover, the proposed procedure is less sensitive to the first step estimator than alternative bootstrap methods.

Several examples are studied in more detail, and the high level conditions under which the general theory is valid, are verified for these examples. They include empirical likelihood based inference for : [1] the mean of interval censored data; [2] the average treatment effect in observational studies; [3] estimating equations with missing data; and [4] censored quantile regression. A simulation study illustrates the generality of the procedure and demonstrates its good finite sample performance compared to competing procedures based on asymptotic normality, bootstrap or two-step (plug-in) empirical likelihood.

In a second step the proposed procedure can be generalized to empirical likelihood based goodness-of-fit tests for semiparametric models that are distribution free.

Keywords: Asymptotic theory; Empirical likelihood; Semiparametric inference; Semiparametric Regression; Stochastic equicontinuity; Wilks' phenomenon.

AMS: 62M10, 62G10.

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Dimension-reduction model-adaptive test for parametric single-index models: A dimension-reduction model-adaptive approach

Xu Guo[1], Tao Wang[2] and **Lixing Zhu**[3]

[1]College of Economics and Management, Nanjing University of Aeronautics and Astronautics
[2] Department of Biostatistics, Yale University
[3] Department of Mathematics, Hong Kong Baptist University

Local smoothing testing based on multivariate nonparametric regression estimation is one of the main model checking methodologies in the literature. However, the relevant tests suffer from typical curse of dimensionality, resulting in slow convergence rates to their limits under the null hypothesis and less deviation from the null hypothesis under alternative hypotheses. This problem prevents tests from maintaining the significance level well and makes tests less sensitive to alternative hypotheses. In this paper, a model-adaption concept in lack-of-fit testing is introduced and a dimension-reduction model-adaptive test procedure is proposed for parametric single-index models. The test behaves like a local smoothing test, as if the model were univariate. It is consistent against any global alternative hypothesis and can detect local alternative hypotheses distinct from the null hypothesis at a fast rate that existing local smoothing tests can achieve only when the model is univariate. Simulations are conducted to examine the performance of our methodology. An analysis of real data is shown for illustration. The method can be readily extended to global smoothing methodology and other testing problems.

Keywords: Dimension reduction; parametric single index models; model-adaption; model checking.

AMS: 62J02, 62G10.

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On a Vasicek's type Goodness-of-Fit test for beta generated distributions and the distribution of order statistics

K. Zografos[1], A. Chaji[2]

kzograf@uoi.gr, chaji85ms@yahoo.com

[1]Department of Mathematics, University of Ioannina, Greece
[2]Shohadaie Hovaizeh University of Technology, Iran

Shannon entropy and its by products is an omnipresent quantity with applications in almost every branch of science and engineering. The maximum entropy principle (MEP) is a general method of inference, it is developed on the basis of Shannon entropy and it provides with a tool to approximate the unknown probabilistic model which is in harmony with some prior information about it. It is successfully applied in statistics and econometrics, among many other fields. Shannon entropy and the MEP have been used to develop goodness of fit tests, among many other applications. The pioneer work by Vasicek (1976) was the starting point for the development of goodness-of-fit tests by means of Shannon entropy. After that, several papers have constructed tests based on Vasicek's entropy estimate while the method is well elaborated in the works by Soofi et al. (1995), Park and Park (2003) and Goria et al. (2005). Based on the above papers and generally speaking, a test statistic of goodness-of-fit could be based on a function, say the difference, between an estimate of the Shannon entropy and the respective parametric estimate of the same index, subject to the null hypothesis.

On the other hand, the last decade is characterized by the definition and the study of broad families of distributions which compose properties of two or more probability distributions and provide with great flexibility in modeling real data. An example of such a type of generalized families of univariate distributions is the beta-generated distributions (BGD) introduced by Eugene et al. (2002) and Jones (2004). Beta-generated distributions extend the distribution of order statistics and they have been further discussed and studied in the paper by Zografos and Balakrishnan (2009), among other.

The aim of this talk is to introduce, at first, a Vasicek's type estimator of Shannon entropy of the beta-generated distributions and the distribution

of order statistics. The proposed estimator is adjusted to the class of beta-generated distributions and it is a consistent estimator of Shannon entropy of this generalized family of univariate distributions. A short simulation study is also performed to examine the small sample behaviour of the proposed estimator and to compare how it implements in comparison with the classic Vasicek estimator of Shannon entropy of the beta-generated distributions. Moreover, a goodness of fit test is developed inside the broad family of the beta-generated distributions. The introduced test procedure is based on the proposed Vasicek's type estimator of Shannon entropy of the beta-generated distributions. A simulation study is performed in order to compare the proposed procedure with competitive tests.

Keywords: Shannon entropy, Vasicek's estimator, Goodness-of-Fit test; Beta-generated distribution, Order statistics.

AMS: 62B10.

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