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A Wealth and Status-Based Model of Residential Segregation

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We extend the classic “Schelling model” (1971, 1978) to incorporate the wealth and status of agents and the desirability and affordability of residences. We analyze the effects of 1) the degree of the status-wealth correlation, and 2) the extent to which the wealth of residents shapes the affordability of residences, on levels of status and wealth segregation. Both factors generally exert a positive effect on both forms of segregation and interact to produce higher levels of segregation. The greater the correlation between status and wealth, the more the agents tend to segregate, either due to choice (for the wealthy and high status) or exclusion (for the poor and low status). We also find that housing price endogeneity is a precondition for status segregation.

Keywords: agent-based models, residential choice, Schelling, segregation, wealth, status

INTRODUCTION

The neighborhoods in which people live affect their quality of life, future life chances, social networks, occupations, and how others judge them (e.g., Wilson, 1996; Bourgois, 2003). Despite a decline in popular consciousness of racial segregation, whites and blacks in the United States remain highly segregated from one another (Massey and Denton, 1993). The segregation of Hispanics and Asian Americans

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from whites is increasing (Charles, 2003). A phenomenon made complex by its myriad causes and effects, residential segregation plays a significant role in processes of continuing socioeconomic inequality (Massey, Gross, and Eggers, 1991).

Developing theory about the causal mechanisms underlying neighborhood self-organization accordingly promises to illuminate a range of stratification outcomes. Explaining the processes underlying residential choice is challenging, because such patterns cannot be practically studied experimentally and likely result from multiple causes (Fossett, 2003). Agent-based models can help develop theories of neighborhood emergence, stability, and change by allowing researchers to perform virtual experiments showing the kinds of residential patterns likely to emerge from a given set of initial conditions and behavioral rules.

One of the earliest and best-known models of residential segregation is that proposed by Thomas Schelling (1971, 1978). In the "Schelling model," agents inhabit a checkerboard-like 8×8 lattice. The agents are divided into two classes that are meant to represent any binary social division that could affect the distribution of agents in space (e.g., blacks and whites, Catholics and Protestants, surfers and swimmers). Agents are initially distributed randomly across the lattice. From that point on, agents observe their Moore neighborhood (the 8-cell combination of the 4 adjacent cells and the 4 diagonal cells), and change locations if the number of agents of the other type exceeds some threshold. Schelling showed that populations could reach high levels of segregation even when agents are willing to remain in neighborhoods in which up to two-thirds of their neighbors are members of the other group.

Since its introduction, researchers have extended the standard Schelling model to investigate a variety of questions. For example, modeling the role of "vision" in the dynamics of the Schelling model has shown that the further agents can see in their environment, the larger the resulting segregated neighborhoods (Laurie and Jaggi, 2003). Pancs and Vriend (2003) showed that segregation could emerge from best-response dynamics even when agents have a strict preference for integrated neighborhoods. Irwin, Jayaprakash, and Warren (2004) examined the segregation patterns that emerge when agents consider other factors alongside race when making residential choices, including neighborhood density and the quality of public and private services. They found that racial segregation can emerge from preferences not specifically concerning race, such as when whites (but not African Americans) prefer low-density housing. Zhang (2004) revisited Schelling's results using a stochastic evolutionary game theoretic model and showed that segregation can emerge and remain stable even when agents prefer living in neighborhoods that are

approximately 50% white and 50% black. Bruch and Mare (2004) examined alternative specifications of Schelling's choice function and found that racial segregation markedly decreases when agents use a probabilistic choice function rather than the deterministic choice function specified by Schelling.

Bruch (2006) developed a semi-realistic version of the Schelling model, basing the grid on Los Angeles census tracts, and the agents' choice functions on the results of housing preference data from the Los Angeles Family and Neighborhood Survey. She generally found that incorporating race into agents' residential choices increased the level of racial segregation, and incorporating economic factors into agents' residential choices tended to diminish racial segregation. Mark Fossett (2003) developed a highly detailed model of residential choice that included many features of urban landscapes and multiple determinants of residential choice. He showed, among other findings, that ethnic preferences can lead to the stability of ethnic segregation even in the absence of housing discrimination. While a complete overview of research using agent-based models to study residential segregation could easily fill one or more separate articles, this sampling does give a sense of some of the most recent and innovative work in the area.

In Schelling's model, and several subsequent models, agents choose neighborhoods based on their desirability, and all neighborhoods are equally accessible. For example, in Schelling's model, if a white agent has eight black neighbors, and would prefer to move to a neighborhood with four black neighbors and four white neighbors, the only constraint placed upon this agent is that such a neighborhood exists and is available. In other words, agents can automatically satisfy their preferences, assuming a neighborhood that meets those preferences is open on the board. This contrasts sharply with observed real-world patterns, in which the most desirable neighborhoods are often the least accessible and agents' incomes limit their residential aspirations. In the model presented here, we relax the assumption of perfect accessibility and examine the implications for patterns of residential choice.

To do so, we focus our analysis on status and wealth, two theoretical properties of agents that shape the desirability and accessibility of neighborhoods, respectively. We do not examine the effects of race in this model, as race has already been extensively examined in a number of models (e.g., Bruch, 2006; Bruch and Mare, 2004; Fossett, 2003, Irwin, Jayaprakash and Warren, 2004; Laurie and Jaggi, 2003; Macy and van de Rijt, forthcoming; Pans and Vriend, 2003; Zhang, 2003; and Zhang, 2004, to name a few recent examples). Substantially less

attention has been paid to segregation based on the relative wealth of individuals and the desirability and accessibility of neighborhoods, i.e., gentrification dynamics (Glass, 1964).¹ Despite this inattention, the push and pull of wealth and status drive much of what is interesting and socially significant about residential segregation.

While residential choice processes are based in complex cultural understandings of what factors make a neighborhood attractive, we believe that much of the dynamics can be captured in terms of wealth and status. These two variables shape the relative desirability of neighborhoods and the relative ability of individuals to access them. In the model that follows, we make an effort to extend the formal modeling of residential segregation to incorporate these key bases of stratification and residential choice.

For the purposes of our model, we operationally define status as the extent to which other agents view an agent as a desirable neighbor. Examples of high-status agents include those that are popular, famous, or respected in their community. We assume that agents perceive communities populated by high-status agents as more desirable places to live than communities populated by low-status agents. As we will explain in more detail in the next section, we “operationally define” the desirability of a cell’s neighborhood as the mean status of the cell’s neighbors.

While agents in our model seek desirable neighborhoods, they are constrained in their abilities to do so by their level of wealth. We define wealth as an agent’s level of resources. In the model, wealth has two important consequences for determining the accessibility of a neighborhood:

1. Wealth determines the range of cells to which an agent can afford to move.
2. Under some conditions of the model, an agent’s wealth affects the housing price of adjacent cells, so that cells surrounded by wealthy agents have higher housing prices than cells surrounded by poor agents.

Our model has antecedents in prior models, notably those introduced by Mark Fossett (2003) and Elizabeth Bruch (2006). Fossett’s “Simseg” model is a highly detailed agent-based model with many parameters; here we describe only the features relevant to our model. In the Simseg model, cells are assigned a level of “quality” (on a 1–99 scale) that indicates the price of a housing unit in that location. The

¹But see Torrens and Nara (2005) for an exception.

quality of housing units is randomly distributed across the lattice. Simseg assigns agents a level of status (also on a 1–99 scale). In Simseg, status indicates socioeconomic status and is an indicator of both prestige and resources. It determines both an agent's contribution to the desirability of a neighborhood and an agent's ability to pay for housing (Fossett, 2005:38–39). Fossett found little segregation based on status under these conditions.

While our model is much simpler than Fossett's, we extend his analysis in several ways. Most importantly, we disaggregate his status variable into status and wealth. Rather than assuming that high-status agents automatically possess higher levels of resources, we conceptualize these variables as separate and systematically explore the effects of different degrees of status and wealth correlation. In another extension, we examine the effect of allowing the wealth of a cell's neighbors to housing prices. In Fossett's model, housing quality (price) is exogenously determined at initialization, meaning that the housing price assigned to each cell remains constant. We relax this assumption and examine the effect of determining housing prices endogenously.

Bruch (2006) also presents a model that shares some features with ours. As in our model, agents in her model possess some level of wealth, and the price of housing is determined by the average wealth of that housing unit's neighbors. However, Bruch assumes that the price and desirability of a housing unit are identical (expensive housing units are also the most desirable, assuming agents can afford them). This is similar to Fossett's assumption that a single variable underlies the desirability and accessibility of residences, although Bruch assumes that this is true at the level of housing units, while Fossett assumes that it is true at the level of agents. In contrast, we allow housing prices and desirability to vary independently. In addition, Bruch was interested in the effects of wealth on racial segregation and did not examine wealth- or status-based segregation. Residential segregation based on status and wealth receives substantially less attention in the literature and is the focus of this article.

Our model makes several contributions to the literature on residential choice, including:

- Treating the accessibility and desirability of housing as analytically distinct concepts and investigating varying degrees of association between them,
- Explicitly comparing the effects of exogenous and endogenous determination of housing prices within a single model, and
- Examining the effects of different decision rules on patterns of segregation.

MODEL

Design

The model is a torus-shaped cellular automata on a 31×31 lattice. In the results reported here, the grid is populated with 800 agents, giving a vacancy rate of approximately 17%.

Agents possess two attributes, status and wealth. Status indicates the extent to which other agents prefer to live near the focal agent. We operationally define this assumption by setting the desirability of a neighborhood to the mean status of a cell's neighbors. In other words, cells become more desirable to agents as they acquire more high-status neighbors and less desirable as they acquire more low-status neighbors. Status is a continuous, normally distributed variable with a mean of 0 and a standard deviation of 1. Thus, a cell with no neighbors has an average status of neighbors of zero. The desirability of this cell as a residence will increase if it acquires a neighbor with positive status and decrease if it acquires a neighbor with negative status. Similarly, if an agent with a status of 0.1 moves next to a cell that already has one neighbor with status of 0.7, the desirability of that neighborhood will decrease from 0.7 (the status of the sole original neighbor) to 0.4 (the average status of both neighbors). This approach to status is consistent with the assumption, widespread in sociological social psychology, that status is relative and zero-sum (e.g., Berger et al., 1977; Ridgeway and Walker, 1995).²

Wealth indicates an agent's level of resources and determines an agent's ability to relocate according to their preferences. Like status, wealth is normally distributed with a mean of 0 and a standard deviation of 1.³ As explained below, wealthy agents enjoy greater latitude in choosing their places of residence than poor agents. In some conditions, an agent's wealth also affects the cost of cells nearby.

We manipulate two key variables: the extent to which wealth and status are correlated and the extent to which housing prices are endogenous. The status-wealth correlation parameter allows us to examine the consequences of different assumptions about the association between wealth and status. Status and wealth, while analytically distinct concepts (Willer et al., 1997), are often correlated in practice (Veblen, 1953 [1899]; Berger et al., 1985; Stewart and Moore, 1992). For example, wealthy neighborhoods are often also higher in status than poor neighborhoods, and wealthy people are often seen as more desirable neighbors than poor people.

²The model was written in NetLogo 3.1 (Wilensky, 1999).

³We also explored the effects of implementing an exponential wealth distribution and found nearly identical results.

At the same time, status and wealth are not always correlated. Wealth does not assure an individual of status, as in the case of wealthy, but often disrespected, “nouveau riche” and “yuppies.” Likewise, poverty does not always guarantee low status, as in the case of poor, but respected, artists. This is illustrated by gentrification dynamics, in which wealthier individuals move into poor urban neighborhoods that have acquired cachet due to a fashionable bohemian or artistic community. Furthermore, the extent to which status and wealth are correlated likely varies substantially across time and space. By manipulating the degree of correlation between status and wealth, we can systematically determine the conditions under which different levels of segregation may occur. The relationship between status and wealth is described by equation 1:

$$S = X(1 - C) + W(C) \quad (1)$$

In equation (1), S is status, X is a random variable distributed $N(0, 1)$, C is the status-wealth correlation parameter, and W is an agent’s wealth. The status-wealth correlation parameter varies between 0 and 1. When $C = 0$, status and wealth are independent. As C increases, wealth increasingly determines status. When $C = 1$, wealth perfectly determines status.

The second key variable is price endogeneity, or the extent to which the price of housing is determined by the wealth of an agent’s neighbors. The cost of a residence may be determined by exogenous factors, such as the quality of existing housing stock or proximity to valued natural features such as mountains or bodies of water. Alternatively, the cost of a residence may be determined by endogenous factors, such as the wealth of nearby agents. Wealthy agents may increase nearby property values by investing in the upkeep and improvement of their own residences and attracting providers of expensive goods and services to the area.

Earlier models have assumed that housing prices are exogenous (Fossett, 2003) and that housing prices are endogenous (Bruch, 2006), but no model directly compares the effects of endogenous and exogenous housing prices within the same design. In the model presented here, we allow the extent to which housing prices are endogenous to vary as show in equation (2), which is calculated separately for each square on the grid.

$$H_i = E \frac{1}{n_i} \sum W_{ij} + R_i(1 - E) \quad (2)$$

In equation (2), H indicates the price of cell i , E is price endogeneity and falls in the range $[0,1]$, n is the number of neighbors for cell i ,

W_{ij} is the wealth of each agent j that is a Moore neighbor to agent i , and R_i is an exogenous housing price assigned to cell i . R_i is determined by randomly assigning a value to each cell, using the same distribution used for wealth.

The model then smoothes price differences across cells by requiring each cell to diffuse a total of 25% of its value to each of the neighboring cells, with the remainder staying on the focal cell. This step prevents housing prices from changing drastically across adjacent cells. When housing prices are perfectly endogenous ($E = 1$), the price of a cell is determined by the mean wealth of the cell's neighbors. When housing prices are perfectly exogenous, the price of a cell is determined by R_i . At intermediate values of E , housing prices are an average of the average wealth of a cell's neighborhood and the exogenously determined price, weighted by E .

In searching for a residence, agents attempt to find the most desirable residence they can afford. The decision rule is as follows: On each round, agents sequentially survey all open spaces on the board and observe the average status of each cell's neighbors. They then move to the most desirable open cell (i.e., the cell with the highest average status neighbors) for which the housing price H is equal or less than the agent's wealth W . Following each round, the desirability and housing price of each cell is recalculated to reflect agents' moves, and the process repeats. After our initial analyses, we will also explore the effects of alternative decision rules.

Measures

The history of segregation measurement research is characterized by diverse proposals for measuring segregation and substantial debate over the relative merits of each proposal (e.g., Duncan and Duncan, 1955; Massey and Denton, 1988; Fossett, 2005). In a recent systematic analysis of this debate, Massey and Denton (1988) review 20 segregation indices and compare their performance when applied to 1980 census data from 60 metropolitan statistical areas. Using factor analysis, Massey and Denton argue that each index of segregation measures one of five underlying dimensions: uneven distribution, isolation and exposure, clustering, centralization, and concentration.⁴

While Massey and Denton's analysis has proven influential in empirical studies of segregation, their measures are not widely adopted by researchers using agent-based models of segregation processes. Instead, modelers tend to introduce measures particular to their own

⁴Subsequent analyses using the 1990 and 2000 censuses show that this conceptual scheme remains valid (Iceland, Weinberg, and Steinmetz 2002).

model (Fossett, 2005). Fossett critiques the failure of agent-based models of segregation to use standard segregation measures. He points out that the use of idiosyncratic measures in simulation studies risks discovering substantive “findings” that are simply artifacts of the measure used, rather than general outcomes from the model. This practice also substantially limits efforts to compare results across models, as well as limiting efforts to compare the results of segregation models with the results of segregation studies using empirical data (Fossett, 2005:1-3). Fossett’s recommendations for segregation indices largely dovetail with the results of Massey and Denton’s analysis.

Massey and Denton’s analysis shows that the uneven distribution and isolation and exposure measures account for most of the variance in segregation. We report results for each type of index. Uneven distribution indices measure the extent to which the distribution of households by some trait in local neighborhoods corresponds to the distribution of households in the population. For example, in a city that is 90% white and 10% African American, a perfectly even distribution of households (zero segregation) under this definition requires that each neighborhood consist of 90% white households and 10% African American households. Departures from this distribution increase the proportion of neighborhoods that are disproportionately white or disproportionately African American, and thus increase the level of segregation in the city.

The Index of Dissimilarity

To measure uneven distribution, Massey and Denton recommend using the index of dissimilarity, commonly referred to as D . D “measures departure from evenness by taking the weighted mean absolute deviation of every unit’s minority proportion, and expressing this quantity as a proportion of its theoretical maximum” (Massey and Denton, 1988:284). The most common substantive interpretation of D is as a measure of the proportion of minority households that would need to relocate in order to achieve an exact even distribution (Massey and Denton, 1988:284). D has a straightforward substantive interpretation and a long history of usage. Massey and Denton and Fossett recommend using D over other measures of uneven distribution to enable comparison with prior research.⁵

⁵ D has also been the subject of critique, with some researchers recommending alternative measures, particularly the variance ratio V (Coleman, Hoffer, and Kilgore, 1982). We also calculated V for all results presented here. We find that V and D are correlated with $\alpha > .99$, and produce virtually identical substantive patterns. We therefore only report results for D .

Calculating D requires the presence of discrete groups in the population. Agents in our model are differentiated by two continuous traits, wealth and status. In order to calculate D for our model, we must create discrete heuristic categories from these continuous traits. We divide the population into high-wealth and low-wealth agents, and high-status and low-status agents, using a median split. We can then compute both wealth-based and status-based D using Fossett's adaptation, which shifts the unit of analysis from census tracts to the Moore neighborhoods used in agent-based models:

$$D = 100 \bullet \sum |q_k - Q| / (2TPQ) \quad (3)$$

In equation (3), q_k refers to the proportion of low-wealth (status) agents in neighborhood k , Q refers to the proportion of low-wealth (status) agents in the entire population, P refers to the proportion of high-wealth (status) agents in the population, and T is the total population size. D varies from 0 to 100, with 0 indicating perfect integration and 100 indicating complete segregation (i.e., 100% of low wealth (status) agents would need to move to achieve perfect integration).

While D is the generally recommended measure of uneven distribution, there is one important technical problem with the index that we must address. Fossett (2005) points out that randomly assigning agents to a grid produces non-zero levels of segregation, because random assignment can produce clusters of locally segregated neighborhoods by chance. This means that measures such as D may register positive levels of segregation, even in the absence of systematic forces producing segregation. For example, under random assignment, it is highly improbable (and in some cases impossible) that the distribution of groups in a neighborhood will perfectly mirror those in the population. In the case of wealth, some neighborhoods will randomly tilt toward a greater proportion of wealthy agents and others towards a greater proportion of poor agents. Each instance like this increases D , because it is registered as a deviation from the population proportions (Fossett, 2005: 28–32).

Fossett shows that levels of D under random assignment can be extremely high, more than 70%, under some conditions. Thus, it is essential to account for the expected levels of segregation under the conditions of the model, or risk inflating estimates of segregation. This is particularly problematic if expected levels of segregation vary with conditions manipulated in the model, as they may lead to spurious inferences about the causal role of those conditions for producing segregation.

To address this problem, we normalize D using the expected value of D under conditions of random assignment (Fossett, 2005: 30–36).

This new measure, D^* , indicates the proportion of low-wealth (status) agents who would have to move in order for D to reach the level of segregation expected under random assignment. To do so, we first calculate Winship's (1977) measure of expected value of D under random assignment, as given in Fossett (2005:32).

$$E[D] = (1/2NPQ) \bullet \sum \beta(i, N, Q) \bullet |q_i - Q| \quad (4)$$

In equation (4), N represents the number of agents in a neighborhood, P represents the population proportion of high-wealth (status) agents, Q represents the population proportion of low-wealth (status) agents, i is a 0 to N index, $\beta(i, N, Q)$ is the binomial probability that i agents in a neighborhood of N agents with proportion Q low-wealth (status) agents are low wealth (status), and q_i is the proportion of low-wealth (status) agents in a given neighborhood. $E[D]$ weights each possible neighborhood configuration's contribution to the index of dissimilarity by the likelihood that it will occur. Once we have calculated $E[D]$, we can calculate D^* , as in Winship (1977).

$$D^* = 100 \bullet (D - E[D]) / (100 - E[D]) \quad (5)$$

By using D^* , we can be sure that the segregation levels we present are the results of systematic forces producing segregation, rather than an artifact of random assignment. We present D^* for all the results of the model shown in the following sections.

The Revised Index of Isolation

While the index of dissimilarity measures the uneven distribution of groups of agents across the grid, indices of exposure and isolation measure levels of potential in-group and out-group contact. Both Massey and Denton (1988) and Fossett (2005) recommend using the revised index of isolation R^* . R^* , along with its parent measure, the index of isolation, was originally presented by Bell (1954). The index of isolation calculates an agent's levels of co-residence with agents of the same group. The measure estimates the minimum level of same-group interaction households experience within their neighborhoods. Highly isolated households live near and interact with members of their own group only.

To calculate the revised index of isolation, we first subdivide our agents into four categories, based on whether their wealth and status is above or below the median. This creates four heuristic classes: high-wealth/high-status agents, high-wealth/low-status agents, low-wealth/high-status agents, and low-wealth/low-status agents. For each group,

we can then calculate the original index of isolation using Fossett's (2005) adaptation of Bell's (1954) formula to accommodate the site-centered neighborhoods characteristic of agent-based models of segregation.

$${}_A P_A = \sum p_i / A \quad (6)$$

In equation (6), ${}_A P_A$ is the index of isolation for some group A (one of the four wealth/status groupings), p_i refers to the proportion of neighbors of group A in the Moore neighborhood of agent i , and A refers to the number of agents of type A in the population. ${}_A P_A$ is the average proportion of same group neighbors for each agent in the population. ${}_A P_A$ is calculated separately for each of the four groups. One drawback of ${}_A P_A$ is that it does not take into account the relative numbers of agents of each type. For example, if wealthy agents constitute 90% of the population, we expect that, on average, 90% of a wealthy agent's neighbors will also be wealthy. This matters for the model developed in this paper because as the status-wealth correlation parameter increases, there will be more high wealth/high status and low wealth/low status agents in the population, and fewer high wealth/low status and low wealth/high status agents in the population. Therefore, it is important to control for relative group size to avoid spurious effects of the status-wealth correlation measure on levels of segregation. The revised index of isolation can be calculated as shown below, again drawing on Bell (1954) and Fossett (2005):

$$R_A = 100 \bullet ({}_A P_A - P) / (1 - P) \quad (7)$$

R_A adjusts ${}_A P_A$ so that it measures isolation occurring beyond what would be expected by chance given the distribution of groups in the population. While R_A is sometimes reported as equivalent to the variance ratio, V , Fossett points out that this does not hold for agent-based models that used site-centered (e.g., Moore) neighborhoods, and that V and R_A should thus be treated separately in these models (Fossett, 2005: 19). We now turn to using these measures to analyze the results of our computational experiments.

RESULTS

In the first computational experiment, we investigate the effects of status-wealth correlation and price endogeneity on levels of segregation based on wealth and status. This experiment crosses the status-wealth correlation (C ranges from 0 to 0.9 in increments of 0.3) with price endogeneity set to low, moderate and high levels

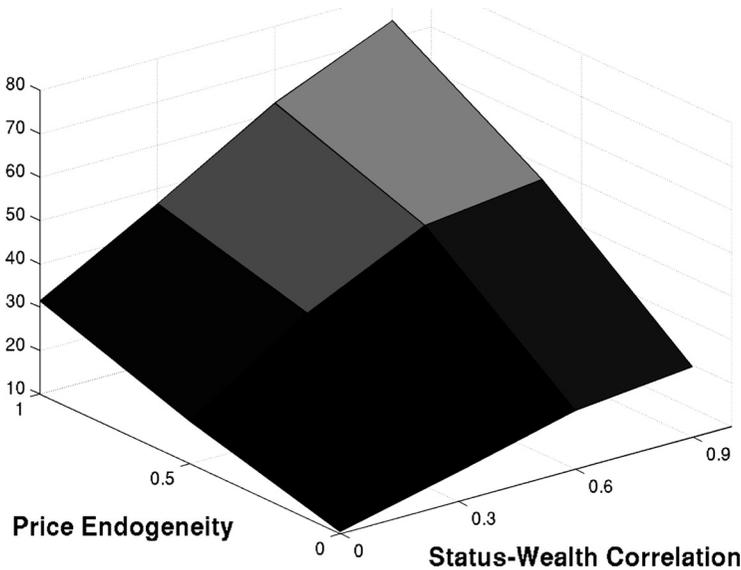


FIGURE 1 Index of dissimilarity for wealth, as a function of the status-wealth correlation and price endogeneity (Wealth distribution = normal, $N = 800$, 500 rounds, 15 repetitions, results average of last 50 rounds).

($E = 0, 0.5$, and 1). The model runs for 500 rounds, and we analyze the results from rounds 450 to 500.⁶ For robustness, the experiment is replicated 15 times and the results are averaged. Figure 1 shows the normed index of dissimilarity D^* for wealth as a function of status-wealth correlation and price endogeneity.

Wealth Segregation

As Figure 1 reveals, agents become progressively more segregated by wealth as both the status-wealth correlation increases and as price endogeneity increases. In the lower left corner of the surface, at $C = 0$ and $E = 0$, segregation by wealth is minimized, reaching a level of roughly 10.5. This means that only 10.5% of low-wealth agents would need to relocate in order for the population to have the level of wealth segregation expected under random assignment. Following the x-axis from this point (status wealth correlation increases as price endogeneity remains constant at zero), we see an absolute increase in the level of wealth segregation of over 15 percentage points, to

⁶We also examined the results using a larger range of rounds and found virtually identical results, suggested the model had converged to a stable point by 500 rounds.

approximately 26.3. Agents are substantially more segregated by wealth when wealth and status are correlated than when they are not.

The positive effect of the status-wealth correlation on wealth segregation derives from the fact that as status and wealth become more correlated, the world becomes increasingly populated by two kinds of agents: those who are high in wealth and status and those who are low in both. Agents who are high in both wealth and status prefer to live near one another and possess the resources necessary to live wherever they choose. They are drawn to live near one another by virtue of their high status and are capable of living in most cells on the board due to their high wealth.

While low-wealth, low-status agents would prefer to live near high-status agents, they are limited in their ability to do so because they must find cells on the lattice that have been assigned prices they can afford. Furthermore, because of their low status, these agents are undesirable neighbors, and thus other agents are motivated to avoid living near them. Despite low-status agents preferring not to live near one another, they are forced to do so when wealth and status are highly correlated, because these agents lack better options. This illustrates that homogenous networks can emerge through processes of both choice (as in the case of high-status, high-wealth agents) and exclusion (as in the case of low-wealth, low-status agents), as discussed by Heckathorn (2002).

Agents also become increasingly segregated by wealth as housing prices become endogenous. If we look at the point on the graph at which D^* is minimized ($C = 0, E = 0$), and then follow the y-axis (price endogeneity increases as the status-wealth correlation remains constant at zero) we see an increase in D^* , from 10.5 when housing prices are exogenous to 31.5 when housing prices are endogenous. Price endogeneity increases wealth segregation because low-wealth agents can now be forced to relocate when wealthier agents move nearby and drive up housing prices. When housing prices are exogenous, low-wealth agents need only find a cell on the lattice that has been assigned a housing price less than or equal to their wealth. In an exogenous housing price world, the wealth of a cell's neighbors do not affect the price of that cell, and so low-wealth agents can live near wealthier agents without incurring greater costs. As housing prices are increasingly determined endogenously, low-wealth agents are increasingly ghettoized in the search for affordable residences.

Agents experience the greatest levels of wealth segregation when wealth and status are highly correlated and housing prices are endogenous. As Figure 1 shows, these two variables interact to produce levels of D^* that reach as high as 74, indicating that nearly three

quarters of low-wealth agents would need to relocate in order for the level of segregation to reach what is expected under random assignment.⁷ Endogenizing housing prices amplifies the effect of the wealth status correlation. As discussed earlier, correlating wealth and status increasingly creates two classes of agents, those high in wealth and status and those low in wealth and status. The high-wealth and high-status agents prefer to live together due to their status and are able to live together due to their wealth. When housing prices become endogenous, high-wealth/high-status agents become even more isolated because, by residing in clusters, they increase the price of nearby housing, effectively pricing out less affluent agents. Their high wealth enables them to afford the high housing prices that they create when they live near one another, but other agents are forced to relocate. Low-wealth, low-status agents cannot move to the high-wealth, high-status neighborhoods because they lack the resources.

Status Segregation

Agents show patterns of segregation by status that are in some ways, but not all, similar to patterns of segregation by wealth. Figure 2 shows the index of dissimilarity, D^* , for status, drawing on the data from the same computational experiment described above. The variables manipulated are price endogeneity and wealth-status correlation.

The clearest substantive difference between the results for status segregation presented in Figure 2 and the results for wealth segregation presented in Figure 1 is that price endogeneity does not exert a positive main effect on status segregation. As Figure 2 shows, when status-wealth correlation and price endogeneity are at zero, levels of segregation based on status are negligible, reaching 4.18. Following the y-axis from this point, with price endogeneity increasing but the status-wealth correlation held constant at zero, one can see that status segregation actually declines slightly (to 3.99) as housing prices are increasingly determined by the wealth of one's neighbors. This slight decline stands in contrast to the substantial increase in wealth segregation found when price endogeneity increases.

It is not surprising that increasing price endogeneity does not affect status segregation. Increasing price endogeneity sorts low- and high-wealth agents by creating a mechanism by which low-wealth agents

⁷This interaction effect, as well as the main effects of status-wealth correlation and price endogeneity, were also examined within a regression framework. We find positive and highly significant results for all three measures. Results are available upon request from the authors.

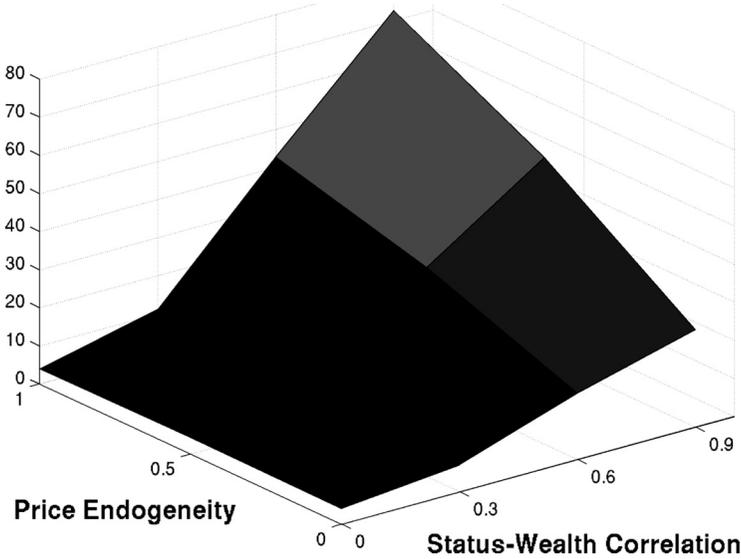


FIGURE 2 Index of dissimilarity for status, as a function of the status-wealth correlation and price endogeneity (Wealth distribution = normal, $N = 800$, 500 rounds, 15 repetitions, results average of last 50 rounds).

are excluded from high-wealth neighborhoods. But as long as wealth and status are uncorrelated, as assumed in this analysis, roughly equal proportions of high- and low-status agents will appear in the resulting high- and low-wealth neighborhoods. This leads to the conclusion that status differences may be insufficient to produce residential status segregation. Instead, resource differences or other structural mechanisms of exclusion must supplement status to restrict members of two or more groups in space.

Returning to Figure 2, agents become more segregated by status as the status-wealth correlation increases. The status-wealth correlation alone can produce levels of D^* of approximately 25. As in the case of wealth segregation, this occurs because increasing the status-wealth correlation increases the extent to which the population is divided into two types of agents—those high in wealth and status and those low in wealth and status. High-wealth and status agents then cluster together by choice, while low-wealth and status agents cluster together as a result of exclusion.

Furthermore, there is an interaction of status-wealth correlation and price endogeneity, as the effects of price endogeneity on status segregation are only expressed as the status-wealth correlation increases.

When both the status-wealth correlation and price endogeneity are high, D^* for status segregation equals approximately 72. Again, the mechanism here is identical to that which produces high levels of wealth segregation: when the status-wealth correlation is high, making housing prices endogenous magnifies the exclusion dynamics by allowing agents high in both status and wealth to create residential zones that are prohibitively costly for low-wealth agents to enter. As status and wealth become increasingly correlated, patterns of status segregation increasingly resemble patterns of wealth segregation.

Isolation

We now turn to examining the revised index of isolation for agents according to four wealth-status groupings of agents: high-wealth and high-status agents, low-wealth and high-status agents, high-wealth and low-status agents, and low-wealth and low-status agents. Figure 3 shows R^* for each of the four wealth/status groupings, when wealth status correlation is low ($C = 0$) and when wealth status correlation is high ($C = .9$).

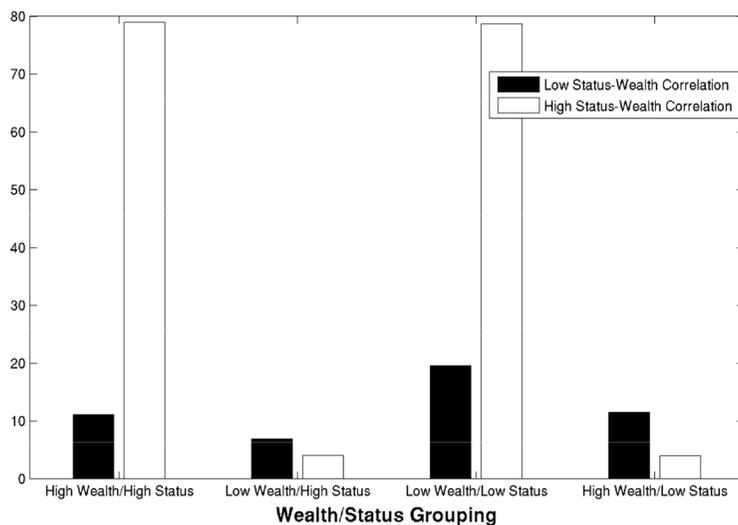


FIGURE 3 Revised index of isolation, as a function of wealth/status grouping and wealth/status correlation (high = .9, low = 0). (Price endogeneity = 1, Wealth distribution = normal, $N = 800$, 500 rounds, 15 repetitions, results average of last 50 rounds).

Looking first at R^* when the status-wealth correlation is low, we see that levels of isolation are low for all four groupings, ranging from a high of approximately 20 for low-wealth/low-status agents to a low of approximately 7 for low-wealth/high-status agents. However, this pattern changes substantially when wealth and status are highly correlated.

When wealth and status are highly correlated, the status-wealth correlation has little effect on the isolation levels of low-wealth/high-status agents and high-wealth/low-status agents, but a substantial effect on levels of isolation for high-wealth/high-status and low-wealth/low-status agents. High-wealth/high-status and low-wealth/low-status agents exhibit R^* values of nearly 80 when the status-wealth correlation is high. These results echo earlier findings, as high-status/high-wealth agents tend to cluster together by choice, while low-status/low-wealth agents cluster together because they are excluded from other types of neighborhoods.

“Hybrid” agents, those with high wealth and low status or low wealth and high status, display low levels of isolation regardless of the degree of status-wealth correlation. When the status-wealth correlation is low, agents high in wealth and low in status experience levels of isolation of approximately 11.5. When the status-wealth correlation is high, R^* for these agents is even lower, at approximately 4. Similarly, R^* for agents low in wealth and high in status is approximately 7 when the status-wealth correlation is low and 4 when it is high. Regardless of the nature of the association between wealth and status, in our model agents who are high in one but not the other tend to live in diverse neighborhoods.

This occurs because hybrid agents’ mixed levels of wealth and status create pressures towards *both* choice and exclusion that prevent them from clustering exclusively with agents of their own type. To understand how this works, first consider the high-wealth, low-status agents. Because of their high wealth, these agents can exercise choice by moving to their preferred neighborhoods. However, because of their low status, other agents will avoid them if possible, including agents of their own type. In other words, although high-wealth, low-status agents have the resources to cluster together, they do not have the interest. This stands in contrast to the case of high-wealth, high-status agents, who form isolated clusters because they prefer and are able to live near one another.

Exclusion and choice combine in a different way to form the diverse neighborhoods of low-wealth, high-status agents. Low-wealth/high-status agents will attract others to live near them because they improve the status of their neighborhood. But because of their low

wealth, they do not create expensive neighborhoods that exclude low-wealth agents, as high-wealth/high-status agents tend to do. Thus, these agents tend to be found in diverse areas, because their presence helps make their neighborhoods both more attractive and accessible to other types of agents.

Sensitivity to Assumptions

Thus far, we have assumed that agents seek to maximize the status of their residence, within the constraints allowed by their resources. However, this is not the only possible decision rule. For comparison, we also briefly consider two additional decision rules:

1. We examine a rule in which only status matters to agents. Here, agents simply move to the highest status neighborhood they can find without consideration of cost.
2. We examine the assumption that agents seek to minimize cost while at least maintaining their current level of status. Under this rule, agents move to the lowest-cost cell they can find that has a desirability (mean status of neighbors) equal to or greater than their current cell.⁸

Results are shown in Figure 4 for the decision rule presented and explored earlier in our model (“Maximize status”) and the two new decision rules (“Status only” and “Minimize cost”). For all analyses, we set price endogeneity to 1 and assume a wealth-status correlation of .9. These conditions maximize status segregation, allowing us to find the set of behavioral assumptions that leads to the greatest levels of status segregation.

As shown in Figure 4, segregation by status is greatest when agents seek to maximize status and are held to resource constraints, the decision rule assumed in our model. In contrast, the population exhibits the lowest levels of status segregation— D^* equals approximately 6—when agents seek to maximize status and wealth is not a consideration. Under these conditions, lower status agents can continually “chase” higher status agents, who are free to move away but are unable to exclude lower status agents from their neighborhoods. As a result, the population never settles into stable neighborhoods of low- and high-status agents. This suggests that choice alone

⁸Note that agents include their own cell in this search process. This prevents agents from moving to a cell that is more expensive than their own but cheaper than the cells on which they do not reside.

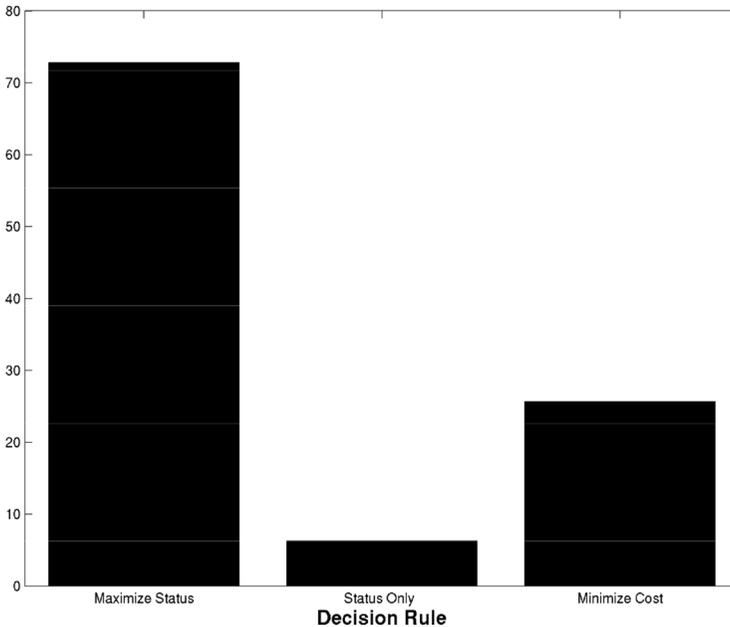


FIGURE 4 Index of dissimilarity based on status for three decision rules. (Price endogeneity = 1, Status-wealth correlation = .9, Wealth distribution = normal, $N = 800$, 500 rounds, 15 repetitions, results average of last 50 rounds).

is unlikely to produce high levels of status segregation. Instead, status segregation requires some level of resource-driven exclusion in order to segregate populations.

Finally, populations characterized by agents seeking to minimize cost without losing status produce moderate levels of segregation, with D^* equal to approximately 25. In these populations, dense clusters of high-wealth/high-status agents are less likely to develop because all agents prioritize lower-cost housing and are likely to move away from wealthy agents. This rule extends to wealthy agents themselves. At the same time, we observe moderate status segregation because agents try to avoid losing status, and therefore are attracted to high-status agents (even when wealthy) and averse to low-status agents (even when poor). This analysis emphasizes that small changes to behavioral assumptions can have substantial impact on levels of segregation. In the absence of empirical data describing how individuals actually weigh various factors, it is important to test the sensitivity of various behavioral rules.

DISCUSSION

The present research extends the classic Schelling model to the study of wealth and status in residential segregation. While scholars have recently begun to examine how wealth and status shape the accessibility and desirability of neighborhoods, many important questions remain (e.g., Fossett, 2005; Bruch, 2006). For example, Fossett's (2005) work models wealth and status as a single variable influencing both the desirability and accessibility of neighborhoods, and Bruch (2006) views desirability and accessibility at the neighborhood level as perfectly correlated. We contribute to this literature by analytically distinguishing wealth and status and modeling the association between them as a factor underlying segregation. We assume in our model that agents wish to live in neighborhoods with relatively high-status neighbors and that their ability to live in neighborhoods is determined in part by their wealth.

We manipulated the degree to which the status and wealth of agents in the model was correlated. We also manipulated price endogeneity, the degree to which the price of housing is determined endogenously (i.e., as a function of the wealth of the residents of the neighborhood). These simulations produced results generally consistent with cultural beliefs about the dynamics of gentrification and the role of wealth and status in residential segregation. We found that as the correlation between wealth and status of agents increased, so did segregation based on wealth and status. This was because, as the high-status/high-wealth individuals increasingly become one and the same, they increasingly desire to live near one another (due to their high status) and are increasingly able to do so (due to their high wealth). Low-wealth/low-status individuals are priced out of these desirable neighborhoods, incidentally clustering with one another. This result highlights an underlying dynamic of gentrification: the clustering of wealthy, high-status agents is primarily a product of choice (and privileged access to the most desirable residences), whereas the clustering of poorer, lower-status agents results from exclusion (and the inability to access the most desirable residences).

These results also suggest that that the emergence of other cultural bases of status besides wealth may decrease residential segregation by weakening the wealth-status correlation. Future research aimed at designing interventions to decrease residential segregation could focus on identifying factors that would decouple the desirability of neighbors from the wealth of those neighbors. That said, such research should not be conducted in isolation from the greater ecology of urban life. The mixed record of urban renewal efforts serves as a warning that

unanticipated consequences may interfere with mechanism-based planning efforts (e.g., Jacobs, 1961).

We also analyzed the effects of randomly determining housing prices versus basing housing prices on the wealth of agents in the neighborhood. When housing prices were endogenized based on the wealth of the residents of the immediate neighborhood, segregation based on wealth increased. This is because, when housing prices are endogenous, poorer agents will be priced out of neighborhoods leaving behind wealthier agents. This pattern was observed even when status/wealth correlation was zero. This extremity of this result underscores the significance of endogenizing housing price in models of this type. Researchers may benefit from exploring the effect of this variable within their own models, or may at least wish to give clear theoretical justifications for adopting one of these modeling strategies.

Wealth segregation was accentuated by the degree of correlation between wealth and status. The reason for this interaction effect is more familiar: the more correlated wealth and status are, the more the wealthy desire to live near one another. This produces an increase in choice homophily among wealthy agents, while the pressures forcing poorer agents out of wealthy, unaffordable neighborhoods are still present.

In our analysis of patterns of status segregation we found an interesting and unanticipated effect. By and large, status segregation effects echoed wealth segregation effects, with status and wealth correlation and price endogeneity tending to increase segregation, and also having interactive effects. However, as opposed to wealth segregation, we found no effect of price endogeneity on status segregation unless wealth and status were at least moderately correlated. This is because when wealth and status are uncorrelated, increasing price endogeneity tends to sort agents into relatively poorer and wealthier neighborhoods that contain roughly equal proportions of high- and low-status agents. These results suggest the utility of treating status and wealth as conceptually distinct and modeling the association between them. This approach may be especially important as status and wealth are often associated with other variables of interest, such as race.

We conducted several ancillary simulations to analyze how sensitive results were to our assumptions, an important but often-neglected practice in agent-based modeling (Macy and Willer, 2002). When agents were assumed to seek high-status neighborhoods in the absence of wealth constraints, the population failed to reach high levels of status segregation. This result suggests that status differences alone are

insufficient to produce status segregation. Status segregation requires both status and resource differences, at least as defined in our model. We obtain higher levels of status segregation by incorporating wealth into the model. In the primary results analyzed here, we showed that a model in which agents seek to maximize the status of their neighborhoods within the constraints allowed by their wealth can produce very high levels of both wealth and status segregation. An alternative model, in which agents seek to minimize housing costs without losing status, produced moderate levels of segregation.

Our model could be extended in future work. For example, we did not analyze how sensitive our results were to agents' "visions," i.e., how broadly agents search in assessing their neighborhood and other prospective neighborhoods. The assumption that agents only consult their immediate "Moore neighborhood" is a reasonable initial assumption, given the resources required for a fuller search. Past research has shown that segregation patterns can be affected by vision, with segregated enclaves tending to clump in larger clusters or swaths when agents attend to larger local neighborhoods in their residential choice process (Lauri and Jaggi, 2003). Thus, the breadth of agents' visions could affect patterns of segregation in the present model.

It might also be interesting to bring our model closer to the original Schelling model by incorporating the effects of race. Here, we did not include race so that we could focus on the dynamics of wealth and status. Future modeling could examine the effects of correlating race, wealth, and status at different levels. Of course, such a model would also be substantially more complex and perhaps more difficult to interpret.

Another possible extension would be to explore the effects of various neighborhood configurations. While Moore neighborhoods are among the most commonly used and defensible neighborhood configurations, many others are possible. For example, one could examine diamond shaped von Neumann neighborhoods or irregularly shaped Voronoi neighborhoods. While models employing these extensions could help explore the generality of the results, they are unlikely to significantly alter the primary substantive patterns discussed here.

By extending the Schelling model to include status and wealth, the present research produced several results consistent with observed real world dynamics. For example, the segregation of wealthy, high-status agents from poor, low-status agents was driven by the differential capacities of wealthy and poor agents to access the most desirable residences. As a result, the homophilous clustering of the wealthy and high status agents was based on these agents' choices and preferences, whereas the clustering of the poor, low-status agents was based on

exclusion from more desirable regions. On the basis of these and other findings, we conclude that the incorporation of accessibility of residences is critical for the modeling of gentrification dynamics.

CONCLUSION

This paper represents an effort to extend agent-based modeling of residential segregation dynamics and patterns. One advantage of the methodology we employed was that we were able to scrutinize alternative worlds that might not naturally occur, to perhaps better understand the effects of factors that are always present in the real world. For example, the factors we focused on were the correlation between status and wealth and the degree to which the cost of living in a neighborhood was a product of the wealth of its residents. These are both factors that are almost always present at non-zero levels in the real world. The wealthy tend to be more respected than the poor, and housing prices tend to be at least minimally associated with the average wealth of the neighborhood. Because these principles tend to naturally occur in the real world, it would normally be quite difficult to systematically analyze their independent (and combined) effects on segregation patterns.

We find it promising that this initial extension was capable of producing some of the basic patterns of gentrification observed in the field. Such modeling efforts can produce theoretical explanations for complex macro-level patterns that might otherwise be quite difficult to disentangle. However, caution must be used in the modeling of social phenomena. One should not assume that a model that produces recognizable patterns has necessarily explained them. Rather, agent-based models are ways to suggest internally logical, potential explanations of their outputs, and should therefore be understood as proposing and logically evaluating potential mechanisms that must also be empirically tested in order to be more completely assessed.

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