The Journal of Mathematical Sociology

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To cite this Article: , 'The Role of Metanetworks in Network Evolution', The Journal of Mathematical Sociology, 31:2, 101 - 119
xxxx:journal To link to this article: DOI: 10.1080/00222500601187884
URL: http://dx.doi.org/10.1080/00222500601187884

Journal of Mathematical Sociology, 31:101–119, 2007

ISSN: 0022-250X print/1545-5874 online
DOI: 10.1080/00222500601187884

The Role of Metanetworks in Network Evolution

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The question of what structures of relations between actors emerge in the evolution of social networks is of fundamental sociological interest. The present research proposes that processes of network evolution can be usefully conceptualized in terms of a network of networks, or "metanetwork," wherein networks that are one link manipulation away from one another are connected. Moreover, the geography of metanetworks has real effects on the course of network evolution. Specifically, both equilibrium and non-equilibrium networks located in more desirable regions of the metanetwork are found to be more probable. These effects of metanetwork geography are illustrated by two dynamic network models: one in which actors pursue access to unique information through "structural holes," and the other in which actors pursue access to valid information by minimizing path length. Finally, I discuss future directions for modeling network dynamics in terms of metanetworks.

Keywords: mathematical sociology, metanetworks, network dynamics, network evolution, social network analysis, social structure

INTRODUCTION

Theories of micro-macro bridging, and the reciprocal relationship between structure and agency, have become a central epistemological issue to sociology (Alexander et al., 1987). The interplay between individual and group levels of social analysis is significant to research in many areas of sociology, including rational choice (Schelling, 1978; Coleman, 1986; Friedman and Hechter, 1988), social psychology (Lawler, Ridgeway and Markovsky, 1993), and agent-based modeling (Macy and Willer, 2002). The case for micro-macro modeling of social processes rests on two fundamental assertions: 1) social structure

The author would like to thank Ko Kuwabara, Michael W. Macy, Brent Simpson, and Arnout van de Rijt for comments and contributions to earlier versions of this paper.
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shapes individuals' behavior and attitudes and 2) individuals create and manipulate social structure (Collins, 1981). While the first claim is fundamental in the history of sociology, the latter remains comparatively less researched and theorized (Coleman, 1990).

In the last decade, network theorists, using social networks as their representation of structure, have begun to research the problem of how individuals create and alter social networks (e.g., Doreian and Stokman, 1997; Bala and Goyal, 2000). Network analysis has focused traditionally on static configurations and how characteristics of social network embeddedness can influence individuals' choices, preferences, and social interactions. Network dynamics research is primarily concerned with how individuals alter network structure. Thus, network dynamics research represents a new field in the study of the relationship between social structure and individual action.

Research on network dynamics in economics and sociology describes how the structure of relations between social actors evolves and changes across time. Much of the network dynamics literature has focused on the identification of networks that are both efficient (i.e., globally satisfactory to actors as compared with alternatives) and stable (i.e., likely to persist) (e.g., Jackson and Wolinsky, 1996; Dutta and Mutuswami, 1997; Bala and Goyal, 2000; Dutta and Jackson, 2002). Others dynamic network research, however, has attempted to model the process of network formation (e.g., Myerson, 1991; Skyrms and Pemantle, 2000; Watts, 2001; Jackson and Watts, 2002).

The present research contributes to the latter focus of the network dynamics literature. I draw upon existing models of network evolution to assess network probability, the rate of occurrence of different network structures, and analyze equilibrium selection, the question of which of several equilibria is most likely to obtain. In particular, I examine the critical role played by "metanetworks," a structural factor that influences the course of network dynamics and observed network probability.

Metanetworks are "networks of networks," theoretical constructs portraying different networks as nodes with connections existing between networks that are one link manipulation different from each other. Network evolution is tantamount to movement around a metanetwork. As actors change the structure of the network they inhabit they move to a different location in the metanetwork. I find that some regions of metanetworks are more attractive to actors than others. Equilibrium and non-equilibrium networks found in these regions are more probable because actors are more likely to find them in the course of network evolution. Thus, a network's probability is not only a function of its inherent desirability to actors, but also of its proximity in the metanetwork to other desirable networks.

The models that follow feature a sequential-move network formation game, employing assumptions from previous models of this type. Actors take turns adding and deleting ties one by one, with turn-taking determined randomly. Rather than assuming extensive calculation abilities, in their evaluation of which tie to add or delete, actors only consider the immediate effects of addition or deletion. The models also allow for specification of the probabilities of both equilibrium and non-equilibrium networks.

In the sections that follow, I further detail my conception of metanetworks, review relevant research on network dynamics, outline the modeling approach, and present the modeling assumptions. I illustrate claims relating metanetwork accessibility and network probability in two models. In Model 1, where network-embedded actors attempt to maximize "structural holes" (Burt, 1992) through strategic network manipulation, the most accessible equilibrium network in the metanetwork is also the most probable. In Model 2, where actors manipulate their network in order to maximize information validity, this result is replicated even when the more probable network equilibrium is Pareto-deficient. Third, for both models I show that on the way to equilibrium, network evolution is more likely to pass through non-equilibrium networks that reside in more accessible neighborhoods of the metanetwork. I conclude by discussing results and exploring future directions for the modeling of network evolution.

METANETWORKS

As mentioned earlier, metanetworks represent networks of possible networks within which networks that are a single link manipulation different from each other are linked. Network evolution can thus be conceptualized as movement around a metanetwork. By way of example, Figure 1 gives the 4-actor metanetwork. The 4-actor metanetwork includes all the possible 4-actor networks as nodes. Connections between nodes represent "neighboring networks," networks that are one link manipulation apart. Thus, as a group of four actors manipulate the network ties connecting them, they trace a path through the metanetwork of Figure 1.

I argue that the geography of metanetworks affects patterns of structural change by determining the accessibility of different networks. These effects occur because the desirability of metanetwork neighborhoods shapes the accessibility of the networks in the neighborhood. More desirable neighborhoods are more accessible because

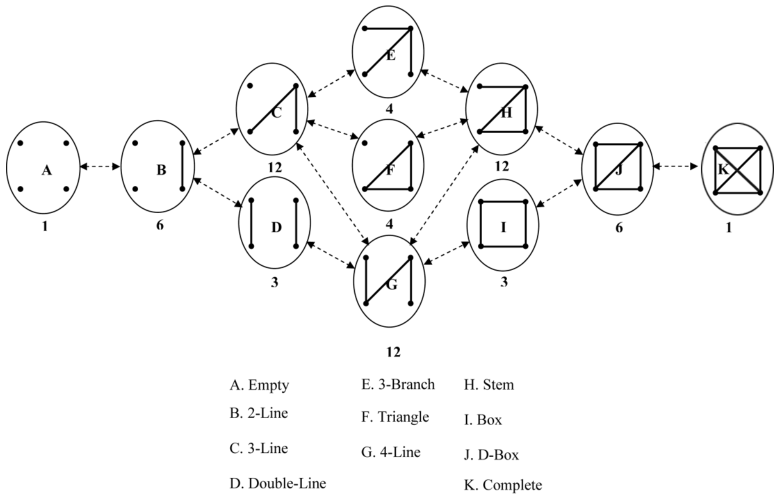

FIGURE 1 The general 4-actor metanetwork. Numbers below each network indicate the number of isomorphs the network has in the full metanetwork. Letters refer to names of networks given above.

they are likely to attract actors as they make decisions to alter the network they inhabit. The result is that network dynamics are more likely to settle on equilibria in or near attractive metanetwork neighborhoods. The desirability of an equilibrium network's neighborhood can even have a larger effect on equilibrium selection than the relative desirability of the equilibria themselves, as shown in Model 2.

Note that in Figure 1 numbered coefficients appear beneath each network. These coefficients indicate the number of isomorphs a given network structure has in the metanetwork. For example, there are 12 isomorphic versions of the 4-line (Network G of Figure 1), but there is only one version of the fully-connected 4-actor network (Network K). To simplify analysis, graphs and tables give only one example of each network structure by summing together isomorphs, rather than results for each individual network. This is possible since all networks of a given structure occupy structurally identical positions in the metanetwork, and therefore operate exactly the same in the models.

Structural forms will be more likely to emerge in network evolution to the extent that they are (1) more desirable to actors and (2) located in more heavily trafficked areas of the metanetwork. While the first of these factors is anticipated by the existing dynamic networks literature, the second is more novel. All things being equal, networks combining high levels of desirability and accessibility will be the most probable in the course of network evolution.

In the model, as actors make locally maximizing decisions to alter their network link by link, they move towards or away from equilibrium networks. In this way, the properties of networks that are distal in the metanetwork, far from the equilibria, may have strong effects on which equilibria actors eventually arrive at. Thus, an equilibrium network's probability is in part a function of its location relative to actors' other options. To determine a network's probability, it must be studied in its appropriate context. Network probability is a function of whether a network is an equilibrium network or not (so that actors will stay there when they reach it), but also of being located in a well-trafficked area of the metanetwork (so that it will be reached by actors to begin with). The layout of networks and their relative attractiveness to actors partially determines which equilibria will be selected and with what probability.

The concept of a metanetwork is not intended as a mathematical innovation. Mathematically, a metanetwork is the transition matrix of a discrete-state Markov process with networks as states.[1] Figure 1 is known as a *state-space diagram*, or *Markov diagram*.

DYNAMIC NETWORKS

Recently researchers in economics and sociology have modeled network dynamics as a utility-maximizing process (see Dutta and Jackson, 2003, for a review).[2] This literature is based on the assumption that different network formations and locations are of different utilities to actors. The central idea in this literature is that if networks are a source of utility, then rational actors should be expected to modify their local network structures to maximize utility. Thus, in this literature,

[1]Note that this not the first model of network dynamics to view network evolution in terms of a Markov process. For example, Jackson and Watts (2002) develop and implement a similar model, though the focus of their analysis is different than the present focus on the properties of metanetworks.

[2]A wide variety of research on network dynamics exists across the social sciences. This research concerns the dynamics of a diversity of network models including models of structural balance (Cartwright and Harary, 1956; Macy, Kitts, and Flache, 2003), group formation (Carley, 1991), social exchange (Leik, 1992; Willer and Willer, 2000; Bonacich, 2003), policy-making (Stokman and Zeggelink, 1996), status hierarchies (Gould, 2002), technological innovation (Podolny and Stuart, 1995; Stuart, 1998), and scientific collaboration (Moody, 2004).

network structure is an endogenous variable and actors exercise agency over the structural forms they inhabit. Some agreement exists in this literature on some standard assumptions for modeling network formation, presented in the next section.

Utility, or value, in these models is defined as a function of network structure. In some models, utility represents information flow or communication potential (Bala and Goyal, 2000, 2003; Dutta and Jackson, 2000; Jackson and Wolinsky, 1996; Johnson and Gilles, 1999; Slikker and van den Nouweland, 2003; Goyal and Redondo, 2004). In Gould (2002), utility is a combined function of the status of the actors that one is connected to and the similarity of the tie strength with them. Elsewhere, ties constitute bargaining or exchange connections (Leik, 1991; 1992; Kranton and Minehart, 2000, 2001; Corominas-Bosch, 1999; Willer and Willer, 2000; Bonacich, 2003; Bloch and Ghosal, 2000). In other models, network ties represent time invested in cooperative projects, utility measuring the returns on these investments (Gehrig, Regibeau, and Rockett, 2000; Jackson and Wolinsky, 1996). Developments in the methodology of dynamic network analysis have grown substantially, and are documented in three edited volumes (Weesie and Flap, 1990; Stokman and Doreian, 2001; Breiger, Carley, and Pattison, 2003).

A central question in the networks dynamics literature is under what conditions *equilibrium* networks are *efficient*? Many of the papers in this literature analyze the relationship between equilibrium and efficiency (for an extensive review see Dutta and Jackson, 2003: 6–11). Equilibria are efficient only under rather restrictive conditions. It has been demonstrated with various models how actors may get stuck in Pareto-inefficient networks.

MODELING ASSUMPTIONS

Among scholars who study network dynamics in game-theoretic terms, a variety of modeling assumptions are made, some more commonly than others. Wherever possible, I have attempted to use network- and actor-level modeling assumptions that are common practice. Networks are assumed to be simple, i.e., loops (ties that go to the same node) are not allowed and each network has no more than one tie between each pair of nodes (Wasserman and Faust, 1994: 95). Ties are also non-valued (140), i.e., of equal strength, and they are undirected (72). Actors, (1) add and delete links one by one, (2) are myopic in that they derive utility from the network that is obtained immediately after their link manipulation, and (3) only have control over links between themselves and others. Link additions are only implemented if they are agreed upon by both actors.[3]

The present analysis is confined to these assumptions. In my models of network formation, network manipulation follows the following iterative procedure:

(1) Pick an actor.
(2) The actor decides to propose a tie, remove a tie, or do nothing, whichever maximizes her utility.
(3) The network changes,
(4) Return to step 1.

In the analysis that follows, I use the concepts of *pairwise stability* and *Pareto efficiency*. Stasis in the models is called *pairwise stability* (Jackson and Wolinsky, 1996). A network is pair-wise stable if there is no pair of actors that can each benefit from initiating a tie between them and no single actor that can benefit from deleting a link. *Pareto efficiency* is my central efficiency concept. A network is Pareto-efficient if there exists no other network in which at least one actor is better off, and no actor is worse off.

DEMONSTRATING METANETWORK EFFECTS

This paper is concerned with the role of metanetworks in network probabilities and network equilibrium selection. My main argument in demonstrating metanetwork effects is that it is not only the desirability of a *network* that determines its probability of being observed, but it is also the desirability of *its neighborhood* in the metanetwork. Specifically, networks that exist in more desirable locations of the metanetwork will accordingly be more probable. To establish the validity of metanetwork effects on network structure in dynamic settings, I proceed by demonstrating the following effects.

- *Equilibrium Network Selection (Model 1)*. Metanetworks can determine which of multiple equilibrium networks actors will arrive at from by shaping the desirability of each equilibrium's location.
- *Pareto-inefficient Equilibrium Selection (Model 2)*. Metanetwork effects on equilibrium selection may even be so strong as to select

[3]Some exceptions to these modeling assumptions include Bala and Goyal (2000) where actors can change all links at once, the simultaneous network formation of the so-called "Myerson game" (Myerson, 1991), Dutta, Ghosal, and Ray (2005) who consider far-sighted actors, and Leik (1992) who considers a "manipulator" proposing links that do not involve himself.

a Pareto-inefficient equilibrium. This is because certain equilibria are located in more desirable locations than other, more desirable, equilibria.

To illustrate the above effects, I designed two models wherein actors manipulate the network they reside in to maximize certain preferences. Both models follow the assumptions introduced above but assume different values of network positions, i.e., different network-based utility functions. To demonstrate the first effect, I create a model of Burt's theory of "structural holes" (Burt, 1992; 2001; 2002; 2004). In the model, each of a set of four actors tries to maximize the value of their network ties in pursuit of structural holes. I am interested in whether actors' equilibrium selection is constrained by the location of the equilibrium networks in the larger metanetwork. I seek to demonstrate that the equilibrium network located in a more attractive location will be more probable.

To demonstrate the second effect, I model a "Valid Information Game." As opposed to the Burt model where actors seek access to unique information, in the Valid Information Game actors try to maximize the accuracy of information by attempting to minimize their path-length from all other actors. Here I show that a relatively less desirable (Pareto-inefficient) equilibrium will be more probable because of the accessibility of its neighborhood.

MODELS

In the first model that follows, the Burt Game, actors attempt to maximize the structurally determined access to relatively high amounts of unique information, as per Burt's conception of "network constraint" (Burt, 1992: 54). Conversely, in the Valid Information Game, actors attempt to maximize the validity of the information to which they have access. Actors in both models exercise agency through utility-maximizing manipulation of their network structure.

The assumptions regarding what constitutes a metanetwork are consistent with the earlier definition. The metanetwork consists of nodes representing all the different network configurations possible between a given number of actors and ties connecting "neighboring networks," i.e., networks that are identical except for the addition or deletion of a single link. The latter assumption corresponds to the earlier actor assumption that only one link can be changed at a time.

Both models are implemented in the 4-actor metanetwork case for a few reasons:

1. It is large enough to see biasing effects,
2. There are a convenient number of equilibria in both models for exploring the above claims,
3. It is analytically tractable, and
4. It can be conveniently displayed in its entirety (see Figure 1).

As above, in figures the metanetwork is given without isomorphic networks separately, though in the models isomorphs are treated separately. Each isomorph will yield the same probabilities in the models that follow because isomorphic networks occupy structurally identical locations in the metanetwork.

Model 1: The Burt Game

The Burt Game is a simple network dynamics model based on Burt's theory of structural holes and the above assumptions. Burt's theory of structural holes says that actors are advantaged by occupying network positions that grant them a relative advantage in access to unique information and control over information flow (Burt, 1992). I extend this theory by claiming that, under circumstances where access to unique information and control over information flow is valued, actors will try to alter their network structure to maximize their relative advantage in access to it.

Actors in the model attempt to minimize their "network constraint," a metric that describes the value of an actor's network position (Burt, 1992: 54). Network constraint represents the extent to which incoming information is redundant, i.e., the extent to which information from the same person is available through multiple channels. High scores represent high redundancy of access to information.

Equation (1) gives the reciprocal of Burt's "network constraint" measure, inverted so that higher scores indicate greater utility to the focal actor.

$$U_i = \frac{1}{\frac{1}{d_i^2}\sum_j \left[1 + \sum_k \frac{1}{d_k}\right]^2} \tag{1}$$

Here, U_i is actor i's utility and d_i is i's "degree," the number of actors i is connected to. j is the index for neighbors of i, and k is the index for neighbors of i that are also connected to j. U_i increases with the number of actor i's ties, but decreases with the number of closed triads actor i is involved in. In making decisions actors calculate the utility of their position in the current network compared with the networks resulting from all possible link manipulations, choosing the link manipulation that maximizes her utility. It is possible that actors may decide not to change their network on their turn, as is the case when the network dynamics reaches equilibrium.

Using the modeling assumptions and taking equation (1) as actors' utility function, I construct a transition matrix that gives the probabilities of moving from all source networks to all destination networks. This transition matrix is displayed in Table 1. Columns are source networks, rows are destination networks, and values in the table represent the probabilities of a transition from the source to the destination network. For example, the entry "1" in the (B, A) cell indicates that when the empty network (A) is the source network, a tie will be added with 100% probability, always resulting in the 2-line (B). This is because it is utility-maximizing to all actors in the empty network to add a tie to one of the three other actors. As a second example, the "1/6" entry in the (D, B) cell is obtained as follows: In order to move from Network B to D the two isolated actors must connect. One of the two isolated actors is chosen to manipulate the network 1/2 of the time. Both isolates are indifferent to whom they connect with as long as they connect to someone, since in each case their resulting utility equals 1, which is greater than utility of 0 if they do not add a tie. Given that they are indifferent to the three options for adding a tie, either isolate will add a tie to the other 1/3 of the time when they are chosen to manipulate the network. Therefore the probability of the transition from the 2-line to the double line (B to D) is $1/2 * 1/3 = 1/6$.

TABLE 1 Transition Matrix for the Burt Game. Columns are Source Networks, Rows are Destination Networks

	A	B	C	D	E	F	G	H	I	J	K
A	0	0	0	0	0	0	0	0	0	0	0
B	1	0	0	0	0	0	0	0	0	0	0
C	0	.83	0	0	0	0	0	0	0	0	0
D	0	.17	0	0	0	0	0	0	0	0	0
E	0	0	.375	0	1	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0	0	0	0
G	0	0	.625	1	0	0	.5	.25	0	0	0
H	0	0	0	0	0	1	0	0	0	0	0
I	0	0	0	0	0	0	.5	0	1	.5	0
J	0	0	0	0	0	0	0	.75	0	.5	1
K	0	0	0	0	0	0	0	0	0	0	0

For two networks, the 3-Branch (E) and the Box (I), the probability of moving from the network to any other is 0%, and the likelihood of remaining in the network is 100%. In both networks neither actor can improve her position by deleting a tie or by adding a tie that is acceptable to the relevant other. Adding a tie to either network involves the creation of a closed triad, while the deletion of a tie would not eliminate a closed triad. It is utility-maximizing for each of the actors to make no change. Thus these are the only two pairwise-stable networks. This means that, if the course of network evolution passes through either of these two networks, it will stay there forever.

Figure 2 shows the corresponding metanetwork. The probabilities from Table 1 can be found above the corresponding arrows in Figure 2. Note that the two pairwise, stable network isomorphs, E and I, have only inward-pointing arrows.

Now each network is given an equal probability of being first in network evolution, which I do by constructing the following row vector which accounts for the numbers of isomorphs corresponding to each network type:

$$v_0 = [1\ 6\ 12\ 3\ 4\ 4\ 12\ 12\ 3\ 6\ 1]/64 \tag{2}$$

The gth element of this vector is the probability of starting network dynamics in the gth network. For example, the 7th element of the vector (12/64) is the number of isomorphs of the 4-line network (network G in Figure 1), divided by the total number of networks.

We can now find the probability of evolution reaching each of the 11 networks in round t by multiplying the probabilities for $t-1$, v_{t-1}, by T:

$$v'_t = Tv'_{t-1} \tag{3}$$

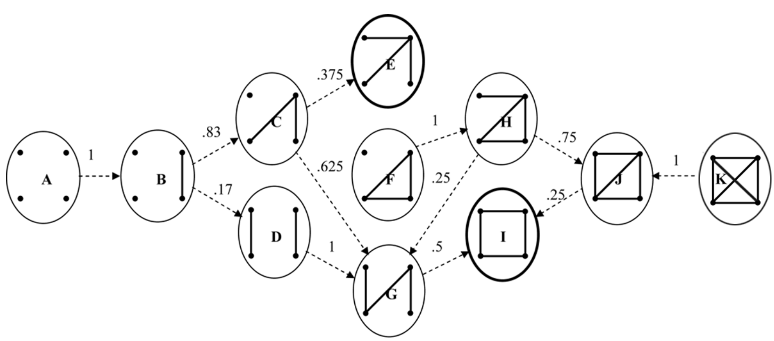

FIGURE 2 The 4-actor metanetwork for the Burt Game. Arrows represent potential transitions between networks and numbers represent the probabilities of these transitions. Bolded circles surround pairwise-stable networks.

As t approaches infinity, the probability of each of the 11 network structures approaches the following values:

$$v'_{\infty} = [0\ 0\ 0\ 0\ .17\ 0\ 0\ 0\ .83\ 0\ 0] \tag{4}$$

Thus, in 83% of the cases the Box network equilibrium obtains, and in 17% of the cases the 3-Branch network obtains.[4] The structure of the metanetwork strongly favors the former over the latter. One can see how the metanetwork creates the bias towards the Box network by analyzing transitions that lead to and away from it.

In the absence of an understanding of the effects of metanetwork neighborhoods one's best guess might be that these two pairwise-stable equilibria would have equal probabilities since both are locally-optimal. However, the very unequal probabilities of the two networks indicate the strength of metanetwork neighborhood desirability. An examination of the arrows portrayed in Figure 2 shows that many more arrows point towards the Box than to the 3-Branch. This indicates that many more transitions in the metanetwork move in the direction of the Box than the 3-Branch. This shows that, in order to anticipate the probability of a given equilibrium network, one must not only know that it is an equilibrium network, but also the characteristics of its location vis-à-vis other networks in the metanetwork.

In the model, network dynamics begin with a random start. Figure 2 shows that if actors initially find themselves in either Network E or I, they will stay there because the two are equilibria. Additionally, if the random start begins in Networks D, G, F, H, J, or K, actors will inevitably wind up in the Box network (Network I) and then stay there, though it may take several transitions for them to get there. If network evolution begins in Networks A, B, or C, it is possible to reach the 3-Branch equilibrium network (Network E). Still, even in these cases, it is more likely that actors will wind up in the Box equilibrium network since 62.5% of transitions leading from Network C are directed towards the Box network's neighborhood. Thus, even in the networks that *can* access the 3-Branch equilibrium network, it is more likely that actors *will* find the Box network.

Because the Box network is embedded in a very desirable location in the 4-actor metanetwork, as compared with the 3-Branch, it is far more likely to be arrived at in the course of network evolution. However, the Box network is also Pareto-efficient and has a greater average utility ($\bar{U} = 2$) than the 3-Branch ($\bar{U} = 1.5$), which is Pareto-inefficient. The

[4]A reader might suspect that the different number of isomorphs for the Box and 3-Branch might be the driving factor making the Box more probable. However, the Box actually has fewer isomorphs (3) than the 3-Branch (4).

effects of network desirability and accessibility are thus confounded. A more impressive demonstration of metanetwork accessibility effects would show that a Pareto-inefficient equilibrium can be more probable by virtue of these effects. To address this, Model 2 is presented to demonstrate how a Pareto-inefficient equilibrium network can be more probable than a Pareto-efficient one by virtue of its superior metanetwork neighborhood location.

Model 2: The Valid Information Game

Model 2 is presented as an illustration of Claim 2: that metanetwork effects are sufficiently strong as to make even a Pareto-inefficient equilibrium the most probable network. I also address the possibility that the effects observed in the previous section might somehow be peculiar to the Burt Game I used to demonstrate them. To address these possibilities, actors in Model 2 are based on a different assumed utility function.

Model 2 was designed to have empirical plausibility while also diverging substantially from the utility function of Model 1. Whereas in the Burt Game actors valued network changes in pursuit of access to unique information and brokering opportunities, in the present model actors manipulate network structure in an effort to improve the validity of the information they gather. I do not investigate here the appropriate scopes of these two models, but it is likely that in some settings individuals seek unique information advantages (e.g., corporate and investment settings), while in other settings valid information is more valued (e.g., disaster warnings, rumor, and history). Thus, both models bear a correspondence with a range of empirical phenomena, as well as being useful for exploring and demonstrating the effects of metanetworks.

The model replicates Model 1 in every way except the utility function that actors use in evaluating networks. In the Valid Information Game, actors try to seek primary or secondary sources of information by maximizing the numbers of actors they are tied to through paths of length 1 or 2, while also trying to minimize cost. Thus, the utility function employed in the Valid Information Game gives an actor a unit benefit for every other actor that is less than three steps away and lets the actors equally share the costs of all ties in the network:

$$U_i = z_i - \frac{1}{2n}\sum_j d_j \tag{6}$$

Here, U_i is i's utility, z_i is the number of actors that actor i can reach in maximally two steps, n is the number of actors, and d_i is actor i's degree.

TABLE 2 Transition Matrix for the Valid Information Game. Columns are Source Networks, Rows are Destination Networks

	A	B	C	D	E	F	G	H	I	J	K
A	0	0	0	0	0	0	0	0	0	0	0
B	1	0	0	0	0	0	0	0	0	0	0
C	0	1	0	0	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0	0	0	0
E	0	0	.5	0	1	0	0	.5	0	0	0
F	0	0	0	0	0	0	0	0	0	0	0
G	0	0	.5	1	0	0	0	.25	0	0	0
H	0	0	0	0	0	1	0	.25	0	.83	0
I	0	0	0	0	0	0	1	0	1	.17	0
J	0	0	0	0	0	0	0	0	0	0	1
K	0	0	0	0	0	0	0	0	0	0	0

As in Model 1, I construct a transition matrix, given in Table 2, based on standard network and actor assumptions and the above utility function. I again use an initial vector that gives each network equal probability as the starting network. I multiply the transition matrix with the initial vector an infinite amount of times thereby obtaining the following network probabilities.

$$v'_\infty = [0\ 0\ 0\ 0\ .44\ 0\ 0\ 0\ .56\ 0\ 0] \tag{7}$$

The 3-Branch and the Box are the only pairwise-stable networks under these model specifications, but this time the 3-Branch is Pareto-efficient while the Box network is not. Also, the 3-branch has a higher average utility ($\bar{U} = 2.25$) than the Box ($\bar{U} = 2$). Since both networks are pairwise-stable, a naïve prediction would be that each network is equally probable. Instead the Box network (56%) was found to be more probable than the 3-Branch (44%).[5] Thus, even though the 3-Branch Pareto-dominates the Box, and even though both are locally optimal equilibria, the structure of the metanetwork makes the Box more likely to evolve.

A closer investigation reveals how the Box network is more probable than the 3-Branch. Figure 3 gives the metanetwork for Model 2. The probabilities from Table 2 can be found above the corresponding arrows in Figure 3. Note that the two pairwise-stable network isomorphs, E and I, have only inward-pointing arrows.

[5]As in Model 1, one might suspect that the number of isomorphs for the two equilibria might be driving the finding that the Box is more probable. However, the Box has fewer isomorphs (3) than the 3-Branch (4).

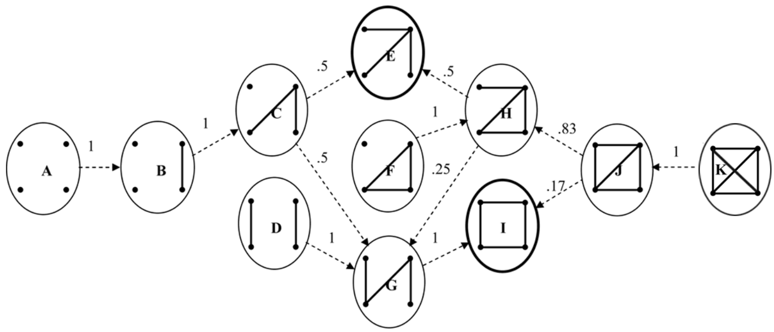

FIGURE 3 The 4-actor metanetwork for the Valid Information Game. Arrows represent potential transitions between networks and numbers represent the probabilities of these transitions. Bolded circles surround pairwise-stable networks.

Again, the initial network in the model is randomly determined. If actors initially find themselves in either equilibrium network (Networks E or I), they will remain there. If the random start begins in Networks A, B, or C, they will face a 50% probability of going to the 3-Branch and a 50% probability of moving to Network G, and then on to the Box network. All other random starts favor one equilibrium or another. If network evolution begins with either Networks G or D, then transitions will lead directly to the Box equilibrium network. If network evolution begins in Networks, F, H, J, or K, resulting transitions may lead to either equilibrium network, with a slight bias towards the 3-Branch equilibrium network. The Box network's overall probability is slightly higher than that of the 3-Branch. This is because the 100% likelihood of going from Networks D and G to the Box offers a greater accessibility advantage than the 3-Branch derives from being a slightly more likely destination for network evolution beginning in Networks F, H, J, or K. This finding, that a Pareto-inefficient network can be more probable than a Pareto-efficient one as a result of metanetwork effects, supports Claim 2.

DISCUSSION

In this paper I have presented a new conceptualization of network evolution in terms of movement around a metanetwork. I further claimed that networks located in more desirable regions of the metanetwork would be more likely to result in the course of network evolution. The preceding modeling sections offer support for two specific claims about the effects of metanetwork neighborhood desirability. The results of

Model 1 supported the claim that the desirability of the metanetwork neighborhoods surrounding equilibrium networks can determine which of multiple equilibria will be more likely. In the case of the Burt Game, one of two equilibrium networks was located in a more desirable neighborhood and was far more probable than the other.

Claim 2 suggested that greater metanetwork neighborhood desirability could make even a Pareto-inefficient equilibrium network the most probable. The demonstration of this claim was intended to address two possible alternative explanations for the results of Model 1: (1) that the earlier observed effects were somehow peculiar to the Burt Game and (2) that the equilibrium selection result from Model 1 was in fact a result of the Pareto efficiency of the more probable network. In the Valid Information Game, I found that a Pareto-inefficient equilibrium was the most probable because it was embedded in a more desirable neighborhood than the other equilibria.

These models were intended to illustrate the central claim of this paper: that a network will be probable to the extent that it is desirable to actors relative to other networks in its neighborhood and is located in a region of the metanetwork that is desirable to actors. Undesirable networks are unable to attract actors to switch to them, or if they do, it is only briefly in a path to a more desirable network. Networks located in undesirable regions of the metanetwork may never be found by actors.

FUTURE DIRECTIONS

The present analysis suggests the possible utility of a structure-centric view of social structure, where the researcher focuses on identifying the various factors promoting structural probabilities, while considering even structural forms that have not been observed. As opposed to earlier attempts to focus analysis on social structure that failed because they neglected the interplay of structure and agency (Elster, 1982; Coleman, 1990), the approach introduced here assumes that agents produce structural change. Nonetheless, the primary units of my analysis were structural forms. Some have argued that the distinguishing feature of sociology is the investigation of social structure. Given this, it may be useful to return social structure to the center of sociological analyses, but with a dual awareness of the agents below and the metastructure above.

It might prove advantageous in future modeling to conceptualize structural forms from a “selfish structure” perspective. It could be that social structural forms thrive upon agent carriers in a way analogous to Dawkins’ (1976) model of selfish genes thriving on human vessels.

Structural forms will be likely to occur to the extent that they can satisfy the agents within them.

To build from existing research, I have followed several assumptions from the network dynamics literature, including the assumption that actors are relatively myopic. In the models I assumed that actors only consider neighboring networks in deciding on a given link manipulation. It is entirely possible that real actors could look deeper into the metanetwork in making network manipulations (e.g., Ray et al., 2004; Page et al., 2005). It could be useful in future modeling to investigate whether actors empowered with the ability to look several transitions ahead, or with the ability to learn from previous tie changes, derive an advantage from these broader horizons. Such research could offer further support for the contention that metanetworks matter, and indicate the practical value of meta-strategies in dynamic settings.

Another assumption I have inherited from past work is that link changes are made one by one rather than actors reconfiguring multiple ties at once. If multiple changes are be made at once, conclusions with respect to equilibrium, efficiency, and accessibility may very well be quite different from those drawn here. Yet metanetworks would still exist under such conditions, they simply will take on a different form, as would their effects on network dynamics.

I have attempted to show a new way in which structural factors may shape the exercise of agency over structure. However, the structural factors that I investigated were of a different order than the effects of straightforward network structure previously studied. I investigated the effects of metastructure, specifically a network of networks, on network dynamics. Whereas network structure may affect individuals' attempts to change structure by constraining the information or resources available to actors, metanetworks affected the network dynamics in a very different way. Metanetworks structure the options available to locally-maximizing agents attempting to change their structure, affecting the likelihood of finding various structures along the way. The effects of metanetwork neighborhood desirability on the probability of networks indicate the need to take into account a broader structural context in modeling or predicting the path of network evolution.

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