Constrained Model Predictive Control using Kinematic Model of Vehicle Platooning in VISSIM Simulator

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Abstract—Driving Heavy Duty Vehicles (HDVs) as a platoon has potential to significantly reduce the fuel consumption, human labor and increase the safety. A suitable controller which can maintain the vehicle movement in a defined topology is essential for HDV platooning. This paper proposes a controller based on the combination of Constant Distance (CD) and Headway Time (HT) topologies using Model Predictive Control (MPC) for a longitudinal HDVs platoon. In addition to this, a MPC controller is compared with conventional PID (Proportional-Integral-Derivative) controller. The controller aims to maintain intervehicular distance and headway time between the vehicles for two cases, namely unconstrained and constrained optimization problem. The predictive control algorithm uses a kinematic model of vehicle platooning. A systematic handling of constraints yields significant improvements in the performance of the proposed MPC strategy over conventional PID controller. A road network of a U.S. freeway I5 has been built in VISSIM for the simulations. The results and discussions are at the end of the paper.

I. INTRODUCTION

The idea of heavy duty platooning is analogous to the concept of railways on a highway road networks. This is a promising solution to increase the safety on roads, reduce human labor and fuel cost. In the last decade, platooning was originally designed for Automated Highway System (AHS) and enables a number of vehicles to drive within a short, acceptable intervehicular distance. The improvements in wireless communication and vehicle control technology make platooning feasible for partially automated vehicles, such as Adaptive Cruise Control (ACC) and Cooperative Adaptive Cruise Control (CACC) vehicles [1]. While the technical feasibility of platooning has been analyzed worldwide under numerous projects, the implementation details of the platooning vary since different objectives and motivations are envisioned. The common part in all is that the platooning strategies depend on a proper controller to guarantee the following vehicles can track the first vehicle quickly without huge overshoot and oscillations [2]. Currently, there are many research works carried out in the design and implementations of controllers for platooning with different topologies [3], [4].

In ACC control, an individual vehicle has a sensor to measure the distance and apply the control strategy to itself in order to maintain the required distance and velocity. This is a decentralized approach and requires larger distance gap between the vehicles for safety aspect [5]. CACC control has shown improved dynamics by regularly monitoring the movement of the platoon vehicles using wireless communication. However, CACC will not be able to provide appropriate control action when the communication between the vehicles is affected severely due to congestion in the network or temporary disruptions due to surrounding road conditions and infrastructure [6]. Hence, a predictive technique plays a significant role in overcoming the effect of packet drops.

In the last few years, MPC schemes can be found in the literature [7], [8], [9], [10] for the design and implementation of ACC or CACC controller considering intersection collision avoidance and other vehicle interventions with V2X communication. CACC control aims for improved throughput and reduced fuel consumption [11]. The control objective is to maintain a desired intervehicular distance between the vehicles. Cruise control is a robust approach which overcomes the effects of packet drops. Also, MPC provides a prediction of the future desired acceleration for a horizon defined which is applied as feedforward action in the preceding vehicle. In this robust predictive design approach, a buffer is used to reduce the control errors during the time intervals of packet drops [12].

In general, MPC application maintains equal velocity for all the vehicles, while focusing on fuel saving [13]. Also, the constraints used to formulate the MPC scheme vary among different works. The proposed control method in this work is also based on MPC, for which the objective is to minimize the control errors over a prediction horizon, given certain constraints. The intended acceleration over a prediction horizon is determined by an MPC using a kinematic model of the platoon in VISSIM using combined CD and HT topologies. Here, VISSIM simulator is invoked with MATLAB and both of them communicate via VISSIM COM interface. Whereas, ACC and CACC have been modeled in Dynamic Link Library coded in C++ [14]. Further, simulation results show that MPC design improves the overall control performance of platoon.

The rest of the paper is organized as follows: Section II indicates the terminology and notation. Section III deals with the modeling and controller design. Section IV shows the simulation results using conventional PID controller and the MPC strategy with 14 vehicles platooning for a U.S. freeway I5 in VISSM. Section V shows the discussion on comparative results of PID controller, unconstrained MPC and constrained MPC. The conclusion and scope of future work are indicated in Section VI.
II. TERMINOLOGY AND NOTATIONS

A. Terminology for Topology

Currently, there exist two major topologies for the inter-vehicular distance, which is CD and HT topology. While the CD topology tries to maintain the same predefined fixed inter-vehicle distance irrespective of the velocity of vehicles, the HT topology tries to maintain a fixed time gap between two adjacent vehicles. That is, the inter-vehicular distance is time variant and a function of current velocity. Recent researchers are using only one of them as the space policy, however, both of them have some advantages and disadvantages. In this paper, a combined CD and HT topology is used to make sure that a considerable gap is maintained at high speed and also the collision of adjacent vehicles is avoided when the speed is slow.

B. Notation for the Platoon

The platoon considered in this paper is formed with \( n \) identical vehicles which can share their position, velocity and acceleration with each other moving in a longitudinal path. The first vehicle which is driven by human and is named the header shown in Fig. 1. The goal of platooning is to make the following unmanned vehicles in the platoon track the velocity of the header with the intervehicular distances being maintained at a certain value. The entire platoon is divided into some same sub-systems which consist of three vehicles.

The three vehicles in the sub-systems are the \( l^{th}, (j-1)^{th}, \) and \( j^{th} \) vehicle in the platoon and for convenience, they are named the header, the leader, and the follower respectively. Hence, the control objective is transferred to make the leader track the velocity of the header and the follower track the velocity of the leader, when the distance between each two of them is maintained according to the CD and HT topology. In the platoon \( x_i^l, v_i^l, \) and \( a_i^l \) represent the position, velocity and acceleration of the \( i^{th} \) vehicle at time \( k \) in the platoon respectively.

III. MODELLING AND CONTROLLER IMPLEMENTATION STRATEGIES

A. Conventional PID Controller

A conventional PID controller can be formulated as:

\[
u(t) = k_p e(t) + k_i \int_0^t e(t) + k_d \frac{de(t)}{dt},\]

where \( k_p, k_i, \) and \( k_d \) represent the proportional, integral and derivative gain of the system and \( e(t) \) is the error signal. Using the concepts mentioned in [5], the force required for moving a platoon of vehicle can be considered as:

\[
m(u) = F_x - mg \sin \theta - f_x mg \cos \theta - \frac{1}{2} C_{\text{air}}(u + u_w).
\]

where \( m \) is the mass of the vehicle, \( u \) is the forward velocity, \( F_x \) is the tractive force, \( f_x \) is the fraction coefficient of the road, \( g \) is the acceleration due to the gravity, \( \theta \) is the angle of inclination, \( C_{\text{air}} \) is the coefficient of the drag force due to the air and \( u_w \) is the velocity of the wind. Here, \( u_w \) is assumed to be zero. Considering the sub-system in the platoon, assume \( a_j = 0 \), the control law for obtaining the desired acceleration of the follower in the subsystem is given by:

\[
a_f = \frac{k_i(x_h - x_j - hd_1 - T_1 v_f + x_l - x_f - hd_2 - T_2 v_f)}{2d} + \frac{k_p(v_h - v_f + v_l - v_f) + k_d(a_h + a_l)}{C_{\text{air}}v^2 + 2k_d}.
\]

where \( hd_1 \) and \( hd_2 \) are the desired constant distances between the header and the leader and the desired distances between the leader and the follower. \( T_1 \) and \( T_2 \) are the headway time between these above vehicles.

B. MPC Strategy

In this section, a kinematic model [15] for the design of MPC controller is given first by:

\[
\begin{align*}
x_{k+1}^l & = x_k^l + v_k^l T + \frac{1}{2} a_k^l T^2, \\
v_{k+1}^l & = v_k^l + a_k^l T.
\end{align*}
\]

where \( T \) is the sampling time.

The task of the platoon is to track the speed of the header, meanwhile maintaining the intervehicular distances according to the CD and HT topology. Formulating the state vector using the error of intervehicular distance and velocity and taking the acceleration as the input of each state vector using the error of intervehicular distance and acceleration can be considered and considering the constraints on the velocity is infeasible. In this paper, to reduce the dimension of the state vector, the vector is formed with the information of three vehicles only. That is, the position and velocity of the header, the \( (j-1)^{th}, \) and \( j^{th} \) acceleration of the header. And the input is the acceleration of \( (j-1)^{th}, \) and \( j^{th} \). This platoon control scheme with MPC for a three-vehicle subsystem is illustrated in Fig. 2.

Now, considering a \( n \) vehicles platoon, no extension is needed for the state vector or the input vector. Also, all the information needed is explicit in the state vector so that the constraints on the velocity and acceleration can be performed.

The discrete-time model formed is given by:

\[
X_{k+1} = AX_k + Bu_k - 1 + B \Delta u_k.
\]

\[
Y_k = \begin{bmatrix} x_{k+1}^l - x_k^l \\ v_{k+1}^l - v_k^l \\ a_{k+1}^l - a_k^l \end{bmatrix} = CX_k.
\]
where:

\[
A = \begin{bmatrix}
1 & T_s & 0 & 0 & 0 & 0 & 1/2 T_s^2 \\
0 & 1 & 0 & 0 & 0 & 0 & T_s \\
0 & 0 & 1 & T_s & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}, \quad
B = \begin{bmatrix}
0 & 0 \\
1/2 T_s^2 & 0 \\
0 & T_s \\
0 & 0 & 0 & 1/2 T_s^2 \\
0 & T_s \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0
\end{bmatrix},
\]

\[
X = \begin{bmatrix} x_k^1 & v_k & x_{k-1}^j & v_{k-1}^j & x_j^0 & v_j^0 & a_k^1 \end{bmatrix}^T,
\]

\[
u_k = a_k^{-1} a_j^0, \quad u_k = u_{k-1} + \Delta u_k.
\]

Using the formulated discrete-time model, the predictions after \(N\) steps are made [17]. Also, here only one-step control is considered, that is \(\Delta u_{k+n} = 0(n > 0)\). Hence, the prediction of \(N\) prediction horizon is given by:

\[
\hat{Y} = \begin{bmatrix} Y_{k+1}^1 \\ Y_{k+2}^1 \\ \vdots \\ Y_{k+N}^1 \end{bmatrix} = \Phi X_k + \Gamma u_{k-1} + \Gamma \Delta u_k.
\]

Then, the reference \(R\) can be generated based on different topology. For CD and HT topology used in the paper, \(R\) is given by:

\[
R = \begin{bmatrix} \hd_1 \ast (j-2) & \hd_2 & 0 & 0 & \cdots & 0 \end{bmatrix}^T.
\]

where:

\[
\hd_1 = \hd + v_{k-1}^j, \quad \hd_2 = \hd + v_{k}^j T.
\]

Then according to the control goal, a cost function is formulated as:

\[
J = (\hat{Y} - R)^T (\hat{Y} - R) + \lambda \Delta u_k^T.
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\[
J = (\hat{Y} - R)^T (\hat{Y} - R) + \lambda \Delta u_k^T.
\]

where \(\lambda\) is the tuning parameter of chosen value 0.1. The optimal solution is obtained by minimizing the cost function \(J\). For an unconstrained MPC, the optimal input can be obtained by calculating the extreme point of the convex cost function \(J\), which is given by:

\[
\frac{\partial J}{\partial \Delta u} = 2(\Gamma^T \Gamma + \lambda I) \Delta u + 2 \Gamma^T (\Phi X_k + \Gamma u_{k-1} - R) = 0.
\]

Hence the control law to obtain the optimal control action, namely the desired acceleration is given by:

\[
\Delta u_k = (\Gamma^T \Gamma + \lambda I)^{-1} \Gamma^T (R - \Phi X_k - \Gamma u_{k-1}) = [\Delta a_{j-1} \Delta a_j]^T.
\]

For a constrained MPC, quadratic programming is used [18]. The constraints are on the velocity and acceleration due to the mechanism of the vehicle, which is represented by:

\[
v_{\text{low}} \leq v \leq v_{\text{up}} \quad (\text{kmph}), \quad 0 < v_{\text{low}} \leq v_{\text{up}},
\]

\[
-a_{\text{low}} \leq a \leq a_{\text{up}} \quad (\text{m/s}^2), \quad a_{\text{low}} \geq 0, a_{\text{up}} \geq 0.
\]

Minimizing a quadratic cost function \(J = \frac{1}{2} x^T H x + f^T x\), subject to linear constraints \(A^T x \leq b\) is a quadratic programming issue, which can be solve by using the MATLAB function \textit{quadprog}(H,f,A,b) [19].

To use this function the cost function of the MPC controller is transferred into the standard form:

\[
J = \frac{1}{2} \Delta u^T H \Delta u + f^T \Delta u.
\]

where:

\[
H = 2 * (\Gamma^T \Gamma + \lambda I), \quad f = 2 * \Gamma^T * (\Phi * X_k + \Gamma u_{k-1} - R).
\]

The linear constrains of velocity and acceleration should also be represented in the standard form like \(A^T \Delta u \leq b\). Here, a matrix \(C^*\) is introduced to formulate these constrains in the standard form:

\[
C^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
A^* \text{ and } b \text{ are obtained as:}
\]

\[
A^* = \begin{bmatrix} C^* B \\ -C^* B \\ \vdots \\ -\sum_{i=1}^{N} C^* A_i^{i-1} B \\ \sum_{i=1}^{N} C^* A_i^{i-1} B \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix},
\]

\[
b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}.
\]
The control law for a constrained MPC is formulated as:
\[ \Delta u = \text{quadprog}(H, f, A^*, b) = [\Delta a_1^{-1} \Delta a^T]. \]

Simulating the proposed MPC controller iteratively on the subsystems, the desired accelerations of each leader and follower are obtained, however, it should be noticed that only the desired acceleration \( \Delta a_j \) is applied to the \( j \)th vehicle and \( \Delta a_j^{-1} \) are discarded.

**IV. SIMULATION AND RESULTS**

**A. Simulation Parameters**

Simulations are conducted using MATLAB and VISSIM 8 to test the performance of both the PID controller and the MPC controller strategies. A road network of a U.S. freeway I5 (Fig. 3) is built in VISSIM 8 and an HDV platoon of 14 vehicles are generated on this road. The simulation consists of testing the performance of PID controller, unconstrained MPC and constrained MPC. The parameters used in the implementation of PID controller are listed in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters Used In PID Controller</th>
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<tbody>
<tr>
<td>Symbols</td>
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</tr>
<tr>
<td>Sampling time</td>
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<tr>
<td>Aerodynamic coefficient</td>
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<tr>
<td>Constant intervehicular gap</td>
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<tr>
<td>Headway time</td>
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<tr>
<td>Proportional gain</td>
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<tr>
<td>Integral gain</td>
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<tr>
<td>Derivative gain</td>
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</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Speeds and Interverhicular Distance for a Headway Time of 0.3 s</th>
</tr>
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<tbody>
<tr>
<td>Speed (in kmph)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>70</td>
</tr>
</tbody>
</table>

**B. Simulation Results of conventional PID controller**

The speed, intervehicular distances, and acceleration profiles of each vehicle in the platoon are shown in Fig. 4 to Fig. 6 respectively.

**Fig. 4.** Speed profile for 14-vehicle platoon in U.S. I5 road network.

From Fig. 4 and Fig. 5, it is seen that the tracking capabilities of speed and intervehicular gap respectively are good in terms of error minimization, if the PID controller is well tuned. However, in the acceleration profile, some oscillation happens and cannot be controlled because of no information on the future states of the system.

**C. Simulation Results of MPC controller**

The predictive algorithm was tested for an unconstrained case with analytic solution and a constrained case using a headway time of 0.3 s, the desired intervehicular distances which the vehicles should maintain during platooning are given in Table II.
quadratic programming respecting to the given constraints. The typical MPC behavior are analyzed with the influence of tuning parameters, mainly length of the horizon on the control response. For the unconstrained MPC with a prediction horizon $N$ of 10, the similar tracking performance is obtained for speed and intervehicular distances when compared with conventional PID controller. But, an overall improvement in the acceleration plot is noted from the profile. It is observed that the oscillation is reduced in the acceleration profile in comparison to PID controller. Further, the efficacy of the predictive algorithm is tested for an increased horizon $N$ of 50. The tracking performance remains the same, however since more horizon is given, the desired acceleration for controlling the platoon is reduced. The acceleration profile of constrained MPC with a prediction horizon $N$ of 10 and proper constraints are shown in Fig. 7.

As the acceleration of the platoon vehicles varies, it is seen that the desired acceleration is fixed in the range and no huge oscillation occurs, which means an overall improvement is performed by the optimization technique.

Following the controllers design, the testing is performed with various setpoint inputs. Table III shows the performance in speed and spacing errors of platoons using metrics like percentage overshoot and IAE, ISE, MSE defined by:

\[
IAE = \int_0^{\infty} |e(t)| \, dt. 
\]

\[
ISE = \int_0^{\infty} [e(t)]^2 \, dt. 
\]

\[
MSE = \frac{1}{N_t} \int_0^{\infty} [e(t)]^2 \, dt. 
\]

where $e(t)$ is the error signal and $N_t$ is the total simulation time.

**V. DISCUSSION**

A well-tuned PID controller is capable of tracking the speed reference and maintaining the desired intervehicular distances with less computational time. However, the tuning process can be tedious, since there are three gain parameters. Also, the usage of constraints on process variables cannot be performed in the design of PID controller. After each simulation, the violation of saturation limits has to be checked manually. Unconstrained MPC controller has the similar constraint issues when compared with conventional PID controller.
Whereas, if a communication failure happens then the controller can still give a proper control action to the vehicles because of future predicted steps. Hence, an unconstrained MPC strategy gives more robustness. In general, tuning process for the MPC controller is much easier because only the prediction horizon and the tuning parameter in the cost function need to be tuned. The simulation results show that the performance of the unconstrained MPC can be guaranteed even though no fine tuning is conducted. A drawback of unconstrained MPC is that the solving process is computationally extensive. A high-performance computer is needed when the sampling frequency is high. Constrained MPC has similar advantages and disadvantages as that of an unconstrained MPC. An obvious improvement is that the constraints can be considered by the controller automatically. The controller is able to limit the process variables according to the set constraints. Also, MPC starts adjusting the control signal ahead of reference changes, while PID cannot start before. It is also seen that control responses are more sluggish and less stable if the control horizon is lower. It can be guaranteed even though no fine tuning is conducted. A high-performance computer is needed when the sampling frequency is high. Constrained MPC has similar advantages and disadvantages as that of an unconstrained MPC. An obvious improvement is that the constraints can be considered by the controller automatically. The controller is able to limit the process variables according to the set constraints. Also, MPC starts adjusting the control signal ahead of reference changes, while PID cannot start before. It is also seen that control responses are more sluggish and less stable if the control horizon is small. Hence, an improvement of the overall performance is shown by considering the constraints systematically.

VI. CONCLUSIONS AND FUTURE WORK

This paper analyzes the feasibility of using PID controller to control an HDVs platoon with the help of simulation and proposes an MPC controller for the constrained issues. When the platoon moves at a speed which is far away from the constraints, conventional PID controller can be used for a higher communication frequency and unconstrained MPC can be used for a lower communication frequency. Doing so, the control objectives are achieved with a minimal cost. When the platoon moves near to either maximum or minimum speed limit, a constrained MPC should be used to ensure the speed does not violate the constraints and also guarantees the safety in case of communication failures. A quantitative comparison between spacing errors and velocities of first and the last vehicle of the platoon is performed for metrics mainly, ISE, IAE, MSE, and percentage overshoot. Similar results are examined for all the controllers tested and the overall performance of MPC is satisfactory when compared with conventional PID controller. The future work can be conducted on testing a shiftable hybrid-controller, which consists of PID, unconstrained MPC and constrained MPC for different conditions. All the conditions given here are qualitative. More tests are needed to have quantitative results. To verify the robustness, a simulation that can include the effects of the communication failure should be carried out. Also, an effective simulation environment for prediction technique can be performed to test the platoon maneuvers which includes splitting, merging and lane changing.

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