Dynamic Traffic Assignment: History, Recent Results and Unanswered Questions

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Outline

- What is dynamic traffic assignment (DTA)?
- Why study DTA?
- Dynamic user equilibrium and its limitations
- Urban freight networks and city logistics
- Coupling to other-than-transport infrastructure networks

Primary Goal of this Talk

- Provide an introduction to a “new” branch of mathematical modeling and numerical computation, called dynamic traffic assignment (DTA), that holds the promise of mitigating congestion on urban road networks.

Other Goals of This Talk

- To refine your understanding of dynamic traffic assignment,
- To show some of the reasoning that goes into the construction of dynamic user equilibrium models
- To present illustrative calculations
- To acknowledge commercial ventures to provide DTA support to metropolitan planning organizations (MPOs)

Still Other Goals, if Time Permits

- To describe how DTA is pertinent to the study urban freight and city logistics.
- To show how DTA plays a role in congestion pricing, commodity pricing and mechanism design.

What is Dynamic Traffic Assignment (DTA)?

- From Wikipedia:

“**Dynamic Traffic Assignment (DTA)** is widely understood to be the prediction of time-varying vehicular traffic flows on an urban network, in a way that is consistent with traffic flow theory and travel demand theory.”

International DTA Meetings

- ORSA/TIMS 1980 Washington, DC (The first DTA paper session?)
- 12th ISTTT 1993 Berkeley: first time there were lots of DTA papers
- DTA2006 Leeds University, England
- DTA2008 Katholieke Universiteit of Leuven, Belgium
- DTA2010 Takayama, Japan
- DTA2012 Martha's Vineyard, USA
- DTA2014 Salerno, Italy
- DTA2016 Sydney, Australia
- DTA2018 Hong Kong, China
- DTA2020 Seattle, US (hopefully!)

A Different Style of Presentation

- I am a game theorist who is also an expert in very large scale computation.
- So my presentation will be a bit theoretical.
- Emphasis on key ideas, expressible via mathematics.
- However, I will be giving a verbal explanation of everything.
- So you may ignore the mathematics !

What is a Mathematical Game?

- A game is a competition among agents to obtain a payoff according to a set of rules.
- A game becomes mathematical when equations, inequalities and extrema are used to express it.
- We will look at dynamic traffic assignment (DTA) from the perspective of differential game theory (also called dynamic game theory).

The Game Theorist's Approach

- What are the inherent types of competition that characterize a game of interest?
- How may that competition may be expressed mathematically?
- From that mathematical representation, how may the game of interest be studied qualitatively and quantitatively?

Definition of DTA Repeated

- From Wikipedia:

Dynamic Traffic Assignment (DTA) is widely understood to be the prediction of time-varying vehicular traffic flows on an urban network, in a way that is consistent with traffic flow theory and travel demand theory.

- The notion of prediction used here is <u>not</u> statistical inference.

Equilibria, Topological Network Design and Mechanism Design

- The equilibria we discussed arise as solutions of games.
- Topological design concerns links and nodes.
- Mechanism design is intervention wherein players are given incentives to approach the welfare optimum.
- Congestion pricing is a type of mechanism design involving potentially dynamic tolls

Game-Theoretic Equilibrium

- Nash equilibrium: each player maximizes utility assuming the strategies of other players are fixed
- User equilibrium: a feasible flow pattern for which no driver may decrease his/her travel cost by changing routes
- Atomic and Non-Atomic forms of the above

Presentation Plan for Game-theoretic DTA

- The flow of the subsequent material is really quite simple; it will be something like this:

  Drivers selfishly optimize own delay
  $\Longrightarrow$ drivers are agents in a noncooperative game
  $\Longrightarrow$ models as equations and inequalities
  $\Longrightarrow$ equations and inequalities are manipulated
  $\Longrightarrow$ recognizable problem categories (to be named)
  $\Longrightarrow$ numerical and qualitative analyses
  $\Longrightarrow$ understanding, vetting and solution of the models.

Congestion Games

- These games see agents as fully noncooperative.
- They may be formulated for static as well as dynamic instances of traffic assignment.
- Let's tell the "story" of user equilibrium …….
- A very important feature of congestion games is the "Price of Anarchy."

Static User Equilibrium, Slide 1

- The simplest network equilibrium is user equilibrium:

$$h_p > 0,\ p \in P_{ij} \Rightarrow c_p = u_{ij}$$

$$c_p > u_{ij},\ p \in P_{ij} \Rightarrow h_p = 0$$

Static User Equilibrium, Slide 2

- Equivalent Variational Inequality (VI):

find $h^* \in \Lambda$ such that

$$\sum_{a \in A} c_p(h^*)(h_p - h_p^*) \geq 0 \text{ for all } h \in \Lambda$$

Static User Equilibrium, Slide 3

- Static user equilibrium (UE) is one type of “congestion game”.
- Dynamic user equilibrium (DUE) is one type of dynamic congestion game.
- Both are games in so-called “normal form” as opposed to “extensive form.”

Static User Equilibria, Slide 4

- Static UE has recently enjoyed a revival as a topic of scholarly inquiry.
- Revival is due to popularity of vehicular traffic analogies in theoretical computer science begun by Roughgarden and Tardos (2002).

Dynamic User Equilibrium: General Remarks

- Path delay operator
- Schedule delay (arrival penalty)
- Effective delay = path delay + arrival penalty
- The delay operator is described by an embedded submodel known as the dynamic network loading (DNL) problem.
- The DNL problem is usually difficult to solve

Dynamic User Equilibrium, Slide 1

- So-called schedule delay:

$$F\left[t + D_p(t,h) - T_A\right]$$

- The effective unit path delay operator:

$$\Psi_p(t,h) = D_p(t,h) + F\left[t + D_p(t,h) - T_A\right] \quad \forall p \in P$$

Dynamic User Equilibrium, Slide 2

- The DNL problem may be formulated in various ways.
- One way is via a complex simulation model.
- Another is through network traffic flow theory.
- It is widely held that DNL should be based in some way on hydrodynamic traffic flow theory. Why?

Dynamic user equilibrium, Slide 3

- Trip matrix: each cell contains the volume of traffic between a given origin and a given destination
- Flow conservation: for every origin-destination pair what departs must ultimately arrive.

Dynamic User Equilibrium, Slide 4

The trip matrix

$$Q = \left(Q_{ij} : (i,j) \in \mathcal{W} \right)$$

$$\begin{aligned} Q_{ij} &\in \Re^1_{++} \text{ fixed travel demand for OD pair } (i,j) \in \mathcal{W} \\ \mathcal{W} &= \text{the set of all origin-destination pairs} \end{aligned}$$

Flow conservation

$$\sum_{p \in \mathcal{P}_{ij}} \int_{t_0}^{t_f} h_p(t)\, dt = Q_{ij} \quad \forall (i,j) \in \mathcal{W}$$

$\mathcal{P}_{ij}$ = subset of paths that connect origin-destination pair $(i,j) \in \mathcal{W}$.

Dynamic User Equilibrium, Slide 5

$$\Lambda_F = \left\{ \begin{array}{c} h \geq 0 : \frac{dy_{ij}}{dt} = \sum_{p \in P_{ij}} h_p(t), \; y_{ij}(t_0) = 0, \; y_{ij}(t_f) = Q_{ij} \\ \forall (i,j) \in \mathcal{W} \end{array} \right\}$$

Definition

Dynamic user equilibrium $DUE(\Psi, \Lambda_F, t_0, t_f)$. A vector of departure rates (path flows) $h^* \in \Lambda_F$ is a dynamic user equilibrium if

$$h_p^*(t) > 0, p \in P_{ij} \Longrightarrow \Psi_p[t, h^*(t)] = \min_h \Psi_p[t, h(t)] = v_{ij}$$

Dynamic User Equilibrium, Slide 6

- With some algebra aided by simple calculus the DUE problem is easily restated as a system of inequalities constrained by flow conservation and the physical requirement that flows of vehicles can never be negative.
- That restatement is called a variational inequality.
- Variational inequalities enjoy a rich literature.

Differential Variational Inequality for DUE

- $DUE\,(\Psi, \Lambda_F, t_0, t_f)$ is equivalent to the following variational inequality (VI) under mild regularity conditions:

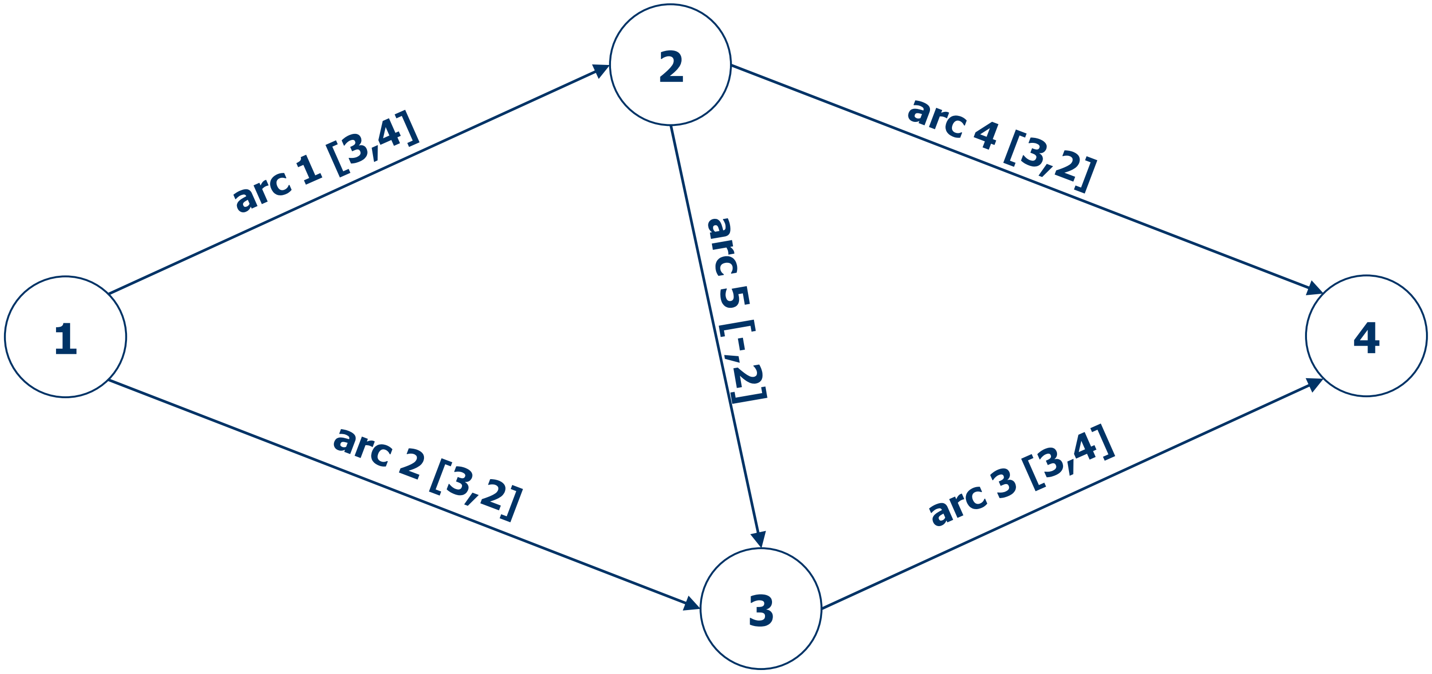
$$\left.\begin{array}{c} \text{find } h^* \in \Lambda_F \text{ such that} \\ \sum_{p\in\mathcal{P}} \int_{t_0}^{t_f} \Psi_p(t, h^*)(h_p - h_p^*)\,dt \geq 0 \\ \forall h \in \Lambda_F \end{array}\right\}$$

The Braess Paradox

- Adding a link can increase congestion.
- Increasing capacity can increase congestion.
- Deleting a link can lessen congestion.
- Decreasing capacity can decrease congestion.
- Modeling and computational implications are <u>very</u> significant.

How? Why?

Example of the Braess Paradox: Arc 5 is Added (LeBlanc, 1973)



[INITIAL FLOW, SUBSEQUENT FLOW]

Example of the Braess Paradox: Numerical Solutions

- Unit Arc Costs

$$A_1 = 40 + 0.5(x_1)^4$$

$$A_2 = 185 + 0.9(x_2)^4$$

$$A_3 = 40 + 0.5(x_3)^4$$

$$A_4 = 185 + 0.9(x_4)^4$$

$$A_5 = 15.4 + (x_5)^4$$

- Result $TotalCost_{Before} = 2030.4 \leq TotalCost_{after} = 2204.4$

The Equilibrium Network Design Problem (ENDP), Slide 1

- Minimize congestion (flow, capacity enhancements)
- Subject to constraints
  - Budget constraint
  - Nonegativity (of capacity enhancements)
  - Flows must obey user equilibrium (non-atomic Nash equilibrium)

The Equilibrium Network Design Problem (ENDP), Slide 2

- The ENDP is a non-convex, bi-level mathematical program.
- It is a specific instance of a mathematical program with equilibrium constraints (MPEC).
- It is a Stackelberg game.
- It is very hard to solve.

Some Comments About the Braess Paradox

- Not really a paradox.
- It <u>may but does not have to occur</u> on any network with congestion externalities.
- Its reliable detection without first solving the equilibrium network design problem (ENDP) is an unsolved problem.
- The “Price of Anarchy” is a partial alternative that can lower computational burden.

The Price of Anarchy (Inefficiency)

- The price of anarchy (Koutsoupias and Papadimitriou, 1999; Papadimitriou, 2001; Roughgarden, 2002; Roughgarden and Tardos; 2002) is the ratio of the worst case equilibrium cost to the system optimal cost:

$$\rho = \frac{\max\left\{\text{cost of equilibrium flow } f^e : e \in S\right\}}{\text{cost of optimal flow } f^*}$$

$S =$ the set of all equilibrium flow patterns

Bounding the Price of Anarchy in Congestion Games

- Non-Atomic Congestion Games Based on User Equilibrium (Han et al, 2008; Roughgarden, 2007)
- Positive, monotonic arc costs
- Regularity Condition:

(potential function) x (constant) $\geq$ (system optimal costs)

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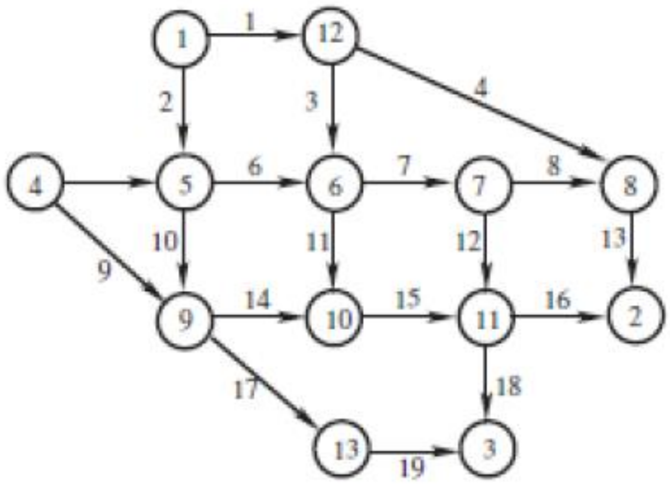
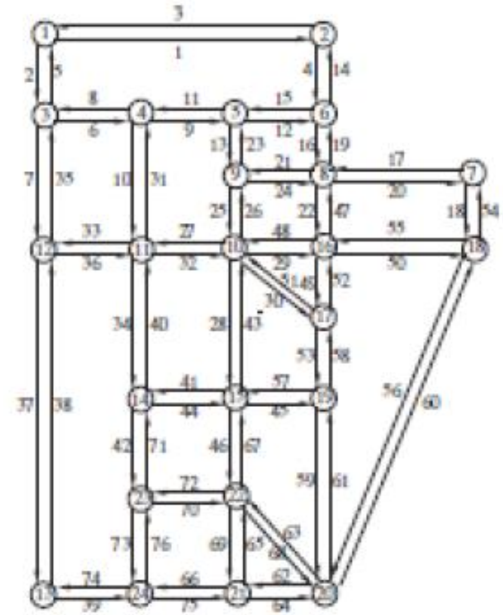
Incomplete Aspects of the Congestion Games Literature

- DTA version of the theory, especially the price of anarchy, is woefully incomplete.
- For non-separable transport cost functions, no potential function exists.
- Exclusively a non-commodity point of view.
- Variational inequality representations needed for realistic commodity networks.

Some Example Networks, Slide 1

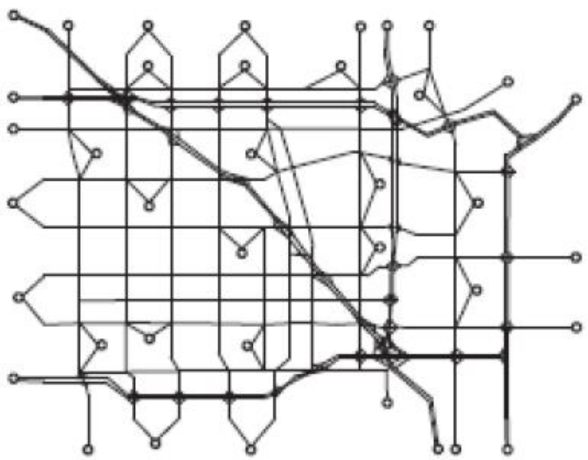


Nguyen network
(13 nodes, 19 links, 4 zones)



Sioux Falls network
(24 nodes, 76 links, 24 zones)

Some Example Networks, Slide 2



Anaheim network
(416 nodes, 914 links, 38 zones)

Chicago sketch network
(933 nodes, 2950 links, 387 zones)

DUE and Other DTA Algorithms

- Fixed point
- Gap function
- Proximal point
- Computational intelligence
- Statistical learning

Computational Burden

	Nguyen network	Sioux Falls	Anaheim	Chicago Sketch
No. of iterations	75	43	28	42
Computational time	2.2s	246s	761s	8543s
Avg. time per DNL	0.04s	4.0s	23.3s	111.2s
Avg. time per FP update	0.006s	1.4s	3.1s	59.1s

8543 seconds = 2.373 hours

Achieved with a single-processor desktop, without sophisticated programming and based on LWR DNL

Status of Dynamic Traffic Assignment

- As a research community, DTA scholars are ready to attempt solving for the time-varying flows of a major metropolitan region.
- The DUE model presented is presently appropriate for a deliberate planning by MPOs.
- Large-scale computation still remains to be demonstrated. My prediction is ……….

Regional and Urban Freight Congestion Games

- Generalization of Network Spatial Price Equilibrium

- Inter-Regional and Urban Congestion Games involving commodities **AND** information technology
- Although the flows calculated are commodity flows, they nonetheless must be assigned.

Simultaneous Conversion of All Northeast Powerplants to Coal

- Original sponsor: U.S. DOE
- Context: energy crisis of the 1970s.
- Was there enough rail capacity to move coal supplies or would other goods be "crowded out"?
- Model was expanded under other sponsors (US AID, World Bank) to encompass all freight modes and 20 commodities.
- Applied to study diverse issues in India, the Levant, Chile, former USSR and Africa (1979 to 2005).

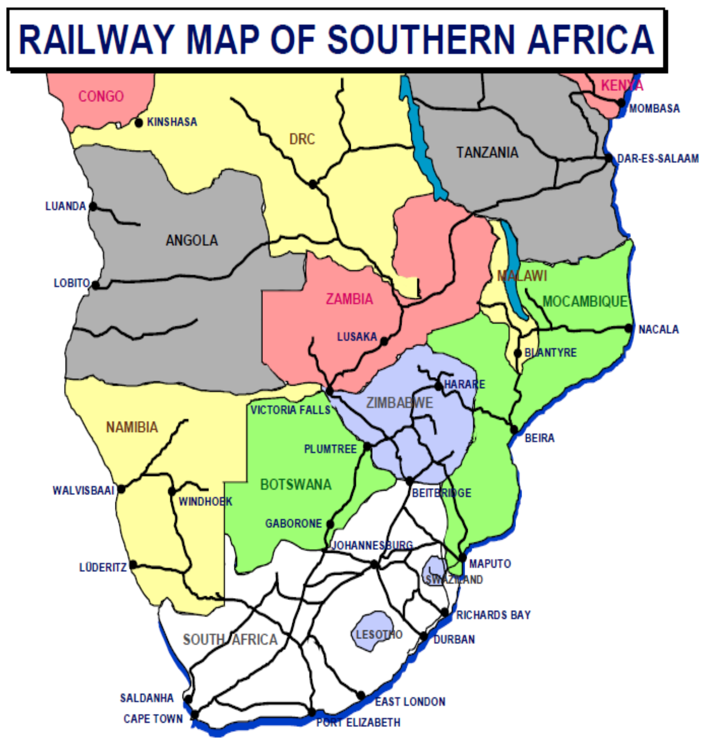
Northeast Coal Transport Network

- Rail network for conversion of powerplants to coal (energy crisis of the 1970s):

Transportation Extortion and Famine Relief in Southern Africa (1984 to 1992)

- Rolling stock owned by the Republic of South Africa
- Rolling stock “withheld” to avoid undesirable political outcomes
- Moral and humanitarian issues
- Network: from sea routes to dirt paths = some of the largest congestion games involving commodity flows ever considered

Southern Africa Rail Extortion



Freight Network Equilibrium, S 1

- Behavior is more complex.
- Shipper-carrier dichotomy (Friesz et al, 1981)

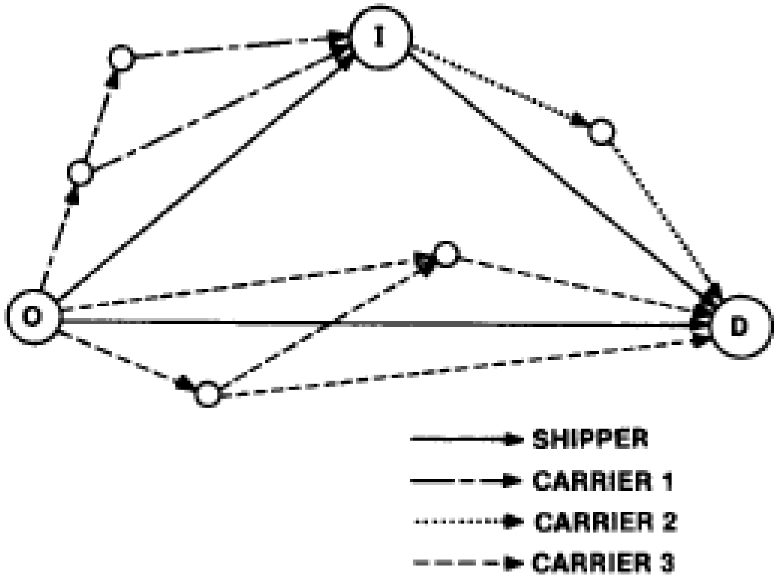


FIGURE 1.: Relationship of Shippers' and Carriers' Networks.

Freight Network Equilibrium, S. 2

- Freight Network is behaviorally more complex, involving multiple shippers, multiple carriers, and multiple modes.
- Freight network equilibrium is notationally complex.
- In this talk, we restrict ourselves to shipper equilibrium.
- The dynamics are relatively easy.

Static Spatial Computable General Equilibrium

- Friesz et al (1998)
- Computable general equilibrium with an embedded freight model
- Explicit treatment of freight networks allows many policies to be studied. There may be congestion but it is not necessarily the most significant.
- Other externalities, beyond congestion: such as vibration and hazardous spills.

Static Spatial Price Equilibrium (1)

- If flow occurs between a given OD pair then delivered price equals local price
- If delivered price exceeds local price for a given OD pair, then path flow is zero

Static Spatial Price Equilibrium (2)

$$h_p > 0,\ p \in P_{ij} \Rightarrow \pi_i + c_p = \pi_j$$

$$\pi_i + c_p > \pi_j,\ p \in P_{ij} \Rightarrow h_p = 0$$

Spatial Oligopolistic Network Competition (2)

- Nash game: each firm maximizes profit
- Decision variables: consumption output, shipments
- Constraints: flow conservation, non-negativity
- Variational inequality representation
- This problem is computable and **very well understood** (see Friesz et al, 2006).

Spatial Oligopolistic Network Competition (3)

- Oligopolistic equilibrium can be computed efficiently.
- Price of anarchy: requires a new approach since there is no potential function.
- **Almost no associated mechanism design literature** for commodity flows stemming from oligopolistic network equilibrium.

Dynamic Oligopolistic Network Competition (1)

- Friesz et al, 2006
- The state dynamics

$$\frac{dI_i^f}{dt} = q_i^f + \sum_{j \in N_f} \sum_{p \in P_{ji}} h_p^f - \sum_{j \in N_f} \sum_{p \in P_{ij}} h_p^f - c_i^f$$

Dynamic Oligopolistic Network Competition (2)

- Objective of a typical firm f:

$$\Phi_f(c^f, q^f, s^f; c^{-f}, q^{-f}) = \int_{t_0}^{t_f} e^{-\rho t} \left\{ \sum_{i \in \mathcal{N}} \pi_i \left(\sum_{g \in \mathcal{F}} c_i^g, t \right) c_i^f - \sum_{i \in \mathcal{N}_f} V_i^f(q^f, t) \right.$$

$$\left. - \sum_{w \in \mathcal{W}_f} r_w(t) s_w^f - \sum_{i \in \mathcal{N}_f} C_i^f(k_i) - \sum_{i \in \mathcal{N}_f} \psi_i^f(I_i^f, t) \right\} dt$$

Note collaboration costs

Dynamic Oligopolistic Network Competition: DVI

find $(c^{f*}, q^{f*}, s^{f*}) \in \Omega$ such that

$$0 \geq \sum_{f \in \mathcal{F}} \int_{t_0}^{t_f} \left[\sum_{i \in \mathcal{N}_f} \frac{\partial \Phi_f^*}{\partial c_i^f} \left(c_i^f - c_i^{f*} \right) + \sum_{i \in \mathcal{N}_f} \frac{\partial \Phi_f^*}{\partial q_i^f} \left(q_i^f - q_i^{f*} \right) \right.$$

$$\left. + \sum_{w \in \mathcal{W}_f} \frac{\partial \Phi_f^*}{\partial s_w^f} \left(s_w^f - s_w^{f*} \right) \right] dt \quad \text{for all} \quad (c, q, s) \in \Omega$$

This constraint set includes inventory and collaboration dynamics

The End