Second-Order Asymptotics of Universal JSCC for Arbitrary Sources and Additive Channels

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Joint work with Vincent Tan and Mehul Motani

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JSCC of Transmitting a GMS over an AWGN Channel

\[ S^k \xrightarrow{f} X^n + Y^n \xrightarrow{\phi} \hat{S}^k \]

- \( S^k \) i.i.d. \( \sim \mathcal{N}(0, \sigma^2) \) and \( Z^n \) i.i.d. \( \sim \mathcal{N}(0, 1) \);
S^k \text{ i.i.d. } \sim \mathcal{N}(0, \sigma^2) \text{ and } Z^n \text{ i.i.d. } \sim \mathcal{N}(0, 1);

An \((k, n, P)\)-code consists of one encoder \( f : S^k \rightarrow X^n \) and one decoder \( \phi : Y^n \rightarrow \hat{S}^k \) s.t. \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq P; \)
JSCC of Transmitting a GMS over an AWGN Channel

- $S^k$ i.i.d. $\sim \mathcal{N}(0, \sigma^2)$ and $Z^n$ i.i.d. $\sim \mathcal{N}(0, 1)$;
- An $(k, n, P)$-code consists of one encoder $f : S^k \rightarrow X^n$ and one decoder $\phi : Y^n \rightarrow \hat{S}^k$ s.t. $\frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq P$;
- Non-asymptotic fundamental limit

$$k^*(n, P, \epsilon, D)$$

$$:= \sup\{k : \exists \text{ an } (k, n, P)-\text{code s.t. } \Pr\{d(S^k, \hat{S}^k) > D\} \leq \epsilon\}.$$
Second-Order Asymptotics

- Kostina and Verdú\(^1\) showed that for any \( \varepsilon \in [0, 1) \),

\[
k^*(n, \varepsilon, D) = \frac{nC(P)}{R(\sigma^2, D)} - \sqrt{\frac{n(R(\sigma^2, D)V(P) + C(P)V(\sigma^2, D))}{(R(\sigma^2, D))^3}} Q^{-1}(\varepsilon) + O(\log n),
\]

where

\[
C(P) = \frac{1}{2} \log(1 + P), \quad R(\sigma^2, D) = \frac{1}{2} \log \frac{\sigma^2}{D},
\]

\[
V(P) = \frac{P(P + 2)}{2(P + 1)^2}, \quad V(\sigma^2, D) = \frac{1}{2}.
\]

Nearest Neighbor Decoding for Additive Non-Gaussian Noise Channels

Amos Lapidoth, Member, IEEE

Abstract—We study the performance of a transmission scheme employing random Gaussian codebooks and nearest neighbor decoding over a power limited additive non-Gaussian noise channel. We show that the achievable rates depend on the noise distribution only via its power and thus coincide with the capacity region of a white Gaussian noise channel with signal and noise power equal to those of the original channel. The results are presented for single-user channels as well as multiple-access channels, and are extended to fading channels with side information at the receiver.
On the Role of Mismatch in Rate Distortion Theory

Amos Lapidoth, Member, IEEE

Abstract—Using a codebook $C$, a source sequence is described by the codeword that is closest to it according to the distortion measure $d_0(x, \hat{x}_0)$. Based on this description, the source sequence is reconstructed to minimize the reconstruction distortion as measured by $d_1(x, \hat{x}_1)$, where, in general, $d_1(x, \hat{x}_1) \neq d_0(x, \hat{x}_0)$.

We study the minimum resulting $d_1(x, \hat{x}_1)$-distortion between the reconstructed sequence and the source sequence as we optimize over the codebook subject to a rate constraint. Using a random coding argument we derive an upper bound on the resulting distortion. Applying this bound to blocks of source symbols we construct a sequence of bounds which are shown to converge to the least distortion achievable in this setup. This solves the rate distortion dual of an open problem related to the capacity of channels with a given decoding rule—the mismatch capacity. Addressing a different kind of mismatch, we also study the mean-squared error description of non-Gaussian sources with random Gaussian codebooks. It is shown that the use of a Gaussian codebook to compress any ergodic source results in an average distortion which depends on the source via its second moment only. The source with a given second moment that is most difficult to describe is the memoryless zero-mean Gaussian source, and it is best described using a Gaussian codebook. Once a Gaussian codebook is used, we show that all sources of a given second moment become equally hard to describe.

Our interest in this paper is in a situation where the distortion measure $d_1(x, \hat{x}_1)$ that best describes the sensitivities of the end user is different from the distortion measure $d_0(x, \hat{x}_0)$ according to which the source is encoded. Such a situation can arise if encoding to minimize $d_0(x, \hat{x}_0)$ is easier to implement than encoding to minimize $d_1(x, \hat{x}_1)$, or when an individual who has his own special sensitivities that are best described by $d_1(x, \hat{x}_1)$, attempts to reconstruct a source sequence that was compressed using a lossy compression algorithm that minimizes $d_0(x, \hat{x}_0)$ and over which he has no control. It should be noted, however, that in our setup we allow the choice of the codebook to depend on the two distortion measures (as well as on the source law) and it need not be optimal for the distortion measure $d_0(x, \hat{x}_0)$. In this respect our problem arises more naturally when the mismatch in distortion measures is introduced to reduce complexity rather than due to a change in the distortion criteria after the source was compressed. We do, however, briefly address the latter case as well by analyzing the performance attained with a random codebook that is drawn according to the optimal distribution for the distortion measure.
Mismatched JSCC: Coding Scheme

Encoder $f$

Arbitrary Source $S^k$

Source Type Partition

Min Distance Encoding

Gaussian Codebook for Channel Input $X^n(I, J)$

Arbitrary Noise $Z^n$

Gaussian Codebook for Source Reproduction $\hat{S}^k(\hat{I}, \hat{J})$

Source Reproduction $(\hat{I}, \hat{J})$

Modified NN Decoding

Reproduction $\hat{S}^k(\hat{I}, \hat{J})$

Decoder $\phi$

Transmit any memoryless source over an additive arbitrary noise channel

Arbitrary memoryless source: $S^k$ i.i.d. according to any $P_S$ s.t.

\[ E[|S|^2] = 2, \quad E[|S|^4] < 1, \quad E[|S|^6] < 1; \]

Arbitrary i.i.d. noise: $Z^n$ i.i.d. according to any distribution $P_Z$ s.t.

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  - spherical codebook: each channel codeword $X^n$ is generated according to a uniform distribution over a sphere with radius $\sqrt{nP}$;
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    - Fix a positive number \(\xi\) and let \(N := \lceil 2k\xi \rceil\);
    - For each \(i \in [N]\), define the power type class
      \[
      \mathcal{T}_i := \left\{ s^k : \gamma(i - 1) \leq \|s^k\|^2/k < \gamma(i) \right\},
      \]
      \[
      \gamma(i) := (1 - \xi + i/k)\sigma^2.
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  - For each \(i \in [N]\), let \(\hat{S}^k(i, \tilde{j})\) be \(i\)-th sub-codebook for source reconstruction.
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    - For each \(i \in [N]\), let \(\{X^n(i, \tilde{j})\}_{\tilde{j} \in [M_i]}\) be \(i\)-th sub-codebook for channel input.
Given $S^k$, if $S^k \not\in \mathcal{T}_i$ for any $i \in [N]$, the encoder $f$ declares an error;
Mismatched JSCC: Minimum Distance Encoding

- Given \( S^k \), if \( S^k \notin T_i \) for any \( i \in [N] \), the encoder \( f \) declares an error;
- if \( S^k \in T_i \) for some \( i \in [N] \), the encoder \( f \) transmits \( X^n(I, J) \) where

\[
J = \arg \min_{\tilde{j} \in [M_i]} d(S^k, \hat{S}^k(I, \tilde{j})).
\]
Decoder $\phi$ employs the modified nearest neighbor decoding
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- For simplicity, let $\mathcal{D} := \{(r, s) \in \mathbb{N}^2 : r \in [N], s \in [M_r]\};$
- Given channel output $Y^n$, the decoder $\phi$ declares $\hat{S}^k(\hat{I}, \hat{J})$ as the source estimate if

$$\begin{align*}
(\hat{I}, \hat{J}) &= \arg\min_{(\tilde{i}, \tilde{j}) \in \mathcal{D}} \|X^n(\tilde{i}, \tilde{j}) - Y^n\|^2 + 2 \log M_{\tilde{i}}.
\end{align*}$$
Mismatched JSCC: Modified Nearest Neighbor Decoding

- Decoder $\phi$ employs the modified nearest neighbor decoding
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    \[
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    \]

- “mismatched” information density\(^2\)
  \[
  \tilde{\varpi}(x^n; y^n) := C(P) + \frac{\|y^n\|^2}{2(P + 1)} - \frac{\|y^n - x^n\|^2}{2},
  \]

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- “mismatched” distortion-tilted information density

$$
\log M_i := -\log \Pr\{d(s^k, \hat{S}^k) \leq D\} + \log k, \|s^k\|^2/k = \gamma(i).
$$

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Mismatched JSCC: Definitions

- Ensemble excess-distortion probability

\[
\overline{P}_{e,k,n}(D) := \Pr \left\{ S^k \notin \bigcup_{i=1}^{N} T_i \right\} + \sum_{i \in [N]} \Pr \left\{ S^k \in T_i, d(S^k, \hat{S}^k(i, \hat{j})) > D \right\}.
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\]

- Fundamental limit: for any \( \varepsilon \in [0, 1) \),

\[
k_{sp,sp}^*(n, \varepsilon, D) := \sup\{ k : \exists \text{ an } (k, n)\text{-code using spherical codebooks for source and channel codebooks s.t. } \overline{P}_{e,k,n}(D) \leq \varepsilon \}.
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- Fundamental limit: for any \( \varepsilon \in [0, 1) \),

\[ k^{*}_{sp,sp}(n, \varepsilon, D) := \sup \{ k : \exists \text{ an } (k, n)\text{-code using spherical codebooks for source and channel codebooks} \text{ s.t. } \overline{P}_{e,k,n}(D) \leq \varepsilon \}. \]

Similarly, we define \( k^{*}_{sp,iid}(n, \varepsilon, D) \), \( k^{*}_{iid,sp}(n, \varepsilon, D) \) and \( k^{*}_{iid,iid}(n, \varepsilon, D) \).
Universal coding scheme
Remarks on the Setting of Mismatched JSCC

- Universal coding scheme
  - extensions of Lapidoth’s coding schemes for mismatched channel coding and rate-distortion problems;
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- **Universal coding scheme**
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  - extension of Csiszár’s coding scheme\(^3\) of transmitting a DMS over a DMC using unequal error protection;

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- Universal coding scheme
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  - extension of Csiszár’s coding scheme\(^3\) of transmitting a DMS over a DMC using unequal error protection;
- Concatenation of source and channel codes\(^4\);
- The choice of values for \(\xi\) and \(M_i\) for \(i \in [N]\) is important.

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Mismatched dispersion functions

\[ V(\sigma^2, \zeta_s) = \frac{\zeta_s - \sigma^4}{4\sigma^4}, \]
\[ V_{sp}(P, \zeta_c) = \frac{P^2(\zeta_c - 1) + 4P}{4(P + 1)^2}, \]
\[ V_{iid}(P, \zeta_c) = \frac{P^2(\zeta_c + 1) + 4P}{4(P + 1)^2}; \]
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Optimal bandwidth expansion ratio

\[ \rho^*(P, \sigma^2, D) = \frac{C(P)}{R(\sigma^2, D)} = \frac{1}{2} \log(1 + P) = \frac{1}{2} \log \frac{\sigma^2}{D}; \]
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Joint source-channel mismatched dispersions: for any \( \dagger \in \{\text{sp, iid}\}, \)

\[ V_{\dagger}(\sigma^2, \zeta_s, P, \zeta_c) := \frac{\rho^*(P, \sigma^2, D)V(\sigma^2, \zeta_s) + V_{\dagger}(P, \zeta_c)}{(R(\sigma^2, D))^2}. \]
Dispersion for Mismatched JSCC: Main Result

- Quantization range: \( \xi = \sqrt{\frac{\log k}{k}} \)

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Dispersion for Mismatched JSCC: Main Result\(^5\)

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Dispersion for Mismatched JSCC: Main Result

- Quantization range: $\xi = \sqrt{\frac{\log k}{k}} \rightarrow N$ and $\mathcal{T}_i$ for all $i \in [N]$;
- For each $i \in [N]$, $M_i$ is chosen s.t.
  \[ \log M_i = -\log \Pr\{d(s^k, \hat{S}^k) \leq D\} + \log k, \]
where $\frac{\|s^k\|^2}{k} = \Upsilon(i)$ and $\hat{S}^k$ is a $\dagger$-Gaussian codeword.

---

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where \( \frac{\|s^k\|^2}{k} = \gamma(i) \) and \( \hat{S}^k \) is a \( \dagger \)-Gaussian codeword.

**Theorem 1 (Zhou-Tan-Motani 2017)**

For any \( \varepsilon \in [0, 1) \) and any \( (\ddagger, \dagger) \in \{\text{sp, iid}\}^2 \),

\[
k_{\ddagger, \dagger}^*(n, \varepsilon, D) = n\rho^*(P, \sigma^2, D) - \sqrt{nV_\dagger(\sigma^2, \zeta_s, P, \zeta_c)Q^{-1}(\varepsilon)} + O(\log n).
\]

---

Theorem 2 (Zhou-Tan-Motani 2017)

For any \( \varepsilon \in [0, 1) \) and any \((\dagger, \dagger) \in \{\text{sp, iid}\}^2\),
\[
k^*_\dagger,\dagger(n, \varepsilon, D) = n\rho^*(P, \sigma^2, D) - \sqrt{nV_{\dagger}(\sigma^2, \zeta_s, P, \zeta_c)Q^{-1}(\varepsilon)} + O(\log n).
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- The second-order asymptotics depend only on the type of the channel codebook; consistent with the results by Scarlett-Tan-Durisi and Zhou-Tan-Motani;
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- Recover the dispersion of transmitting a GMS over an AWGN channel by Kostina and Verdú (TIT 2013) when setting $P_S = \mathcal{N}(0, \sigma^2)$ and $P_Z = \mathcal{N}(0, 1)$;
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- As a corollary, for any \( \varepsilon \in [0, 1) \) and any \( (\dagger, \dagger) \in \{sp, iid\}^2 \),

\[
\lim_{n \to \infty} \frac{k_{\dagger, \dagger}^*(n, \varepsilon, D)}{n} = \rho^*(P, \sigma^2, D).
\]
Summary

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  - Universal coding scheme using UEP, modified minimum distance encoding and modified nearest neighbor decoding;

Future work
- Large deviations for mismatched JSCC;
- Mismatched JSCC over fading channels;
- Extensions to multi-terminal problems.
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  - Universal coding scheme using UEP, modified minimum distance encoding and modified nearest neighbor decoding;
  - Second-order asymptotics depend only on the type of the channel codebook;
  - Same is also true for moderate deviations (arXiv 1711.11206).

- **Future work**
  - Large deviations for mismatched JSCC;
  - Mismatched JSCC over fading channels;
  - Extensions to multi-terminal problems.