DCALTA/IPC Research Paper

Why Defined Contribution Plans Need Private Investments

The Benefits of Private Equity and Venture Capital in Diversified and Time-varying Portfolios

Abstract

We examine the impact of including private investment funds into diversified (e.g., balanced and target date fund) portfolios that otherwise hold only public stocks and bonds. Our analysis utilizes a comprehensive sample of 2,515 U.S. private equity funds to create simulated portfolios for 1987-2017 that invest part of their overall equity allocation in these funds. We find that investing in private funds always increases average portfolio returns and reliably increases Sharpe ratios (return per unit of risk). The results are robust when accounting for the inclusion of higher fees for the private portfolio and while randomly selecting a few funds from each vintage year (e.g., 10), suggesting that the results are feasible in practice for many investor types.

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A Key Partnership: DCALTA & Institute for Private Capital

DCALTA and the Institute for Private Capital have collaborated on this research study to analyze the viability of replacing a portion of public equity allocations with private markets investment in diversified and time-varying portfolios. We will be furthering our research to analyze other alternative investments and strategies where quality data exists. Our collaboration is based on an alignment of views realizing the importance of defined contribution plans and ultimately, participants, benefiting from the same opportunity set of available investments utilized by defined benefit plans, endowments and other pools of capital.

Institute for Private Capital

We improve public understanding of the role of private capital in the global economy. Academic and industry experts work together to generate new knowledge about private capital markets based on objective academic research. IPC mission is to promote a deep understanding of the role of private capital markets in the global economy. IPC brings together academic and industry experts who work together to generate new knowledge about private capital markets based on objective academic research. Findings are disseminated through academic publications, research symposia and education outreach. Central to the institute's mission is creating new data resources which can be utilized by academic researchers. This effort fills a significant gap in the knowledge base of financial economics. Privately held capital constitutes the vast majority of global capital, yet little academic research focuses on it because so little data are available.

Defined Contribution Alternatives Association

Founded in 2015, DCALTA is a 501(c)(6) non-profit organization composed of industry leaders from nearly all segments of the DC community, allowing us to represent the collective voice of the industry with a balanced view. Our members include a diverse range of plan sponsors, asset management firms, consultants, asset servicers, recordkeepers and other stakeholders in the DC community who support our mission. Our mission is to enhance and secure participant outcomes through education, research and advocacy on the benefits of private equity, hedge funds, and other alternative assets within a defined contribution framework. The benefits to defined contribution (DC) participants are clear: the inclusion of a modest allocation of diversified, professionally managed alternative assets within a multi-asset class portfolio, such as a target date suite or balanced fund, will likely enhance their retirement security and increase their chances of achieving retirement adequacy. We are dedicated to finding solutions to help overcome market bias and operational impediments that may limit the use of alternative investments within DC plans.

DCALTA Founding Research Partners

Cambridge Associates
FEV Analytics
ICMA-Retirement Corporation
Neuberger Berman
Northern Trust
StepStone Group
VOYA Financial

Foreword

I would like to thank the Institute for Private Capital, the DCALTA Board of Directors and DCALTA Founding Research Partners for their contributions to this impactful study. Before the formal research process began, DCALTA analyzed defined contribution plans (schemes) across the globe to better understand the types of alternative asset classes and structures being used, the amounts allocated to those alternative asset classes and the operational structures associated with them. Our research confirmed our hypothesis that the U.S. defined contribution plan space was operationally capable of including illiquid investments in DC plans but lagged in actual implementation relative to other countries (i.e., Australia, Mexico). DCALTA believes this to be attributed to litigation concerns due to a lack of regulatory and/or legislative clarity, product liquidity limitations and a lack of knowledge on the opportunity set of available investments.

The retirement industry has seen tremendous growth in the number of defined contribution plan sponsors using target date funds on their plan menus to help enhance participant outcomes. This asset growth is attributed to several factors; the Pension Protection Act of 2006, plan design using QDIAs, contribution flows, and the inclusion of a broad array of investments and strategies to enhance risk-adjusted returns. Our study analyzes the viability of using private market investments, and not the operational implementation of such. We will continue our research on other asset classes and look forward to sharing it with the industry.

Jonathan Epstein

DCALTA
President and Founder

Insights from Cambridge Associates

Why do US defined benefit (DB) plan sponsors hire professional investment advisors and diversify into alternative asset classes? For many reasons. Hiring professional investors—through an advisor or an in-house investment team—can enable these DB plan sponsors to construct portfolios designed to enhance risk-adjusted returns, particularly compared to those available to DC plan participants. Illiquid alternative assets such as private equity and hedge funds—if implemented successfully—can offer diversification benefits, access to alpha through manager selection in less efficient markets, and a potential illiquidity premium. US-based DC plans have not yet widely adopted best practices from their DB counterparts, in part due to structural impediments and in part due to litigation risk. DC Plans in other countries, however, have structures that address these issues. The superannuation funds scheme found in Australia and New Zealand is a prime example.

In Australia alone, there are over 200 superannuation funds (SAFs or supers) with more than four members, holding A$2.1 trillion and covering 26.8 million accounts[1]. Supers are DC plans in the key sense that employers (and employees) contribute a prescribed amount to a member's account where all investment risk and responsibility falls on the member. However, two important features of supers distinguish them from US-based DC plans and enable them to invest in alternative asset classes: a stable asset base and focus on multi-asset, professionally-managed pools, rather than providing a menu of asset class options. Superannuation funds typically have a much more stable asset base than US-based DC plans due to a greatly reduced reliance on the employer as plan sponsor. Originally the supers were each tied to a particular industry or region and their names may still resemble that heritage. Today, workers can open an account with any provider—in fact, there is competition among supers for participant assets. Since supers are not tied to the employer, members can change jobs without having to liquidate a retirement program. With greater account "stickiness," supers have found it quite feasible to add illiquid investments in general and private equity in particular. Mimicking this scheme for US single-employer DC plans would require new legislation, however it could be argued that multiemployer DC plans in the US already enjoy similar asset stability to supers.
A second key difference, and one that is not dependent on legislation, is that supers tend to be more "sponsor-directed" than the common "participant-directed" scheme in the US. This means that super members select from a menu of investment programs focused on specific objectives or risk levels, rather than from a menu of individual asset classes. In the US, professional management is almost exclusively focused in the target date fund (TDF) space, where structural, liquidity, and legal issues have resulted in a focus on traditional, liquid assets. Additionally, TDFs are just one option in the plan, alongside individual asset classes. Only a handful of DC plans in the US employ a similar structure to the supers, whereby the plan sponsor manages the entire portfolio. In our experience, these plans are able to leverage this structure and include illiquid alternative asset classes, which can result in better risk-adjusted returns.

Cambridge Associates works with a number of supers in Australia and New Zealand, and we have chosen the following examples from among that set. In one case, the super offers members a menu of six risk-based portfolios, from Secure to High Growth. In the most aggressive of these portfolios, there is a 15% target

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[1] Superannuation Statistics – August 2019, Association of Superannuation Funds of Australia (ASFA)

allocation to alternatives (ranging up to 20%), which is primarily comprised of private equity. Further, this super recently added co-investments to its private equity allocation. Another super has cumulatively committed over A$2 billion to private equity in the past decade and carries an A$400 million hedge fund allocation.

One challenge the supers face with respect to adding more alternative assets is that the total expense ratio for the investment options is highly regulated and transparent. While this can limit the alternatives allocation because of the high fees involved, the competition for member assets makes earning higher risk-adjusted returns after fees a primary concern.

Sponsor-directed investment programs and non-employer-linked plans are not unique to Australia and New Zealand. A prominent example in the Western Hemisphere is the Mexican AFORE scheme, which allows participants to choose from 11 different regulated firms with DC offerings. The investment programs are managed by professional investors and participants can switch firms if they suffer poor performance. There is no tie to the employer, so changing jobs does not create a liquidity need. In an effort to further enhance diversification and boost return potential, these AFOREs have recently accelerated their push into global private equity as the regulatory authority has increased the allowable allocation to international assets. While the US lags other countries in adding alternatives to DC plans, we have seen some signs of promise. One US multiple employer plan sponsor, for example, launched white label funds and limited participants to monthly liquidity because Cambridge Associates analysis showed this could greatly expand their investible universe. Relaxation of the daily liquidity constraint expanded asset class options and alpha opportunities in two ways. First, the sponsor was able to give its participants access to top-tier managers in illiquid and diversifying asset classes such as hedge funds. Second, this structure enhanced alpha opportunities within traditional asset classes. While many top-tier institutional managers offer daily vehicles (such as CITs or mutual funds), there are many more that are only accessible on a monthly basis. For example, the plan was able to add a high-alpha 130/30 strategy to plan, which would be very difficult in a pure daily liquidity environment. Further, by embedding high alpha and high active share managers within a white label structure, the sponsor had greater confidence that the participants would be protected from making poor short-term decisions by trading among individual managers.

DC plan sponsors in the US seeking to improve outcomes for participants may find it useful to look at the examples set by Australia, New Zealand, Mexico and others to make a long-term case for updated legislation. In the meantime, there are incremental steps they can take to bring best practices from defined benefit plans to the growing world of defined contribution plans. The results presented in this paper support the benefits that can accrue to plan participants through the introduction of private asset classes.

An Introduction by Northern Trust

Private assets play a crucial role in the diversification of investment portfolios. These assets, such as hedge funds, real estate, infrastructure and private equity, provide the dual benefits of diversification and low correlation to traditional asset class performance. Alternative investments are utilized in investment portfolios with long liability streams, such as defined benefit (DB) pension plans. In fact, DB plans in the top 200 saw assets in private equity, specifically, increase by 10.7% to $367.2 billion as of Sept. 30, 2018[2]. U.S. defined contribution plans, however, have not followed suit, being virtually shut out of the benefits of diversification that alternative assets offer investment portfolios.

While the U.S. pioneered the concept of the DC plan corporate benefit in the 1970s, it was as a supplement to the rich DB plans employers also provided. Today other countries, notably Australia, have made the greatest strides in improving upon the DC formula. However, the typical U.S. DC plan structure and opportunity sets of available investments have not kept pace with other global DC systems (schemes). Further, and in most cases, DC plans have become the single largest retirement asset many U.S. working people have.

One of the greatest differences between the U.S. DC plan and the Australian DC plan, or Superannuation Scheme, is that many Australian Supers incorporate alternatives into their multi-manager, diversified portfolio funds. Frequently the Australian Super`s default fund, which may be a balanced fund, will include upwards of a 15% allocation to PE, real estate and infrastructure. The investment fiduciaries on these balanced funds believe that private investments unlock exposure to new opportunities which are increasingly unlikely to be found in the public markets, where the number of public companies has been declining. The most interesting piece of information found during our initial research was that most Australian Supers find the vast majority of their alternative investments here in the U.S., the largest economy and most diverse public and private market in the world. This is interesting because U.S. DC plans are basically shut out from investing and benefiting from the performance and diversity of their home country market.

As many global governments seek to improve their retirement systems, they are considering the move forward to include alternatives in their developing DC solutions. Their focus is on incorporating the best technology and operating model to efficiently include alternative assets in the daily accounting and valuation processes of retirement investments.

As DC plans replace DB plans across the globe, it has become evident that it is difficult to replace one with the other without improved infrastructure and the inclusion of a long term, broad based asset allocation investment model. We must transition from a supplemental DC savings mindset to a long term, compensation replacement mindset which includes institutionalizing DC investments to be more like DB investments. The research presented in this paper makes it clear that including private assets as an option in DC plans is crucial to this transition.

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[2] https://www.pionline.com/article/20190204/PRINT/190209959/largest-funds-top-11-trillion-assets-up-6-4

1. Introduction to the Research

The ultimate goal of investors is to maximize risk-adjusted portfolio returns. This simple premise of Modern Portfolio Theory (MPT) led to the creation of sophisticated mathematical and econometric methods in an attempt to master the craft of portfolio management. With liquid assets such as publicly traded stocks and bonds, MPT methodology can be implemented in a straightforward way. However, closed-end drawn-down fund structures like private equity (PE) buyout and venture capital (VC) funds, pose challenges for traditional portfolio optimization and asset allocation methods. In particular, the lack of available data due to infrequent reporting, illiquidity, and lack of measurable return characteristics creates fundamental challenges for established portfolio management methodology. Furthermore, actual investment decisions are delegated to fund managers, so investors only commit to funding a future set of unknown investments with both entry and exit dates uncertain. As a consequence, *ex ante* risk-return analysis of private investments must contend with additional uncertainty in the investment process. However, empirical evidence suggests that private fund investors have historically earned a premium to public markets, perhaps as compensation for the illiquidity and uncertainty associated with private investments.[3]

The private market investment universe has experienced massive growth from its beginning in the 1980s to a total of about $6 trillion in committed capital today. Despite their relative complexity and higher costs, the vast majority of institutional investors today hold some type of "alternative asset" in their portfolio. University endowments, in particular, were early adopters of private investments. The success of the so-called "Yale Model" popularized investing into alterative assets and has resulted in the strategy adoption, to varying degrees, by most other types of investors. Binfare, Brown, Harris, & Lundblad (2019) show that portfolios of large endowments significantly outperform portfolios consisting of traditional assets. Over the last 20 years, other large institutional investors like pension funds have struggled to meet their mounting obligations by investing in public markets alone. As a consequence, investors have increased their allocations to private funds, most prominently to buyout funds. A wide-spread belief is that such investments continue to perform better than traditional investments in public stocks and bonds and therefore private investments unlock exposure to new businesses which are increasingly unlikely to be found in public markets.

Surprisingly, the challenges of portfolio management in private markets have not been carefully examined in the academic literature and so little is known about the investment properties of portfolios that include alternative investments. Given this gap, we believe that a logical next step is to outline practical solutions for portfolio management with assets beyond stocks and bonds. This paper provides a realistic simulation of historical performance of diversified portfolios including allocations not only to publicly-traded securities but also to private funds assets. Using a comprehensive database on private market funds, we are able to make claims not only about return characteristics and diversification benefits of private investments in diversified investor's portfolios, but also about feasible allocation strategies that can help guide investors to make better investment decisions. Overall, our analysis allows for a much deeper understanding of diversified portfolio allocation strategies as well as risk and returns properties.

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[3] See Brown and Kaplan, 2019.

In this paper, we limit the analysis to buyout and VC funds. While there are many other types of illiquid investment strategies such as real estate, infrastructure, hedge funds, and private credit, there is a 30-year history of buyout and VC investments which facilitates a long-term analysis and provides more confidence that we can understand important issues through multiple investment cycles. In addition, there is substantial heterogeneity among just buyout and venture capital funds with respect to their risk and return characteristics. Focusing on buyouts and VC allows us to better understand portfolio management with private funds in a fairly simple setting that can then be easily extended to include a broader set of investment funds.

Our primary methodology is very straightforward. We conduct historical simulations where part of the public equity portfolio is replaced with allocations to private equity funds. We use actual private fund cash flows and rebalance public assets on a quarterly basis back to target allocations for equity and fixed income. All aspects of our simulations are feasible and do not rely on making secondary market transactions in private funds. We then examine the risk and return characteristics of the portfolios that include private fund allocations paying careful attention to the fact that returns of the portfolio are artificially smoothed by the lack of observed price data.

We obtain three main results:

1. Returns are consistently higher for portfolios that incorporate private equity funds.
2. Risk is consistently lower for portfolios that include buyout funds. In contrast, VC funds tend to increase portfolio risk but much of this risk is related to positive skewness in returns and thus represents "upside" risk. In addition, Sharpe ratios are consistently higher for portfolios with buyout funds and with a combination of buyout and VC funds.
3. It is challenging to hit specific allocation targets in all years with only primary commitments to private equity funds. We estimate a simple parametric model that improves asset allocation for dynamic strategies such as target date funds.

In the remainder of the paper, we provide a short overview of related literature on private fund return and diversification characteristics. Next, we discuss the data and portfolio simulation methodology. Finally, we report our primary results on portfolio characteristics with private funds both for fixed and dynamic allocations.

2. A Brief History of Private Investment Research

The growing demand by investors for private market funds has helped fuel a growing academic literature on the return characteristics of private investments, and for buyout and VC funds in particular. Most of the recent literature agrees that buyout and VC fund investments have outperformed public market benchmarks. While there is nearly a consensus on the return benefits of private funds, there is less agreement on risk attributes and the implications for optimal asset allocation.

Kaplan & Schoar (2005) find that average private market fund returns (net of fees) equal those of its public benchmarks. However, more recent work leverages the vastly improved fund-level data assembled since then. Harris, Jenkinson, and Kaplan (2014), use Burgiss data and show that buyout and VC funds outperform the S&P 500 by an average of more than 3% annually between 1987 and 2010. Brown, Harris, Jenkinson, Kaplan,

& Robinson (2015) demonstrate that while there is heterogeneity in the data provided by different databases, the similarity of performance estimates strengthens confidence in the conclusions derived from recent studies about superior PE returns. Brown and Kaplan (2019) update results from previous work and find continued outperformance of buyout funds through 2018.

Risk estimates, though, are much more sensitive to the quality and timespan of the data as well as the methods used for evaluation. The standard approach for public equities, which involves a regression analysis of returns on various risk factors, is difficult to apply to alternative assets due to the lack of observable market prices. Korteweg (2019) provides a survey of various empirical methods to assess risk for PE and documents that risk of the typical buyout fund is generally somewhat higher than for the market as a whole with an equity market beta in the range of 1.1-1.5. VC funds appear to have significantly higher market betas, typically documented in the range of 1.5-2.0.

Although the literature has not yet provided an agreed-on statistical framework to evaluate the risk of alternative assets in a broadly diversified portfolio, some work has attempted to estimate the potential diversification benefits. Goetzmann, Gourier, & Phalippou (2018) show how allocations to private funds extend the efficient frontier. In particular, large venture capital, buyouts, infrastructure, and debt funds have a significant impact in expanding an investor's efficient frontier. Surprisingly, the authors show that investing in large funds achieves better diversification than investing in small funds.

Overall, existing evidence suggests that investors may benefit from private fund investments by both gaining higher returns as well as better diversification. These potential diversification benefits are at the core of the analysis in this paper and are likely to have grown in recent years as the changing market landscape limits public investment opportunities. According to Doidge, Kahle, Karolyi, & Stulz (2018), the total number of publicly listed firms in the US has dropped significantly from its high of around 8,000 at the end of the last century to around 4,000 in recent years. In contrast, the number of companies owned by buyout and VC funds has increased over the same period from around 2,000 to more than 10,000. This development is associated with increasing buyout and corporate M&A activity, fewer IPOs and an increasing number of private funding rounds for private companies. While this trend does not mean that the universe of investable companies is shrinking, it does indicate that there is a substantial movement away from public and toward private market ownership.

This trend also implies that the available public equity universe is changing with respect to its demography. Public companies today tend to be bigger and older, and increasingly tilted towards low exposure to traditional risk factors like size and value.[4] Consequently, the available risk premia are lower and offer fewer diversification possibilities than a few decades ago. Hence, investors have increasing incentives to turn to private market funds to obtain exposure to risk premia.

3. Data and Methodology

Similar to Harris, Jenkinson, & Kaplan (2014) we take advantage of the comprehensive fund-level data of Burgiss.[5] A primary feature of the Burgiss dataset is that the data are sourced directly from Limited Partners (LPs) and therefore include the complete transactional and valuation history of their investments to private market funds. As such, the data provided includes the entire array of timed cash flows net-of-fees to LPs and

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[4] See Bartram, Brown, and Stulz, 2018.
[5] Currently, the Burgiss universe has detailed data on 9,486 funds across asset classes with a total capitalization of about $6.89 trillion. For a detailed discussion about the strength and weaknesses of the Burgiss database see Harris et al. (2014).

enable us to construct portfolios from an investor's perspective. Having a complete set of cash flows for a large sample of funds is essential for our analysis.

We utilize data on 1,121 buyout funds and 1,394 VC funds all with a U.S. focus with vintage years from 1987 to 2017. Buyout funds represent the largest market segment of private funds, are widely accessible. Buyout funds account for more than 40% of the capital invested to private market funds, more than 80% of the capital in our sample, and are typically the largest average allocation among closed-end draw-down funds in institutional portfolios.[6] VC funds are generally smaller funds and considered riskier because of the young innovative companies typically in a VC portfolio and higher market betas. Top VC funds are also difficult for most investors to access because of excess demand for these funds and the tendency for VC general partners to limit the size of their funds.

Table 1 reports the performance distributions of these funds by category. Panels A and B of Table 1 report statistics for the internal rate of return (IRR) and total value to paid-in capital (TVPI). Over our sample period, buyout funds had an IRR of 12.2% and VC funds had a slightly higher IRR of 13.5%. Despite the similar means, IRRs for VC funds are very positively skewed relative to buyout funds with a lower return by the median fund but much higher returns for the top 5% of funds. These values are consistent with the prior evidence on the distribution of buyout and VC fund IRRs. Reported values for TVPI in Panel B reveal a similar pattern. Buyout funds have somewhat lower average multiples but significantly lower variation in multiples as compared to VC funds.

Panel C of Table 1 reports Kaplan-Schoar (2005) public market equivalent (PME) performance measures with the S&P 500 as the reference index. The PME method discounts fund cash flows using realized rates of return from a public market index so that the values represent a "market-adjusted multiple." The mean value of 1.12 for buyout funds indicates that returns over the life of the average fund were 12% higher than returns from a similar investment pattern in the S&P 500. The PME of 1.18 for VC funds suggests slightly higher market-adjusted returns. The distribution of PMEs also shows the well-documented spread in performance with VC funds having more variation in performance than buyout funds. In addition, venture capital PMEs are also highly positively skewed with a median PME of less than 1.0, but substantially higher returns above the 95th percentile.

As noted above, having complete and precisely-dated cash flows for all funds is essential for our analysis. Table 2 shows net asset values (NAVs), contributions, and distributions of buyout and VC funds by year of a fund's life as a percentage of fund commitments. Panel A tables values for buyout funds. The median NAV values show why there is a well-established need to over-allocate to funds in order to hit a target allocation level: In no year is the median NAV higher than 0.76 of committed capital despite the fact that the median fund does eventually call all its capital (as shown by contributions). Most importantly, the differences between the 5th and 95th percentiles shows substantial cross-sectional variation in all years of fund life for NAVs, contributions, and

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[6] Burgiss data, as of May 2019, provides historical capitalization of buyout funds as $2.678 trillion, venture capital as $644 billion, and real estate as $737 billion. Binfare et al. (2019) show that university endowments reporting to NABUCO allocate an average of 4.4% of assets to buyout compared to only 1.57% to VC and less than 1% to real estate.

distributions. For example, in year 5 of fund life, NAVs range from as low as 33% of committed capital at the 5th percentile up to 133% of committed capital for the 95th percentile. Likewise, in year 5 of fund life, funds in the 5th percentile have called only 58% of capital whereas funds in the 95th percentile have recycled capital from early exits and allocated 110% of committed capital. Distributions exhibit even more variation. These facts show that the valuation and cash flow experiences of investors in buyout funds can vary widely across funds and through time. Consequently, most investors will want to hold diversified portfolios of funds.

Panel B of Table 2 provides statistics for VC funds and documents even more time-series and cross-sectional variation in NAVs and distributions. For example, with VC funds the highest median level of NAV is just 67% of committed capital (in year 6). It is worth noting that the median buyout and venture funds still have an NAV greater than 30% of commitment value in year 10 of fund life suggesting that the typical fund invests a significant part of its portfolio well beyond the common contractual term of 10 years. Together these results shed light on why private fund allocations can vary so much through time even when there is a strategy of fixed annual commitments.

As our proxy for public market portfolio returns in our subsequent analysis, we use performance data for popular Vanguard funds to mimic the (net-of-fee) performance of investible public asset benchmarks for U.S. equity and U.S. fixed income. Table 3 summarizes the Vanguard funds used in our analysis: the Vanguard Total Stock Market Fund (VTSMX) and the Vanguard Total Bond Market Fund (VBMFX).[7]

Portfolio Construction Methodology

The starting point for our analysis is an all-public benchmark portfolio with a 60% allocation to U.S. equity (VTSMX) and a 40% allocation to U.S. fixed income (VBMFX).[8] We choose these weights because they are commonly cited in the portfolio management literature as a benchmark for diversified portfolios. We subsequently examine other (dynamic) allocations. Our analysis centers on investing part of the equity allocation in private funds and comparing the portfolio properties to a benchmark that is all public assets.

The investment into private market funds is modeled as a systematic annual commitment process. At the beginning of each year, commitments to available funds of that vintage year are made based on a specific mechanical rule (we consider several alternatives described throughout the paper). For simplicity, all subsequent cash flows within a quarter are assumed to accrue at the end of each quarter for the private funds (this assumes a portfolio borrows or lends as needed during the quarter at low cost or to/from themselves). Cash flows for private investments in buyout and VC funds come into and out of the public equity account. Also, at the end of each quarter, the portfolio is rebalanced to return the allocation to target fixed income weights (e.g., 40%) so that the only variation in portfolio performance (relative to the all-public alternative) should stem from the differences in returns between the public equity and private equity funds.

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[7] Our simulations date back to 1987 which is a few years before the inception of VTSMX in April 1992, so we extrapolate the fund return series to 1987 by using the time-series of CRSP value-weighted index returns and subtract a fixed management fee of 30 basis points (bps) annually which is the approximate level of fees for that fund at inception.
[8] In the financial planning industry, a portfolio that allocates 60% to public equities and 40% to fixed-income products has been viewed historically as an efficient way to construct diversified portfolios for long-term investors.

We conduct our analysis by targeting an allocation of one-third (33.3%) of the equity portfolio to private markets funds (or 20% of the total portfolio in the base case). We start by naively assuming a 5-year investment period with a full draw-down of the commitment and no return of capital during the investment period. Therefore, we commit one fifth of one-third, or 4%, of the equity portfolio each year (1/5 x 1/3 x 60% = 4% of total assets) to private investments. To ensure that our portfolio strategy is feasible, and we can examine the range of possible outcome, we allocate to a finite number of buyout funds per vintage (e.g., 10) which are randomly drawn from the available Burgiss fund universe. Consequently, this assumes no skill in selection, but that our hypothetical investor does have access to all funds. The advantage of this method is that simulating this random selection process 1,000 times provides good estimates of the range of possible outcomes an unskilled investor could realistically expect to have with this strategy. Of course, a skilled investor with good access to all types of funds would necessarily do better than what our results indicate.

In a later section, we also consider time-varying allocations to funds such as would be experienced by an institutional portfolio with varying risk or investment horizons (e.g., a legacy pension plan liquidating) or a target date fund (TDF).

Figure 1 plots the resulting evolution of average allocations to each asset class for the 1,000 simulations. There are two important features. First, from 1987 to 1994, there is a steady increase in private fund allocation from zero to about 14% indicating that the allocation is building up slowly as funds draw (and return) capital. From 1995 to 2017, the allocation is more stable though it does step up around the financial crisis to about 16% of portfolio value. Given the long period needed to obtain a more stable allocation, we call the period from 1995 forward the "steady state" and conduct our performance and diversification analysis using this time period. Second, the allocations are always too low using the naïve allocation rule. This is because of the well-known tendencies for GPs to draw less than 100% of commitments and to return some capital prior to the end of the investment period. Consequently, in our analysis we consider some "over-allocation" strategies that generate average private fund weights close to the one-third target.

We also note in Figure 1 that the quarterly rebalancing of portfolios relies on valuing the private assets at net asset values (NAVs). To the extent that NAVs are systematically biased through time (e.g., not marked up enough during up markets or down enough during down markets), this will lead to actual economic exposures that may differ more from the 60/40 allocation targeted in the analysis. In general, smoothed returns would lead to portfolios being under-exposed to equities after market declines and over-exposed to equities after market rallies.

Adjusting Smoothed Returns

The issue of smoothed NAVs poses another challenge for our analysis. Although our portfolio construction methodology allows us to extract a time-series of returns for the diversified portfolio, using traditional metrics to characterize risk properties will be inaccurate. Allocating to private funds incorporates reported (smoothed) NAVs into portfolio returns and will result in underestimation of risk. As a result, illiquid assets pose a fundamental challenge that needs to be addressed in order to make proper comparisons of portfolio characteristics.

To adjust such smoothed returns, we follow the framework of Getmansky, Lo, and Makarov (2004). We assume that observed (appraisal-based) returns of funds are a weighted average of true (unobserved) economic returns,

and that the latter should be used to evaluate risk. True economic returns can be recovered after estimating the historical averaging weights. As an example of the effect of smoothing, Table 4 shows the return and risk properties recovered from a portfolio that committed to 10 buyout funds per year and compares it to popular benchmarks. The adjustment causes the (annualized) volatility of buyout funds to roughly double from 8.42% to 15.66%. A similar near-doubling of volatilities is observed for VC funds. As compared to public benchmark indices over the same period, buyout funds are somewhat less volatile and VC funds are somewhat more volatile than the S&P 500 and Total Stock Market Index (Panel C).

As we use Sharpe ratios as the main risk-adjusted performance measure in our analysis, we also rely on the additional adjustment outlined in Getmansky et al. (2004). Specifically, volatilities of the return series also account for the covariances with the (weighted) component returns when annualizing quarterly Sharpe ratios thus addressing the effects of spurious autocorrelation from NAVs. In the following analyses, we report both adjusted volatilities and adjusted Sharpe ratios.[9]

4. Results

Diversified Portfolios by Asset Class

To investigate the effect of including private investments in a diversified portfolio, we start by examining four different types of portfolios over the steady state period of 1995-2017. The first portfolio is a base case benchmark comprised of only public market assets. The other three types of portfolios substitute private funds for part of the public equity. We consider portfolios with (i) just buyout funds, (ii) just venture capital funds, and (iii) a combination of buyout and venture capital funds by investing in 10 randomly sampled funds per vintage in each case. For the combination of buyout and VC the 10 funds are drawn randomly from the universe of funds in each vintage, so allocations vary (on average) with the mix of new funds.

Table 5 summarizes the performance, risk, and other properties of these portfolios. We consider two allocation rules. First is the naïve 4% per year which leads to significant under-allocation. Second is an "over-allocation" that results in approximately the 20% target for private funds on average. The over-allocation strategy commits 5.4% per year for buyout, 4.7% per year for venture funds, and 5.1% for portfolios with both buyout and venture funds.

Columns (1) and (2) in Table 5 show the results for buyout funds following the naïve 4% allocation and simple overallocation strategies, respectively. The results suggest that including private market funds in the portfolio both improves performance and has diversification benefits that lower overall portfolio risk. Buyout funds improve overall return (by 64-86bps per year) but yield a much higher risk-adjusted return. Specifically, the adjusted annual standard deviations fall from 10.39% in the base case to 9.47% and 9.22% with the buyout fund allocation. These results suggest that diversification benefits may be as large as return benefits. Overall, the adjusted Sharpe ratios for the portfolios that include buyout funds are notably higher (0.67 and 0.72) than for the base case (0.55).

Columns (3) and (4) in Table 4 show that allocating to VC funds yields larger excess returns over the all-public benchmark (9.68% and 9.89%, respectively). However, the allocation to VC funds also results in less

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[9] Adjusting smoothed returns - Further research that examines behavioral-based factors – for example those described by Lo's Adaptive Market Hypothesis (2004) – may challenge the assumption that smoothed returns are systematic or behave as a weighted average.

diversification benefit as would be expected given the higher beta of VC funds documented in previous studies (see Korteweg, 2019, for a review). Specifically, both the 4% allocation and the over-allocation portfolios with VC result in an increase in adjusted risk to 12.52% and 12.99% respectively.[10]

Consequently, allocating to only VC results in more modest improvements in adjusted Sharpe ratios. These results are consistent with findings of Goetzmann, Gourier, & Phalippou (2018) who document less diversification benefit from VC than from buyout funds. An important caveat to this result is that the higher risk in VC funds is primarily the result of upside risk. Table 5 also reports (downside) semi-deviation which only measures risk of outcomes worse than the mean return and shows that VC portfolios have only slightly more down-side risk than the all-public benchmark despite having higher adjusted standard deviations. Likewise, skewness of the portfolios with VC funds (which represents the asymmetry in returns) switches to slightly positive (upside) from slightly negative (downside) for the all-public benchmark. However, kurtosis (degree of fat tails) is much higher for the portfolios with VC funds.

The last 2 columns of Table 4 show the results from including both buyout and VC funds in the diversified portfolio. As would be expected, these results are between those just discussed for buyout only and VC only. Returns are higher than the all-public benchmark and adjusted standard deviation is about the same as the all-public benchmark. The adjusted Sharpe ratios are higher than for the all-public benchmark, but inferior to the portfolios with only buyout funds. These results suggest that on a risk-adjusted basis the largest benefits are obtained from the portfolios with just buyouts.

We also examine the reliability of the risk and return metrics presented in Table 5. To gauge this, we calculate the percentage of the private fund portfolio simulations that are better than the public-only benchmark. The last four rows of Table 5 report these findings. In 100% of simulations, the returns with private funds outperform the public-only benchmark. This suggest that despite the wide dispersion of returns in private funds, the ability to diversify by investing in multiple funds is sufficient to have nearly guaranteed superior returns historically.

Figure 2 shows this result graphically by plotting the histograms of simulated portfolio returns.[11] The plots show the clear risk-return trade-off between buyouts and VC. Portfolios with just VC funds have a higher average return but also a significantly higher dispersion in returns. The portfolio with buyout funds and VC lies in the middle as expected.

The third-to-last row of Table 4 shows that results for standard deviation are highly dependent on the investment: buyout fund alone always lower risk but including VC funds almost always increases risk. The results for Sharpe ratios are show the relative importance of higher returns: including private funds leads to reliably higher adjusted Sharpe ratios.

The median portfolio allocations for the over-allocation portfolios are presented in Figure 3. The plots show that during the post-1995 (steady-state) period, there are persistent deviations from the 20% target allocation.

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[10] Much of this increase may be the result of the higher beta of the equity portfolio. For example, if we assume that VC funds have an average beta of 1.7 then a 20% allocation to VC implies an overall beta of 1.23 for the equity portfolio. In analysis not reported, we create "beta-neutral" portfolios by adjusting the fixed income weights so as to keep the overall equity portfolio beta constant and find lower volatilities for portfolios with VC funds.

[11] In these graphs, we plot normal distributions with the means and standard deviations for the over-allocation strategies provided in Table 4 specifications (2), (4), and (6).

Buyout funds (light blue line) tend to experience smaller allocation deviations. VC funds (red line) experience the largest swings around the target allocation. This suggests that if allocating to VC alone, an investor will find it difficult to achieve stable portfolio weights and the shifts can occur fairly quickly. For example, there are times of significant over-allocation (above 30%) as well as under-allocation (9%) within a few years of each other. The spike around 2000 reflects the high valuation during the dotcom bubble and is likely to reflect a large overweight to technology stocks. When valuations dropped in the early 2000s, the portfolios with VC quickly became quite under-allocated to private funds. For the last five years of our sample, portfolios with VC funds have again become over-allocated because of strong relative returns of VC. As expected, the private fund median allocations for the portfolio that invests in both buyout and VC funds (green line) lies between the other two.

In sum, allocations made with only primary commitments to private funds can vary considerably through time, especially when portfolios include a significant VC component. Despite this variation, the overall risk of the portfolios (discussed above and presented in Table 4) does not change much with the addition of private funds because the rebalancing back to an overall constant equity allocation absorbs much of the fluctuation in private fund values. Overall, buyout funds provide a more stable source of excess return and diversification. For this reason, because of the skewed return properties of VC funds, and because not all investors have access to top-performing VC funds, we focus the remainder of the analysis on only buyout funds.

Cash Flow and Valuation Dynamics

We now consider other practical aspects of investing in private funds and the impact these have on overall portfolio risk and return. We extend the analysis by examining the effects of (1) the number of fund commitments in each vintage year, and (2) additional costs incurred when managing a portfolio of private funds.

First, we explore the effect of the number of annual commitments on portfolio risk and return. In practice, the number of fund commitments affects the operational costs of the investor (time spent doing research and due diligence, monitoring & reporting, etc.). We consider allocating to 2, 5, 10, 20, or all funds available in each vintage year on an equally-weighted basis.[12] We still target an overall 20% allocation to buyout funds. The simulation results are summarized in Figure 4. As would be expected, the range of outcomes declines as the number of funds per vintage increases. Even a strategy that invests in only 2 funds per vintage has a good chance of underperforming the all-public benchmark whereas all portfolios with 5 or more funds per vintage outperform. The best risk-reward is for the portfolio with 20 funds. It has high average returns and low dispersion in returns. As a practical matter, there may be significant additional costs associated with investing in such a large number of funds per vintage. Overall, it appears that the decline in dispersion of returns is limited with more than 10 funds per vintage.

As discussed above, a challenge investors face in private funds is how to achieve and maintain their desired level of allocation. Given the range of NAVs documented in Table 2, the reliability of being close to a target allocation likely depends on the number of funds in the portfolio. Figure 5 displays the 5th and 95th percentiles of the distribution by the number of fund commitments per vintage year. While there is a tendency for portfolios with more commitments per vintage to have a tighter range of allocations, there is a large amount of co-movement in all cases. Given the similarity of the times-series features to the median allocations plotted in Figure 3, these results suggest that the dominant effect on allocations is market-wide changes in valuations of private funds relative to public market returns.

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[12] If there are not enough funds for our particular portfolio in a given vintage, we invest in all funds of that vintage.

Additional Costs for Managing Private Funds

The cash flow data provided by Burgiss are reported from an LP's perspective (e.g. distributions received after accounting for management and performance fees) so that no additional assumptions about the fee structure of any individual fund need to be made. However, investing in private funds typically requires additional resources for selection and monitoring. For example, the costs associated with managing alternative investments are often higher because of the need to build and manage relationships with fund managers and the additional burden of sourcing private investment opportunities. We estimate these effects by simulating additional costs incurred by the private fund portfolio. Specifically, we consider two cases: an additional 25bps or 50bps on the capital allocated to private funds. The results are provided in Table 6 and show that the diversified portfolio returns are dampened slightly by fees (as expected) but continue to outperform the all-public benchmark portfolio.

The results for returns indicate that introducing an additional fee of 25bps, the overall mean portfolio return declines by 5 basis points. This is consistent with our expectation given the target of a 20% allocation of private equity (0.20 x 25bps). The introduction of an additional fee of 50bps has a proportional impact of reducing portfolio returns by 10 basis points (0.20 x 50bps). Given the fees represent roughly a constant shift down in returns, the standard deviation and other higher-order moments of the distributions hardly change.[13]

Dynamic Allocations

In contrast to the previous results with a fixed target allocation to private funds, we now examine portfolios that have time-varying allocations. Specifically, we seek to understand the properties of a portfolio strategy that provides an investor exposure to more private funds early in the investment life-cycle but then shifts to increasingly lower-risk and more liquid assets over time. Many investors face these types of life-cycle issues with their portfolios. These include pension and insurance funds with time-varying liabilities as well as individuals seeking higher returns from private funds but facing the need for liquid assets in retirement.

We retain our framework of augmenting a portfolio of investible public assets with private funds. In particular, we simulate the impact of including private market funds into target-date funds (TDFs) that typically have a defined change in asset allocation over several decades.[14] As a set of benchmark portfolios, we use the asset allocations of Morningstar's Lifetime Index Funds. We approximate the "glide-paths" of these funds using a four-parameter logistic model that mimics the target allocations to fixed-income and equities. The four parameters characterize i) the beginning target allocation of private equity, ii) the ending target allocation of private equity, iii) the inflection point (e.g., target date), and iv) the steepness of the glide-path. We model specific funds, but the method can be extended to approximate nearly any smooth glide path. In our simulations, we again target a private fund allocation equivalent to one-third of the total equity allocation. We examine three different Morningstar Lifetime Index Funds target dates (2005, 2020, and 2035) but focus on the 2005 target date since it has the most dynamic allocations over our sample period.

The allocation strategy needed to replicate the glide-path of a TDF introduces an additional layer of complexity. As we demonstrated above, targeting a fixed allocation of buyout and VC funds results in an actual allocation with substantial variation and a slight upward trend (i.e., opposite the trend in TDFs). In addition, the delay

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[13] The average allocation to private equity increases marginally by 0.1% for any additional 25 basis points fee as in our model the liquidity needed for the additional fee is effectively taken out of the public equity account.

[14] Today, TDFs are the most common default investment option in US 401(k) plans with more than $1 trillion in AUM. Its characteristic glide path ensures a transition from a return-seeking portfolio allocation (high allocation to equities in early years) towards a more conservative allocation mix (high allocation to bonds and money-market funds) around the target date or retirement.

between when commitments are made and when capital is called complicates the commitment strategy. As shown in Table 2, fund contributions regularly occur for as long as 7 years despite a common contractual investment period of 5 years. Consequently, a simple over-allocation strategy can mitigate underinvestment (as shown previously) but is less practical for achieving the time-variant target allocation of TDFs. Due to the nature of the glide-path, an investor would either be (1) over-allocated late in the investment term if implementing a simple over-allocating strategy, or (2) under-invested early in the investment term if targeting late term allocations.

For example, Figure 6 shows the range of buyout fund allocations with the naïve commitment strategy of targeting one-fifth of the overall equity allocation. In this case, the fund never reaches its allocation target. Figure 7 shows the range of buyout fund allocations with a simple, over-allocation of 0.27 times the contemporaneous target allocation (equivalent to the over-allocation for buyouts presented in Table 5). In this case, the portfolio reaches its target in the early 1990s. However, the position does not scale down quickly enough and leaves the portfolio significantly over-invested in private funds starting in the early 2000s. In both cases, the allocations to private funds do not decline in magnitude nearly as much as the TDF equity allocations. In short, committing a fixed percentage of the contemporaneous target allocation is insufficient for matching the glide path dynamics. Consequently, a superior allocation strategy must follow a more dynamic commitment process to be able to closely match the asset allocation of TDFs.

To this end, we propose a simple model that incorporates the dynamic aspects to allocations while remaining simple and feasible. Referring back to the NAVs and cumulated cash flows reported in Table 5, we assume the optimal commitment strategy follows a time-weighted average of contemporaneous and future target allocations. We then optimize the expected allocations based on the logistic model glide-path and the historical data on commitments, distributions, and NAVs assuming public market returns to be constant (at the average values within our sample period). This effectively allows us to forecast (on an expected value basis), the value of both public and private capital investments and infer the portfolio allocation. This provides all necessary inputs for a deterministic model that allows us to estimate the weights of an optimal commitment function by minimizing the difference between targeted and modeled allocations.

The method allows for allocations to be a function (e.g., weighted average) of each future year's target allocations based on a specific glide-path. We find in the case of the logistic TDF that a good approximation to the general weighting function is simply targeting a fixed over-allocation strategy referencing the TDF target 4 years in the future ($Target_{t+4}$).[15] In particular, the resulting commitment strategy for buyout funds at time t follows

$$Commitment_t = NAV_{t-1} * 0.30 * Target_{t+4} \quad . \tag{1}$$

One shortcoming of this approach is that it utilizes average historical returns, NAVs and cash flows. Applying this strategy to historical quarterly data reveals that the variation in returns and cash flows can be large and cyclical. In particular, the realized allocations to private market funds still decay at too slow of a rate. In addition, for TDFs early in their life-cycle (e.g., a 2035 TDF), this strategy results in underinvestment although less severe than with the naïve contemporaneous allocation strategy. To incorporate historical cyclical variation into the estimation, we introduce an additional time-varying weight which allows for further augmenting the dynamic allocation strategy. Specifically, we include an exponential weighting function that increases allocations prior to the target date and decreases allocations after the target date (relative to equation (1)). We optimize the

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[15] More precisely, every solution to our minimization problems placed at least 95% of the overall weight on the allocation target four years out. For better applicability we generalize without loss of precision to a single weight function. Also, the weight placed on the year four target allocation varied marginally around our final estimate of 0.295 for various target dates.

allocations for our historical sample of buyout funds for the three target dates. We find better allocations are obtained using

$$Commitment_t = (NAV_{t-1} * 0.21 * Target_{t+4}) * e^{0.04*(Target-t)} \quad . \tag{2}$$

While the time-weighted approach outlined above provides a better dynamic commitment strategy, its simplicity comes at the cost of forgoing a period-by-period estimation that takes current conditions into account (e.g., use other information such as recent investment rates and NAVs of funds already invested in). For example, an investor could define their own specific ramp-up schedule and commit to funds in a way that minimizes deviations from the target each year. However, such a setting requires solving a complex optimization problem and could generate a very volatile commitment schedule (which raises concerns along the lines of liquidity management and diversification among vintage years). A nice feature of our calibrated model is that the commitment schedule is smooth and predictable. As shown in Figure 8 for the 2005 TDF, following the commitment strategy shown in Equation 2 leads to fund allocations much closer to the targets. Compared to the naïve and over-allocation strategies, the calibrated model still overweight's PE in some years and underweights in others but is within a few percent of the target in almost all years.

In Table 7, we present return and risk measures for the calibrated model (Equation 2) for the hypothetical TDF with target date 2005. The average returns of the TDF portfolio are higher than for the all-public benchmark and the calibrated portfolio outperforms in 82% of the 1,000 simulations. The adjusted standard deviation falls considerably from 9.89% to 8.50% suggesting that the diversification benefits from adding buyout funds remain substantial. This fall in risk drives the Sharpe ratio up for the calibrated portfolio so that in 100% of simulations, it is higher than for the all-public benchmark. This is the first evidence we are aware of that an investor can invest their portfolios with private market funds and achieve substantial diversifications benefits while at the same time manage dynamic allocation targets within reasonable bounds.

5. Conclusions

Allocations to private market funds have grown dramatically over the course of the last decades. Portfolios of institutional investors, university endowments and selected individual investors have exposure to such private investment; yet the traditional portfolio management methods in place have only very limited power in guiding asset allocation and portfolio optimization decisions.

To date, the academic literature has offered few practical solutions for how to incorporate illiquid assets into diversified portfolios. We make use of a comprehensive historical dataset to document properties of portfolios that include significant allocations to private funds. Traditional buyout funds lead to both higher returns and lower risk suggesting a widespread benefit. In contrast, VC funds increase returns, but also change the nature of risks more by increasing volatility primarily through positively skewed returns (i.e., "upside" risk).

Overall, we demonstrate the superior historical performance of portfolios with private funds and document diversification benefits from buyout funds especially. In the process, we extend the traditional portfolio management frameworks and risk assessment methods to account for data limitations arising from illiquid private market funds. Our results suggest that typical diversified portfolios benefit from allocations to private investments but that the nature of the benefit depends on the type of private investment.

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Tables and Figures

Table 1. Fund and Benchmark Summary Statistics

This table reports summary statistics for the buyout funds (BO) and venture capital funds (VC) in our sample which covers 1987-2017 vintage funds. Performance characteristics are represented by three measures: Panel A reports the Internal Rate of Return (IRR, in percent), Panel B reports the Total Value Paid In (TVPI) multiple and Panel C reports the Public Market Equivalent (see Kaplan and Schoar (2005) for further discussion) using the S&P 500 as the benchmark return.

	N	Mean	Min	1%	5%	10%	25%	50%	75%	95%	99%	Max
Panel A: IRR												
Buyout	1,121	12.2	-100.0	-39.8	-12.3	-5.6	4.9	12.2	20.2	38.4	58.7	127.9
Venture Capital	1,394	13.5	-100.0	-40.2	-17.1	-11.6	-3.0	6.5	17.5	60.3	203.1	516.2
Panel B: TVPI												
Buyout	1,121	1.59	0.00	0.28	0.68	0.89	1.15	1.48	1.93	2.83	3.75	6.08
Venture Capital	1,394	1.88	0.00	0.13	0.38	0.54	0.90	1.28	1.88	5.21	14.81	42.41
Panel C: PME												
Buyout	1,121	1.12	0.00	0.15	0.48	0.67	0.87	1.06	1.31	1.95	2.49	4.37
Venture Capital	1,394	1.18	0.00	0.10	0.23	0.35	0.62	0.88	1.21	2.84	9.11	23.12

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Table 2. Valuation and Cash Flow Lifecycle Dynamics

This table reports the median net asset values (NAVs) and cash flows for our sample funds by years since inception as a[...] are the accumulated payments by the LP to the fund (capital calls). Distributions are the accumulated payoffs the LP [...] buyout funds and Panel B reports values for Venture Capital funds. P5 is the 5th percentile of all funds and P95 is the 9[...]

	Year	1	2	3	4	5	6	7	8	9	10
Panel A: Buyout Funds											
	P5	0.01	0.11	0.22	0.29	0.33	0.29	0.21	0.10	0.06	0.0?
NAV	Median	0.18	0.38	0.57	0.70	0.76	0.76	0.69	0.57	0.44	0.3?
	P95	0.54	0.90	1.11	1.26	1.31	1.33	1.30	1.22	1.09	0.9?
	P5	0.02	0.15	0.28	0.45	0.58	0.70	0.77	0.79	0.81	0.8?
Contributions	Median	0.20	0.40	0.61	0.77	0.89	0.95	0.98	0.99	1.00	1.0(
	P95	0.55	0.86	0.97	1.05	1.10	1.16	1.18	1.20	1.21	1.2?
	P5	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.09	0.21	0.2?
Distributions	Median	0.00	0.00	0.05	0.14	0.28	0.45	0.66	0.88	1.08	1.2?
	P95	0.06	0.19	0.41	0.76	1.14	1.56	1.82	2.05	2.34	2.5?
Panel B: Venture Capital Funds											
	P5	0.06	0.14	0.22	0.24	0.23	0.17	0.14	0.10	0.05	0.0?
NAV	Median	0.20	0.41	0.55	0.66	0.66	0.67	0.64	0.56	0.45	0.3(
	P95	0.78	1.15	1.63	1.73	1.86	1.88	1.54	1.55	1.29	1.2(
	P5	0.09	0.20	0.33	0.48	0.61	0.72	0.80	0.85	0.88	0.8?
Contributions	Median	0.22	0.44	0.63	0.78	0.87	0.93	0.97	0.99	1.00	1.0(
	P95	0.66	0.90	1.00	1.00	1.00	1.00	1.01	1.02	1.03	1.0?
	P5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.0?
Distributions	Median	0.00	0.00	0.01	0.06	0.13	0.23	0.36	0.48	0.59	0.7(
	P95	0.01	0.22	0.55	1.29	2.17	2.90	3.65	4.88	5.21	5.5?

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Table 3. Public Market Funds

This table lists the public benchmark indices used for our analysis. Only the benchmark that applies to the majority of return data for our time-series is reported in this table. For detailed construction of fund benchmark, we refer to the Vanguard fact sheets.

Index Fund	Benchmark	Ticker
Vanguard Total Stock Market	CRSP US Total Market Index	VTSMX
Vanguard Total Bond Market	Bloomberg Barclays US Aggregate Bond Index	VBMFX

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Table 4. Risk and Return in Private and Public Markets

This table reports first and second moments of the return time-series' for private and public market funds for the years 1995 to 2017 (corresponding to the steady state in our analysis). We extract the (unadjusted) return series from our portfolio simulations by making use of the linearity in expectations; that is the overall return in a given quarter is the weighted average of the individual asset returns. For the private market funds, we also report adjusted moments following the procedure outlined by Getmanksy et.al (2004). In Panel A to C, we report results from simulations were 10 funds of Buyout (Panel A) or Venture Capital (Panel B) were randomly selected per vintage year. Panel C reports results for portfolios that invest all funds in the respective index only.

	Return (arithmetic)	Return (geometric)	Volatility
Panel A: Buyout Funds			
Unadjusted	11.49%	11.15%	8.42%
Adjusted	11.52%	10.30%	15.66%
Panel B: Venture Capital Funds			
Unadjusted	14.09%	13.51%	11.02%
Adjusted	14.02%	12.17%	20.41%
Panel C: Benchmark Indices			
S&P 500	11.02%	9.58%	16.25%
Total Stock Market Index	10.66%	9.28%	16.52%

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Table 5. Diversifying Portfolios with time-invariant Target Allocation to Private Funds

This table presents mean values for the results of 1000 simulations targeting a constant allocation of 20%. All reported moments are annualized. Furthermore, we report adjusted standard deviations and Sharpe ratios following the approach outlined in Getmansky et.al. (2004) to account for smoothed reporting of NAVs. Moments of the return distributions and measures of risk are reported for the steady state period. In addition, we report the average allocation to each asset class in the steady state and the percentage of simulations that outperformed the base case benchmark (portfolio with 60% public equity and 40% fixed income allocation). Specifications (1) and (2) report results with US buyout funds only, (3) and (4) report results with US venture capital funds only, and (5) and (6) report results where equal allocations are made to both buyout and venture capital funds. Specifications (1), (3), and (5) allocate 4% of total portfolio value annually to private funds. In contrast, specifications (2), (4), and (6) over-allocate such that the target allocation is closer to the 20% target, on average.

	All Public Benchmark	With Buyout Funds		With VC Funds		Buyout and VC Funds	
		(1)	(2)	(3)	(4)	(5)	(6)
Return	8.05%	8.69%	8.91%	9.68%	9.89%	9.26%	9.49%
Standard Deviation	9.56%	8.19%	7.78%	9.72%	9.84%	8.90%	8.76%
Standard Deviation (adjusted)	10.39%	9.47%	9.22%	12.52%	12.99%	10.83%	10.95%
Sharpe Ratio	0.60	0.78	0.85	0.76	0.77	0.78	0.82
Sharpe Ratio (adjusted)	0.55	0.67	0.72	0.59	0.58	0.64	0.66
Semi-Deviation	11.59%	9.77%	9.22%	10.37%	10.18%	10.22%	9.91%
Semi-Deviation (adjusted)	12.59%	11.29%	10.93%	13.36%	13.43%	12.45%	12.39%
Skewness	-0.67	-0.65	-0.62	0.17	0.37	-0.33	-0.18
Kurtosis	0.68	0.70	0.72	3.22	4.04	1.58	2.04
Average Allocation Fixed Income	40.0%	39.7%	39.7%	39.6%	39.6%	39.7%	39.6%
Average Allocation Public Equity	60.0%	45.6%	40.6%	43.2%	40.6%	44.4%	40.7%
Average Allocation Private Equity	0.0%	14.7%	19.7%	17.1%	19.8%	15.9%	19.7%
Average Deviation from Target	-	-5.3%	-0.3%	-2.9%	-0.2%	-4.1%	-0.3%
Std. Dev. (PE allocation - PE target)	-	2.6%	3.4%	6.0%	7.0%	3.9%	4.7%
Probability (Return Sim > Return Base Case)	-	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Probability (Adj. SD Sim < Adj. SD Base Case)	-	100.0%	100.0%	1.3%	1.3%	0.7%	0.6%
Probability (Adj. SR Sim > Adj. SR Base Case)	-	100.0%	100.0%	98.3%	97.3%	100.0%	100.0%
Probability (Adj. Semi-D Sim < Adj. Semi-D Base Case)	-	100.0%	100.0%	1.8%	1.9%	72.5%	76.6%

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Table 6. Portfolio Characteristics with Additional Fees for Managing Alternatives

This table reports the impact of an additional fee for managing the exposure to private market funds for the steady-state sample period of 1995-2017. Specification (1) re-tabulates the previous results in which no additional fee is paid and corresponds to specification (2) in Table 5 that includes buyout funds only. Specifications (2) and (3) impose an additional fee of 0.25% and 0.50%, respectively. All fees are paid from the public equity account.

DCALTA/IPC Research - 2019	All Public Benchmark	(1) No Fee	(2) +25bps	(3) +50bps
Return	8.05%	8.91%	8.86%	8.81%
Standard Deviation	9.56%	7.78%	7.78%	7.78%
Standard Deviation (adjusted)	10.39%	9.22%	9.22%	9.21%
Semi-Deviation	11.59%	9.22%	9.22%	9.21%
Semi-Deviation (adjusted)	12.59%	10.93%	10.93%	10.92%
Skewness	-0.67	-0.62	-0.62	-0.62
Kurtosis	0.68	0.72	0.72	0.72
Sharpe Ratio	0.60	0.85	0.84	0.84
Sharpe Ratio (adjusted)	0.55	0.72	0.71	0.71

Table 7. Diversifying Portfolios with time-varying Allocation to Private Equity (TDFs)

This table presents the results for N=1,000 simulations targeting a defined allocation according to a glide path for a TDF with target date 2005. All reported moments are annualized. Furthermore, we report adjusted standard deviations and Sharpe Ratios following the approach outlined in Getmansky et.al. (2004) to account for smoothed reporting of NAVs. Moments of the return distributions and measures of risk are reported for the steady state period (1995-2017). We also report the average deviation from target allocation and its standard deviation in the steady state. The bottom 3 rows report the percentage of simulations that outperformed the all-public benchmark. The calibrated portfolios are constructed by allocating annually to private equity funds following the commitment strategy shown in Equation (2).

DCALTA/IPC Research - 2019	All Public Benchmark	Calibration
Return	8.29%	8.37%
Standard Deviation (annualized, adjusted)	9.89%	8.50%
Semi-Deviation (adjusted)	9.98%	8.50%
Sharpe Ratio (adjusted)	0.59	0.75
Average Deviation from Target		0.2%
SD (PE allocation - PE target)		1.2%
Probability (Return Sim > Return Bench)		82%
Probability (Adj. SD Sim < Adj. SD Bench)		100%
Probability (Adj. SR Sim > Adj. SR Bench)		100%
Probability (Adj. Semi Sim < Adj. Semi Bench)		100%

## Figure 1. Average End-of-Quarter Portfolio Allocation
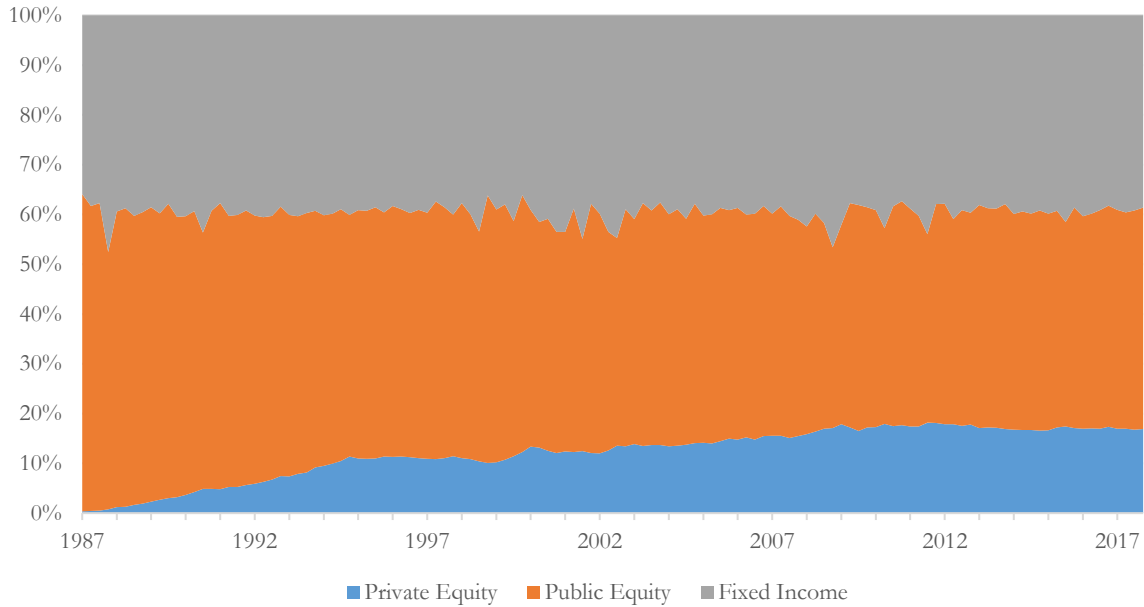
Figure 1. Average allocation per asset class. This figure shows the time-series of average alloactions for diversified portfolios for a naïve allocation strategy (N=1000 simulations**).** Generated portfolios have investment in 10 randomly selected buyout funds per vintage. DCALTA/IPC Research – 2019
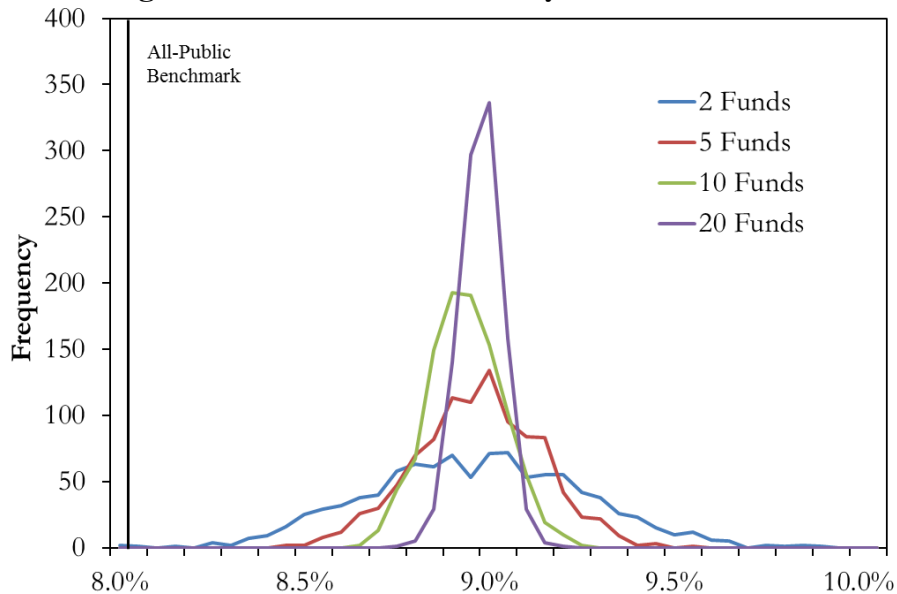
Figure 2. Portfolio Return Distribution by Asset Class
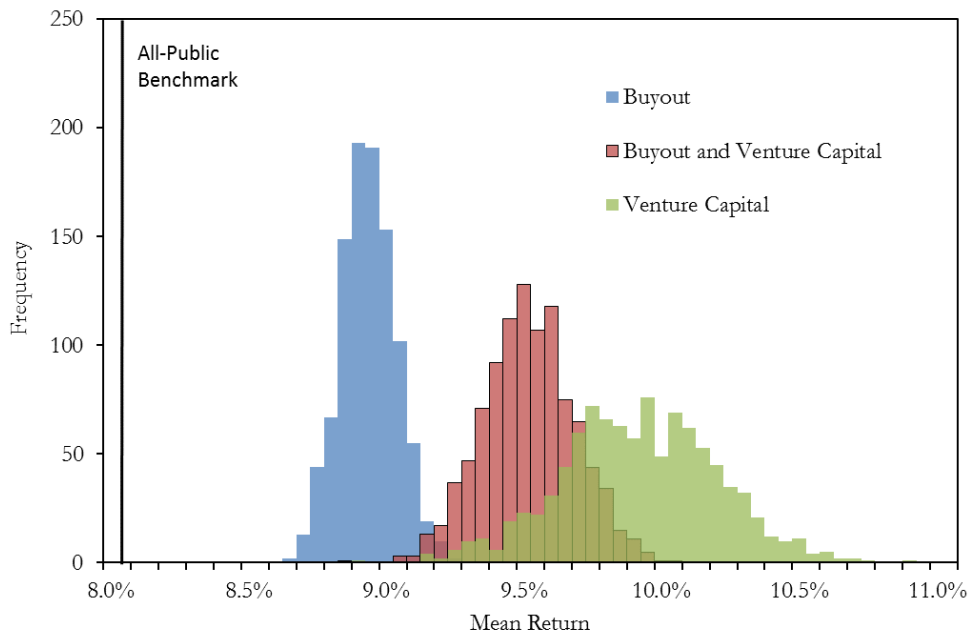
Figure 2. Mean return by asset class. This figure shows the return distributions for diversitfied portfolios with 20% allocation to private market funds, on average (N=1000 simulations**).** Generated portfolios have investment in 10 randomly selected funds (either buyout, venture capital or both) per vintage. DCALTA/IPC Research – 2019

Figure 3. Median Portfolio Allocation by Asset Class
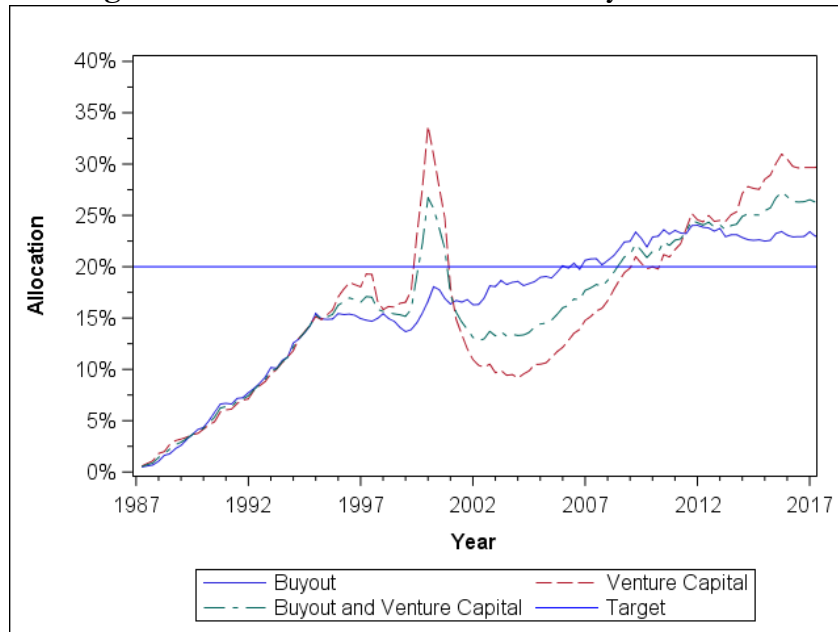
Figure 3. Median allocation by asset class. This figure shows the median allocation for diversitfied portfolios with a targeted 20% allocation to private market funds, on average (N=1000 simulations). Generated portfolios have investment in 10 randomly selected funds (either buyout, venture capital or both) per vintage. DCALTA/IPC Research – 2019

Figure 4. Return Distributions by Number of Funds

Figure 4. Return distribution by number of funds selected. This figure shows the return distributions for diversitfied portfolios with 20% average allocation to buyout funds (N=1000 simulations). Generated portfolios have investment in 2, 5, 10, 20 or all availvale buyout funds per vintage, randomly selected per vintage. DCALTA/IPC Research – 2019

**Figure 5. Allocation Distribution by Number of Funds**
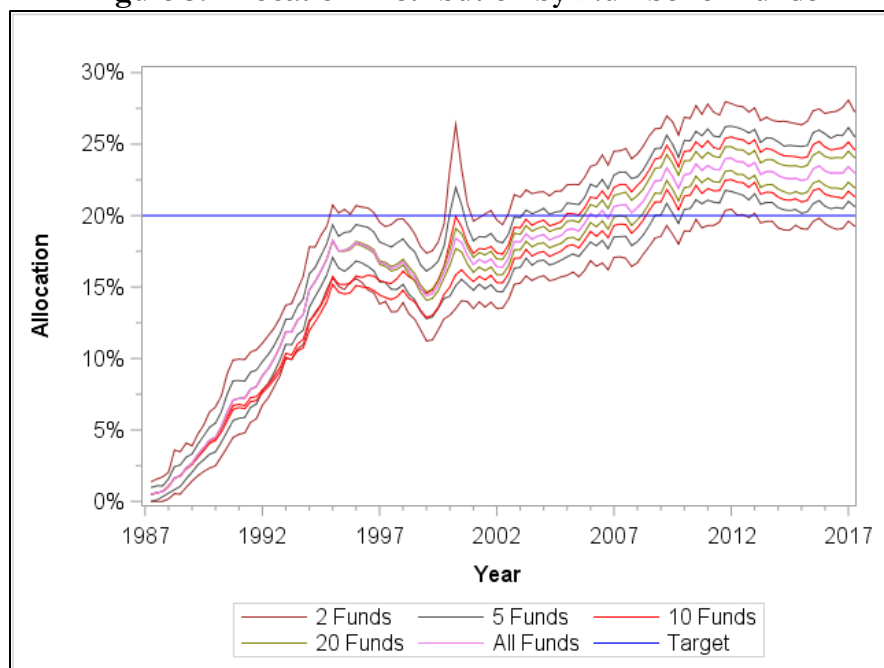
Figure 5. Allocation distribution by number of buyout funds selected. This figure shows the 5th and 95th precentile of the allocation distribution for diversitfied portfolios with a targeted 20% allocation to randomly selected buyout funds (N=1000 simulations). Generated portfolios have investment in 2, 5, 10, 20 or all availvale buyout funds per vintage, randomly selected per vintage. DCALTA/IPC Research - 2019

## Figure 6. 2005 TDF with Naïve Allocation Strategy
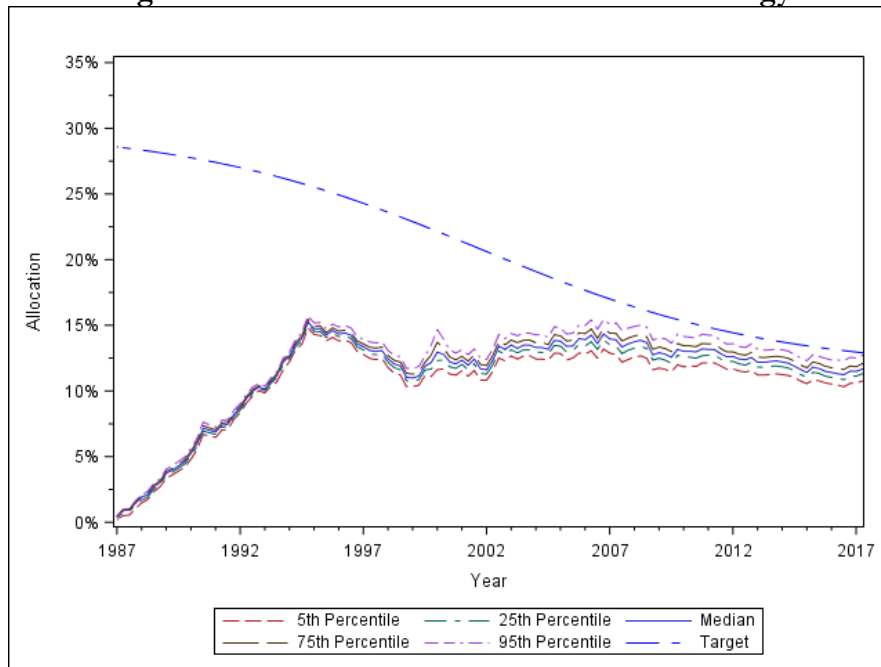
Figure 6Error! Not a valid bookmark self-reference.. 2005 target date fund without overalloaction. This figure shows the allocation distribution if a portfolio is constructed by simply allocating a fixed percentage of the contemporenaous target alloaction to buyout funds. DCALTA/IPC Research - 2019

Figure 7. 2005 TDF with Over-Allocation
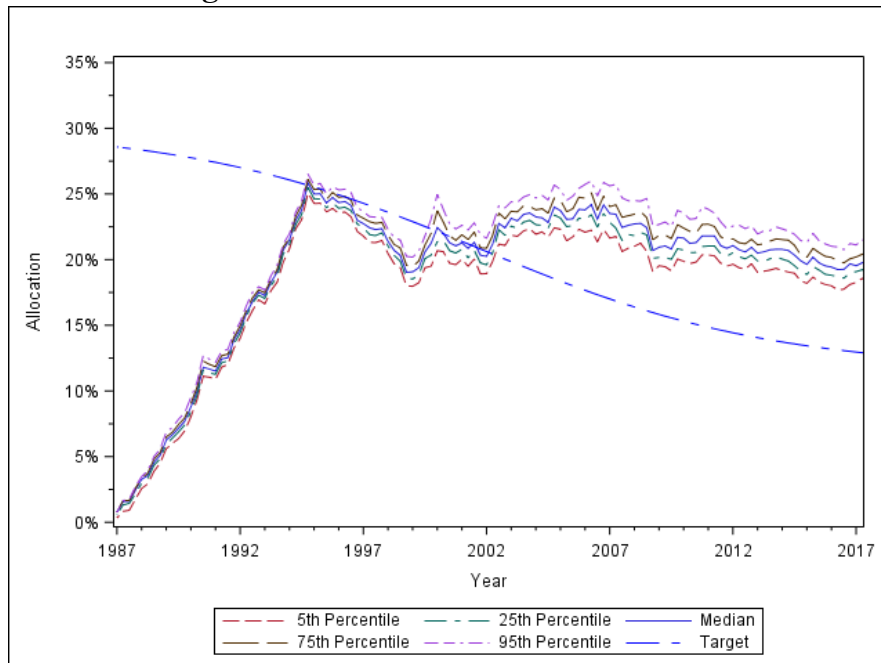
Figure 7. 2005 target date fund with overalloaction. This figure shows the allocation distribution if a portfolio is constructed by simply overallocating a fixed percentage of the contemporenaous target alloaction to buyout funds. DCALTA/IPC Research - 2019

Figure 8. 2005 TDF with Calibrated Allocation
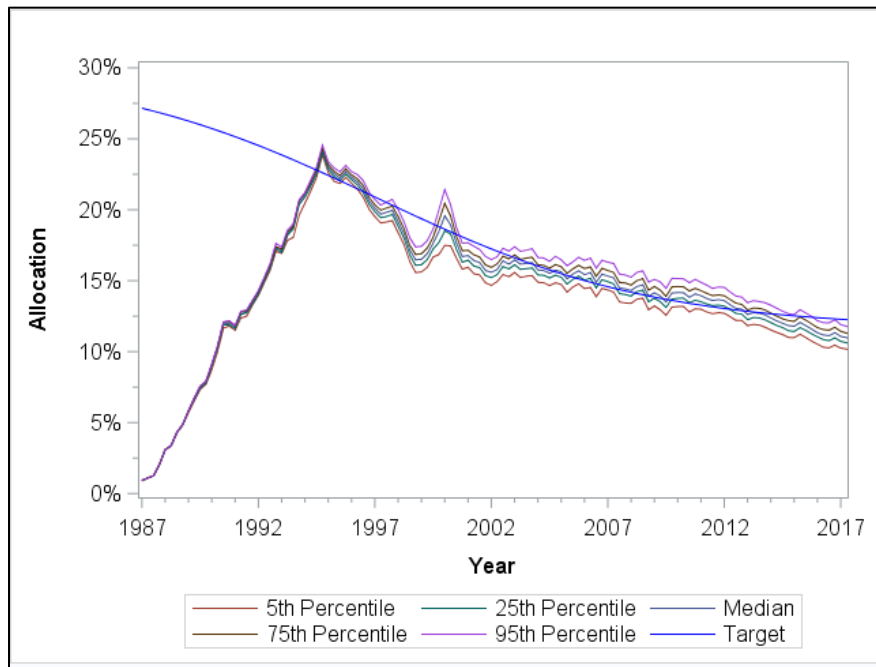
Figure 8. 2005 target date fund with calibrated allocation strategy. This figure shows the allocation distribution if a portfolio is constructed following the calibrated overallocating strategy. DCALTA/IPC Research - 2019