Circular functions and trigonometry

1. The circle: radian measure of angles. Length of an arc; area of a sector.

- <u>Be able to move from radians to degrees and vice versa:</u>

I always start with $360° = 2\pi\ radians$. This is easy to remember because a whole revolution of a circle is $360°$, and the path to cover the whole circle (circumference) is $2\pi r$. So think of "r" in this case as not a radius, but radians to help you remember.

Then the rest of the angles can be derived easily.

<u>Be able to calculate length of arc and area of sector, in terms of radians:</u>

$$Arc\ length = \frac{\theta}{2\pi} \times 2\pi r = \theta r$$

$$Arc\ length = \frac{\theta}{2\pi} \times \pi r^2 = \frac{\theta r^2}{2}$$

Note that $\frac{\theta}{2\pi}$ is just expressing what fraction of the circle we are interested in.

2. Definition of cosθ , sinθ and tanθ in terms of the unit circle. Exact values of sin, cos and tan of 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. Definition of the reciprocal trigonometric ratios secθ, cscθ and cotθ. Pythagorean identities: $\cos^2\theta + \sin^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$.

- <u>Be able to know how to use sinθ ,cosθ and tanθ:</u>

All you need to remember is SOHCAHTOA.

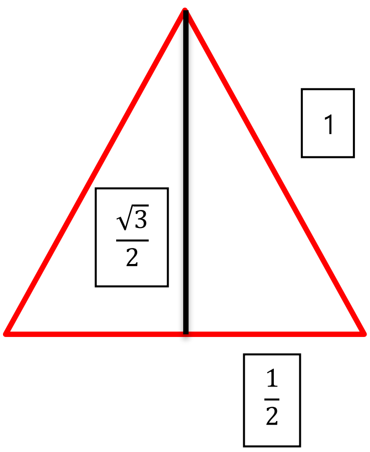
$$Sin\theta = \frac{O}{H}$$

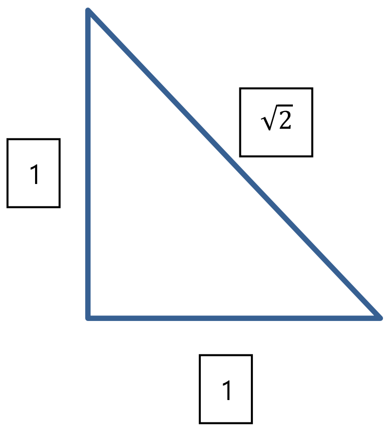
$$Cos\theta = \frac{A}{H}$$

$$Tan\theta = \frac{O}{A} = \frac{Sin\theta}{Cos\theta}$$

<u>Be able to know exact values of</u> *0*, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$:

You just need to know two triangles: An equilateral triangle with side magnitude "1" that is divided in half and an isosceles with two sides with magnitude "1". Deduce the angles!





Be able to use reciprocal ratios:

This are nothing too difficult, but is more of knowing the terms.

$$csc\theta = \frac{1}{sin\theta}$$

$$sec\theta = \frac{1}{cos\theta}$$

$$cot\theta = \frac{1}{tan\theta}$$

Be able to use the Pythagorean identities:

These are provided in the data booklet and the formulas are all derived from Pythagoras's theorem.

3. Compound angle identities. Double angle identities.
- Be able to use complementary angles:

Complementary angle essentially allows you to move between sin and cos.

Consider $\alpha + \beta = \frac{\pi}{2}$.

Then $sin\alpha = cos\beta$ because you will notice that they are defined so that they are always complementary to each other. Thus we can also express like this by substitution:

$$sin\alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$$

Be able to use compound angle identities and double angle identities:

This is a way to expand expressions like $sin(A + B)$ and $sin(2C)$**.** No proof is needed and it is included in the formula booklet.

If you try to expand the expression mentioned above, $\cos\left(\frac{\pi}{2} - \alpha\right)$, you will indeed get that it equals to $sin\alpha$.

4. Composite functions of the form f(x) = asin(b(x + c)) + d.

- Be able to transform $f(x) = asin(b(x + c)) + d$:

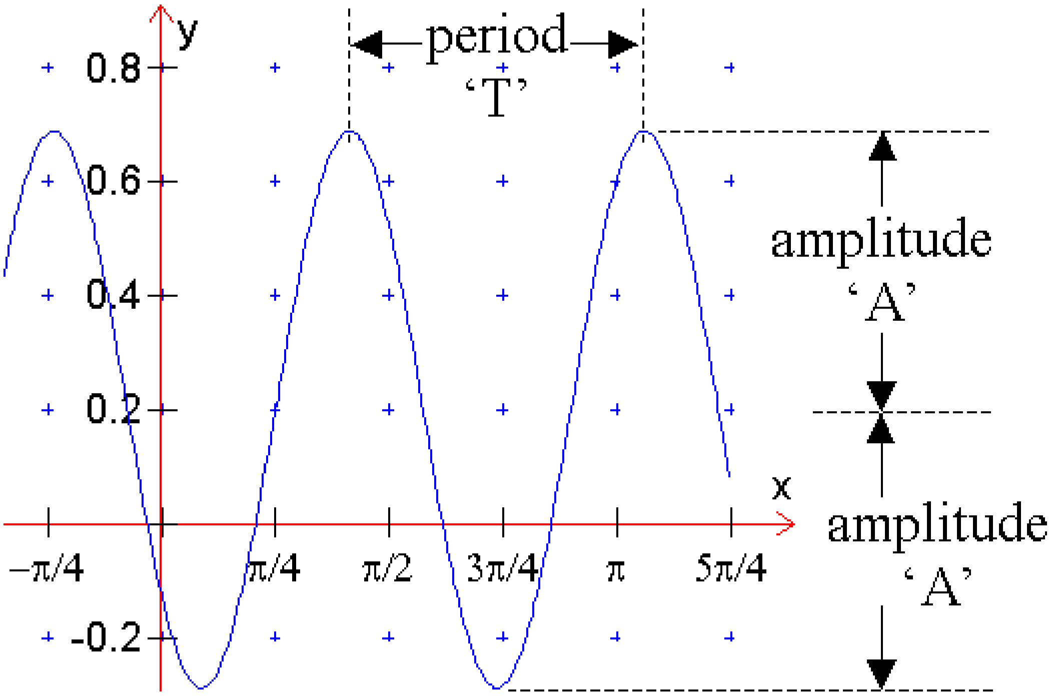
a = amplitude

b = angular velocity. If $b > 0$ it will increase oscillation and if $b < 0$ it will decrease.

c = phase shift or the shift horizontally. It works in the same way as regular f(x) function.

d = vertical shift like regular linear f(x).

T = period, also known as wavelength.



How do we find the values using a specific graph? The formulae are rather intuitive.

$a = \frac{max-min}{2}$, because we want to take half of the total range the graph is oscillating.

$b = \frac{2\pi}{T}$, because basically we are asking how fast does the graph oscillate? We are basically looking at how many whole revolutions (2π) we can fit in one T (our wavelength). If our wavelength is very large, it means we can fit many whole revolutions, which means our angular velocity must be very slow.

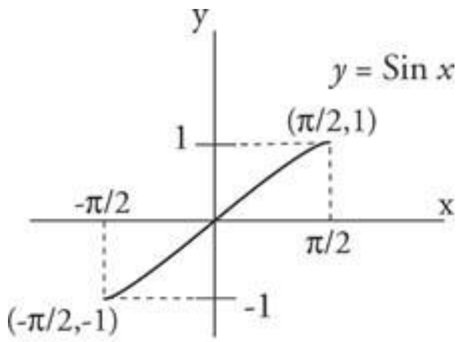
$d = \frac{max+min}{2}$, because now we are looking at the midpoint or the "average".

$c = set\ f(0)\ or\ any\ convenient\ input\ and\ solve\ for\ c.$

5. The inverse functions x→arcsinx, x→arccosx, x→arctanx; their domains and ranges; their graphs.

- <u>Be able to graph inverse trig functions:</u>

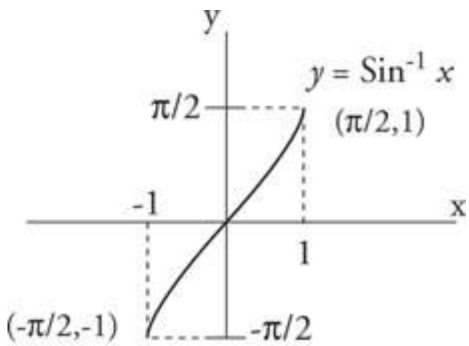
$f(\theta) = sin(\theta)$



Domain: $-\pi/2 \leq x \leq \pi/2$

Range: $-1 \leq y \leq 1$

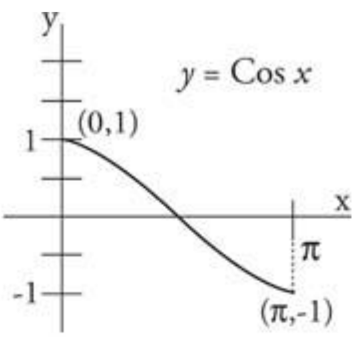
$f^{-1}(\theta) = arcsin(\frac{O}{H})$



Domain: $-1 \leq x \leq 1$

Range: $-\pi/2 \leq y \leq \pi/2$

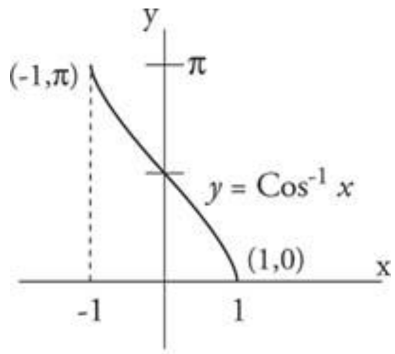
$f(\emptyset) = cos(\emptyset)$



Domain: $0 \leq x \leq \pi$

Range: $-1 \leq y \leq 1$

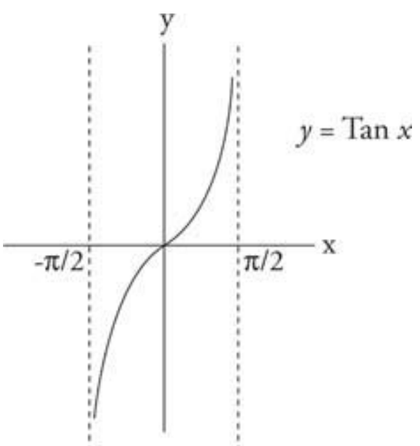
$f^{-1}(\emptyset) = arccos(\frac{A}{H})$



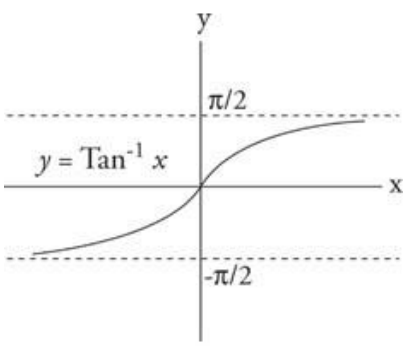
Domain: $-1 \leq x \leq 1$

Range: $0 \leq y \leq \pi$

$f(\beta) = tan(\beta)$



$f^{-1}(\beta) = arctan(\frac{O}{A})$



Domain: All real numbers

Range: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

6. Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization.

- Be able to solve trig equations with finite intervals:

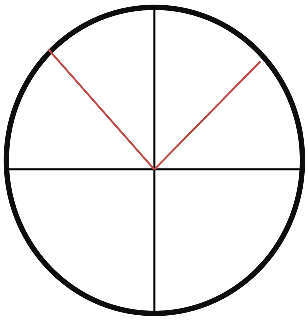
The key is to identify which trig identity to use.

$$sin\theta = sin\emptyset$$

$$Solution\ 1: \theta = \emptyset \pm 2\pi k$$

$$Solution\ 2: \theta = \pi - \emptyset \pm 2\pi k$$

Why $\pi - \emptyset$? Well, if you draw the unit circle you will see that sin will have same values for $\pi - \emptyset$ because it is an odd function. The diagram below shows that by definition (opposite over hypotenuse) is indeed identical for both angles.

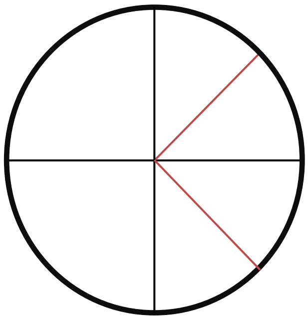


$$cos\theta = cos\emptyset$$

$$Solution\ 1: \theta = \emptyset \pm 2\pi k$$

$$Solution\ 2: \theta = -\emptyset \pm 2\pi k$$

Why only $-\emptyset$? If we draw again, cos is an even function so its negative angle will always be identical to its positive angle.



For tan, the only important thing to remember is that its period is πk and not $2\pi k$.

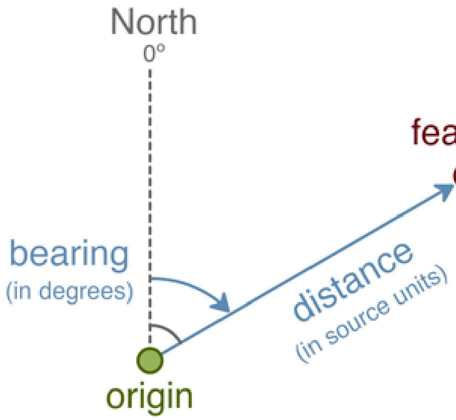
Always make sure that your answers are within the given boundaries in the question!

7. The cosine rule. The sine rule including the ambiguous case. Area of a triangle as $\frac{1}{2}absinC$.

- <u>Be able to use compound angle transformation(?):</u>

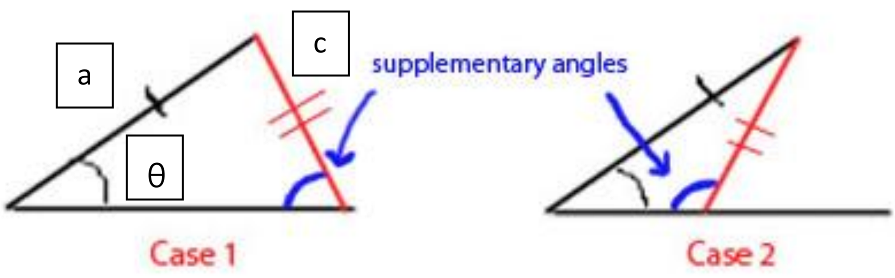
<u>Be able to solve problems involving bearings:</u>

Bearing is the angle in reference from the north direction of viewer's perspective, rotated clockwise.



<u>Be able to use sine and cosine rules:</u>

Both formulae are in the formula booklet. However, there may be ambiguous cases where cos rule and sin rule give different values.



When given side a, c and angle θ, there can be two possible solutions. So when using sine rule, remember that there is always another solution with its complementary angle $(\pi - \theta)$.

So when there is a disagreement between cosine rule and sine rule, check that you are using the correct angle and length values.

<u>Be able to calculate the area of triangle using sin:</u>

This is in the formula booklet too so you need to be able to only plug in the numbers.