Online Appendix: Treating Time With All Due Seriousness*

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Grant and Lebo have raised concerns about the properties of the GECM in a variety of cases. In our paper, "Treating Time with All Due Seriousness," we reassert the mathematical equivalence between the ADL and GECM and its appropriateness for stationary time series and weakly exogenous regressors. Here we use simulations to further buttress our claims.

Grant and Lebo perform simulations to test the performance of the GECM in six cases. In all cases the data in question are unrelated; they are testing for spurious relationships. In case 1 both the dependent and independent variables are integrated but the data are not cointegrated such that error correction is inappropriate. In case 2, the dependent variable is a bounded unit root. These two cases are beyond the scope of our discussion and we have no issue with their evidence on this score. In case 3, the dependent and independent variables are stationary and importantly, *white noise* processes. We discussed this case in our response, noting that the long run relationship has a unique character because the dependent variable exhibits no inertia, it responds immediately to any shocks in X_t (a case Grant and Lebo did not consider) or unmodeled shocks more generally. In effect, the long run equilibrium is not dynamic in the sense that the effects of X_t are not carried forward into the future through lagged Y_t. Thus the error correction model is not the right one in this case, but once again, neither is an ADL where the coefficient on the lagged dependent variable is zero. We argue that this case is both uninteresting and unlikely to occur because few — in our careers we have yet to encounter any — political time series are purely white noise. In case 4, the dependent variable is stationary but the independent variable is a unit root. In this case the left and righthand sides of the equation are unbalanced and no model relating the untransformed variables is appropriate. In case 5, the dependent variable is fractionally integrated. We addressed our concerns with this case in the response, noting that estimates of fractional integration are highly uncertain and that this uncertainty propagates to the GECM. In case 6, the dependent variable is explosive. The equation is unbalanced in this case as well and no model involving the untransformed data will be appropriate.

In this appendix we demonstrate that when the dependent and independent variables are stationary with varying degrees of autocorrelation in X_t, the GECM performs as asserted. In particular, the estimates of short and long run effects are unbiased, we reject the true null that the data are unrelated at conventionally accepted rates, and we reject the false null at least as often as convention accepts. All simulations were conducted in R and the code is available on request.

In the first set of experiments we simulate two autoregressive, stationary time series that are unrelated to each other and estimate a GECM. We consider a range of autoregression parameters for X_t and fix Y_t as moderately autoregressive. The latter decision simply fixes the value of the error correction coefficient across the experiments. We simulated:

$$Y_t = 0.5Y_{t-1} + e_{1t} \tag{1}$$

$$X_t = \rho X_{t-1} + e_{2t} \tag{2}$$

where $e_{1t}, e_{2t} \sim N(0, 1)$ and ρ ranged from 0 to 0.9 by increments of 0.10. We then estimated the GECM:

$$\Delta Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \beta_3 \Delta X_t \tag{3}$$

for sample sizes = 25, 50, 75, 100, 250, 500, 750, and 1000. The true value of $\alpha = \beta_2 = \beta_3 = 0$. The true value of β_1, the error correction rate, is $0.50 - 1 = -0.50$, even though there is no long run relationship between X_t and Y_t. In this case, the error correction rate tells us how quickly Y_t adjusts to unmodeled shocks and how much inertia the process contains.

The key question for the simulations is whether, as Grant and Lebo maintain, we incorrectly find evidence of a long run relationship between X_t and Y_t. Our focus is thus on the rejection rates and biases in β_2 and the long run multiplier, $\beta_2 / -\beta_1$. Across the 80 experiments we conducted, rejection rates on these two null hypotheses hover around the nominal 5.0% rate in samples of 50 or greater, approaching 10% when the sample sizes drops to 25 and $\rho = 0.90$.[1] See Table 1. The mean bias on the LRM for X_t (whose true value is 0) is always less then 0.037, which occurs in a sample of size 25; the magnitude of the bias averages less than 0.006.[2] See Table 2.

[1] The average rejection rate on these null hypotheses were 5.93 and 5.89%, respectively, with a standard deviation of 1.46. They were largest when $T = 25$, reaching 9.80% and 11.40% when $\rho = .90$.

[2] The value of ρ has little to no impact on the magnitude of these biases.

The comparative values for β_2 are smaller still. There is no evidence that analysts would falsely conclude that there is a long run relationship between X_t and Y_t at unacceptable levels in this scenario. We wish to make clear that this occurs even in the presence of significant error correction rates. Rejection rates on the error correction coefficient hit 1.0 once the sample size hits 75; we are very precisely estimating the autoregressive nature of Y_t and thus the rate it returns to its long run equilibrium. But as the simulations demonstrate, the conclusion an analyst would draw in this case – finding a significant error correction but nonsignificant effects of X_t – are that all the dynamics in Y_t are working through Y_t and are not conditional on X_t. Further, the distribution of the t-statistic is standard in this case.

In the second set of experiments we examine the behavior of the GECM when the ADL model describes the DGP – when X_t has a long run effect on Y_t. We allow X_t to be generated in the same manner as the previous experiment and now let:

$$Y_t = 0.5Y_{t-1} + 0.25X_t + 0.50X_{t-1}. \tag{4}$$

We once again estimate the GECM given in equation 3. Following the algebraic equivalences between the ADL and GECM, this implies that the true GECM values are $\alpha = 0$, $\beta_1 = -0.50$, $\beta_2 = 0.25$ and $\beta_3 = 0.75$. The long run relationship between X_t and Y_t can be described by the long run multiplier, which is $\beta_2 / -\beta_1 = 1.5$. In this set of 80 simulations, the biases are again small, across all estimated coefficients and the LRM. See Table 3. Those on the effects of X_t average about 0.012 or about 1.5% of the true value. The LRM bias is of similar magnitude, averaging about 0.016 or about 1%. The rejection rates on the coefficients are presented in Table 8.

Finally, we conduct a third set of experiments related to the utility of fractional integration methods in political science. These methods, including the FECM method proposed by Grant and Lebo, rely critically on the estimation of the fractional difference parameter d. We conduct a series of experiments using the default maximum likelihood procedure included in standard statistical software packages. We simulate an ARFIMA(0,d,0) where the AR (ϕ) and MA (θ) parameters are set to zero, an ARFIMA(1,d,0) model where $\phi = 0.6$, an ARFIMA(0,d,1) model with $\theta = 0.6$, and an ARFIMA(1,d,1) model where $\phi = 0.5$ and $\theta = 0.3$ with a range of fractional differencing parameters ($d = 0, .1, .2, .3, .4, .45$) and sample sizes ($t = 50, 100, 250, 500, 1,000, 1,500$). The results are presented in tables five, six, seven, and eight. They show that the default maximum likelihood procedure performs poorly in small to medium samples and that these problems are exacerbated as the models become more complex.

References

Table 1: Rejection Rates for Estimated Coefficients for Selected Autocorrelation and Sample Sizes. X_t and Y_t are Unrelated

T	$\hat{\phi}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_2/-\hat{\beta}_1$	$\hat{\beta}_2$ Coverage Rate
25	0	0.964	0.07	0.049	0.930
50	0	1.000	0.056	0.044	0.944
75	0	1.000	0.064	0.052	0.936
100	0	1.000	0.076	0.067	0.924
250	0	1.000	0.039	0.038	0.961
500	0	1.000	0.052	0.05	0.948
750	0	1.000	0.048	0.047	0.952
1000	0	1.000	0.051	0.051	0.949
25	0.1	0.976	0.068	0.045	0.932
50	0.1	1.000	0.085	0.071	0.915
75	0.1	1.000	0.052	0.05	0.948
100	0.1	1.000	0.051	0.044	0.949
250	0.1	1.000	0.041	0.04	0.959
1000	0.1	1.000	0.051	0.051	0.949
25	0.5	0.977	0.097	0.096	0.903
50	0.5	1.000	0.066	0.074	0.934
75	0.5	1.000	0.061	0.062	0.939
100	0.5	1.000	0.056	0.051	0.944
250	0.5	1.000	0.047	0.046	0.953
1000	0.5	1.000	0.047	0.047	0.953
25	0.7	0.975	0.09	0.099	0.910
50	0.7	1.000	0.071	0.075	0.929
75	0.7	1.000	0.076	0.081	0.924
100	0.7	1.000	0.052	0.052	0.948
250	0.7	1.000	0.06	0.06	0.94
1000	0.7	1.000	0.047	0.046	0.953
25	0.9	0.974	0.098	0.114	0.902
50	0.9	1.000	0.087	0.105	0.913
75	0.9	1.000	0.078	0.089	0.922
100	0.9	1.000	0.082	0.094	0.918
250	0.9	1.000	0.046	0.053	0.954
1000	0.9	1.000	0.042	0.043	0.958

The data generating processes are given by $Y_t = 0.5Y_{t-1} + e_{1t}$; $X_t = \rho X_{t-1} + e_{2t}$; and $e_{1t}, e_{2t} \sim IN(0,1)$. The estimated GECM is given by $\Delta Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \beta_3 \Delta X_t$.Results are for 1000 simulations. The true value of $\alpha = \beta_2 = \beta_3 = 0$. The true value of β_1, the error correction rate, is $0.50 - 1 = -0.50$. The true value of the long run multiplier is 0.

Table 2: Average Biases in Estimated Coefficients for Selected Autocorrelation and Sample Sizes. X_t and Y_t are Unrelated

T	$\hat{\phi}$	$\hat{\alpha_0}$	$\hat{\beta}_1$	$\hat{\beta}_3$	$\hat{\beta}_2$	$\hat{\beta}_2/-\hat{\beta}_1$
25	0	-0.001	-0.109	-0.006	0.012	-0.016
50	0	0.008	-0.050	0.003	0.004	-0.008
75	0	0.006	-0.028	-0.005	-0.001	0.001
100	0	-0.003	-0.024	0.000	-0.003	0.007
250	0	-0.002	-0.010	-0.002	-0.004	0.007
1000	0	0.000	-0.002	0.000	0.000	0.000
25	0.1	-0.008	-0.114	-0.016	-0.012	0.019
50	0.1	0.006	-0.055	-0.001	0.011	-0.023
75	0.1	0.004	-0.033	0.002	0.008	-0.012
100	0.1	0.002	-0.027	0.003	0.004	-0.009
250	0.1	-0.001	-0.012	-0.001	-0.001	0.001
1000	0.1	-0.001	-0.003	0.001	0.000	-0.001
25	0.5	-0.011	-0.140	0.007	0.006	-0.004
50	0.5	-0.001	-0.070	-0.006	-0.006	0.010
75	0.5	0.006	-0.043	-0.001	0.000	-0.001
100	0.5	0.005	-0.034	0.000	-0.004	0.009
250	0.5	-0.002	-0.011	-0.003	-0.001	0.002
1000	0.5	-0.001	-0.002	-0.001	-0.001	0.001
25	0.7	-0.012	-0.148	-0.012	-0.003	-0.005
50	0.7	0.010	-0.065	0.002	0.005	-0.010
75	0.7	-0.002	-0.050	0.003	0.002	-0.004
100	0.7	-0.004	-0.034	-0.003	0.002	-0.003
250	0.7	-0.001	-0.013	0.001	0.001	-0.001
1000	0.7	-0.001	-0.003	0.000	-0.001	0.002
25	0.9	0.008	-0.152	-0.009	-0.004	0.019
50	0.9	0.003	-0.072	0.000	-0.005	0.010
75	0.9	-0.004	-0.050	0.002	-0.001	0.002
100	0.9	0.002	-0.041	-0.003	-0.001	0.002
250	0.9	0.002	-0.014	0.001	-0.001	0.001
1000	0.9	0.001	-0.004	-0.001	0.000	0.001

The data generating processes are given by $Y_t = 0.5Y_{t-1} + e_{1t}$; $X_t = \rho X_{t-1} + e_{2t}$; and $e_{1t}, e_{2t} \sim IN(0,1)$. The estimated GECM is given by $\Delta Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \beta_3 \Delta X_t$.Results are for 1000 simulations. The true value of $\alpha = \beta_2 = \beta_3 = 0$. The true value of β_1, the error correction rate, is $0.50 - 1 = -0.50$. The true value of the long run multiplier is 0.

Table 3: Average Biases in Estimated Coefficients for Selected Autocorrelation and Sample Sizes. X_t and Y_t are Related

T	$\hat{\phi}$	$\hat{\alpha_0}$	$\hat{\beta}_1$	$\hat{\beta}_3$	$\hat{\beta}_2$	$\hat{\beta}_2/-\hat{\beta}_1$
25	0	-0.006	-0.078	-0.008	0.006	0.068
50	0	-0.003	-0.041	-0.006	0.005	0.043
75	0	0.004	-0.021	-0.007	-0.002	0.024
100	0	0.002	-0.016	-0.003	-0.004	0.026
250	0	-0.001	-0.009	0.001	0.002	0.013
1000	0	-0.001	-0.001	0.001	0.000	0.001
25	0.1	0.000	-0.089	-0.004	0.032	0.058
50	0.1	0.005	-0.036	0.002	0.004	0.037
75	0.1	-0.003	-0.024	0.000	0.008	0.022
100	0.1	-0.002	-0.022	0.002	0.008	0.022
250	0.1	-0.004	-0.008	0.001	0.001	0.013
1000	0.1	0.000	-0.002	-0.002	0.002	-0.001
25	0.5	0.002	-0.075	-0.004	0.043	0.029
50	0.5	0.002	-0.038	-0.003	0.024	0.024
75	0.5	-0.002	-0.024	-0.002	0.018	0.012
100	0.5	0.003	-0.017	0.000	0.015	0.006
250	0.5	-0.001	-0.005	0.004	0.004	0.002
1000	0.5	0.000	-0.002	-0.001	0.002	0.002
25	0.7	0.003	-0.085	-0.011	0.048	0.057
50	0.7	-0.004	-0.035	0.001	0.037	0.004
75	0.7	0.001	-0.022	-0.004	0.018	0.014
100	0.7	-0.002	-0.018	-0.006	0.010	0.020
250	0.7	0.001	-0.008	0.002	0.006	0.007
1000	0.7	0.001	-0.002	-0.001	0.002	0.002
25	0.9	0.002	-0.088	-0.011	0.069	0.057
50	0.9	-0.003	-0.038	-0.006	0.037	0.022
75	0.9	0.003	-0.024	-0.003	0.026	0.013
100	0.9	-0.002	-0.018	-0.001	0.018	0.011
250	0.9	-0.002	-0.007	0.003	0.008	0.003
1000	0.9	-0.001	-0.002	0.000	0.002	0.001

The data generating processes are given by $Y_t = 0.5Y_{t-1} + 0.25X_t + 0.50X_{t-1}$; $X_t = \rho X_{t-1} + e_{2t}$; and $e_{1t}, e_{2t} \sim IN(0,1)$. The estimated GECM is given by $\Delta Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \beta_3 \Delta X_t$.Results are for 1000 simulations. The true value of the parameters are: $\alpha = 0$, $\beta_1 = -0.50$, $\beta_2 = 0.25$ and $\beta_3 = 0.75$. The long run relationship between X_t and Y_t is $\beta_2/-\beta_1 = 1.5$. The true value of β_1, the error correction rate, is $0.50 - 1 = -0.50$.

Table 4: Rejection Rates for Estimated Coefficients for Selected Autocorrelation and Sample Sizes. X_t and Y_t are Related

T	$\hat{\phi}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_2/-\hat{\beta}_1$	$\hat{\beta}_2$ Coverage Rate
25	0	0.988	0.639	0.478	0.927
50	0	1.000	0.914	0.869	0.938
75	0	1.000	0.984	0.978	0.945
100	0	1.000	1.000	1.000	0.935
250	0	1.000	1.000	1.000	0.950
1000	0	1.000	1.000	1.000	0.942
25	0.1	0.99	0.718	0.550	0.947
50	0.1	1.000	0.955	0.911	0.948
75	0.1	1.000	0.995	0.993	0.939
100	0.1	1.000	0.999	0.999	0.938
250	0.1	1.000	1.000	1.000	0.944
500	0.1	1.000	1.000	1.000	0.942
750	0.1	1.000	1.000	1.000	0.963
1000	0.1	1.000	1.000	1.000	0.956
25	0.5	0.994	0.835	0.764	0.938
50	0.5	1.000	0.990	0.986	0.933
75	0.5	1.000	1.000	1.000	0.955
100	0.5	1.000	1.000	1.000	0.947
250	0.5	1.000	1.000	1.000	0.957
1000	0.5	1.000	1.000	1.000	0.943
25	0.7	0.992	0.902	0.858	0.921
50	0.7	1.000	0.998	0.999	0.933
75	0.7	1.000	1.000	1.000	0.946
100	0.7	1.000	1.000	1.000	0.944
250	0.7	1.000	1.000	1.000	0.943
1000	0.7	1.000	1.000	1.000	0.945
25	0.9	0.996	0.943	0.930	0.916
50	0.9	1.000	1.000	1.000	0.943
75	0.9	1.000	1.000	1.000	0.943
100	0.9	1.000	1.000	1.000	0.951
250	0.9	1.000	1.000	1.000	0.940
1000	0.9	1.000	1.000	1.000	0.952

The data generating processes are given by $Y_t = 0.5Y_{t-1} + 0.25X_t + 0.50X_{t-1}$; $X_t = \rho X_{t-1} + e_{2t}$; and $e_{1t}, e_{2t} \sim IN(0,1)$. The estimated GECM is given by $\Delta Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_{t-1} + \beta_3 \Delta X_t$.Results are for 1000 simulations. The true value of the parameters are: $\alpha = 0$, $\beta_1 = -0.50$, $\beta_2 = 0.25$ and $\beta_3 = 0.75$. The long run relationship between X_t and Y_t is $\beta_2/-\beta_1 = 1.5$. The true value of β_1, the error correction rate, is $0.50 - 1 = -0.50$.

Table 5: ARFIMA(0,d,0) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.075	[−.297,.145]	−.530	−.156	−.077	.030	.197
.0	100	−.037	[−.191,.117]	−.344	−.109	−.037	.029	.160
.0	250	−.021	[−.117,.075]	−.134	−.064	−.022	.023	.098
.0	500	−.012	[−.081,.055]	−.101	−.041	−.011	.010	.087
.0	1,000	−.005	[−.053,.042]	−.076	−.021	−.005	.008	.049
.0	1,500	−.003	[−.042,.035]	−.038	−.016	−.003	.008	.063
.1	50	−.000	[−.221,.221]	−.545	−.068	.005	.079	.307
.1	100	.055	[−.099,.210]	−.189	.007	.052	.117	.270
.1	250	.071	[−.025,.167]	−.091	.039	.078	.111	.204
.1	500	.083	[.015,.152]	−.000	.055	.082	.839	.156
.1	1,000	.095	[.047,.144]	.037	.083	.094	.110	.161
.1	1,500	.095	[.056,.135]	.055	.081	.097	.111	.138
.2	50	.090	[−.130,.312]	−.213	−.002	.097	.211	.361
.2	100	.156	[.001,.311]	−.081	.098	.162	.218	.366
.2	250	.191	[.094,.288]	.043	.161	.184	.229	.309
.2	500	.186	[.118,.255]	.080	.168	.187	.210	.285
.2	1,000	.191	[.143,.239]	.112	.177	.194	.210	.238
.2	1,500	.196	[.157,.236]	.139	.183	.195	.210	.239
.3	50	.187	[−.033,.409]	−.192	.107	.190	.282	.424
.3	100	.244	[.098,.399]	.003	.187	.248	.311	.410
.3	250	.283	[.186,.379]	.123	.256	.286	.317	.419
.3	500	.284	[.215,.352]	.189	.257	.288	.308	.398
.3	1,000	.293	[.245,.342]	.239	.274	.291	.310	.350
.3	1,500	.291	[.252,.330]	.247	.278	.292	.303	.355
.4	50	.263	[.042,.485]	−.051	.193	.273	.347	.453
.4	100	.336	[.182,.491]	.011	.287	.352	.398	.475
.4	250	.382	[.285,.478]	.258	.352	.389	.415	.460
.4	500	.384	[.316,.452]	.322	.369	.386	.402	.446
.4	1,000	.387	[.338,.435]	.319	.371	.391	.404	.432
.4	1,500	.395	[.355,.434]	.350	.383	.396	.409	.438
.45	50	.315	[.093,.536]	.009	.262	.333	.394	.466
.45	100	.360	[.205,.515]	.126	.310	.369	.416	.481
.45	250	.413	[.317,.510]	.275	.388	.417	.446	.483
.45	500	.431	[.362,.499]	.343	.412	.436	.453	.485
.45	1,000	.442	[.394,.491]	.386	.428	.444	.461	.482
.45	1,500	.444	[.405,.484]	.391	.431	.446	.455	.484

The data generating process is ARFIMA(0,d,0).The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 6: ARFIMA(1,d,0) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.195	[−.630,.239]	−.546	−.325	−.203	−.089	.428
.0	100	−.199	[−.508,.109]	−.530	−.316	−.207	−.089	.174
.0	250	−.121	[−.371,.127]	−.470	−.236	−.122	.002	.232
.0	500	−.102	[−.300,.096]	−.436	−.176	−.099	−.016	.143
.0	1,000	−.073	[−.231,.084]	−.390	−.139	−.064	−.010	.140
.0	1,500	−.042	[−.176,.091]	−.313	−.093	−.045	.011	.169
.1	50	−.093	[−.490,.302]	−.497	−.250	−.095	.033	.356
.1	100	−.096	[−.401,.207]	−.442	−.192	−.088	−.004	.369
.1	250	−.025	[−.278,.228]	−.376	−.130	−.017	.074	.362
.1	500	.023	[−.176,.223]	−.233	−.052	.036	.089	.250
.1	1,000	.064	[−.103,.231]	−.222	−.013	.058	.127	.243
.1	1,500	.052	[−.081,.186]	−.110	−.003	.052	.108	.228
.2	50	−.073	[−.410,.263]	−.446	−.166	−.059	.038	.385
.2	100	.000	[−.281,.282]	−.465	−.089	.021	.100	.335
.2	250	.069	[−.156,.296]	−.243	−.045	.057	.159	.371
.2	500	.102	[−.098,.303]	−.194	.013	.110	.200	.340
.2	1,000	.149	[−.018,.316]	−.110	.096	.151	.214	.327
.2	1,500	.176	[.041,.310]	−.030	.125	.186	.228	.315
.3	50	.010	[−.311,.332]	−.360	−.096	−.002	.152	.345
.3	100	.054	[−.210,.318]	−.301	−.010	.046	.146	.296
.3	250	.141	[−.079,.363]	−.134	.037	.143	.232	.423
.3	500	.169	[−.006,.345]	−.111	.069	.173	.266	.435
.3	1,000	.243	[.086,.401]	.008	.200	.242	.311	.441
.3	1,500	.268	[.139,.398]	.048	.221	.273	.324	.417
.4	50	.097	[−.214,.409]	−.330	−.002	.094	.218	.402
.4	100	.137	[−.127,.401]	−.182	.039	.134	.227	.420
.4	250	.196	[−.006,.398]	−.078	.113	.202	.283	.431
.4	500	.275	[.099,.451]	−.051	.190	.298	.366	.472
.4	1,000	.331	[.187,.475]	−.001	.296	.344	.389	.468
.4	1,500	.333	[.206,.460]	.144	.282	.333	.385	.468
.45	50	.141	[−.155,.437]	−.153	.044	.146	.256	.400
.45	100	.167	[−.069,.404]	−.174	.080	.182	.262	.422
.45	250	.245	[.043,.446]	−.018	.175	.244	.337	.455
.45	500	.310	[.138,.481]	.011	.240	.319	.396	.473
.45	1,000	.365	[.228,.502]	.072	.319	.380	.424	.486
.45	1,500	.387	[.271,.503]	.180	.342	.394	.437	.480

The data generating process is ARFIMA(1,d,0) with $\phi = 0.6$. The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 7: ARFIMA(0,d,1) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.379	[−.972,.212]	−.999	−.586	−.358	−.189	.183
.0	100	−.227	[−.659,.204]	−.844	−.360	−.227	−.076	.140
.0	250	−.132	[−.425,.160]	−.613	−.283	−.151	.002	.311
.0	500	−.041	[−.263,.179]	−.416	−.143	−.039	.036	.296
.0	1,000	−.019	[−.193,.154]	−.246	−.091	−.037	.048	.230
.0	1,500	−.024	[−.157,.108]	−.233	−.069	−.023	.022	.174
.1	50	−.325	[−.877,.225]	−1.00	−.499	−.293	−.117	.261
.1	100	−.173	[−.621,.273]	−.700	−.323	−.173	−.026	.344
.1	250	−.046	[−.336,.243]	−.386	−.167	−.048	.048	.396
.1	500	.022	[−.197,.243]	−.285	−.064	.022	.094	.367
.1	1,000	.061	[−.106,.229]	−.126	.003	.071	.116	.241
.1	1,500	.060	[−.078,.198]	−.108	.001	.067	.107	.256
.2	50	−.323	[−.852,.204]	−.958	−.532	−.302	−.062	.282
.2	100	−.095	[−.519,.327]	−.800	−.229	−.100	.071	.308
.2	250	.069	[−.202,.342]	−.333	−.047	.059	.188	.451
.2	500	.106	[−.114,.327]	−.167	.022	.111	.173	.398
.2	1,000	.151	[−.011,.315]	−.089	.098	.152	.193	.440
.2	1,500	.156	[.020,.293]	−.023	.109	.163	.211	.334
.3	50	−.191	[−.723,.339]	−.999	−.327	−.172	.000	.299
.3	100	−.009	[−.425,.406]	−.526	−.139	−.001	102.	.332
.3	250	.125	[−.164,.416]	−.205	.004	.145	.247	.391
.3	500	.195	[−.024,.415]	−.062	.119	.199	.275	.435
.3	1,000	.251	[.096,.251]	.035	.199	.268	.313	.434
.3	1,500	.272	[.142,.402]	.074	.223	.272	.314	.468
.4	50	−.127	[−.639,.385]	−.884	−.293	−.115	.063	.410
.4	100	.071	[−.324,.467]	−.464	−.028	.077	.206	.414
.4	250	.237	[−.019,.494]	−.214	.138	.269	.338	.434
.4	500	.301	[.106,.496]	−.015	.224	.322	.374	.465
.4	1,000	.342	[.197,.487]	.113	.302	.343	.388	.469
.4	1,500	.355	[.233,.478]	.213	.315	.361	.396	.481
.45	50	−.069	[−.577,.439]	−.633	−.232	−.050	.078	.385
.45	100	.128	[−.262,.519]	−.465	.007	.168	.272	.398
.45	250	.246	[−.014,.506]	−.086	.143	.266	.366	.454
.45	500	.345	[.159,.530]	.112	.289	.366	.408	.480
.45	1,000	.382	[.248,.516]	.204	.357	.397	.435	.482
.45	1,500	.398	[.282,.514]	.249	.373	.401	.431	.484

The data generating process is ARFIMA(0,d,1) with $\theta = 0.6$ .The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.

Table 8: ARFIMA(1,d,1) Simulation Results

d	T	$\bar{\hat{d}}$	95 % CI	Min	25 %	Med	75%	Max
.0	50	−.490	[−1.04,.063]	−.999	−.777	−.490	−.310	.355
.0	100	−.530	[−.938,−.121]	−.999	−.772	−.530	−.354	.271
.0	250	−.316	[−.627,−.005]	−.926	−.659	−.316	−.059	.378
.0	500	−.131	[−.443,.179]	−.838	−.192	−.131	.001	.213
.0	1,000	−.056	[−.291,.178]	−.776	−.108	−.056	.025	.287
.0	1,500	−.064	[−.258,.128]	−.842	−.074	−.064	.014	.345
.1	50	−.382	[−.993,.167]	−.999	−.635	−.437	−.078	.412
.1	100	−.486	[−.908,−.063]	−.999	−.701	−.545	−.380	.267
.1	250	−.265	[−.560,.029]	−.865	−.619	−.210	.066	.390
.1	500	−.059	[−.372,.253]	−.800	−.171	.017	.130	.411
.1	1,000	−.023	[−.245,.198]	−.835	−.053	.024	.095	.381
.1	1,500	.025	[−.159,.211]	−.822	.007	.055	.111	.349
.2	50	−.350	[−.871,.170]	−.999	−.617	−.392	−.130	.392
.2	100	−.442	[−.882,−.002]	−.999	−.632	−.495	−.293	.293
.2	250	−.354	[−.628,−.081]	−.847	−.611	−.493	−.114	.407
.2	500	−.002	[−.278,.273]	−.691	−.076	.069	.146	.355
.2	1,000	−.127	[−.294,.040]	−.754	−.620	.063	.150	.435
.2	1,500	.041	[−.159,.242]	−.685	.093	.148	.206	.425
.3	50	−.345	[−.893,.202]	−.999	−.544	−.361	−.187	.297
.3	100	−.379	[−.770,.011]	−.772	−.545	−.437	−.272	.423
.3	250	−.339	[−.600,−.079]	−.722	−.557	−.467	−.196	.420
.3	500	.112	[−.185,.410]	−.569	.037	.187	.286	.469
.3	1,000	−.135	[−.287,.017]	−.630	−.559	.058	.232	.468
.3	1,500	.035	[−.140,.212]	−.669	−.510	.241	.294	.424
.4	50	−.274	[−.802,.253]	−.733	−.452	−.322	−.125	.428
.4	100	−.328	[−.716,.059]	−.824	−.443	−.347	−.239	.449
.4	250	−.328	[−.580,−.076]	−.607	−.487	−.402	−.234	.462
.4	500	.093	[−.163,.350]	−.489	−.348	.272	.360	.482
.4	1,000	−.203	[−.335,−.071]	−.567	−.475	−.417	.275	.455
.4	1,500	.001	[−.138,.140]	−.544	−.437	.262	.001	.452
.45	50	−.260	[−.804,.282]	−.857	−.403	−.270	−.117	.402
.45	100	−.250	[−.666,.165]	−.655	−.384	−.289	−.118	.435
.45	250	−.309	[−.556,−.061]	−.566	−.432	−.363	−.237	.471
.45	500	.123	[−.123,.371]	−.453	−.242	.258	.384	.466
.45	1,000	−.160	[−.290,−.031]	−.529	−.440	−.372	.271	.470
.45	1,500	.001	[−.120,.123]	−.504	−.428	.270	.385	.482

The data generating process is ARFIMA(1,d,1) with $\phi = 0.5$ and $\theta = 0.3$.The true values of d are: 0, .1, .2, .3, .4. and .45. Results are for 100 simulations.