Constructing mathematical arguments using definitions with precision in middle-grades teacher education in the USA

Sybilla Beckmann

The University of Georgia

ICMI Study 23 Conference
Limited time to satisfy competing demands in teacher education

We want mathematics teachers to:

- have opportunities to learn mathematical forms of argumentation, including using definitions in constructing arguments (CCSS; Common Core State Standards Initiative, 2010);

- study the mathematics they will teach in depth, from the perspective of a teacher (Conference Board of the Mathematical Sciences, 2012).

Can we satisfy the former within the latter?
The multiplicative conceptual field (e.g., Vergnaud 1988) encompasses multiplication, division, fraction, ratio, and proportional relationships and is a foundation for critical topics including linear functions, rates of change, and slope.
A definition of multiplication in terms of quantities

Beckmann & Izsák, 2015

\[ M \cdot N = P \]

\[
\text{(\# of groups)} \cdot (\text{\# of units in 1 group}) = (\text{\# of units in } \text{\# of groups})
\]
A type of fertilizer is made by mixing nitrogen and phosphate in an 8 to 3 ratio.

Suppose you will use $N$ kilograms of nitrogen and $P$ kilograms of phosphate, . . .

derive and explain an equation of the form

$$(\text{fraction}) \cdot P = N$$

. . .
How-many-total-amounts method

\[ P \cdot \frac{2}{3} \cdot P = N \]

- \( P \) = number of phosphate kg, 1 group P
- \( N \) = number of nitrogen kg, 1 group of N

\(? \cdot P = N\) is asking how much of group P is equal to 1 group of N. With my parts lined up as pictured above, I can see that 2 whole groups of P and an additional \( \frac{2}{3} \) group of P "fits" or makes up 1 group of N. So,

\[ \frac{2}{3} \cdot P = N \]
Connections

To Kaur’s paper: The solution method just presented and another solution method presented in Beckmann, Izsák, and Ölmez’s paper use the Model Method.

To Venkat’s paper:
“attention to representational competence can provide a bridge that allows for concurrent attention to teachers’ learning of mathematics and their teaching of mathematics” (p. 587).

To further mathematics: the case of slope.
Another view of slope (Beckmann & Izsák, 2014)

Using definitions

Sybilla Beckmann (UGA)

ICMI Study 23
Another view of slope (Beckmann & Izsák, 2014)
Another view of slope (Beckmann & Izsák, 2014)

\[ y = \frac{3}{2} \cdot x \]

1 group

3/2 groups

3/2 groups

1 group
Conclusion

Using a definition of multiplication provides opportunities to

- build skill in constructing viable mathematical arguments;
- deepen understanding of the mathematics teachers will teach;
- lay a foundation for understanding slope, rate of change, equations and functions.
References


