Preservice Teachers’ Emerging Capacity to Reason with the Variable Parts Perspective on Proportional Relationships

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University of Georgia

NSF Awards: DRL 0903411 and DRL 1420307
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- Dean Stevenson
- Sheri Johnson
- Merve Kursav
- Eric Siy

Other sessions:
- **Thurs 2-3pm, Salon 7:** *Future Teachers' Use of the Definition of Multiplication when Interpreting "Standard" Proportion Equations*
- **Friday 8-10am, Salon 5:** *Reasoning With the Variable Parts and Multiple Batches Perspectives on Proportional Relationships*
- **Friday 10:15-11:30, Salon 7:** *The Development and Negotiation of Sociomathematical Norms for Drawings*
OVERVIEW OF THIS SESSION

1. Give an overview of how to use the variable parts perspective to solve proportional relationship tasks

2. Share a video clip of PSTs learning to use this perspective

3. Share work samples of PSTs using this perspective
AN OVERVIEW OF THE VARIABLE PARTS PERSPECTIVE
A bit of background of our project ...
QUANTITATIVE MEANING OF MULTIPLICATION
QUANTITATIVE MEANING OF MULTIPLICATION

MULTIPLICAND
number of base units in 1 group

MULTIPLIER
number of equal-size groups

PRODUCT
number of base units in M groups

\[ M \cdot N = P \]
QUANTITATIVE MEANING OF MULTIPLICATION

MULTIPLICAND
number of base units in M groups

MULTIPLIER
number of equal-size groups

PRODUCT
number of base units in M groups

M \cdot N = P

“how many groups” division

? \cdot N = P
QUANTITATIVE MEANING OF MULTIPLICATION

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number of base units in M groups

$M \cdot N = P$

“how many groups” division

$? \cdot N = P$

“how many units in 1 group” division

$M \cdot ? = P$
VARIABLE PARTS PERSPECTIVE
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.

- Blue paint: 1 gallon + 1 gallon
- Yellow paint: 1 gallon + 1 gallon + 1 gallon
VARIABLE PARTS PERSPECTIVE

Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.

**Size of Parts**

<table>
<thead>
<tr>
<th>Size of Parts</th>
<th># Gal. Blue Paint</th>
<th># Gal. Yellow Paint</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
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**VARIABLE PARTS PERSPECTIVE**

- **Blue paint**: 2 parts • 1 gallon
  - 1 gallon
  - 1 gallon

- **Yellow paint**: 3 parts • 1 gallon
  - 1 gallon
  - 1 gallon
  - 1 gallon
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15 gallons</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>15 gallons</td>
</tr>
<tr>
<td>3 parts • 15 gallons</td>
<td>2 parts • 15 gallons</td>
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<td>15</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>2/5</td>
<td>4/5</td>
<td>6/5</td>
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Multiply Total Amount
(“Holistic” Method)

Multiply One Part
(“Going Through One Part” Method)
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3. If you use 50 pails of yellow paint, how many pails of blue paint will you need?
Multiply Total Amount
(“Holistic” Method)
Multiply Total Amount
("Holistic" Method)
Blue paint

Yellow paint

50 pails

Multiply Total Amount
(“Holistic” Method)
Multiply Total Amount
(“Holistic” Method)
50 pails per group

Multiply Total Amount
("Holistic" Method)
50 pails per group

Blue paint

? pails

Yellow paint

50 pails

Multiply Total Amount
("Holistic" Method)
$\frac{2}{3}$ groups • 50 pails per group

Multiply Total Amount
(“Holistic” Method)
VARIABLE PARTS PERSPECTIVE

Multiply Total Amount  
(“Holistic” Method)

Multiply One Part  
(“Going Through One Part” Method)
**VARIABLE PARTS PERSPECTIVE**

\[ \frac{2}{3} \text{ groups} \times 50 \text{ pails per group} \]

![Diagram showing blue and yellow paint with groupings and numbers of pails](image)

- **Multiply Total Amount**
  - (“Holistic” Method)
- **Multiply One Part**
  - (“Going Through One Part” Method)
Multiply One Part
(“Going Through One Part” Method)
Blue paint

Yellow paint

50 pails

Multiply One Part
(“Going Through One Part” Method)
Blue paint

Yellow paint

Multiply One Part
("Going Through One Part" Method)
Blue paint

Yellow paint

50\%/3 pails per group

? pails

1 group

50 pails

Multiply One Part
("Going Through One Part" Method)
Blue paint

Yellow paint

1 group

50 pails

Multiply One Part
(“Going Through One Part” Method)
50/3 pails per group

Blue paint

50/3 pails

50/3 pails

2 groups

Yellow paint

50/3 pails

50/3 pails

50/3 pails

1 group

50 pails

Multiply One Part
(“Going Through One Part” Method)
2 groups • $\frac{50}{3}$ pails per group

Blue paint

$\frac{50}{3}$ pails $\frac{50}{3}$ pails

2 groups

Yellow paint

$\frac{50}{3}$ pails $\frac{50}{3}$ pails $\frac{50}{3}$ pails

1 group

50 pails

Multiply One Part

("Going Through One Part" Method)
**VARIABLE PARTS PERSPECTIVE**

\[ \frac{2}{3} \text{ groups} \cdot 50 \text{ pails per group} \]

\[ \Rightarrow 2 \text{ pails} \]

Multiply Total Amount

(“Holistic” Method)

Multiply One Part

(“Going Through One Part” Method)
Multiply Total Amount ("Holistic" Method)

\[
\frac{2}{3} \text{ groups} \cdot 50 \text{ pails per group}
\]

Multiply One Part ("Going Through One Part" Method)

\[
2 \text{ groups} \cdot \frac{50}{3} \text{ pails per group}
\]
A VIDEO CLIP OF PSTs LEARNING TO USE THIS PERSPECTIVE
Algebra and Statistics Course (Spring 2015)
Variable Parts Perspective
January 7
January 7

Lucy’s hot chocolate is made of 2 cups of chocolate and 3 cups of milk with the same amount of liquid in each cup. Make a little more hot chocolate that is the same flavor and color.
Lucy’s hot chocolate is made of 2 cups of chocolate and 3 cups of milk with the same amount of liquid in each cup. Make a little more hot chocolate that is the same flavor and color.

“Say these first two are chocolate ... and then these are milk ... so we started with the same amount in every one ...”
January 7

Lucy’s hot chocolate is made of 2 cups of chocolate and 3 cups of milk with the same amount of liquid in each cup. Make a little more hot chocolate that is the same flavor and color.

“So when we put a smidge in ... there’s still the same amount in every cup but there’s more in each separate cup now but the ratio is still 2 to 3, we still have 2 chocolates and 3 milks. And then if you put more in it will be the same thing as long as you continue to put the same amount in each cup.”
January 7 - 26
January 7 - 26

Explain what it means for volumes of blue and yellow paint to be mixed in a 3 to 5 ratio from variable parts perspective.

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</tr>
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<td>9</td>
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January 7 - 26

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A company makes punch by mixing peach and grape juice in a 3 to 5 ratio. How many liters of peach juice and how many liters of grape juice will the company need to make 320 liters of that punch?

Variable parts: 40 40 40

3 peach juice (120 liters) per part

40 40 40 40 40 5 grape juice (200 liters)
January 26

Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.

a. If you use 41 pails of blue paint, how many pails of yellow paint will you need?

b. If you use 50 pails of yellow paint, how many pails of blue paint will you need?

c. If you use 52 pails of green paint, how many pails of blue paint and how many pails of yellow paint will you need?
Suppose a certain shade of green paint is made by mixing blue paint with yellow paint in a ratio of 2 to 3.

a. If you use 41 pails of blue paint, how many pails of yellow paint will you need?

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January 26

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Given the little bit that I’ve shared about our course ...

- What did you notice or find notable in the video clip?
- What are your initial thoughts about how the PSTs were introduced to the variable parts perspective in our course?
WORK SAMPLES OF PSTs USING THIS PERSPECTIVE
Perhaps keep an eye out for ...

- How the PSTs use the meaning of multiplication and definition of division
- Which quantities in the given problem the PSTs include/represent in their drawings
Questions? Comments! Concerns😊

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