Why should future middle-grades and secondary teachers study multiplication?

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Rutgers University 2017
The Teachers’ Multiplicative Reasoning Research Group

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InPReP$_2$: Investigating Proportional Relationships from Two Perspectives

We thank the National Science Foundation (DRL-1420307) for supporting our work. The opinions expressed are those of the authors and do not necessarily reflect the views of NSF.

We thank our research participants for sharing their thinking with us.
We are investigating how future teachers reason about interconnected topics in the multiplicative conceptual field: multiplication, division, fraction, ratio, and proportional, inversely proportional, and linear relationships.

The future teachers are enrolled in:

- a grades 4 – 8 certification program
- a grades 6 – 12 certification program
The need for coherence

From
*The Mathematical Education of Teachers II* (CBMS, 2012)

“...many incoming undergraduates are not used to seeing the discipline as a coherent body of connected results derived from a parsimonious collection of assumptions and definitions. One necessary ingredient to breaking this cycle is the next generation of teachers, who must have a coherent view of the structure of mathematics in order to develop reasoning skills in their students.” (p. 56)
Courses for future middle-grades and secondary teachers at UGA

We use definitions of multiplication and fractions to organize and bring coherence to the “multiplicative conceptual field”

- Multiplication
- Division
- Fractions
- Ratio
- Proportional and linear relationships

Why multiplication?
A course for future secondary teachers

Our future secondary teachers take a semester-long content course focused on multiplicative reasoning.
Developed by Andrew Izsák.

About 5 weeks each on:

- Multiplication
- Division
- Proportional relationships
How to define multiplication?

- Repeated addition?
- Area of a rectangle?
- Equal groups?
How to define multiplication?

- Repeated addition?
  What about fraction multiplication?

- Area of a rectangle?
  How does that connect to word problems?

- Equal groups?
A quantitative definition of multiplication

\[ N \cdot M = P \]

**Multiplicand:** number of base units in 1 group

**Multiplier:** number of groups in the Product Amount

**Product:** number of base units in the Product Amount

This version: multiplicand first, then multiplier.
Situations are not commutative

$3 \times 5$

$5 \times 3$
Consequences of a non-commutative definition

- Two types of division:
  - How many groups?
  - How many units in 1 group?

- Two ways to view covariation in a fixed proportional relationship (Beckmann & Izsák, 2015):
  - Multiple-batches perspective
  - Variable-parts perspective (previously overlooked in research)
Problem: A package of cheese is $\frac{1}{4}$ pound, but that is only $\frac{2}{3}$ of what you need for a recipe. How much cheese do you need for the recipe?

What kind of problem is that? Why?
Using the definition of multiplication in constructing arguments

- Explaining why we can divide fractions by multiplying by the reciprocal of the divisor.

- Solving proportion problems by reasoning about multiplication and division.

- Developing and explaining equations that relate covarying quantities.
Line from a *variable-parts* perspective
Line from a *variable-parts* perspective
Line from a *variable-parts* perspective
Line from a *multiple-batches* perspective

Why multiplication?
Can future teachers develop equations for lines using a variable-parts perspective?

Data: Interviews with 6 future secondary teachers enrolled in Izsák’s course Spring 2016.

Timing of interview:
- After some instruction on proportional relationships including an introduction to the variable-parts perspective.
- Instruction did not include generating equations in two variables.
Interview 5, Task 3, the line task

Task 3A: [Show the Geogebra sketch.] What do you notice?
Task 3B: Can you use features of the drawing to express a relationship between $x$ and $y$?
Can future teachers develop equations from the variable-parts perspective?

All 6 of the future teachers developed valid equations for the line by reasoning from the variable-parts perspective. Equations included:

\[ x = \frac{4}{3}y \quad y \cdot \frac{4}{3} = x \]

\[ y = \frac{3}{4}x \quad x \cdot \frac{3}{4} = y \]

\[ \frac{1}{3}y = \frac{1}{4}x \]

\[ \frac{1}{3}y \cdot 4 = x \quad \frac{1}{4}x \cdot 3 = y \]
"... Y is partitioned into 3 pieces, so each of this is $\frac{1}{3}$ size pieces [points to one part of the Y strip]."
“There’s, and they’re the same size [points back and forth to a part in the Y strip and a part in the X strip],”
“So there’s 4 of these $\frac{1}{3}$ size pieces in X [indicates the 4 parts in X].”
“And then like I did over here and over here [refers to previous work on another page], \( \frac{4}{3} \) of \( Y \) is \( Y \) times \( \frac{4}{3} \) equals \( X \)."
The \textit{variable-parts} perspective as a foundation for understanding the derivative?

\[ \Delta y \]

\[ \Delta x \to 0 \]

How to think about slope?

Let \( \Delta x \to 0 \)
The *variable-parts* perspective as a foundation for understanding the derivative?

Variable parts:

slope of secant line is 1.25 because

\[ \Delta y \text{ is } 1.25 \text{ of } \Delta x \]
The *variable-parts* perspective as a foundation for understanding trigonometry?

Let the radius be 1 part. It can be any size.

This is 1/2 of a part.

This arc is $\pi/6$ parts.
The variable-parts perspective as a foundation for understanding probability?

The probability of winning a game is 7%. What does that mean? What happens if you play a lot of times?

**Multiple-batches thinking:** Every 100 times you play, you expect to win about 7 games.

100 games → about 7 wins
100 games → about 7 wins
100 games → about 7 wins

*Variable-parts thinking:* Think of 100 parts, 7 representing wins. When you play, the game falls randomly into a part.
The *variable-parts* perspective as a foundation for understanding probability?
Thank you! Questions or comments?

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