Goldbach-Euler-Systems.

Abstract. We answer the Goldbach-Euler question.

Goldbach and Euler asked a question equivalent to "is every even integer greater than 6 always the sum of two primes ?" The primes are : $p_1{=}3, p_2{=}5, p_3{=}7, p_4{=}11, p_5{=}13, p_6{=}17, p_7{=}19, p_8{=}23\ldots$.

For every $k \geq 1$, let P_k be the set of primes $\{p_1, p_2, \ldots p_k\}$. For every even integer $N \geq 6$, $P(N)$ is the set of primes in $[3, N-3]$. Hence $P(N)$ is some P_k and let $\gamma(N)$ be k. For example $P(98) = \{3, 5, \ldots, 89 = p_{23}\}$ and $\gamma(98) = 23$. When some p is a prime factor of some n, we write $p|n$. Assume that Goldbach and Euler were wrong for N with $\gamma(N){=}k$ like $N{=}98$ and $\gamma(N){=}23$. Then, for every $p_i \in P_k$, there exists a non prime integer m_i such that $N{=}p_i{+}m_i$. For example, $98{=}3{+}m_1{=}3{+}5.19\ldots 98{=}89{+}m_{23}{=}89{+}3.3$. Since N is even and p_i is odd, m_i must be odd too. Any $q|m_i$ cannot be larger than $N/3$ because another $r|m_i$ is at least 3. Hence $m_i{=}q_i.r_i.(1{+}2x_i)$ with q_i, r_i in $P(N)$ and $x_i \in \mathbb{N}$ and $N{=}p_i{+}q_i r_i{+}2q_i r_i x_i$. Now we consider the inverse problem : from some P_k, we look at the k^{2k} possible systems for every choices of the q_i, r_i.

For $P_1{=}\{3\}$, we have one possible system $\{N{=}3{+}3.3{+}2.3.3.x_1\} \Longrightarrow N{\geq}12 \Longrightarrow \gamma(N){\geq}3{>}1$.

For $P_2{=}\{3, 5\}$, among the 16 possible systems of equations, two have solutions :

$\{N{=}3{+}3.3{+}2.3.3.x_1,\ N{=}5{+}5.5{+}2.5.5.x_2\} \Longrightarrow N{\geq}30 \Longrightarrow \gamma(N){\geq}8{>}2$

$\{N{=}3{+}5.5{+}2.5.5.x_1,\ N{=}5{+}3.3{+}2.3.3.x_2\} \Longrightarrow N{\geq}28 \Longrightarrow \gamma(N){\geq}8{>}2$

We generalize this with *Goldbach-Euler-Systems.*

Definition (system). *For every $k{\geq}1$, a k-GES is a finite set of equations :*

$$\{N{=}p_1{+}\pi_1{+}2.\pi_1.x_1,\ N{=}p_2{+}\pi_2{+}2.\pi_2.x_2,\ \ldots,\ N{=}p_k{+}\pi_k{+}2.\pi_k.x_k\}$$

where each π_i is the product of two primes in P_k.

By this way, any conter-example N to the Goldbach-Euler question must satisfy a k-GES where $\gamma(N){=}k$. But...

Claim. *For every $k{\geq}1$, every k-GES has no solution or has solutions N where $\gamma(N){>}k$.*

We leave the details, the verifications and the celebrity to other mathematicians.

We have shown that it is easier to verify that some solution is good than find it. Here, we give a solution. Please verify that it is correct. Hence Goldbach and Euler were right and that is not important but we want to write this.

Claim. *Some adults should better take care of their children instead of working hard on some unreal problems. A problem is* easy *or* difficult*. An* easy *problem is solvable with a finite sequence of trivialities. A* difficult *problem is not an easy problem. Hence, a* difficult *problem is not solvable at all. If you do not believe in this, look at the* easy *but deep Gödel's theorem or try hardly to solve the still standing open problem : "is your answer negative ?". The real problems today do not concern mathematics, physic, medecine, politic... The real problem for Mankind is to preserve Life on Earth. The risk is that all our sciences disappear in the same time than all Life on Earth ! The first point is that H_2O is lighter than $O_2, CO_2, N_2, \ldots$: our atmosphere. That is why water always evaporates. Oceans made of water are not sufficient. Enough life in oceans is necessary so that water comes back. Forests are necessary in order to capture evaporating water from grounds. Not enough life in oceans or not enough life in forests implies that water evaporates and does not come back. That is as simple as death. Mankind has to preserve Life. For this, we must take care of our oceans and our forests. Like Paul Erdös would say : "Even babies know that !".*

Listen babies again. They cry rivers : "Save our planet !......Please !"

(S.I.G.L.E.) Special Investigation Group for Life on Earth. www.sigle.space