Multi-objective robust optimisation of the maintenance scheduling of nuclear power plants

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Angers, GdR GOTHA 2016:
July 8th 2016
1 Problem statement

2 Deterministic formulation

3 Robust optimisation

4 Multi-Objective Optimisation
Plan

1. Problem statement
2. Deterministic formulation
3. Robust optimisation
4. Multi-Objective Optimisation
Nuclear reactors must be shut down periodically for maintenance and refueling.

A two level problem:
- Main decisions: dates of outages, refueling quantities.
  - Coupling constraints on outages.
- Second Level: production and stocks variables, to compute the economic cost of the main decisions, fulfilling the technical constraints.

Stochastic problem modeled for the Challenge EURO/ROADEF 2010.
Production constraints for nuclear power plants

Problem statement
Deterministic formulation
Robust optimisation
Multi-Objective Optimisation

imposed decreasing power profile

Pmax

outage
modulation
x>BO

x<BO

outage

time

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Multi-objective robust optimisation for the Challenge EURO/ROADEF 2010
Problem Constraints

- Power demand constraints coupling all production units
- Production constraints for generating units:
  - Production bounds for T1 and T2 units
  - Decreasing profile when fuel level of nuclear units is low
- Fuel constraints:
  - Bounds on fuel stocks and refueling levels
  - Maximal threshold of fuel to operate an outage
- Scheduling constraints for outages: resource constraints, different spacing constraints, maximal capacity offline ...
Best results: simple aggressive and frontal local search heuristics with local moves on outages dates. (Gardi et al and Kuiper et al).

Heuristics and matheuristics iterating in the natural 2 level structure were not efficient.

Approaches based on exact methods required model simplifications and reduction sizes (scenarios and time steps aggregated, hierarchical approaches, relaxed constraints and heuristic fixations in postprocessing).

Only one exact approach did not aggregate the scenarios (Lusby et al for a Benders decomposition approach).
Plan

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Simplifications to the 2010 EURO/ROADEF Challenge

With following (light) hypothesis, the problem can be formulated in a MIP where the only binary variables are the outage decisions:

- Decreasing production profile not imposed. We can relax it totally or use the concavity and enforce an upper-bound on production.
- Maximal modulation per cycle constraint: not a good formulation of fine grain technical constraints.
Variable definition

- **Binaries**: Outage decisions $d_{i,k,w}$ for outage $(i,k)$ and week $w$: $d_{i,k,w} = 1$ if the outage $i,k$ began before $w$. (for efficient standard branching)

- **Continuous variables**:
  - $r_{i,k}$: refueling quantities for the refueling in outage $(i,k)$.
  - $p_{i,k,s,t}$ nuclear (T2) productions T1.
  - $p_{j,s,t}$ non nuclear (T1) productions levels.
  - Fuel stocks $x_{i,s,t}$ (required only for CT6 stretch constraints)
  - Residual stocks $x_{i,s}^f$ (for cost function, to avoid end-of-side effects)

- **Dependent variables**: fuel stocks $x_{i,k,s}^{init}, x_{i,k,s}^{fin}$.

⇒ Formulation with similarities with the one of Lusby et al, main difference in the definition of variables $d$, SOS natural definition for Lusby et al.
Définition des variables, illustration
Constraints related to the variables definitions

\( \forall i, k, w, \quad d_{i,k,w-1} \leq d_{i,k,w} \quad (1) \)

\( \forall i, k, \quad d_{i,k,T_{o,i,k-1}} \leq 0 \quad (2) \)

\( \forall i, k, \quad d_{i,k,T_{a,i,k}} \geq 1 \quad (3) \)

\( \forall i, s, \quad x_{i,0,s}^{init} = X_i \quad (4) \)

\( \forall i, k, s, \quad x_{i,k,s}^{fin} = x_{i,0,s}^{init} - \sum_t D_t P_{i,k,s,t} \quad (5) \)

\( \forall i, k, s, \quad x_{i,s}^f \leq x_{i,k,s}^{fin} + \bar{S}_i(d_{i,k,w} - d_{i,k+1,w}) \quad (6) \)
Minimiser la somme des coûts suivants:

- coûts de rechargement: $\sum_{i,k} C_{i,k}^{rld} r_{i,k}$
- coûts de prod flex: $\sum_{j,w} C_{j,w}^{prd} D_{j,w} p_{j,w}$
- Valorisation fuel final: $- \sum_i C_i^{val} x_i^{fin}$
Production constraints

\[ \forall s, t, \quad \sum_{i,k} p_{i,k,s,t} + \sum_j p_{j,s,t} = \text{Dem}^{t,s} \quad (7) \]
\[ \forall j, s, t, \quad \text{Pmin}^j \leq p_{j,s,t} \leq \text{Pmax}^j \quad (8) \]
\[ \forall i, k, s, t, \quad 0 \leq p_{i,k,s,t} \leq \text{Pmax}^{i,t}(d_{i,k,w_t - Da_{i,k} - d_{i,k+1,w_t}}) \quad (9) \]

CT12 modulation constraints: relaxed in our study no real global modulation constraints, but fine grain limitations to modulate

CT6 decreasing profile:

\[ \forall i, k, s, t, m > 0, \quad \frac{p_{i,k,s,t}}{\text{Pmax}^i} \leq \frac{c_{i,k,m-1} - c_{i,k,m}}{f_{i,k,m-1} - f_{i,k,m}}(x_{i,s,t} - f_{i,k,m}) + c_{i,k,m} \quad (10) \]
\[ \forall i, k, w, \quad x_{i,s,t} \leq x_{i,k,s}^{\text{init}} - \sum_{t' \leq t} \text{D}^{t'} p_{i,k,s,t'} + M_i (1 - d_{i,k,t} + d_{i,k-1,t}) \quad (11) \]
Fuel constraints

- Stock initial: \( x_{i,-1}^{\text{init}} = X_i \).
- Bornes de rechargement:
  \[
  R_{\text{min}i,k} d_{i,k,W} \leq r_{i,k} \leq R_{\text{max}i,k} d_{i,k,W}
  \]
- Stock maximal: \( \forall i, k, 0 \leq x_{i,k}^{\text{init}} \leq S_{\text{max}i,k} \)
- Anticipation maximale:
  \[
  x_{i,k}^{\text{fin}} \leq A_{\text{max}i,k+1} + (S_{\text{max}i,k} - A_{\text{max}i,k+1}) (1 - d_{i,k+1,W})
  \]
- Loi de rechargement:
  \[
  x_{i,k}^{\text{init}} - B_{o_i,k} = r_{i,k} + \frac{Q_{i,k-1}}{Q_{i,k}} (x_{i,k-1}^{\text{fin}} - B_{o_i,k-1})
  \]
Scheduling/spacing/overlapping constraints

\[ \sum_{i,k} \text{Pmax}_{i,w}(d_{i,k,w} - d_{i,k,w} - D_{a_{i,k}}) \leq I_{max} \]

\[ \sum_{(i,k) \in A_{20,w}} (d_{i,k,w} - d_{i,k,w} - D_{a_{i,k}}) \leq N_{20,w} \]

\[ \sum_{(i,k) \in A_{19}} (d_{i,k,w} - L_{19_{i,k}} - d_{i,k,w} - L_{19_{i,k}} - T_{u19_{i,k}}) \leq Q_{19} \]

\[ \sum_{(i,k) \in A_{18}} (d_{i,k,w} - d_{i,k,w} - S_{e18} + d_{i,k,w} - D_{a_{i,k}} - d_{i,k,w} - D_{a_{i,k}} - S_{e18}) \leq 1 \]

\[ \sum_{(i,k) \in A_{17}} (d_{i,k,w} - D_{a_{i,k}} - d_{i,k,w} - D_{a_{i,k}} - S_{e18}) \leq 1 \]

\[ \sum_{(i,k) \in A_{16}} (d_{i,k,w} - d_{i,k,w} - S_{e16}) \leq 1 \]

\[ \sum_{(i,k) \in A_{15}} (d_{i,k,w} - d_{i,k,w} - (D_{a_{i,k}} + S_{e15})^+) \leq 1 \]

\[ \sum_{(i,k) \in A_{14}} (d_{i,k,w} - d_{i,k,w} - (D_{a_{i,k}} + S_{e14})^+) \leq 1 \]
Frontal resolution characteristics

- LP relaxation: not computable in 1h for difficult instances (B8-B9) even with deterministic problem and aggregated production time steps to weeks with Cplex 12.3. OK with 12.5
- LP relaxation not useful to built primal solutions for difficult instances.
- With restricted time windows, good convergence structure
- These difficulties are emphasized adding stretch constraints CT6.

⇒ Strong limiting factor: the sizes of instances
Convergence structure

Convergence sur $B6_3.120$
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Matheuristic resolution

- Constructive matheuristics: relax-and-fix strategies efficient, build primal solutions for all instances.
- Local Search matheuristic: Variable Neighbourhood Descent (VND) iterating with MIP neighborhoods;
  - $\mathcal{N}_{\text{units}}^I$ unit selection: only units $i \in I$ are reoptimised. Successive neighborhoods defining a partition of $I$ are computed iteratively.
  - $\mathcal{N}_{\text{TW}}^{[w, \overline{w}]}$: all outages are reoptimised in the time window $[w, \overline{w}]$.
  - $\mathcal{N}_{\text{cycles}}^{k, k'}$: variables relative cycles $k''$ with $k \leq k'' \leq k'$ are reoptimised.

$\Rightarrow$ Very efficient matheuristic resolution for deterministic problems aggregating production time steps to weeks.
Algorithm 1: POPMUSIC VND with MIP neighbourhoods

Input: an initial solution, a set and order of neighbourhoods to explore

Initialisation: currentSol = initSolution, \( \mathcal{N} \) = initial neighbourhood.

while the stopping criterion is not met
    define the MIP with incumbent currentSol and the neighbourhood \( \mathcal{N} \)
    define currentSol as warmstart
    currentSol = solveMIP(MIP, timeLimit( \( \mathcal{N} \) ))
    \( \mathcal{N} \) = nextNeighborhood(\( \mathcal{N} \))

end while

return CurrentSolution
### VNS to analyse neighbourhoods

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<th>RL v1</th>
<th>RL v3</th>
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Uncertainty set

- Uncertainty: delay in maintenance operations inducing outage prolongations
- Originality: discrete uncertainty. The uncertainty set is enumerated as a set of prolongation scenarios.
- A prolongation scenario defines prolongations on the whole set of outage prolongations.
- We handle a reasonable number of scenario, so that it can be enumerated. Ex: Scenarios with only one prolongation, having the maximal prolongation, while the others have standard prolongations.

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Multi-objective robust optimisation for the Challenge EURO/ROADEF 2010
Robustness definition

Robustness: We want to face off the worst case in the uncertainty set $\Omega$, minimizing the worst expected cost.

In our bilevel problem, second level decisions can be adjusted after the uncertainty outcomes. It leads to a Min-Max-Min scheme.

\[
\min_d c_D.d + Q(d) \quad \text{with:} \\
A.d \leq b
\]

\[
Q(d) = \max_{\delta \in \Omega} Q(d, \delta) \quad \text{and} \quad Q(d, \delta) = \min_x c_p.p \quad T(\delta).d + W(\delta).p \leq h(\delta)
\]

Linearization with discrete scenarios $\delta \in \Omega$, MIP to be solved with Benders decomposition:

\[
\min_{d \in \{0, 1\}^n \times \mathbb{R}_+^m, p_\delta \geq 0} cd + C^{rob} \\
A d \leq b \\
\forall \delta \quad T_\delta x + W_\delta p_\delta \leq h_\delta \\
\forall \delta \quad c_p p_\delta \leq C^{rob}
\]
Variable definition

- **Binaries**: Outage decisions $d_{i,k,w}$ for outage $(i,k)$ and week $w$: $d_{i,k,w} = 1$ if the outage $i,k$ began before $w$. (for efficient standard branching)

- **Continuous variables**:
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  - $p_{i,k,s,\delta,t}$ nuclear (T2) productions T1.
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  - Residual stocks $x_{i,s,\delta}^{fin}$ (for cost function, to avoid end-of-side effects)

- **Dependent variables**: fuel stocks $x_{i,k,s,\delta}^{init}, x_{i,k,s,\delta}^{fin}$. 

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Problem statement

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\[ \min \sum_{i,k} C_{i,k}^{rld} r_{i,k} + \sum_{i,w} C_{i,k,w}^{pen} (d_{i,k,w} - d_{i,k,w-1}) + C^{robust} \]  
(12)

\[ \forall i, k, w, \]  
(13)

\[ \forall i, k, \]  
(14)

\[ \forall \delta, \]  
(15)

\[ \forall i, s, \delta, \]  
(16)

\[ \forall j, t, s, \delta, \]  
(17)

\[ \forall i, t, s, \delta, \]  
(18)

\[ \forall i, t, s, \delta, m, \]  
(19)

\[ \forall s, t, \delta, \]  
(20)

\[ \forall i, k, \delta, \]  
(21)

\[ \forall i, t, s, \delta, \]  
(22)

\[ \forall i, k, \delta, \]  
(23)

\[ \forall i, t, s, \delta, \]  
(24)

\[ \forall i, k, \delta, \]  
(25)

\[ \forall w, \delta, \]  
(26)
Variable partitionning: first level variables 'z': $d_{i,k,w}$, $r_{i,k}$ and $C^{robust}$. Other variables 'y' depend on the scenarios.

\[
\begin{align*}
\min_{z,y \geq 0} & \quad cz \\
\text{s.t.} & \quad Az \geq a \\
& \quad Tz + Wy \geq b 
\end{align*}
\]  

(27)  
(28)  
(29)

Master Problem min $cz$ s.t $Az \geq a$ and cuts generated by the subproblems. (projection of the constraints $Tz + Wy \geq b$ in the space of z variables).
Problem statement
Deterministic formulation
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Generating Benders cuts

- Here just feasibility cuts: $z^0$ given, is it possible for all the prolongation scenarios to have a production planning with cost at most $C^{robust}$?

Transforming into optimization problem and using duality:

$$\min \eta$$
$$Wy + \eta \geq b - T.z^0$$

$$= \max(b - T.z^0).v > 0?$$ (30)
$$W^T.v \leq 0$$ (31)
$$\sum_i v_i \leq 1$$ (32)
$$\eta, y \geq 0$$
$$v \geq 0$$ (33)

- Benders Reformulation: for all extreme ray $v$ of $W^T$, we have cuts $(b - T.z^0).v \leq 0$. 
\[
\sum_{i,k,w} \alpha_{i,k,w}^{(1)} P_{\text{max}}^w(d_{i,k,w} - Da_{i,k}, \delta - d_{i,k+1,w}) + \sum_{i,k} \alpha_{i,k}^{(4)} \left( q_{0,k-1}X_i + \sum_{l=0}^{k-1} q_{l+1,k-1}(r_{i,l+1} - Bo_{i,l}) \right) \\
+ \sum_{i,k} \alpha_{i,k}^{(2)} \left( S_{\text{max},k} (1 + d_{i,k+1,W} - d_{i,k,W}) + q_{0,k-1}X_i + \sum_{l=0}^{k-1} q_{l+1,k-1}(r_{i,l+1} - Bo_{i,l}) \right) \\
+ \sum_{i,k} \alpha_{i,k}^{(3)} \left( S_{\text{max},k} - (S_{\text{max},k} - A_{\text{max},i,k+1}) d_{i,k+1,W} - q_{0,k-1}X_i - \sum_{l=0}^{k-1} q_{l+1,k-1}(r_{i,l+1} - Bo_{i,l}) \right) \\
+ \sum_{i,k} \alpha_{i,k}^{(5)} \left( S_{\text{max},k} - q_{0,k-1}X_i - \sum_{l=0}^{k-1} q_{l+1,k-1}(r_{i,l+1} - Bo_{i,l}) \right) \\
+ \sum_{w} \alpha_{w}^{(17)} \left( 1 - \sum_{(i,k) \in A_{17}} (d_{i,k,w} - Da_{i,k}, \delta - d_{i,k,w} - Da_{i,k}, \delta - Se_{17}) \right) \\
+ \sum_{w} \alpha_{w}^{(18)} \left( 1 - \sum_{(i,k) \in A_{18}} (d_{i,k,w} - d_{i,k,w} - Se_{18}) + (d_{i,k,w} - Da_{i,k}, \delta - d_{i,k,w} - Da_{i,k}, \delta - Se_{18}) \right) \\
+ \sum_{w} \alpha_{w}^{(20)} \left( N_{w}^{20} - \sum_{(i,k) \in A_{20}^{w}} (d_{i,k,w} - d_{i,k,w} - Da_{i,k}, \delta) \right) + \sum_{w} \alpha_{w}^{(21)} \left( I_{\text{max}}^{w} - \sum_{(i,k)} P_{\text{max}}^w(d_{i,k,w} - d_{i,k,w} - Da_{i,k}, \delta) \right) \\
- \sum_{w} \beta_{w}^{(1)} P_{\text{min}}^{w} + \sum_{w} \beta_{w}^{(2)} P_{\text{max}}^{w} + \sum_{w} \beta_{w}^{(3)} Dem^{w} + \sum_{w} \beta_{w}^{(4)} (P_{0,w}^{\text{max}} - Dem^{w}) + \gamma C^{\text{rob}} \geq 0
\]
Implementation points

Challenging difficulty: very large scale problem.

- Numerical instability due to propagation of rounding errors in Benders cuts.
- Too conservative approach: scheduling constraint induce tight planning in summers, there is frequently no 100% robust solution

⇒ Needs for another definition of robustness, for modeling and resolution issues
Specific case of constraints CT14 and CT15

Former robust constraints for CT14 and CT15:

\[ \forall \delta, w, \sum_{(i,k) \in A_{14}} (d_{i,k,w} - d_{i,k,w} - (D_{a_{i,k,\delta} + S_{e14}})^+) \leq 1 \]  \hspace{1cm} (34)

\[ \forall \delta, w \in [d_{15}, f_{15}], \sum_{(i,k) \in A_{15}} (d_{i,k,w} - d_{i,k,w} - (D_{a_{i,k,\delta} + S_{e15}})^+) \leq 1 \]  \hspace{1cm} (35)

The robustness of CT14 and CT15 is equivalent to Soyster’s approach Robust CT14 and CT15 constraints are equivalent to the deterministic constraints with \( D_{a_{i,k}} = \overline{D_{a_{i,k}}} \), using Soyster’s results:

\[ \forall w, \sum_{(i,k) \in A_{14}} (d_{i,k,w} - d_{i,k,w} - (\overline{D_{a_{i,k}}} + S_{e14})^+) \leq 1 \]  \hspace{1cm} (36)

\[ \forall w \in [d_{15}, f_{15}], \sum_{(i,k) \in A_{15}} (d_{i,k,w} - d_{i,k,w} - (\overline{D_{a_{i,k}}} + S_{e15})^+) \leq 1 \]  \hspace{1cm} (37)

\[ \Rightarrow \] As CT14 and CT15 predominant to design good robust solutions, robust constraints with deterministic solution still infeasible

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Lighter robustness approach

For all constraints $c \in CT14$ and $c \in CT15$, continuous variables $z_{c,w}^{(14)}, z_{c,w}^{(15)} \geq 0$ are introduced to penalize robust violations, paying cost $\text{Cpen}^{rob}$ for violations.

We minimize $\sum_w \text{Cpen}^{rob}(z_{c,w}^{(14)} + z_{c,w}^{(15)}) + f_{\text{det}}$ with $f_{\text{det}}$ the previous objective.

We add to the previous deterministic formulation the constraints:

$$\forall w, c \in CT14, \sum_{(i,k) \in A^{14}_c} (d_{i,k,w} - d_{i,k,w} - (\bar{D}_{d_{i,k}} + \text{Se}^{14})^+) \leq 1 + z_{c,w}^{(14)}$$

$$\forall c \in CT15, w \in [d_{15}^c, f_{15}^c], \sum_{(i,k) \in A^{15}_c} (d_{i,k,w} - d_{i,k,w} - (\bar{D}_{d_{i,k}} + \text{Se}^{15})^+) \leq 1 + z_{c,w}^{(15)}$$
Robustified MIP problem with a similar size than the deterministic one.

Same feasible solution set than than the deterministic MIP: furnish robustified solutions for all instances

Same resolution characteristics than than the deterministic MIP. Robust trade-off can help MIP solvers to cut off solutions, difficulties in deterministic problem that lots of solutions have similar cost which is a bottleneck for B&B resolution.

VNS with MIP neighborhoods applies also, and is very efficient.

Computation of Pareto fronts of the best compromise solutions to trade off cost/robustness, using VND

⇒ Simple approach, but efficient in a resolution and operational standpoint.
Plan

1. Problem statement
2. Deterministic formulation
3. Robust optimisation
4. Multi-Objective Optimisation
Objective functions

- Financial costs: minimizing financial costs
- Robustness: minimizing robust violations
- Stability of the planning: minimizing distance from baseline solution
- Security: minimizing stretch occurrences
- Sustainable development: minimizing nuclear wastes

⇒ Simple approach, but efficient in a resolution and operational standpoint.
Stability: minimizing distance to the solution baseline

The baseline solution has impacts on the newly optimised solution (complex operations and contracts that cannot be desorganized)

Stability objective: minimizing distance to the solution baseline $W_{i,k}^0$

Let $C_{i,k,w}^{pen}$ the penalisation to move outage $(i, k)$ in $w$.

Stability costs:

$$C_{desorg} = \sum_{i,k,w} C_{i,k,w}^{pen}(d_{i,k,w} - d_{i,k,w-1}) \quad (38)$$

For example $C_{i,k,w}^{pen} = |w - W_{i,k}^0|$, $C_{i,k,w}^{pen} = (w - W_{i,k}^0)^2$, or $C_{i,k,w}^{pen} = 1_{w \neq W_{i,k}}$. 
Security/modulability: minimizing stretch occurrences

Let $b_{i,k}$ indicating if cycle $(i,k)$ is a stretch

Minimizing stretch occurrences: $\min \sum b_{i,k}$

$(b_{i,k}$ is characterized by $B_0_{i,k} \geq x_{i,k}^{\text{fin}}$):

$\quad (b_{i,k} = 0) \implies B_0_{i,k} \leq x_{i,k}^{\text{fin}}$

Linearization:

$\forall i, k, w, \quad B_0_{i,k} \leq x_{i,k}^{\text{fin}} + b_{i,k} B_0_{i,k}$

(39)

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Sustainable development: minimizing nuclear wastes

Minimizing nuclear wastes: \( \sum x_{i,k}^{\text{fin}} d_{i,k+1,w} \)

Non linear, linearization introduces waste
\[ z_{i,k} = \max(0, x_{i,k}^{\text{fin}} - B_{o,i,k}) d_{i,k+1,w} = (x_{i,k}^{\text{fin}} - B_{o,i,k})(1 - b_{i,k}) d_{i,k+1,w} \]

\[ \forall i, k, w, \quad z_{i,k} \leq (A_{i,k+1} - B_{o,i,k}) d_{i,k+1,w} \] (40)

\[ \forall i, k, w, \quad z_{i,k} \geq x_{i,k}^{\text{fin}} - B_{o,i,k} - (A_{i,k+1} - B_{o,i,k})(1 - d_{i,k+1,w}) \] (41)

\[ \forall i, k, w, \quad z_{i,k} \geq 0 \] (42)
VND to built Pareto fronts

Algorithm 2: Pareto front computation with VND and MIP neighbourhoods

Initialisation: compute bestObjective with VND resolution $C_{pen}^{rob} = 0$.
compute currentObjective, robustViolations with VND resolution with a high value of $C_{pen}^{rob}$.
Let robustViolations the number of violations of robust constraints of the solution of currentObjective.
Pareto front <- (currentObjective, robustViolations)
while currentObjective > bestObjective
    define currentObjective, robustViolations as warmstart for VND
    currentObjective = solve VND with at most 1+robustViolations robust violations allowed
    robustViolations = 1+robustViolations
    Pareto front <- (currentObjective, robustViolations)
end while
return Pareto front

⇒ Similarities with TPM (Two phases Method)
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Courbe de Pareto Cout/Robustesse

Écart relatif à la solution optimale sans robustesse

Nombre de violations robustes

Courbe de Pareto coût/Robustesse de solution sur dataB7_3_120
Problem statement
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Courbe de Pareto coût/Robustesse de solution sur X12_3_120

Ecart relatif à la solution optimale sans robustesse

Nombre de violations robustes

0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
5 10 15 20 25 30 35
Courbe de Pareto coût/Robustesse de solution sur X11_3_120
Problem statement
Deterministic formulation
Robust optimisation
Multi-Objective Optimisation

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Conclusions and perspectives

Conclusions:

- Better knowledge and more efficient resolution for a deterministic MIP formulation. Efficient VNS resolution defining MIP neighborhoods.
- Robust optimization with an hypothesis significantly different from the state of the art: discrete uncertainty.
- Benders approach: numerical difficulties.
- Simple robustified approach: efficient, based on deterministic resolution, consistent to give feasible solutions. Robustness is a trade-off to cut off non robust solutions as lots of solutions have similar cost.
- Efficient construction of Pareto Fronts with VNS

Perspectives:

- Extension with stochastic scenarios of the challenge
- Multi-objective optimization with other sustainable development objectives (CO₂ emissions)
References