A 2-stage robust optimization model for planning nuclear maintenances with uncertain durations on the maintenance

Nicolas Dupin$^{1,2}$

$^1$University Bordeaux1, 351 cours de la libération, 33400 Talence, France
$^2$EDF-R&D, dep. OSIRIS, 1 avenue du Général de Gaulle, 92140 Clamart, France

Abstract. This paper presents a model for scheduling nuclear power plant maintenance. It must obey scheduling constraints and leave sufficient electricity production capacity to meet demands. The duration of a maintenance period is uncertain in our model. We seek for solutions that are robust to variation of maintenance durations selected in an appropriately defined uncertainty set. Once the realization of these random data is observed, a recourse is possible by modifying the power production levels. This gives rise to a 2-stage robust optimization mathematical programming model, with discrete uncertainty sets and an integer second level problem.

Nuclear reactors must be shut down periodically for maintenance and refueling. A stochastic model for this scheduling problem was proposed in [1]. However, it did not consider uncertainty on maintenance duration that are due to frequently observed delays in such operations. We will focus here on developing a robust approach dealing with such uncertainty. For the sake of simplifying the presentation, we do not include all the complexities of the model of [1].

1 Modeling of the problem

1.1 Definition of the simplified model

We denote nuclear units with index $i$, and weekly time steps with index $w$. A production cycle is defined as a period beginning with an outage (for maintenance and refueling) and followed by a production campaign. We index the cycles of unit $i$ with $(i, k)$ for $k = 1, \ldots, K$. The duration of outage $(i, k)$ is denoted $DA_{i,k}$ and counted in weeks.

For all time steps $w$, the global power production must meet exactly the demands $Dem_w$. Nuclear units cannot produce during outages. During production campaigns, nuclear units can be shut down, or produces power $P_{\text{max}}_w$. Other generating units are aggregated in one unit, producing continuously in $[0, P_{\text{flex}}_w]$, with a production cost $C_{w,\text{flex}}$ proportional to the power generated.

We do not model the levels of fuel as in [1]. Instead, we introduce minimal and maximal duration $L_{\text{min}}_{i,k}$ and $L_{\text{max}}_{i,k}$ for the production campaigns. The refueling costs associated to an outage are $C_{r,\text{fl}}^{i,k}$. 
1.2 Deterministic ILP formulation

We define binary variables \( d_{i,k,w} \) for cycle \((i,k)\) and week \(w\): \( d_{i,k,w} = 1 \) if the start of cycle \((i,k)\) arises at the beginning of week \(w\) or earlier. Set up binary variables \( x_{i,k,w} \) indicates if a nuclear reactor is producing in week \(w\). Observe that demand covering constraints impose that the production of the aggregated power plants is \( \text{Dem}^w - \sum_{i,k} \text{Pmax}^w_i x_{i,k,w} \). With these definitions, the planning problem admits an Integer Linear Programming (ILP) compact formulation:

\[
\min \sum_{i,k} C_i d_{i,k,W} + \sum_w C_w \text{flex} P \left( \text{Dem}^w - \sum_{i,k} \text{Pmax}^w_i x_{i,k,w} \right) \quad (P_{det})
\]

\[
\forall k, i, w, \quad d_{i,k,w-1} \leq d_{i,k,w} \quad (1)
\]

\[
\forall i, k, w, \quad x_{i,k,w} \leq d_{i,k,w} - DA_{i,k} - d_{i,k+1,w} \quad (2)
\]

\[
\forall i, k, \quad L_{min_{i,k}} \leq \sum_w x_{i,k,w} \leq L_{max_{i,k}} \quad (3)
\]

\[
\forall w, \quad 0 \leq \text{Dem}^w - \sum_{i,k} \text{Pmax}^w_i x_{i,k,w} \leq \text{Pflex}^w \quad (4)
\]

\[
\forall w, \quad \sum_{i,k} \text{Pmax}^w_i (d_{i,k,w} - d_{i,k,w} - DA_{i,k}) \leq \text{Imax} \quad (5)
\]

Constraints (2) impose no production on outages. Constraints (5) limit the off-line production capacity to \( \text{Imax} \). Coupling constraints on outages considered in [1] are mathematically similar (see [7]). Like it is shown in [7], this ILP has a convenient block structure for a Dantzig-Wolfe reformulation. This leads to a Branch&Price approach following [6].

1.3 Uncertainty sets

We define here the uncertain realizations considered for the outages prolongations, because of maintenance delays. We note \( \delta_{i,k} \) the outage prolongations expressed as an integer numer of weeks, i.e., outage durations are now \( DA_{i,k} + \delta_{i,k} \). We assume first that \( \delta_{i,k} \in [0, \overline{\delta}_{i,k}] \).

A first type of uncertainty sets are discrete sets \( \Omega_{worst} = \mathbb{Z} \cap [0, \overline{\delta}_{i,k}] \). Similarly with [3], we can assume that all extreme cases are not simultaneously met and restrict with a maximal budget \( \Gamma \), \( \Omega_{BS}^\Gamma = \left\{ \delta \in \Omega_{worst} \mid \sum_i \delta_{i,k} \leq \Gamma \right\} \). Alternatively, we can restrict the number of prolongations using a cardinality constraint: \( \Omega_{card}^N = \left\{ \delta \in \Omega_{worst} \mid \exists (\epsilon_{i,k}) \in \{0,1\}, \sum_i \delta_{i,k} \leq \overline{\delta}_{i,k} \epsilon_{i,k} \text{ and } \sum_i \epsilon_{i,k} \leq N \right\} \).

2 A robust optimization model

Our problem is bilevel, the production is adapted after the realization of delays. Our robust problem is minimizing a planning cost, with outages decisions before the outcome of the uncertainties, and decisions on production level being decided after the realizations. It leads to 2-stage robust optimization program.
2.1 A 2-stage approach as a min-max-min problem

Considering a fixed delay $\delta$ in the uncertainty set $\Omega$, the deterministic ILP can be reformulated with all the uncertainty in the RHS, leading to following ILP:

\[
\begin{align*}
\min_{d,x} \quad & c_D.d + c_X.x \\
A.d & \leq b \\
T.d + W.x & \leq h(\delta) \\
d,x & \in \{0,1\}
\end{align*}
\]

where matrix $A$ define the constraints (1) specific to the outages decisions $d$, while matrices $T$, $W$ reformulate the constraints (2), (3), (4) and (5) linking decisions $d$ to the production decisions $x$. The 2-stage robust problem is similar to a game theory problem, we want to face the best strategy in $\Omega$ of a fictive adversary choosing the most penalizing prolongations. It leads to the following min-max-min scheme:

\[
\begin{align*}
\min_d \quad & c_D.d + Q(d) \quad \text{with:} \\
A.d & \leq b \\
Q(d) = \max_{\delta \in \Omega} Q(d,\delta) & \quad \text{and} \quad Q(d,\delta) = \min_x \quad c_X.x \\
T.d + W.x & \leq h(\delta) \\
T.d + W.x & \leq h(\delta)
\end{align*}
\]

This is analogous to [2], [4] and [5], the difference is that $\delta$ and $x$ are not discrete in their applications. In these continuous cases, Kelley’s algorithm can be applied, as described in [2], leading to a cutting planes approach.

2.2 Restriction of the uncertainty sets

Let $d$ a fixed planning of the outages. Then, if we consider two delay scenarios $\delta^1, \delta^2 \in \Omega$, such as $\delta^1 \leq \delta^2$, i.e. for all $(i,k)$ $\delta^1_{i,k} \leq \delta^2_{i,k}$, our constraints imply that a production planning $x$ feasible for $(d,\delta^2)$, is always feasible for $(d,\delta^1)$. It implies $Q(d,\delta^1) \leq Q(d,\delta^2)$. The min-max min problem can thus be restricted on the subset of delays of non dominated vectors of $\Omega$.

For the uncertainty set $\Omega_{\text{worst}}$, the robust problem is thus deterministic, with $\delta_{i,k} = \delta_{i,k}$. For $\Omega_{\text{card}}$, we have to enumerate only the subset of $\Omega_{\text{worst}}$, with $N$ maximal delays, which is reasonable for small values of $N$.

2.3 ILP formulation with scenario enumeration

Consider the min-max-min problem on a reasonable enumeration of scenario $\Omega^0$, either resulting from the above restrictions or assuming that this input data has been generated from probabilistic scenarios. One can linearize the robust min-max-min problem by defining recourse variables $x_{i,k,w,\delta}$ after $\delta$ is realized for considering for all $\delta \in \Omega^0$. To linearize the maximization in $\delta$, we introduce a variable $C$ representing the maximum production cost considering all the delays in $\Omega^0$. This transformation has similarities with the decomposition presented in
This leads to following ILP with a matrix block structure convenient for a Bender’s decomposition.

\[
\min \sum_{i,k} C^{i,k}_{r,f} d_{i,k,w} + C \\
\forall \delta \in \Omega^0, \sum_{w} C^{i,k}_{r,f} \left( DEM^w - \sum_{i,k} P_{MAX}^i x_{i,k,w,\delta} \right) \leq C \quad (P_{robust})
\]

\[
\forall k, i, w, \quad d_{i,k,w-1} \leq d_{i,k,w} \quad (9)
\]

\[
\forall i, k, w, \delta \quad x_{i,k,w,\delta} \leq d_{i,k,w} - DA_{i,k} - d_{i,k} - \delta_{i,k} - d_{i,k+1,w} \quad (10)
\]

\[
\forall i, k, \quad L_{min,i,k} \leq \sum_{w} x_{i,k,w,\delta} \leq L_{max,i,k} \quad (11)
\]

\[
\forall \delta \in \Omega^0, w, \quad 0 \leq DEM^w - \sum_{i,k} P_{MAX}^w x_{i,k,w,\delta} \leq P_{flex}^w \quad (12)
\]

\[
\forall \delta \in \Omega^0, w, \quad \sum_{i,k} P_{MAX}^w (d_{i,k,w} - d_{i,k+1,w} - DA_{i,k} - \delta_{i,k}) \leq I_{max} \quad (13)
\]

\[
\forall \delta \in \Omega^0, w, \quad \sum_{i,k} P_{MAX}^w (d_{i,k,w} - d_{i,k+1,w} - DA_{i,k} - \delta_{i,k}) \leq I_{max} \quad (14)
\]

Acknowledgment

This work is part of my PhD Thesis on a partnership between EDF-INRIA-DGA, under the supervision of A. Miller, R. Sadykov and F. Vanderbeck (INRIA Bordeaux team RealOpt), P. Bendotti and M. Porcheron (EDF R&D - OSIRIS), and E. Talbi (INRIA Lille team Dolphin).

References


