7.2 – Geometric Sequences

Goals:

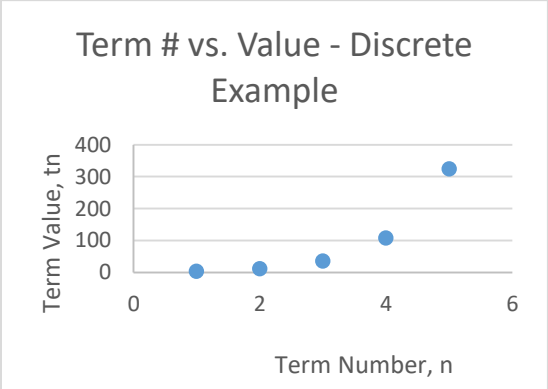
- Be able to determine if a sequence is arithmetic, geometric, or something else entirely.
- Be able to determine the general term, recursive formula, and any term value for a geometric sequence.
- Be able to solve applications of geometric sequences – including exponential growth and decay.

Geometric sequences have a constant ratio (or common difference *factor*) between terms.

Example a) 4, 12, 36, 108, 324, …

Derivation of Geometric Sequences (based on the example above):

Term #	Value	In terms of the initial value	As a multiple of 3	Exponential Form
1	4			
2	12			
3	36			
4	108			
5	324			
.	.			
.	.			
.	.			
n				



Graphing the values of this sequence shows a *discrete* function - the function, t_n, only exists at specific values. Notice that it is also an ***exponential growth*** relationship (increases to the right).

Since n is the term #, the value of the n^{th} term is the last term in this sequence is $t_n = 4(3)^{n-1}$ where:
the initial value is 4 and
the common difference *factor* is 3.

In general, **a = initial value**
r = common difference *factor* (ratio)

General Term	$\boldsymbol{t_n = a(r)^{n-1}}$	$\boldsymbol{n \in \mathbb{N}}$
Recursive Formula (relates each term to the previous term)	$\boldsymbol{t_1 = a,\ t_n = rt_{n-1}}$	$\boldsymbol{n \in \mathbb{N}, n > 1}$

These are important ***restrictions*** and should always be included!

For Example a), state the general term and recursive formula.

Example b) State the general term, recursive formula and t_7 for the following sequence.

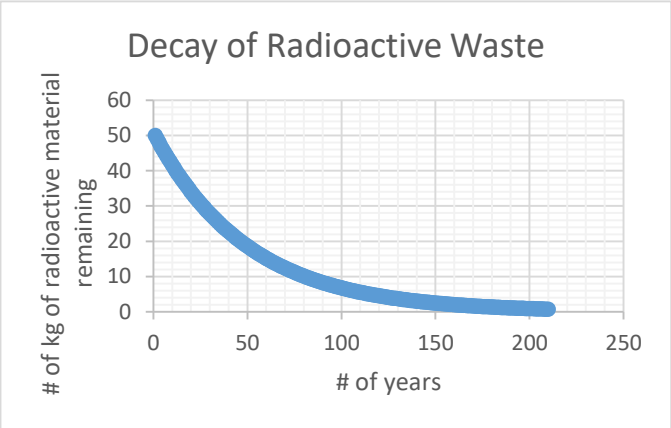
-243, 81, -27, 9, -3, …

Note first:

Example c) Application

A company has 50 kg of radioactive material. It must be stored until it is safe to dispose of. After 1 year, 98% of the material remains radioactive. The material continues to lose 98% of its radioactivity each year.

a) How much of the material is radioactive after 200 years?



The graph above represents this application question. Notice that it is an exponential relationship. Since it is getting *lower to the right*, this is called an ***exponential decay***. If it *rises to the right*, this is called an exponential *growth*.

b) The law requires that less than 1% of a material to be radioactive when disposed. Is 200 years long enough?