Computational modeling of material failure

Nguyễn Vĩnh Phú*

in collaboration with

Stéphane Bordas, Oriol Lloberas-Valls, Amin Karamnejad, Erik Lingen, Martijn Stroeven, Bert Sluys

*School of Civil, Environmental & Mining Engineering University of Adelaide
Outline

• **Computational models for fracture**
  - Continuum mechanics: LEFM, Cohesive zone models
  - Peridynamics
  - Continuous/discontinuous description of failure
    (Damage models, XFEM, interface elements)

• **Multiscale modeling of fracture**
  - Hierarchical, semi-concurrent and concurrent methods
  - Computational homogenization models for fracture

• **Image-based modeling**
  - Conforming mesh methods
  - Level Set/XFEM, Finite Cell Method (non-conforming)
  - Voxel based methods
Continuum mechanics theories

Cauchy, Euler, Lagrange...

\[ \sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i \]
\[ \sigma_{ij} = C_{ijkl} \epsilon_{kl} \]
\[ \epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \] +

Fracture Mechanics
Damage Mechanics
PDE

Spatial derivatives of displacements:
do not exist at discontinuities (cracks)

S. Silling 2000

Peridynamics is a formulation of continuum mechanics that is oriented toward deformations with discontinuities, especially fractures.

Integral equation

\[ \rho(x) \ddot{u}(x, t) = \int_{H_x} f(u(x', t) - u(x, t), x' - x) dV_{x'} - b(x, t) \]

No spatial derivatives of displacements
Continuous/discontinuous description of fracture

Displacement

Strain

Continuous

Weak discontinuity

Strong discontinuity

Discontinuous

Damage Mechanics

Isotropic damage models
Softening plasticity models
Damage-plastic models

Fracture Mechanics

LEFM, EPFM, CZM
Fracture mechanics models

Linear Elastic Fracture Mechanics (LEFM):
- brittle materials
- ductile materials under Small Scale Yielding (SSY) condition
- an existing crack is required

Elastic Plastic Fracture Mechanics (EPFM):
- ductile materials
- an existing crack is required

Cohesive Zone Models (CZMs):
- quasi-brittle materials (concrete)
- ductile materials
- no existing crack is needed
Linear Elastic Fracture Mechanics (LEFM)

\[ \sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{H.O.T} \]

SIF

Remeshing is a key point.

\[ \frac{1}{\sqrt{r}} \]

crack must locate on element edges

Barsoum element [1970s]

Very useful for fatigue life estimation

\[ \frac{da}{dN} = C(\Delta K)^m \]
Cohesive Zone Models (CZMs)

Barrenblatt 1962
Dugdale 1960
Hilleborg, 1976

Constitutive equations

\[ \dot{\sigma} = D \dot{\varepsilon} \quad \longrightarrow \quad \text{deformation} \]
\[ \dot{t}^c = T \dot{\mathbf{u}} \quad \longrightarrow \quad \text{separation} \]

\[ G_f = \frac{K^2 I_c}{E} \]

\[ f_t \quad \text{tensile strength} \]
\[ G_{Ic} \quad \text{fracture energy} \]

[Tension Softening Relationship In Planar Process Zone]

\[ \sigma_{1 \text{max}} \geq f_t \]

[Extrinsic] Cohesive law
[Initially rigid] TSL (Traction Separation Law)

Crack direction criterion
Cohesive crack model

Governing equations (strong form)

\[
\nabla \cdot \sigma + \rho b - \rho \ddot{u} = 0 \quad x \in \Omega \\
n \cdot \sigma = \bar{t} \quad x \in \Gamma_t \\
u = \bar{u} \quad x \in \Gamma_u \\
n_d^+ \cdot \sigma = t_c^+; \quad n_d^- \cdot \sigma = t_c^-; \quad t_c^+ = -t_c = -t_c^- \quad x \in \Gamma_d
\]

Constitutive equations

\[
\dot{\sigma} = D \dot{\varepsilon} \quad \rightarrow \text{deformation} \\
\dot{t}_c = T[\dot{u}] \quad \rightarrow \text{separation}
\]
Cohesive crack model

Weak form

\[
\delta W^{\text{kin}} = \delta W^{\text{ext}} - \delta W^{\text{int}} - \delta W^{\text{coh}}
\]

where

\[
\delta W^{\text{kin}} = \int_{\Omega} \delta \mathbf{u} \cdot \rho \ddot{\mathbf{u}} d\Omega
\]
\[
\delta W^{\text{int}} = \int_{\Omega} \nabla^s \delta \mathbf{u} : \mathbf{\sigma} d\Omega
\]
\[
\delta W^{\text{ext}} = \int_{\Omega} \delta \mathbf{u} \cdot \rho \mathbf{b} d\Omega + \int_{\Gamma_t} \delta \mathbf{u} \cdot \mathbf{t} d\Gamma_t
\]
\[
\delta W^{\text{coh}} = \int_{\Gamma_d} [\delta \mathbf{u}] \cdot \mathbf{t}^c d\Gamma_d
\]
Crack discretization techniques

Zero-thickness interface elements, 1968

PUM FEM, 1999

Embedded strong discontinuity, 1987

Meshless/Meshfree methods, 1994
Interface elements

(+ easy to implement 2D/3D
(+ available in ABAQUS, LD-DYNA

- preprocessing: GMSH
- solver: jem/jive (C++)
Interface elements

Inter-granular fracture of polycrystalline material

Failure of a fiber reinforced composite
Interface elements with discontinuous Galerkin
Partition of Unity Methods

Melenk and Babuska 1996

Sum of shape functions is equal to one

\[ \sum_{J} N_J(x) = 1 \]  \text{(PUM)}

\[ \sum_{J} N_J(x) \Phi(x) = \Phi(x) \]

Approximation of the displacement field

\[ u^h(x) = \sum_{I \in S} N_I(x) u_I + \sum_{J \in S^c} N_J(x) \Phi(x) a_J \]
Extended FEM (XFEM)

Belytschko et al., 1999

nothing but an instance of PUM for crack problems

\[ u^h(x) = \sum_{I \in S} N_I(x)u_I + \sum_{J \in S_c} N_J(x)H(x)a_J + \sum_{K \in S^t} N_K(x) \left( \sum_{\alpha=1}^4 B_\alpha b^K_\alpha \right) \]

Enrichment functions

\[ H(x) = \begin{cases} +1 & \text{if } (x - x^*) \cdot n \geq 0 \\ -1 & \text{otherwise} \end{cases} \]

\[ [B_\alpha] = \begin{bmatrix} \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \frac{\theta}{2} \sin \theta, \sqrt{r} \cos \frac{\theta}{2} \sin \theta \end{bmatrix} \]
Sub-triangulation for numerical integration

Homogeneous LEFM

Interfacial LEFM

two cracks

Matlab code
XFEM for cohesive cracks

\[ u^h(x) = \sum_{I \in S} N_I(x) u_I + \sum_{J \in S^c} N_J(x) H(x) a_J \]

Wells, Sluys, 2001

No good crack tip solution is known, no tip enrichment!!!

not enriched to ensure zero crack tip opening!!!

\[ \dot{\sigma} = D \dot{\epsilon}, \quad \dot{t} = T [\dot{u}] \]

\[ H(x) = \begin{cases} +1 & \text{if } (x - x^*) \cdot n \geq 0 \\ -1 & \text{otherwise} \end{cases} \]
XFEM/Cohesive zones

Size effect

Usual lab tests (10 cm)

“Usual” structures (10m)
... convincing examples

M. Duflot

Northwestern Univ.

F.P. van der Mer, TU Delft
XFEM for material interfaces

Sukumar et al. 2002

Across interface, strain field is discontinuous

Abs-enrichment function

\[ \psi = |\phi(x)| \]

Signed distance function

\[ \phi = \|x - x_c\| - r_c \]

\[ \phi(x) = N_I(x) \phi_I \]

\[ u^h(x) = \sum_{I \in S} N_I(x) u_I + \sum_{J \in S^c} N_J(x) \Psi(x) a_J \]

\[ S^c \quad \phi_{\text{min}} \leq \phi \leq \phi_{\text{max}} < 0 \]
trabecular bone, PhD thesis, Tran, NUS
holes = extremely soft inclusions
Continuum damage mechanics

Kachanov, 1958, Rabotnov 1969, Hult 1979

CDM is a constitutive theory that describes the progressive loss of material integrity due to the initiation, coalescence and propagation of microcracks, microvoids etc. These changes in the microstructure lead to the degradation of the material stiffness at the macroscale.

\[ \sigma = (1 - \omega) E \varepsilon \]

nominal stress \( \sigma \)

effective stress \( \bar{\sigma} \)
damage variable \( \omega \)

damage variable \( \bar{\omega} \)

\[ \sigma = \frac{\bar{\sigma}}{1 - \bar{\omega}} \]

\[ \bar{\omega} = 1 - \frac{\bar{A}}{A} \]
Local damage model

**Isotropic damage model**

\[
\sigma = (1 - \omega) \mathbf{C} \varepsilon \\
\omega = f(\varepsilon_{eq}) \\
\varepsilon_{eq} = g(\varepsilon)
\]

\(\mathbf{C}\) : elasticity tensor  
\(\varepsilon_{eq}\) : equivalent strain \([-\]

**Damage evolution law**

\[
\omega = \begin{cases} 
0 & \text{if } \kappa < \kappa_i \\
1 - \frac{\kappa_i}{\kappa} \frac{\kappa_c - \kappa}{\kappa_c - \kappa_i} & \text{if } \kappa_i \leq \kappa \leq \kappa_c \\
1 & \text{if } \kappa > \kappa_c
\end{cases}
\]

**Irreversibility of failure**

\[\kappa = \max \varepsilon_{eq}\]

**Tensile failure**

\[\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} \langle \varepsilon_i \rangle^2}\]

**Mazars**

Irreversibility of failure  
Linear softening  
Stress update: explicit and simple
Local damage model

In the early 1980s it was found that FE solutions of softening damage do not converge upon mesh refinement, Z. Bazant, 1984.

Isotropic damage model

\[ \sigma = (1 - \omega) C \varepsilon \]

\[ \omega = f(\varepsilon_{eq}) \]

\[ \varepsilon_{eq} = g(\varepsilon) \]

No energy dissipation !!!

Softening plastic models: also suffer from mesh sensitivity.
Nonlocal damage model

\[ \sigma = (1 - \omega)C\varepsilon \]

\[ \omega = f(\bar{\varepsilon}_{eq}) \quad \text{with} \quad 0 \leq \omega \leq 1 \]

\[ \bar{\varepsilon}_{eq}(x) = \int_{\Omega} \alpha(x - \xi)\varepsilon_{eq}(\xi) d\xi \]

\[ \alpha(r) = \exp \left( -\frac{r^2}{2l^2} \right) \]

\( l \) is the length scale

Cabot and Bazant, 1987
Gradient damage model

\[ \sigma = (1 - \omega)C\varepsilon \]
\[ \omega = f(\bar{\varepsilon}_{eq}) \quad 0 \leq \omega \leq 1 \]
\[ \bar{\varepsilon}_{eq}(x) = \int_\Omega \alpha(x - \xi)\varepsilon_{eq}(\xi)\,d\xi \]
\[ \alpha(r) = \exp\left(-\frac{r^2}{2l^2}\right) \]

\( \dot{\sigma} = (1 - \omega)C\dot{\varepsilon} \)
\[ \dot{\omega} = \frac{\partial \omega}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{\varepsilon}_{eq}} \dot{\bar{\varepsilon}_{eq}} \]
\[ = \frac{\partial \omega}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{\varepsilon}_{eq}} N_\varepsilon \dot{\bar{\varepsilon}_{eq}} \]

\[ \begin{bmatrix} K_{\varepsilon u} & K_{\varepsilon \varepsilon} \end{bmatrix} \delta d^{(i)} \begin{bmatrix} \delta d^{(i)} \\ \delta \varepsilon_{eq}^{(i)} \end{bmatrix} = \begin{bmatrix} f_{int,u}^{(i-1)} \\ f_{int,\varepsilon}^{(i-1)} \end{bmatrix} - \begin{bmatrix} f_{ext,u}^{(i-1)} \\ 0 \end{bmatrix} \]

Implicit GD model

\[ \bar{\varepsilon}_{eq} - c \nabla^2 \bar{\varepsilon}_{eq} = \varepsilon_{eq} \]
\[ c = \frac{l^2}{2} \]

Microplane Damage Models (Z. Bazantz)

Peerlings et al., 1996
Continuous vs. discontinuous description

+ easy to implement (2D/3D)
+ one single constitutive law
+ standard elements
- incorrect final stage of failure
- evolving length scale

- hard to implement (3D)
- two separate constitutive laws
- enriched elements
+ correct final stage of failure

bests of both worlds: combined continuous-discontinuous approaches
(Dr. Nguyen Dinh Giang, Univ. Sydney)
Solution strategies

For a quasi-static analysis of softening solids, one encounters cases...

- **snap-through**
- **load control**
- **snap-back**
- **disp control**

Incremental-iterative procedure
Path-following methods

Riks 1972

\[
\begin{bmatrix}
    f^{\text{int}}(u) - \lambda g \\
    \phi(u, \lambda)
\end{bmatrix} = 0 \quad \Rightarrow \quad f^{\text{ext}} = \lambda g
\]

\[\lambda \quad \text{load factor}\]

\[f^{\text{ext}} = \lambda g \quad \Rightarrow \quad \text{reference load vector}\]

Newton-Raphson \quad \phi(u, \lambda) \quad \text{arc-length/constraint function}

\[
\begin{bmatrix}
    f^{\text{int}}(u^{(k)}) - \lambda^{(k)} g \\
    \phi(u^{(k)}, \lambda^{(k)})
\end{bmatrix} + \begin{bmatrix}
    K & -g \\
    v^T & w
\end{bmatrix}^{(k)} \cdot \begin{bmatrix}
    \Delta u \\
    \Delta \lambda
\end{bmatrix} = 0
\]

where

\[
K = \frac{\partial f^{\text{int}}}{\partial u}, \quad v = \frac{\partial \phi}{\partial u}, \quad w = \frac{\partial \phi}{\partial \lambda}
\]

\[
\begin{bmatrix}
    \Delta u \\
    \Delta \lambda
\end{bmatrix} = \begin{bmatrix}
    u_I \\
    0
\end{bmatrix} - \frac{v^T u_I + \phi}{v^T u_{II} + w} \begin{bmatrix}
    u_{II} \\
    1
\end{bmatrix}
\]

\[
\text{correction} \quad \left[\begin{array}{c}
    u \\
    \lambda
\end{array}\right]^{(k+1)} = \left[\begin{array}{c}
    u \\
    \lambda
\end{array}\right]^{(k)} + \left[\begin{array}{c}
    \Delta u \\
    \Delta \lambda
\end{array}\right]
\]

\[u_I = K^{-1} r, \quad u_{II} = K^{-1} g\]
Energy control

\[ \epsilon = B a \]

\[ f^{\text{int}} = \int B^T \sigma \]

\[ V = \frac{1}{2} \int_{\Omega} \epsilon^T \sigma = \frac{1}{2} \int_{\Omega} a^T B^T \sigma = \frac{1}{2} a^T f^{\text{int}} = \frac{1}{2} \lambda a^T g \]

Energy release rate

\[ \ddot{V} = \frac{1}{2} \lambda \ddot{a}^T g + \frac{1}{2} \lambda a^T g \]

Energy release rate

\[ G = \frac{1}{2} \lambda \dot{a}^T g - \frac{1}{2} \lambda a^T g \]

Predefined amount of energy to be released [Nm]

\[ \phi = \frac{1}{2} \left[ \lambda^{(n)} (a_{(n+1)}^T - a_{(n)}^T) - \Delta \lambda^{(n)} a_{(n)}^T \right] g - \Delta \tau = 0 \]

Forward Euler
Energy based arc-length control

![Graph showing reaction force vs. 2u (mm)]

![Graph showing load step number vs. number of iterations per step)]
... and here we are complex failure mechanism in composites

- matrix cracking
- delamination of plies

F.P. van der Mer
EFM, 2008
Peridynamics

S. Silling 2000

\[ \rho_i \ddot{u}_i^n = \sum_{p \in \mathcal{F}_i} f(u_p^n - u_i^n, x_p - x_i)V_p - b_i^n \equiv \tilde{f}_i^n \]

time integration: Verlet integration

continuum version of MD (molecular dynamics)

XFEM can do this as well but would require genius programmers (2D)

See phase field models for similar capacities

Bobaru, 2010

glass

crack branching
Multiscale methods

multiple length scales

Multiscale models:
- better constitutive models
- design new materials
After Ted Belytschko

\[ \langle \sigma \rangle = C : \langle \epsilon \rangle \]

hierarchical methods

semi-concurrent

concurrent method

ARLEQUIN method

pile installation

Wriggers, 2011
Arlequin method

[H. Ben Dhia, 1998]

- partition of unity for energy in gluing zone
  \[ \alpha^M + \alpha^m = 1 \]
- Lagrange multipliers to glue two models

a multi-scale/multi-model method

Implementation of the Arlequin method into **ABAQUS:**

Mortar Method
Arlequin

FEM

work in progress
Heterogeneous materials

Macroscopically homogeneous but microscopically heterogeneous

Macroscopic length scale

Microscopic length scale

Macroscopic behavior depends on:
- Size, shape
- Spatial distribution
- Volume fraction
- Mechanical properties of the constituents.

Phenomenological constitutive models:

\[ \sigma = f(\varepsilon, \alpha) \]

Two many params

The identification of these parameters is generally difficult.
Due to the high numerical effort and memory demand of DNS, it is, in general, not possible to simulate the full structure on the micro-/meso-scale with the computational power available nowadays.
Homogenization = replace a heterogeneous material with an equivalent homogeneous material.

Voigt, 1910
Reuss 1929
Hill 1965

RVE = Representative Volume Element

heterogeneous material

homogeneous material

Macro sample
RVE

l_M

l_M >> l_m >> d

separation of scales
Homogenization

\[ \langle \sigma \rangle = C : \langle \varepsilon \rangle \]

bottom-up approach

RVE

\[ \langle \sigma \rangle = \frac{1}{|\Omega_m|} \int_{\Omega_m} \sigma_m d\Omega \]
\[ \langle \varepsilon \rangle = \frac{1}{|\Omega_m|} \int_{\Omega_m} \varepsilon_m d\Omega \]

1 FEM

6 independent loads are needed to determine 36 constants

\[ \begin{bmatrix} \langle \sigma_{11} \rangle_{\Omega} \\ \langle \sigma_{22} \rangle_{\Omega} \\ \langle \sigma_{33} \rangle_{\Omega} \\ \langle \sigma_{12} \rangle_{\Omega} \\ \langle \sigma_{13} \rangle_{\Omega} \\ \langle \sigma_{23} \rangle_{\Omega} \end{bmatrix} = \begin{bmatrix} E_{1111}^* & E_{1122}^* & E_{1133}^* & E_{1112}^* & E_{1123}^* & E_{1113}^* \\ E_{2211}^* & E_{2222}^* & E_{2233}^* & E_{2212}^* & E_{2223}^* & E_{2213}^* \\ E_{3311}^* & E_{3322}^* & E_{3333}^* & E_{3312}^* & E_{3323}^* & E_{3313}^* \\ E_{1211}^* & E_{1222}^* & E_{1233}^* & E_{1212}^* & E_{1223}^* & E_{1213}^* \\ E_{2311}^* & E_{2322}^* & E_{2333}^* & E_{2312}^* & E_{2323}^* & E_{2313}^* \\ E_{1311}^* & E_{1322}^* & E_{1333}^* & E_{1312}^* & E_{1323}^* & E_{1313}^* \end{bmatrix} \]

\[ \begin{bmatrix} \langle \varepsilon_{11} \rangle_{\Omega} \\ \langle \varepsilon_{22} \rangle_{\Omega} \\ \langle \varepsilon_{33} \rangle_{\Omega} \\ 2\langle \varepsilon_{12} \rangle_{\Omega} \\ 2\langle \varepsilon_{13} \rangle_{\Omega} \\ 2\langle \varepsilon_{23} \rangle_{\Omega} \end{bmatrix} \]

\[ \mathbf{C} \] effective properties

\[ \text{to constitutive model} \]

\[ \text{simple plasticity} \]
Artificial microstructures

Real microstructures: hard to obtain and not meshable

Statistically equivalent to real microstructures
Easy to discretized into finite elements

Build tailor made materials
Computational Homogenization

FE² method

+ nonlinear, large deformation
- computationally expensive
- 2D problems at laboratory scale
- not always robust!!!

Micro problems are solved in parallel.
Troubles with softening RVEs

- RVE does not exist for softening materials
- CH cannot be applied for softening materials
Failure zone averaging

\( \Omega_e \): elastic domain
\( \Omega_d \): active damaged domain
\( \Omega_{du} \): inactive damaged domain

\( \Omega_d = \{ \mathbf{x} \in \Omega_m \mid \omega(\mathbf{x}) > 0, f(\mathbf{x}) = 0 \} \)

\[ \langle \sigma \rangle_{\text{dam}} = \frac{1}{|\Omega_d|} \int_{\Omega_d} \sigma_m d\Omega_d, \quad \langle \epsilon \rangle_{\text{dam}} = \frac{1}{|\Omega_d|} \int_{\Omega_d} \epsilon_m d\Omega_d \]

\( t_M = \frac{1}{h} \sigma_M \cdot n \)
\( u_{\text{dam}} = \langle \epsilon \rangle_{\text{dam}} \cdot (ln), \quad l = |\Omega_d|/h \)
\[ [u]_M = u_{\text{dam}} - \hat{u}_{\text{dam}} \]
RVE does exist for softening materials by using the failure zone averaging technique
Discontinuous CH model

MACRO

\[ \dot{\sigma}_M = D_0 \dot{\varepsilon} \]
\[ \dot{t}_M = T_M [\dot{u}]_M \]

MICRO

localization band

discrete crack

Nguyen et al, 2011
Example
Dynamic discontinuous CH model

- macro: implicit dynamics
- micro: quasi-static

A. Karamnejad, Nguyen, Sluys, 2012
More information

Archives of Computational Methods in Engineering
March 2009, Volume 16, Issue 1, pp 31-75

Multiscale Methods for Composites: A Review

P. Kanouté, D. P. Boso, J. L. Chaboche, B. A. Schrefler

VINH PHU NGUYEN, MARTIJN STROEVEN, and LAMBERTUS JOHANNES SLUYS, J.
Multiscale Modelling 03, 229 (2011). DOI: 10.1142/S1756973711000509

MULTISCALE CONTINUOUS AND DISCONTINUOUS MODELING OF HETEROGENEOUS MATERIALS: A REVIEW ON RECENT DEVELOPMENTS

VINH PHU NGUYEN
Corresponding author.
Delft University of Technology, Faculty of Civil Engineering and Geosciences, P. O.
Box 5048, 2600 GA Delft, The Netherlands

MARTIJN STROEVEN
Delft University of Technology, Faculty of Civil Engineering and Geosciences, P. O.
Box 5048, 2600 GA Delft, The Netherlands

LAMBERTUS JOHANNES SLUYS
Delft University of Technology, Faculty of Civil Engineering and Geosciences, P. O.
Box 5048, 2600 GA Delft, The Netherlands
Image-based modeling
Traditional FE analysis

Geometry (CAD) → Mesh → FE solver
There are many cases in which such CAD geometries are not available. However, image data are so ready: medicine, material sciences...

(1) Industry

See also the FREE program OOF2, NIST, USA
(2) universities
voxel based method

microstructure of cement paste

each voxel = one finite element

- incorrect volume fraction
- images with high resolution are required
- too large problem size!!!
Finite Cell Method (FCM) (Fictitious Domain Methods)

Level set/XFEM
Tools

Matlab is not enough. Consider Fortran, C++, Python.

Move to Ubuntu Linux to make your programming life much easier.

- **Preprocessing**: GMSH, GID, ANSYS, **ABAQUS**

- **Solvers**:
  - FEM: FEAP, OOFEM, libMesh, KRATOS, **Code Aster**, TRILINOS, PERMIX, OpenSees (earthquake, structures), TRILINOS, PERMIX, OpenSees (earthquake, structures)
  - DEM: LAMMPS, KRATOS, YADE…
  - CFD: OpenFOAM, KRATOS…

- **Postprocessing**: GMSH, PARAVIEW, **MATLAB**, TECPLOT

domain decomposition
Habanera develops jem/jive C++ library
Thank you for your attention